Basic Electric Circuits
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Module 7: Circuit Analysis Using Laplace Transform
Lecture 35: Network Stability and Network Synthesis

Namaskar! In the last class, we started discussion on the topic called Network Stability. And we will continue our discussion today about the network stability and also, we will discuss about the network synthesis.

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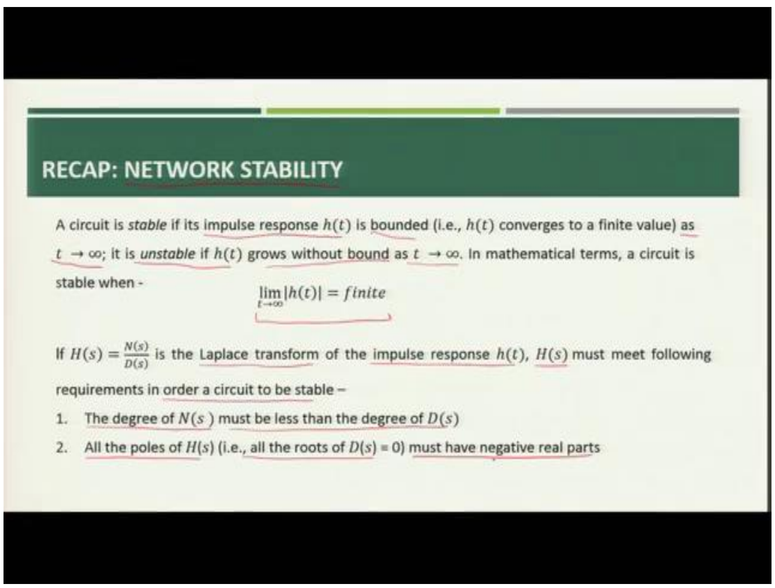

In the last class we discussed about the network stability, so what we discussed that, the circuit is stable if its impulse response $h(t)$ is bounded as time $t \to \infty$. That means the impulse response will converge to a finite value when time $t \to \infty$.

It will be unstable if $h(t)$ grows without bound as $t \to \infty$. We also discussed that in mathematical term, we can write it as $\lim_{t\to\infty} |h(t)| = finite$. Now if $H(s)$ is the Laplace Transform of $h(t)$, we discussed that it can be represented as $H(s) = \frac{N(s)}{D(s)}$ in the Laplace Transform domain. The impulse response $h(t)$, $H(s)$ that is the Laplace Transform of the impulse response must meet the following requirement in order for the circuit to be stable.

What was those requirements, one requirement, which we discuss was the degree of $N(s)$ must be less than the degree of $D(s)$, that is degree of numerator expression should be less than the degree of expression in denominator. Next requirement is that all poles of $H(s)$ must have negative real parts, all poles means, all roots of the denominator expression must have negative real part.

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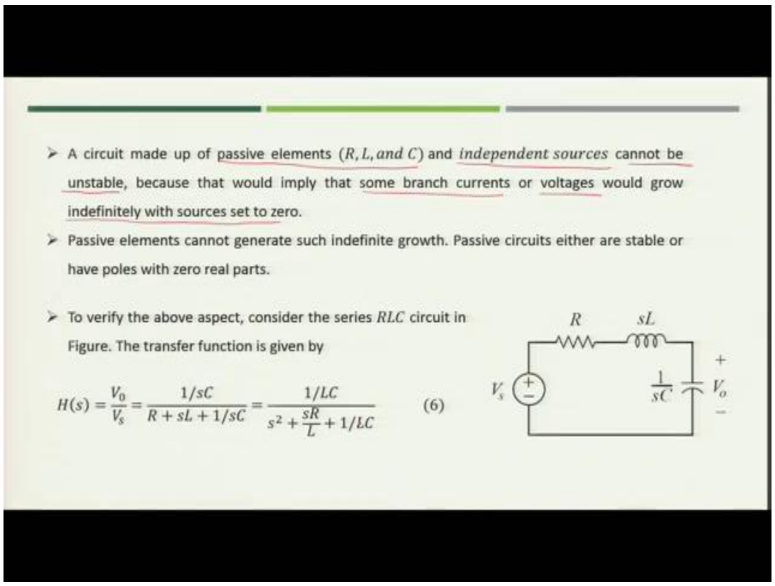

Now we also discussed that, the circuits which are made up of passive elements that is R, L, C elements and independent sources will not be unstable because if we say that these can be unstable that means that there would be some branch currents or voltages which would grow indefinitely with sources set to zero, which generally is not the case, when we analyze the R, L, C circuits. So, let us verify our saying that the passive elements and independent sources, elements network will not be unstable.

Let us see the example that is what we also discussed in the previous lecture, that if it is a series R, L, C circuit, we can say that,

$$\frac{V_0}{V_s} = \frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} = \frac{\frac{1}{LC}}{s^2+\frac{sR}{L}+\frac{1}{LC}}$$

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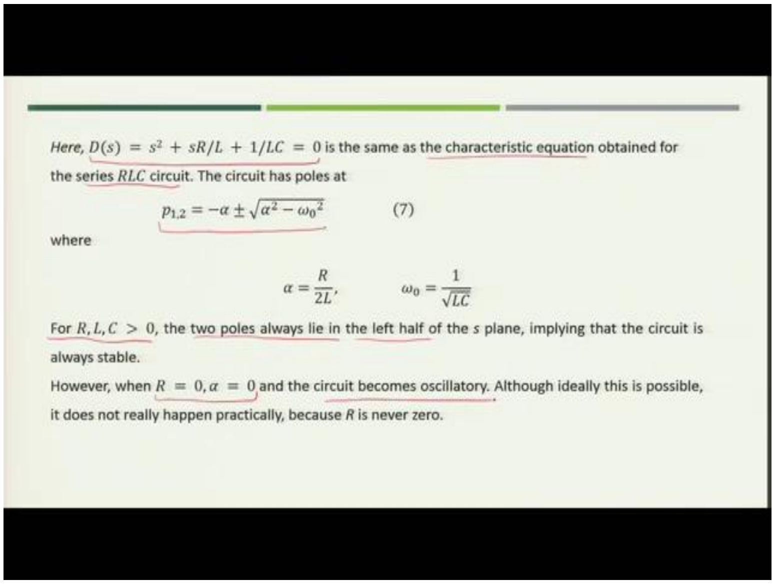

And we also discussed that the denominator component that is the denominator expression that is $s^2 + \frac{sR}{L} + \frac{1}{LC} = 0$ is already we have discussed when we were discussing about the series R, L, C circuit. So, this would be the characteristic equation which we have already discussed in the past.

We can say that the poles of the denominator expression will have the roots of the denominator expression will have value $p_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$. And we also established that $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{LC}$. Now if you see that the R, L, C is greater than 0, when, if this is the condition the 2 poles will always write in the left half of the plane. So that means that, in the network which contains R, L and C component will not be unstable because the pole which we get from the given transfer function of circuit will have always negative real part. The critical condition which we discussed was that when R is equal to 0 the damping coefficient would be 0. In that case the circuit will become oscillatory.

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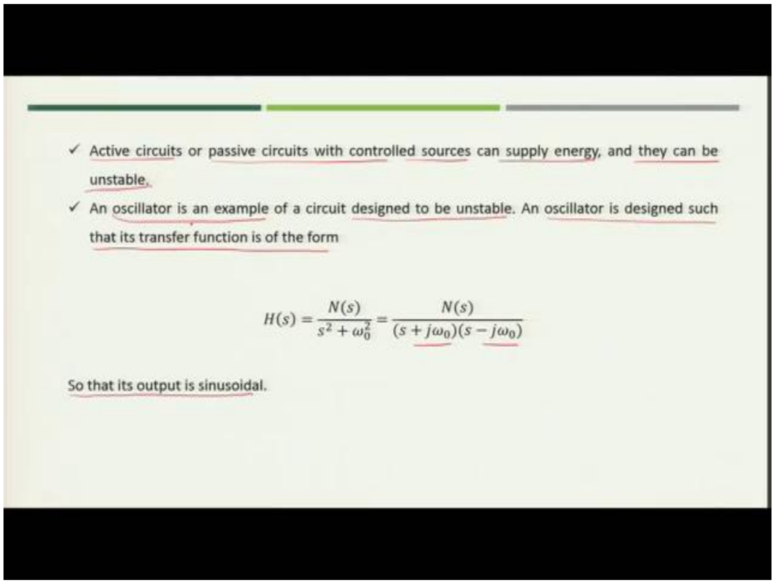

Now there are certain circuits like active circuit and passive circuit which contains the controlled sources which can supply energy and that is why they can be unstable. The oscillator is another example of such circuit, which is designed to be unstable because it will not die out to some constant value when the time $t \rightarrow \infty$. So, when you design the oscillator the transfer function of the oscillator can be given as,

$$H(s) = \frac{N(s)}{s^2 + \omega_0^2} = \frac{N(s)}{(s + j\omega_0)(s - j\omega_0)}$$

You will see that poles of this transfer functions are lying at the imaginary axis. The poles are $-j\omega_0$ and $j\omega_0$, so when you have the poles lying on the imaginary axis that means that the circuit is oscillatory. The output which you will get in this case is sinusoidal. So, when you have this kind of situation then you will say that the circuit is oscillatory.

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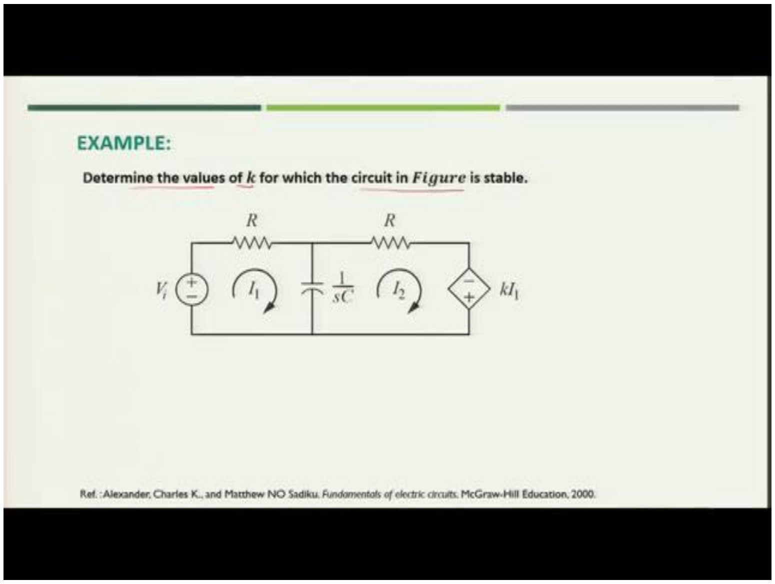

Now let us take one example so that we can understand the concept. We need to determine the value of k for which the circuit which is given in figure is stable. So now here you see that you have one energy storage component that is capacitor, $\frac{1}{sC}$ is the value of the capacitance in s domain and we have 2 resistances we have one dependent voltage source which depends upon the current flowing in the loop. So, we need to find out the value of k for which this figure will be stable.

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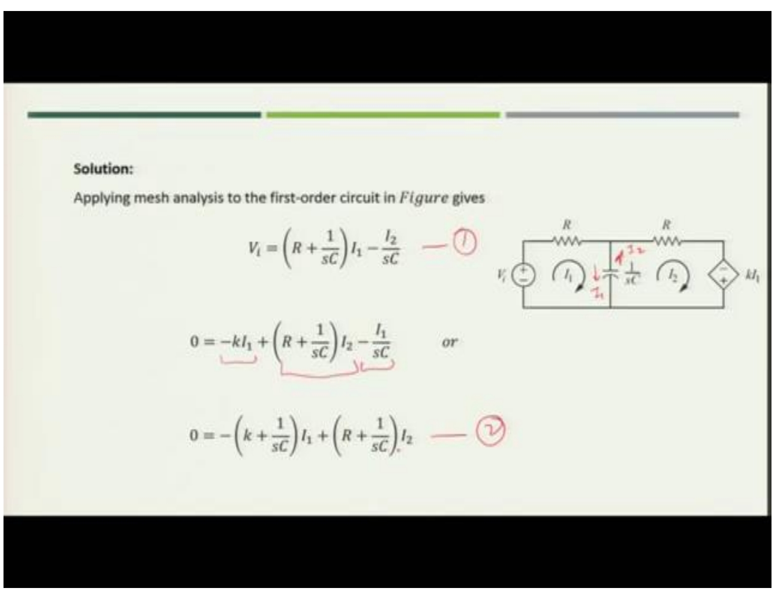

We have to first use the mesh analysis and write the equation for these 2 loops.

$$V_i = \left(R + \frac{1}{sC}\right) I_1 - \frac{I_2}{sC}$$

$$0 = -kI_1 + \left(R + \frac{1}{sC}\right) I_2 - \frac{I_1}{sC} \qquad or$$

$$0 = -\left(k + \frac{1}{sC}\right) I_1 + \left(R + \frac{1}{sC}\right) I_2$$

So now you have these 2 equations, if you represent them in matrix form, how you will write;

$$\begin{bmatrix} V_i \\ 0 \end{bmatrix} = \begin{bmatrix} \left(R + \frac{1}{sC}\right) & -\frac{1}{sC} \\ -\left(k + \frac{1}{sC}\right) & \left(R + \frac{1}{sC}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

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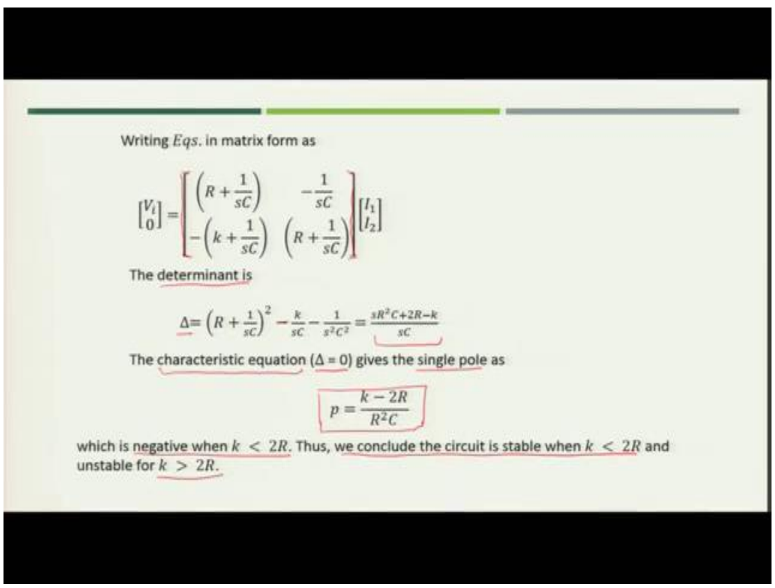

So, when you compile the matrix you will see that if you want to find out the characteristic equation of this circuit, governing characteristic equation then what you have to do? You must find out the determinant of the matrix which we have just compiled. The value of the determinant is

$$\Delta = \left(R + \frac{1}{sC}\right)^2 - \frac{k}{sC} - \frac{1}{s^2C^2} = \frac{sR^2C + 2R - k}{sC}$$

That would be nothing but characteristic equation then what will you get, you will get the value of single pole because here this is the numerator component is only containing the degree of s as 1. So, you will have only a single pole, so the value of pole,

$$p = \frac{k - 2R}{R^2 C}$$

Now if you see, this value of pole P you will say that the pole would be negative when $k < 2R$. We can say that the circuit will be stable when $k < 2R$ otherwise for $k > 2R$ the circuit would be unstable. This you can verify, you can find out the value of k for which your circuit will be stable.

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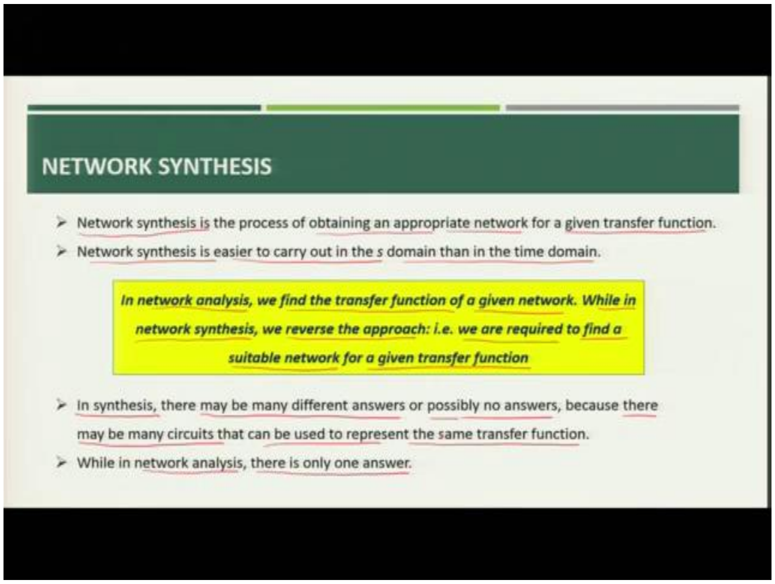

Now let us talk about another concept called Network Synthesis, so what is Network Synthesis? Network Synthesis is the process of obtaining an appropriate network for a given transfer function. Now this Network Synthesis is just opposite to our Network Analysis, where we get the circuit, and we try to find out the transfer function of the given circuit. Network Synthesis is easier to carry out in the s domain than in the time domain.

When we differentiate between Network Synthesis and Network Analysis, we can say that in Network Analysis we find the transfer function of a given network while, in case if Network Synthesis we reverse the approach. That means we are required to find a suitable network for a given transfer function. So, in case of Network Synthesis what we will do? We will get one transfer function and from that we try to find out the suitable network which satisfy the transfer function.

Now in Network Synthesis there may be many different answers to or possibly no answers because there may be many circuits that may be used to represent the same transfer function. We need to find out the most suitable answer for given transfer function. While in Network Analysis the things are opposite, there will be only one answer. Means for Network Analysis we will always get one single transfer function but in case of Network Synthesis one transfer function can represent multiple circuits.

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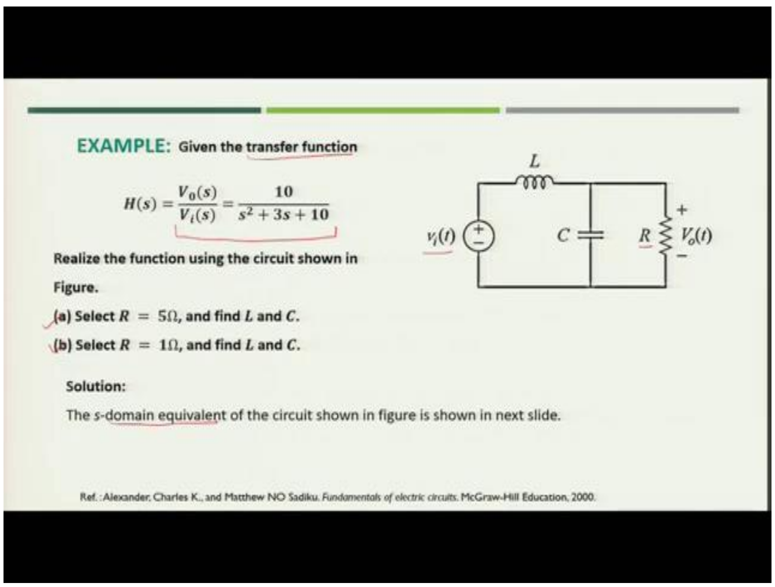

Let us understand these concepts with the help of examples because that would be easier to understand the Synthesis aspect of the circuit. Let us see that there is one transfer function given as

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{10}{s^2 + 3s + 10}$$

Now let us say that this is the circuit where L and C are presented and C is in parallel with R and ratio $\frac{V_0(s)}{V_i(s)}$ is nothing but the ratio between voltage across resistance R divided by input voltage. So now we have got this transfer function and we have the idea that this would-be circuit. What we must do? Since we have 3 unknowns here R, L and C, we will fix one value R. Why we will fix that value, we will see that when we will progress with the solution, that why we are fixing value of R and finding out value of C. We will take 2 cases where R is 5 ohm and R is 1 ohm and we will try to find out the value of L and C. Now first, what we must do? We must find out the s minus domain equivalent of the circuit.

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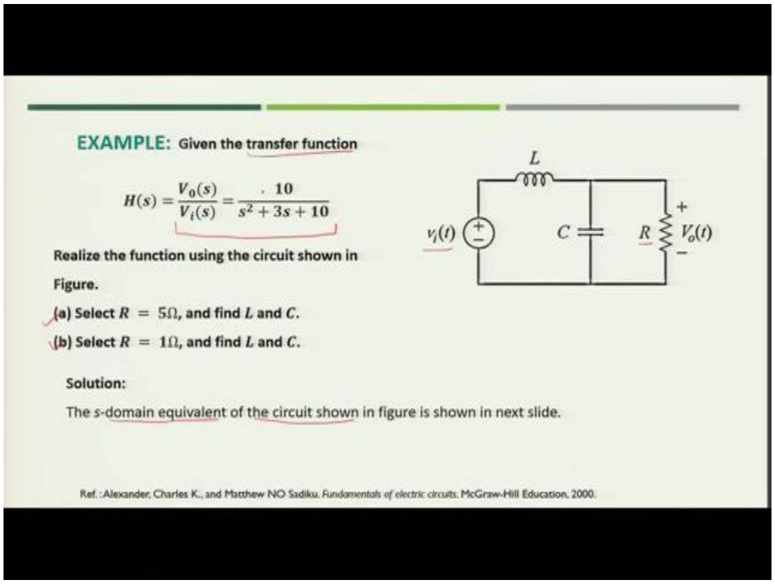

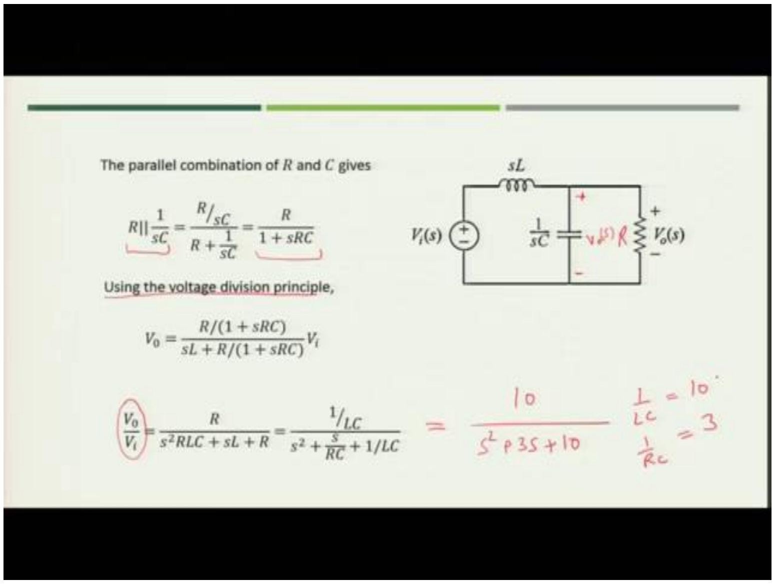

When you create the s minus domain equivalent of the circuit, it will look like as shown in the figure. This is $V_i(s)$ that is input voltage, $V_i(s)$ is across the resistance R and then the Inductor will become sL and the capacitance will be $\frac{1}{sC}$. Now the capacitance and resistance are in parallel and this voltage would be applied across both components because both are in parallel. So, we can find out the equivalent impedance of this parallel combination of $R||\frac{1}{sC}$. When we solve that we will get equivalent impedance in s domain as $\frac{R}{1+sRC}$. Now we can utilize the voltage division principle, says this $V_0$is also across the capacitance, so basically the parallel combination of capacitance and resistance will have the voltage $V_0(s)$ across them. So, you can utilize the voltage division principle and find out the value of V_0 as,

$$V_0 = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} V_i$$

Now the transfer function which we get is,

$$\frac{V_0}{V_i} = \frac{R}{s^2RLC + sL + R} = \frac{1/_{LC}}{s^2 + \frac{s}{RC} + 1/LC}$$

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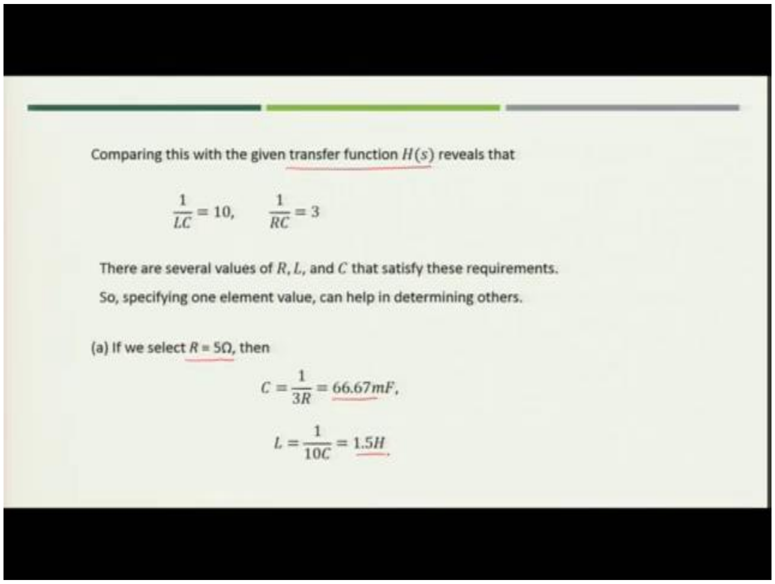

So these are the two values which we will get when we compare the transfer function. Now as we have only 2 equations and 3 unknowns we cannot find the solution, until unless we fix one quantity. So that is what we have discussed in the last slide that we need to fix at least one so that we can write, we can get the answer. So what we had in our question is its fix value as R equal to 5ohm. So when you put R equal to 5ohm, the value of C you will get is 66.67millifarad and L is 1.5 henry.

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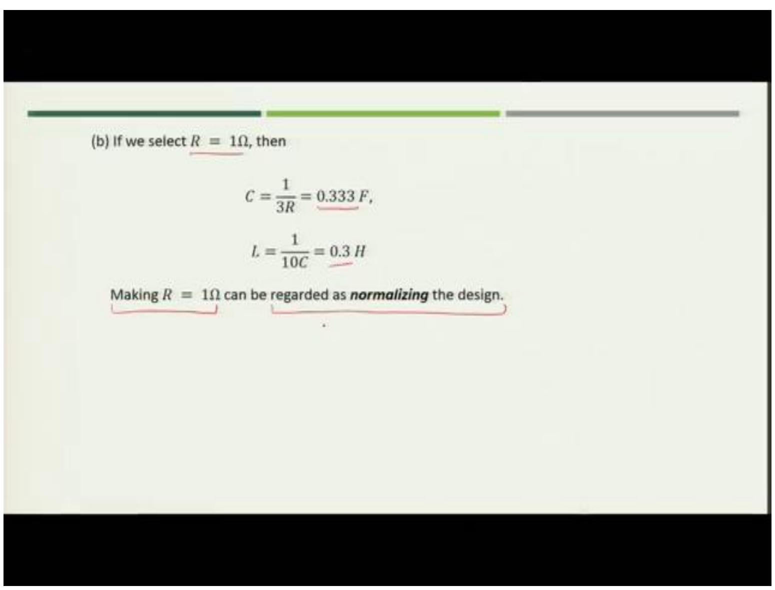

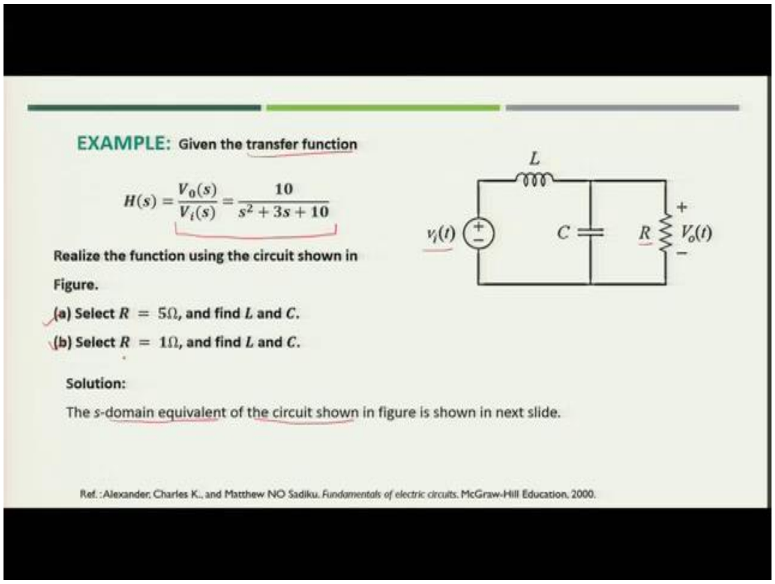

Now when you have R is equal to 1 ohm then C will be 0.333 farad and L is equal to 0.3 henry, now making R equal to 1 ohm generally is said that it is normalizing the design. Because you are fixing R as 1 ohm means you are normalizing the design of the circuit. So with this you can find out the value of unknowns that is L and C. But you have to fix at least one component, so in this case we fixed the value of Resistance R.

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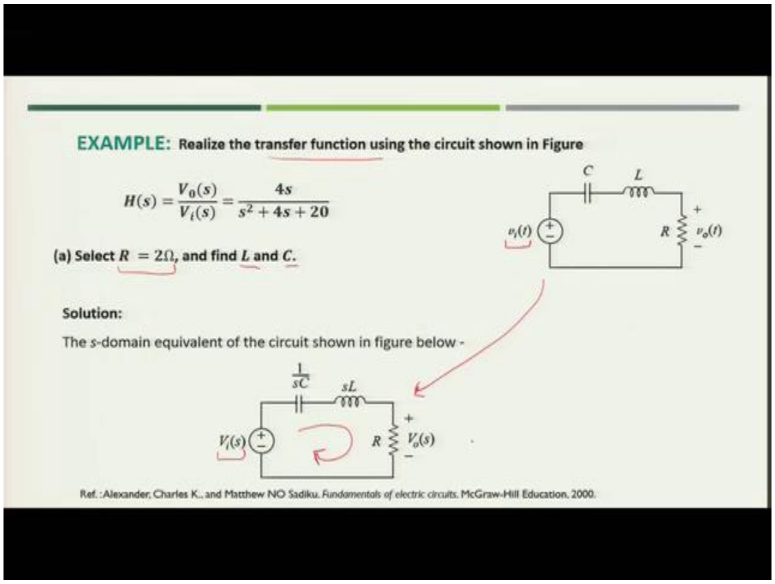

Now let us take another example, in this case we transfer function is given and we need to find out the value of L and C again when the value of R is given that is 2 ohm. So first task which is required is that convert the given circuit into s minus domain. So here you see that R, L and C, both all all of 3 are in series, that is equivalent s minus domain of the given circuit would be; C will be replaced by $1/sC$ and in place of L we will write sL, $V_0(t)$ will become $V_0(s)$ s domain, R will remain same and $V_i(t)$ will become $V_i(s)$. So, what we must do next, now we have to write the loop equation and represent the voltage across R as part of Vi, so that we get the transfer function.

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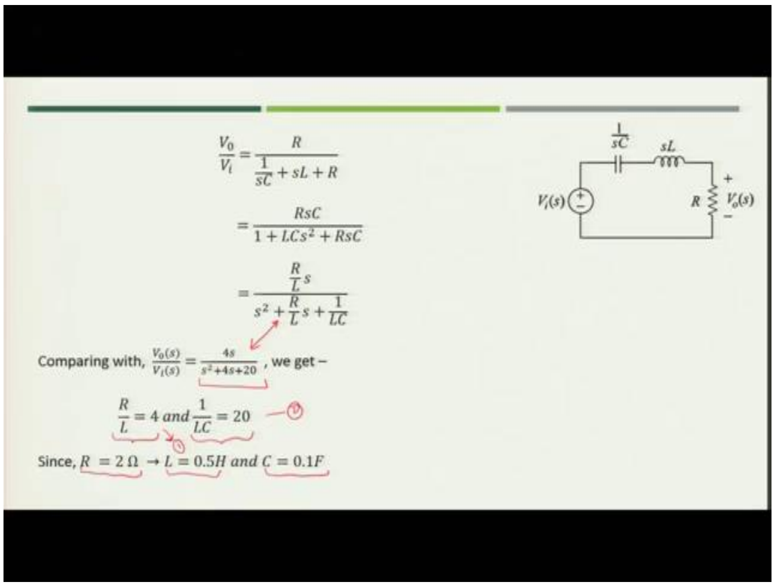

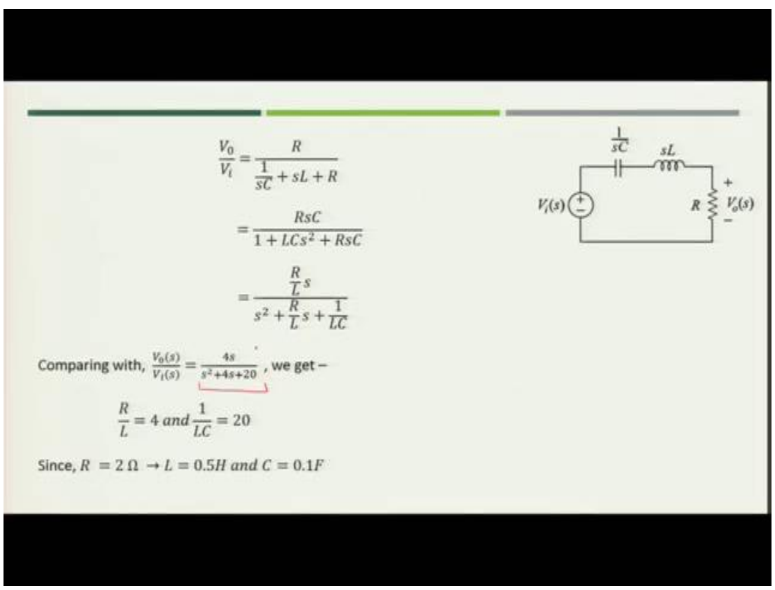

When we write the loop equation and rearrange the voltage V_0 is across R as in terms of V_i, we can write $\frac{V_0}{V_i}$ is nothing but R because voltage across R divided by all 3 series components. So, we can write $R + \frac{1}{sC} + sL$, when we rearrange you can simply write,

$$\frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Now you have got this transfer functions less based on the unknowns which you have in the circuit.

The transfer function which is given to us is 4s upon s square plus 4s plus 20, so if you compare both of them you can simply say that R by L is nothing but equal to 4 and 1 by LC is equal to 20. So here again, you have 2 equations, but you have 3 unknowns. We fix the value of R as 2 ohm, so when you fix, when you fix the value of R as 2 ohm. You can simply get the value of L as 0.5 henry and value of capacitance C as 0.1 farad. With this particular process, you can easily find out the values of the unknowns. So, in this case the unknowns were C and L and we fix the value of R. So, this is how you progress when you are asked to do the Network Synthesis.

With this we can, close our today's session. In this session, we discussed about the Network Synthesis in detail and also carried out our discussion on Network Stability. And particularly in this week, we discussed about the application of Laplace Transform in various circuit analysis. We close this week's session with this note that we will discuss the 2 port network in next week. Thank you.