Basic Electric Circuits
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Module 7: Circuit Analysis using Laplace Transform
Lecture 32: Transfer Function

Namaskar! In this class we will discuss about the transfer function and we will see what we discussed in our previous class first and then we will proceed to transfer function.

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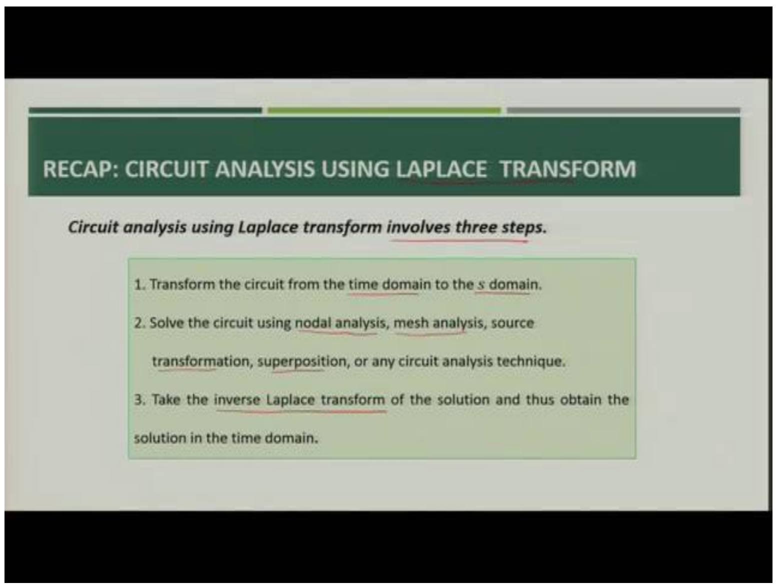

In the last class we were discussing about this circuit analysis using laplace transform and we discussed that these are circuit analysis using laplace transforming involves. The first step was the transformation of this circuit from time domain into s domain. Then we solve this circuit laplace using any circuit analysis technique that maybe nodal analysis or mesh analysis, source transformation superposition and so on. Then finally what result we get that is not as s domain will take the inverse laplace transform to find the solution in time domain.

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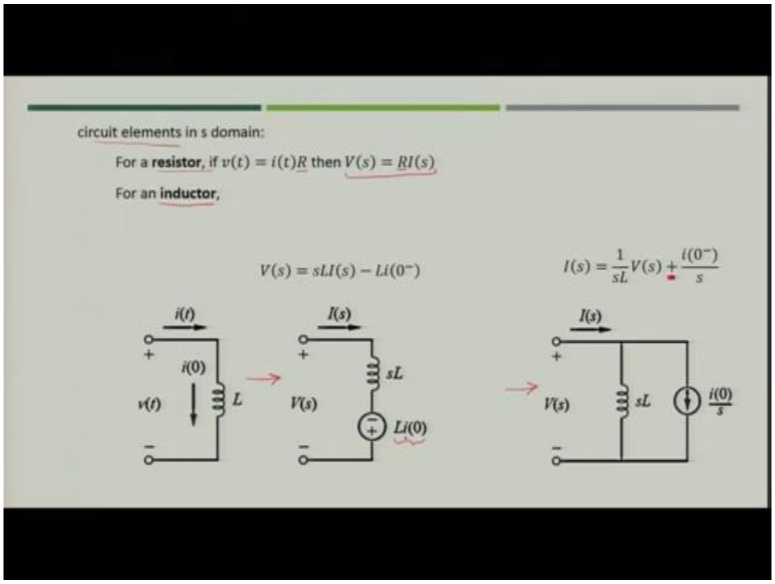

Now, while doing this circuit analysis using laplace transform how we will transform, these are circuit elements from time domain to s domain for register it, there will not be any change. A value of R in time domain will remain R in s domain also. So, $v(t) = i(t)R$, that is the Ohms law or will become $V(s) = RI(s)$ into s domain. These changes would be there in case of inductor and capacitor because these are the energy storage elements.

We will have some initial conditions like in case of inductor, we may have initial current $i(0^-)$. We discussed that we can transform those time domain circuit for inductor into s domain. So this was for the voltage, where you see, this would be applied in case of Kirchhoff's voltage law. Similarly, for current, you will have the circuit as shown in the figure.

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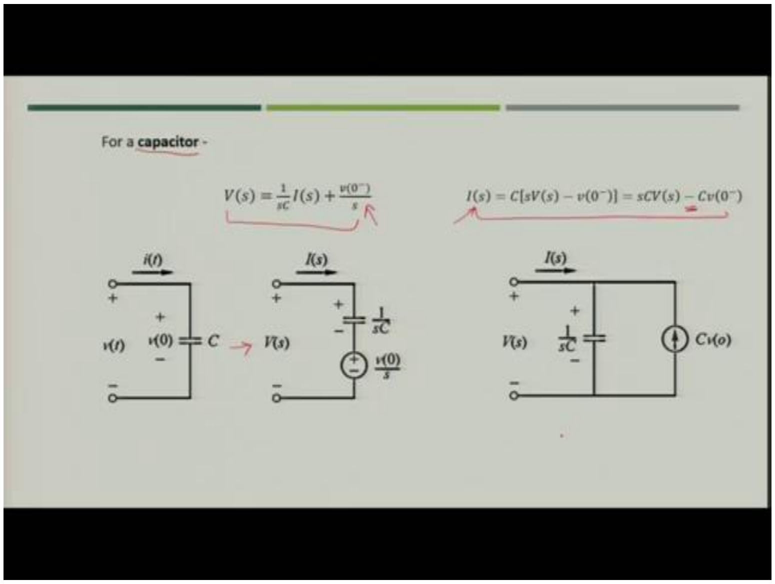

This is what we discussed in the previous class that the direction of the current source that is the fictitious current source which we added because of the initial condition of a current $i(0^-)$ flowing through the inductor. So, the direction is conforming to a plus sign here. And similarly, for here, the initial condition in terms of current will be $-Li(0^-)$. That is why it was having the opposite sign for voltage.

So, this we discussed the previous class. Similarly, for capacitor also we discussed that we will have the two equations one for voltage and another for current. For voltage if you transform the time domain circuit into s domain, this will be as soon in the figure, where $\frac{v(0^-)}{s}$ would be the initial condition. That means the voltage across capacitor at time t is equal to 0.

Similarly, the equivalent current source will also be represented when you are finding out the current flowing through the capacitor in s domain and $Cv(0^-)$ that is the virtual current source will have the direction of current opposite to the flowing from the voltage source that is $V(s)$. So, this is because you have the negative sign here. So, these things we discussed in our previous class and then we, utilized these, equivalent circuits and we created the circuit in s domain and solved it.

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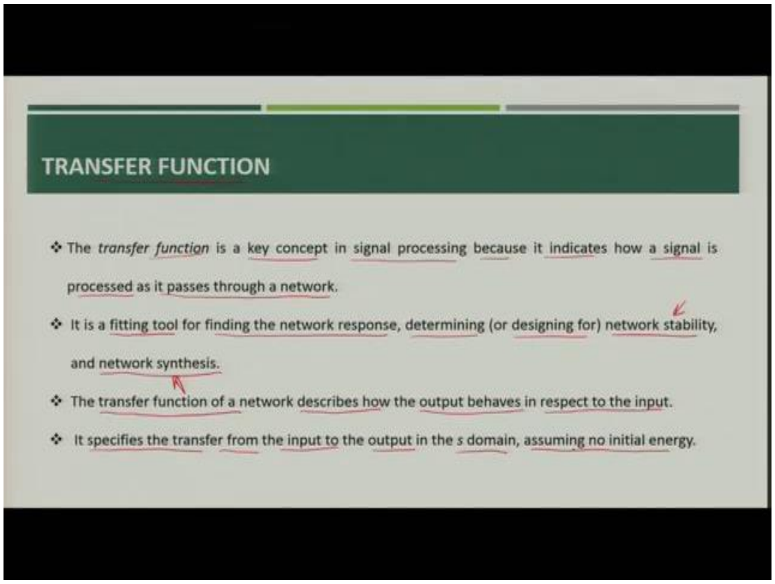

Now, let us proceed forward with the transfer function. What is transfer function? Transfer function is a key concept in signal processing because it indicates how a signal is processed as it passes through a network. So, basically you can say that this is a fitting tool for finding the network response. Determining or designing for the network stability and network synthesis.

So, we will discuss the application of the transfer function when we discuss the network stability and network synthesis concepts later in the course. Now, the transfer function of the network describes how the output behaves with respect to a particular input, and how it will have, we will see in the next slide. It specifies the transfer from input to the output in as domain as you mean no initial energy.

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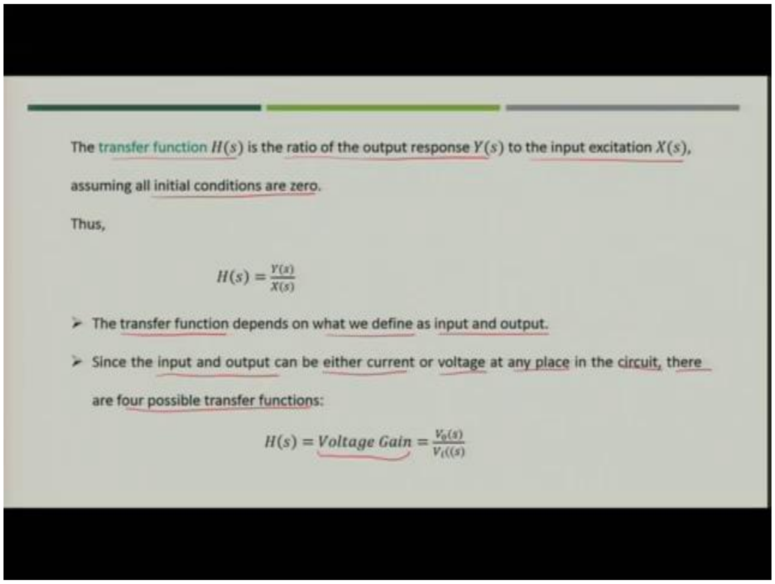

So, how we will define the transfer function transfer function, we generally call it as $H(s)$, which is nothing but the ratio of output response to the input excitation with all initial conditions as zero. So, you can represent transfer function as

$$H(s) = \frac{Y(s)}{X(s)}$$

$Y(s)$ is the output response divided by the input excitation that is $X(s)$ so the transfer function depends on what we defined as input and output.

Now, since the input and output can either current or voltage at any place in this circuit, so therefore, we can have four possibilities in case of the transfer function. First is $H(s)$ in terms of voltage gain where you correlate the output voltage that is output response of the circuit in terms of voltage divided by input excitation given in the form of voltage.

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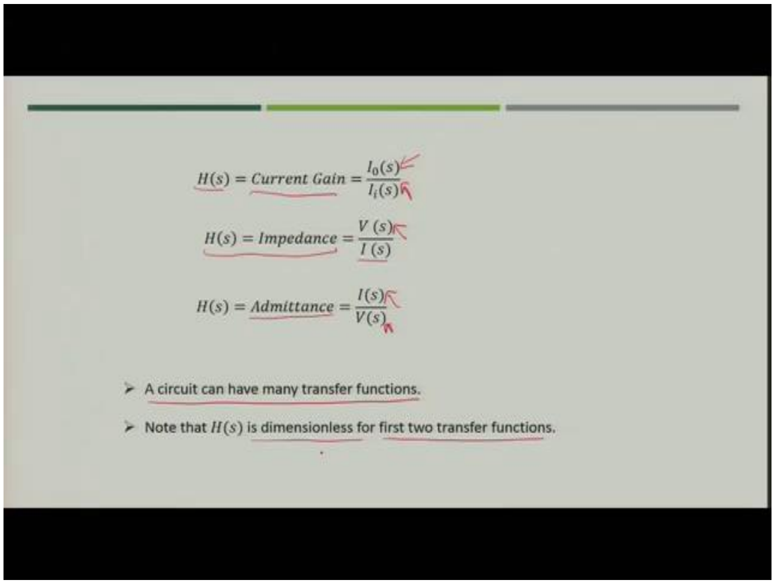

Next is current gain that is again the transfer function which correlates the output current with input current. So, $I_0(s)$ is the output current of this circuit divided by this $I_i(s)$ that is input current for the circuit. Now, next transfer function can be impedance, where output is in the form of voltage, while input is in the form of current. Similarly, it can be admitted also where output is in the form of current and input is in the form of voltage.

$$H(s) = Admittance = \frac{I(s)}{V(s)}$$

In that way we can say a particular circuit can have many transfer functions. Now if you see the first two transfer function, that is voltage gain and the current gain, these are dimensionless because the output an input quantities are the same. So, for these two cases the transfer function will be dimensionless.

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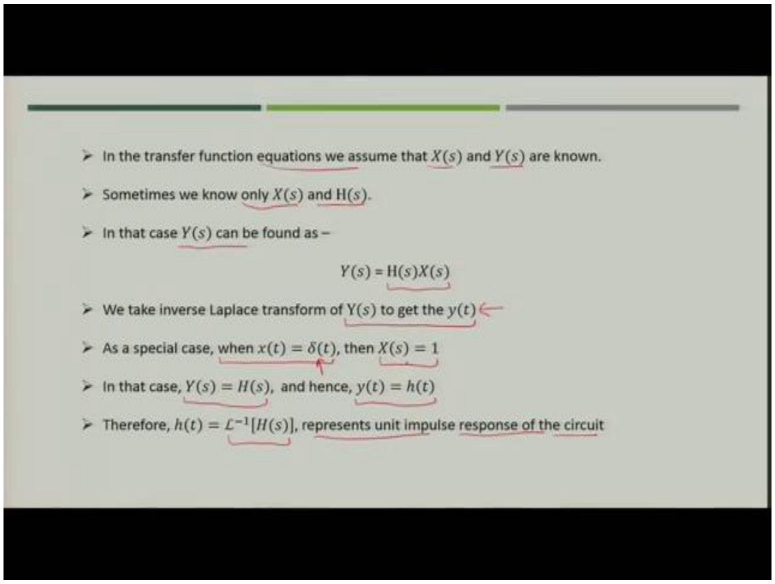

Now, in the transfer function equation we assume that $X(s)$ and $Y(s)$ are known to us, but sometimes it can happen that you know only $X(s)$, $H(s)$ at just that means you know only the input voltage and the transfer function of the circuit. In that case you can find the value of $Y(s)$ that is output response of this circuit with respect to given input, so you will say that $Y(s)$ is nothing but $H(s)X(s)$. Now, you can take the inverse Laplace transform of $Y(s)$ to get the value of output response of this circuit.

As a special case if we say that $\delta(t)$ that is the input is unit impulse function, we will say that access is nothing but equal to one. In that case you will say $Y(s) = H(s)$ and therefore, the, it was Laplace transform at both sides. We will give you the $y(t) = h(t)$. So,

$$h(t) = \mathcal{L}^{-1}[H(s)]$$

This represents unit impulse response of the circuit. When it is excited by a unit impulse function, the transfer function will give you the unit impulse response of the circuit.

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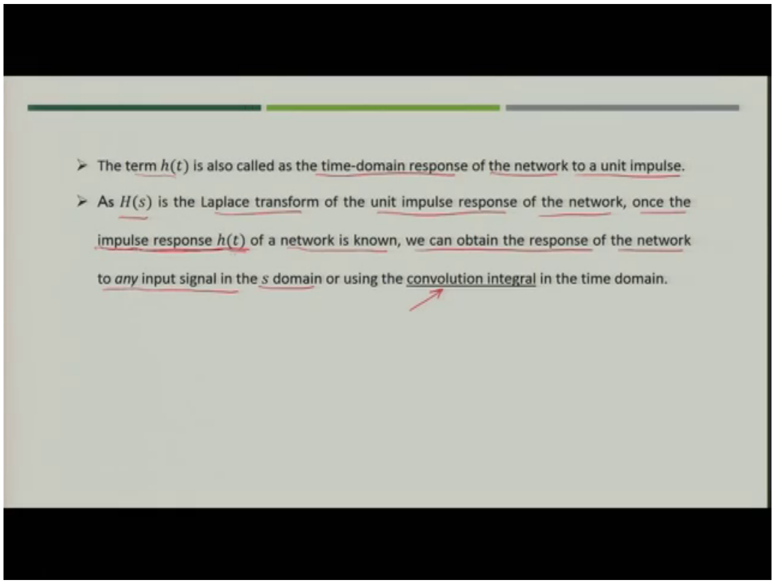

We can also say that $h(t)$ is nothing but the time domain response of the network when unit impulse is given as an input to the network. Now, as $H(s)$ is the Laplace transform of the unit impulse response of the network, once the impulse response entity of the network is known, that means that we know the response of the circuit when a unit impulse input is provided, then we can obtain the response of the network to any input signal in s domain using convolution integral in time domain or corresponding the s domain analysis for convolution integral.

This, we will discuss in the next session where we will see what we mean by the convolution integral and how it can help in finding out the output response of a circuit where we know the input impulse response of the network. So, this we will analyze when we will discuss about convolution integral.

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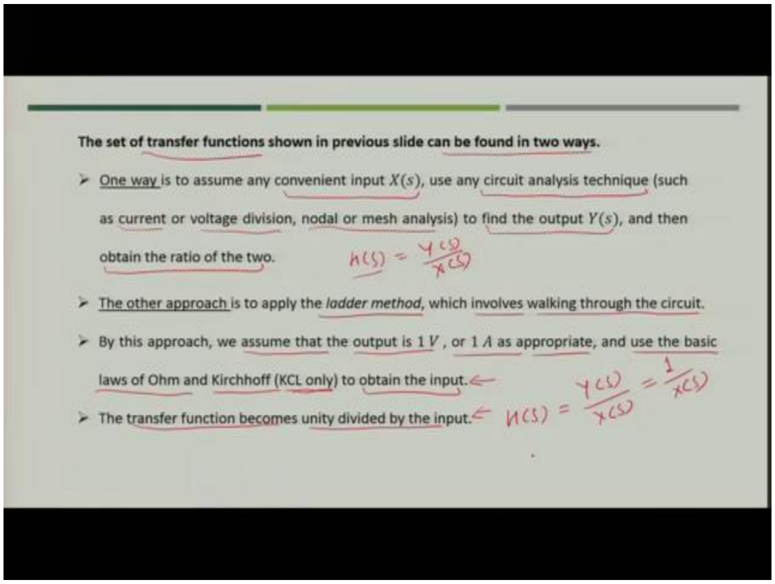

Before going into that detail, let us try to understand that what are the various approaches to find the output response with the help of transfer functions? Basically, we have two ways to process the information and a find finding out the output response. So, first method is that let us assume that the, we have one convenient input access. This access can be either unit impulse or maybe unit step function, unit ramp, whatever it is, you can assume it convenient input. Now, you will use the circuit analysis techniques, what we discussed in our previous lectures.

So, that is like a current or voltage division or nodal or mesh analysis. So, you can apply those techniques and find the output $Y(s)$. Now when you get the output $Y(s)$, then you will try to find out the ratio of the two and when you find out the ratio that is a $H(s) = \frac{Y(s)}{X(s)}$, you know $X(s)$, you can apply the circuit analysis and find the output and then you obtain the ratio of the two to find the transfer function. Or if you know the transfer function, you can straight away find the output response of the circuit.

Now, another approach which you can follow is called as a ladder method. So, what does ladder method means? Basically it involves walking through this circuit. In this approach we assume that output is one volt or maybe one ampere or as appropriate. So, the difference between first approach and second approach is that – in first approach we assume input as known and we take some assumption that the input given to the circuit is maybe unit step or something like that.

But in case of ladder method we first assume output, so it means that we will assume say one volt or one ampere as an output, which we have got from this circuit and then we use the basic laws of ohms and Kirchhoff's law. Generally, we use only the Kirchhoff's current law because it is more convenient in walking through this circuit and then we obtained the input. It means that what we are doing, we are starting from output and using the circuit we are trying to find out what would be the corresponding input for the output given to the circuit.

So, this we can find out the input and then you can find out easily the transfer function by giving the value of, transfer function would be $Y(s)/X(s)$, here $Y(s)$ would be unit so $1/X(s)$ would be the transfer function for this case.

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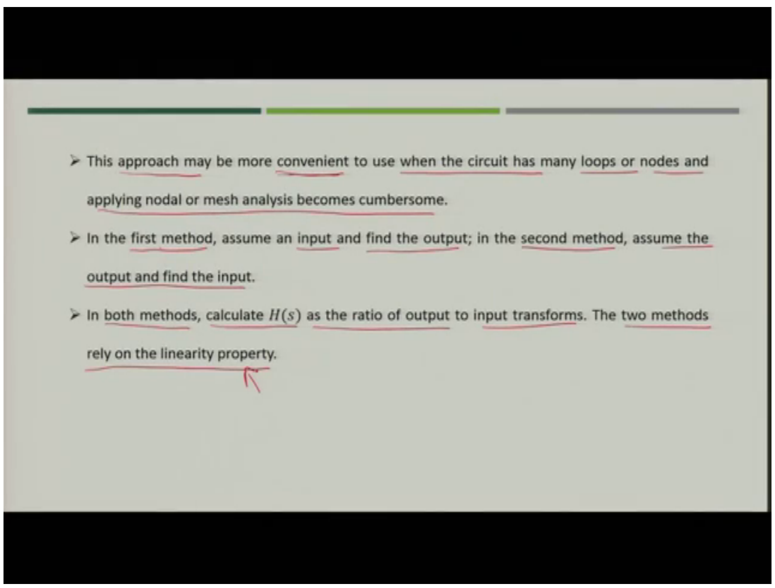

Now these are the two methods you can use either of them based on your convenience. But this approach, what is the ladder approach we discuss is more convenient because when the circuit has many loops or nodes, applying nodal or mesh analysis become cumbersome. If you see the first case you, if you have a complex network where you have multiple mesh and nodes of level, then applying the node nodal or mesh analysis would be a little bit cumbersome.

So that is why compared to first method, second method that is the ladder method is more appropriate in case of complex networks. So, we will use this as a convenient method for finding out the transfer function of the network. Now, in the first method, as we discussed, we assume input and we found the output while in second method, we assume the output and then find the input. Now, in both methods you calculate $H(s)$ as the ratio of output at output to input transform.

And we must keep in mind that these two methods will only be applicable when you have the linear circuit. So, the circuit which follows the linearity property that is a circuit can be used to find out the transfer function using a either the conventional method where you analyze the circuit with the help of various circuit analysis technique or you can use the ladder method. So, now to understand these, the finding out the transfer function let us take a few examples so that you can understand the concept more clearly.

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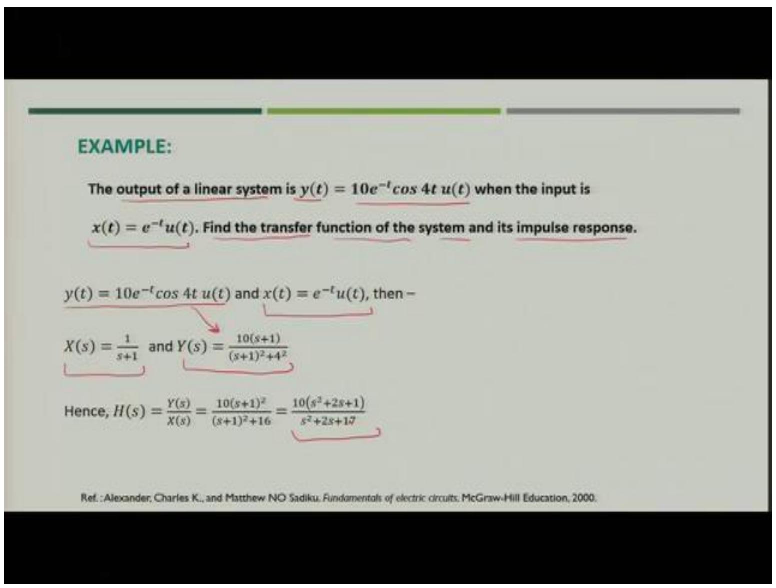

Let us take first example. In this case the output of a linear system is $y(t) = 10e^{-t}cos\ 4tu(t)$ and input given is $x(t) = e^{-t}u(t)$. Now, what we must do, we have to find the transfer function of this system and its impulse response. So, the transfer function you can find out with the help of the s domain analysis. So, you must first convert $y(t)$ into s domain and $x(t)$ into s domain.

When you convert $y(t) = 10e^{-t}\cos 4tu(t)$ into s domain, you will get,

$$Y(s) = \frac{10(s+1)}{(s+1)^2 + 4^2}$$

Here ω=4. Now, the input function is $x(t) = e^{-t}u(t)$, the Laplace transform of $u(t) = 1/s$.

Now, since H s is equal to Y s upon X s, you will put Y s in numerator and the X s in denominator. And when you simplify, you will get the value 10 into s square plus 2 s plus 1 upon s square plus 2 s plus 17

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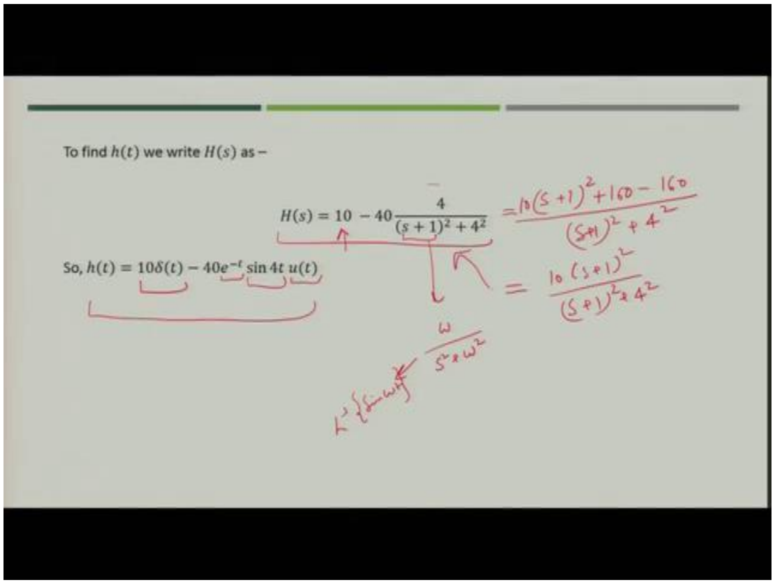

Now, this particular function which you have got, if you rearrange these terms, you can simply write it as,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10}{(s+1)^2 + 16} = \frac{10}{s^2 + 2s + 17}$$

Now if you rearrange it will become,

$$H(s) = \frac{10}{4}\frac{4}{(s+1)^2 + 4^2}$$

When you arrange this in this particular form, what you can say that the inverse Laplace transform

$$h(t) = 2.5e^{-t}\sin 4t$$

So, finally what you get is the transfer function adjust in this form and then you can say $h(t)$ that is the time domain value of $H(s)$, which is the impulse response of the transfer function means you can say a 10 delta t minus 40 e to the power minus t 40 and then you multiply with unit function to say that this is applicable for time t greater than zero. So this would be nothing but the unit impulse response of the circuit. This will help you to understand how you will find out the transfer function and then the unit impulse response of the system.

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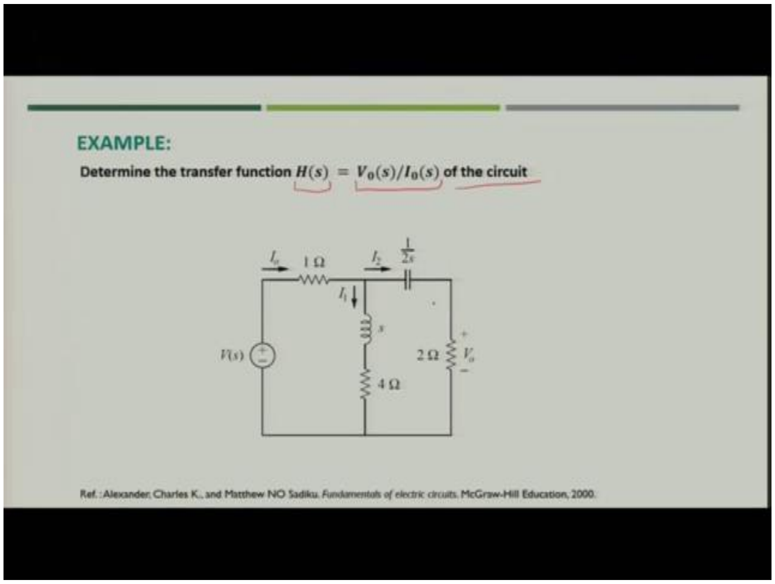

Now, let us take another example here. We need to find out the transfer function $H(s) = \frac{V_o(s)}{I_o(s)}$ for the circuit which is given in the figure. So, in this figure you will see inductor and capacitor both are there, and you have been given the current as $I_o(s)$**,** what you must find out the ratio between $V_o(s)/I_o(s)$. That would be the transfer function for the circuit.

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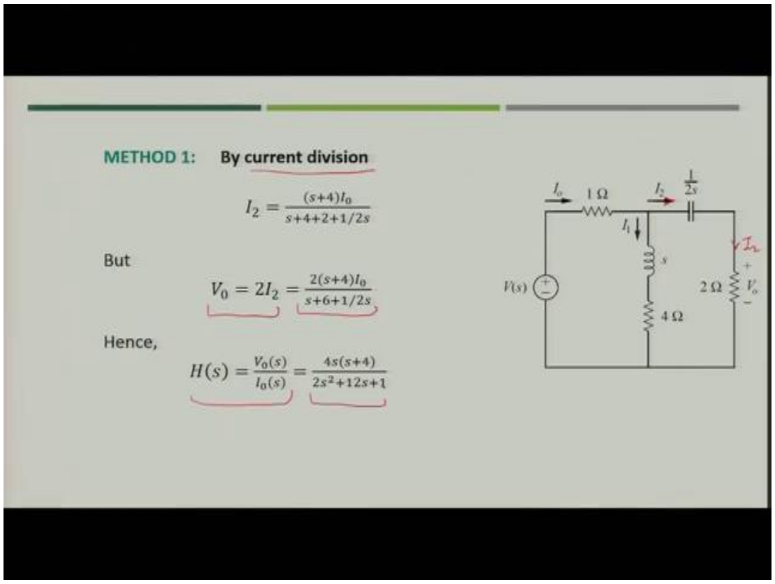

Now, if you see this circuit, you will see that I_2 can be found with the help of current division method. Now in case of current division method, you get,

$$I_2 = \frac{(s+4)I_0}{s+4+2+1/2s}$$

Now when you see this current, and this current is I_2, when you are asked to find the value of V_0. V_0 would be nothing but I2 into the 5 ohm of resistance. So, you will say that

$$V_0 = 2I_2 = \frac{2(s+4)I_0}{s+6+1/2s}$$

So, here in this particular case we used current division method. That is our first method of finding out the transfer function where we use various circuit analysis techniques. So, with this we get the transfer function of this circuit.

$$H(s) = \frac{V_0(s)}{I_0(s)} = \frac{4s(s+4))}{2s^2+12s+1}$$

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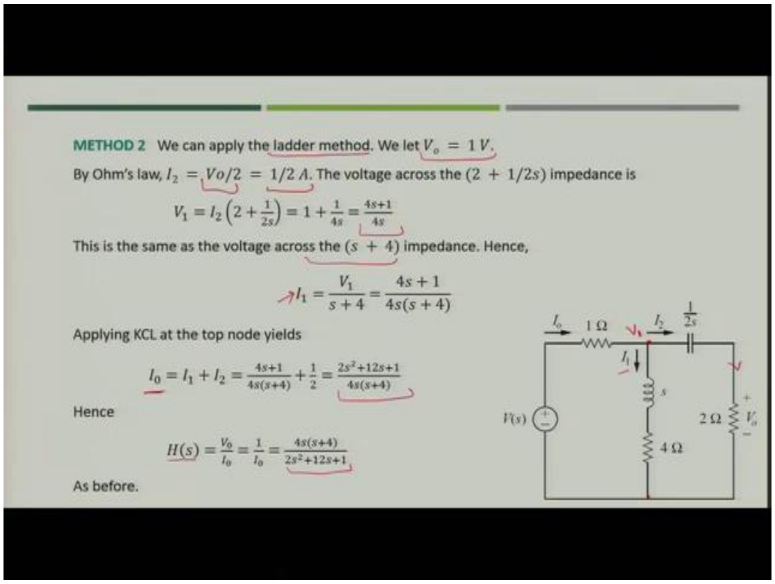

Now, next is that let us try to apply ladder method and we find we will try to find out the value of transfer function when we apply the ladder method. In case of ladder method, what we assume, we assume output voltage as known. So, what we will do here, we will say that the output voltage is given is $V_o = 1\,V$. When you say that $V_o = 1\,V$, the current I_2 which is flowing in this lag will become $V_o/2 = 1/2\,A$. Now, when you know the value of current I_2 you can find out the voltage V_1 which is across this leg.

The voltage across this as well as across this leg will remain same because both are in parallel. We then find out the value of V_1. So V_1 we can find in the terms in terms of I_2 as

$$V_1 = I_2\left(2+\frac{1}{2s}\right) = 1+\frac{1}{4s} = \frac{4s+1}{4s}$$

You can then find out the value of current I_1. I_1 would be,

$$I_1 = \frac{V_1}{s+4} = \frac{4s+1}{4s(s+4)}$$

Now we have I_1 and I_2 both in terms of the transfer function. So, what we can do, we can find out

$$I_0 = I_1 + I_2 = \frac{4s+1}{4s(s+4)} + \frac{1}{2} = \frac{2s^2+12s+1}{4s(s+4)}$$

Then transfer function is given by,

$$H(s) = \frac{V_0}{I_0} = \frac{1}{I_0} = \frac{4s(s+4)}{2s^2 + 12s + 1}$$

So, whether you use first method or second method, you will get the same result.

So, with this week we close today's session and this session we discuss about the transfer function. And we also said that the transfer function will help in identifying the various circuit syntheses, properties. So, we will see in next few sessions that the, how will apply transfer function method in those cases. And particularly in next session, we will first try to understand, what do you mean by convolution theorem and convolution integral, so that we can apply both techniques to further analyze the circuits. Thank you.