Basic Electric Circuits
Module 01
Basic Circuit Elements and Waveforms
Lecture-03
Circuit Elements Part-1
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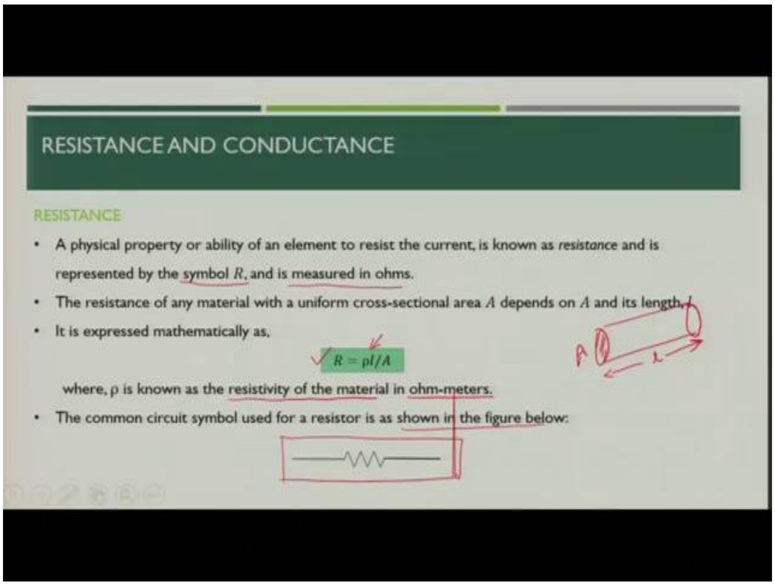

Namaskar, so today we will discuss about the various circuit elements in this lecture. Basically, the major passive elements in our electrical circuits are resistor, capacitor, and inductor. So, today we will discuss the various properties of resistors, capacitors, and inductors. Let us start with the first element called resistor. So, let see what see what is resistance? Resistance is a physical property or ability of an element to resist the current. So, basically whenever the current flows in a particular element it tries to oppose the flow of current. So, that property of the element is called resistance.

Now, how you will represent this resistance? The symbol is R. So, generally you will see that in the literature, the symbol R is most commonly used for the resistance and it is measured in ohms. Resistance of any material is dependent on the area as well as length of the element. So, basically if you see cross section of a particular element of area A and length l, the resistance of that particular element is given by $R = \rho l/A$. It means ρ would be the resistivity of the material which is represented in ohm meters, the length of the element, and cross-sectional area of that element. So, these two are the properties of that element which together with the resistivity of the material will define what is the value of resistances of that element. So, in common circuit elements, resistance is one of the most common and you will see that in the various literature and books, you will see this kind of figure represented for the resistance.

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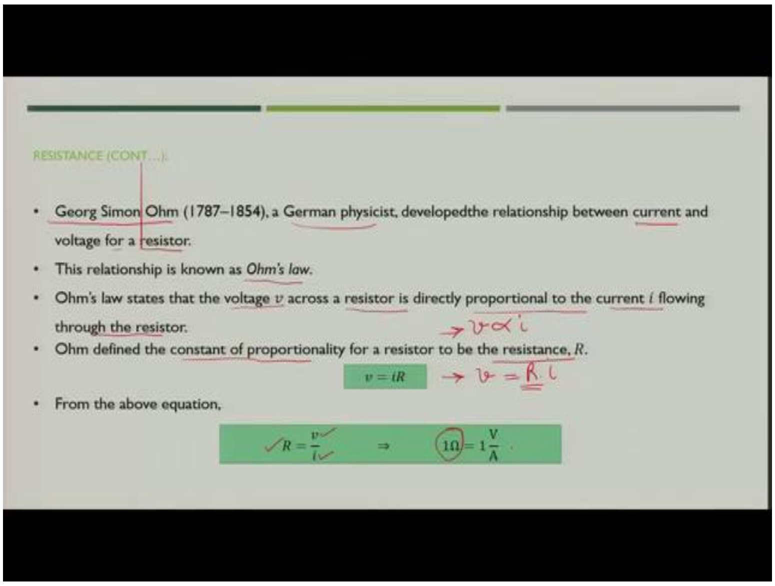

How did the resistance property come into the literature? George Simon Ohm was the German physicist who developed the relationship between current and voltage for a resistor. This relationship is called as Ohms law, which was given by the German physicist. Now, what does Ohm’s law say? Ohm’s law says that, the voltage V across a particular resistor is directly proportional to the current I, which is flowing through that resistor. So, you can simply represent that V is directly proportional to current I.

Now, Ohm define one constant, which is called proportionality constant for this particular equation and that was represented by R and from there the equation came $V = iR$. So, this proportionality constant was the resistance of that element through which the current is flowing. So, now from the above equation you can simply say that resistance R is nothing but, voltage V divided by current. So, for 1 Ohm, what you will say? You will say 1 Ohm is nothing but 1 Volt per Ampere.

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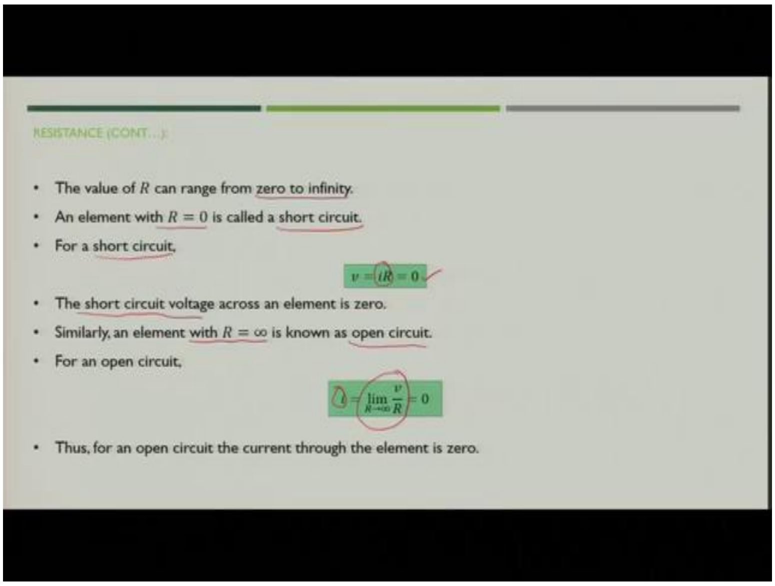

Now, the value of resistance R can change from zero to infinity. Now, if resistance value R is zero it means that, that particular element is short circuit. So, ideally if you take electrical wire you assume that the value of R is zero. Because if you connect that wire through between 2 nodes then the 2 nodes will be short circuited. However, this is only in the ideal condition, because in practical scenario all electrical wires have their own resistance. Although it is small as compared to the other resistive element, but it is still need to be considered. But under ideal condition we assume that the value of that element is zero. So, when the value of R is zero, we generally call it as a short circuit condition. So, what will happen under a short circuit condition? The value of V that is I R will be equal to zero. So, it means that under short circuit condition the voltage across the element would be zero.

Similarly, if the value of R is infinity, because R can range from zero to infinity, under next boundary condition that is R is equal to infinity, you will consider the circuit as open circuit. Under that open circuit condition what would be the current I? You can find out the value of I is nothing but $\lim_{R \to \infty} \frac{v}{R}$ and the value of this is equal to zero. So, it means that under open circuit condition the current flowing through that element would be zero.

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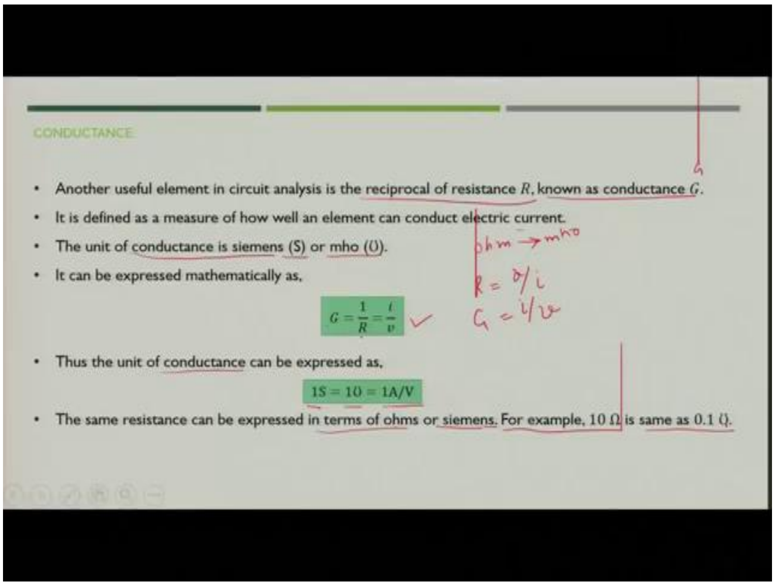

Now, another useful element in the circuit analysis is reciprocal of the resistance which is known as conductance and represented by G. Now, it is defined as a parameter that identifies how well an element can conduct the electric current. So, it is just opposite to the resistance R. R is nothing but the property of any element to resist the flow of current while, conductance says how well that particular element can allow the flow of current.

So, the unit of conductance is given in siemens or alternatively you can say as mho. Mho is nothing but the opposite of ohm. Now, it can be expressed mathematically as,

$$G = \frac{1}{R} = \frac{i}{v}$$

So, the conductance now can be expressed as 1 siemen or 1 mho or as 1 ampere per volt. Now, same resistance can be expressed in terms of ohm or resistance and Ohm or Siemens. That means if 1 resistance is having 10 Ohm as resistance in the same way you can say, that element is having 0.1 mho as conductance.

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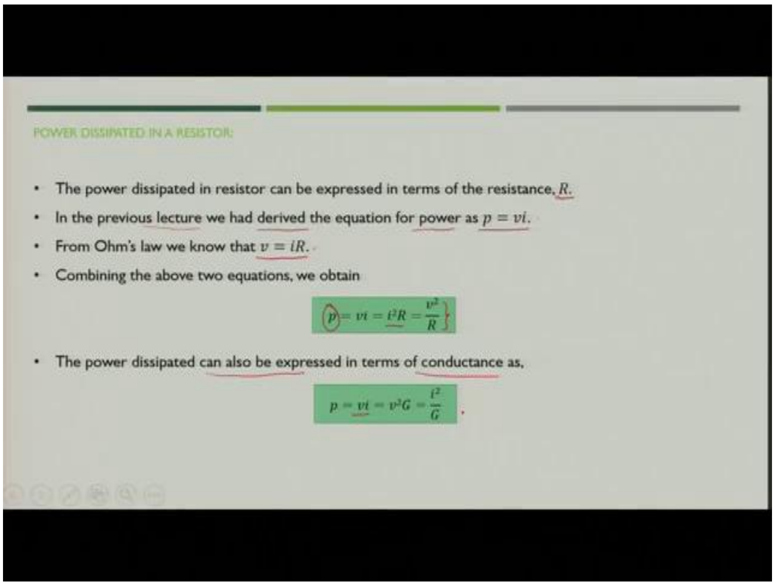

Now, let us see, how you will calculate the power dissipated in a resistor. The power dissipated in resistor can be expressed in term of the element like resistance R and the voltage, and current. So, now in first lecture we derived the equation of power that was $p = vi$. Now form ohms law what we have got, we got $v = iR$. So, if you combine these you will get the value of power

$$p = vi = i^2R = \frac{v^2}{R}$$

So, alternatively the power dissipated can also be expressed in term of conductance. So, wherever you see R you just replace R by R with 1/G, you will get the value of power in term of conductance as,

$$p = vi = v^2G = \frac{i^2}{G}$$

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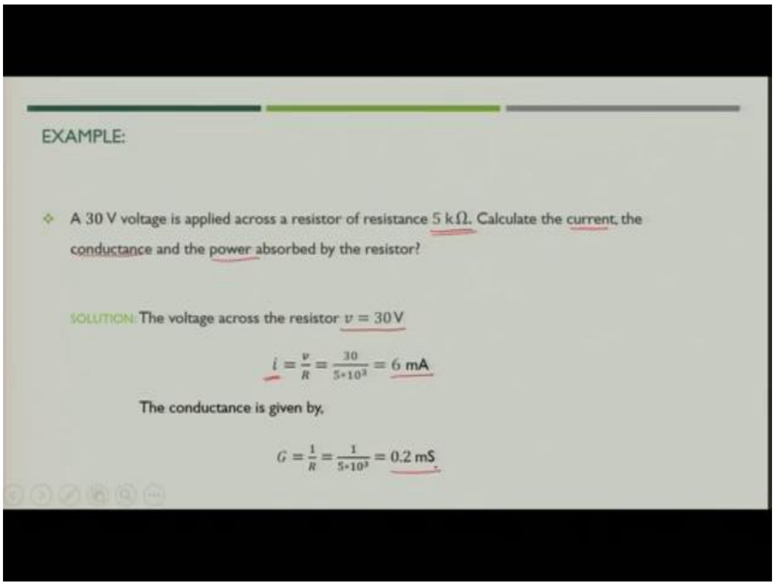

Now, let us take an example to understand better the theory which we just discussed. That is suppose a 30 volt voltage is applied across a resistor of resistance 5 kilo ohm. Calculate the current, conductance and power absorbed by the resistor. So, here voltage across the resistor V is 30 volt then current

$$i = \frac{v}{R} = \frac{30}{5*10^3} = 6 \text{ mA}$$

Now, what would be the conductance?

$$G = \frac{1}{R} = \frac{1}{5*10^3} = 0.2\ mS$$

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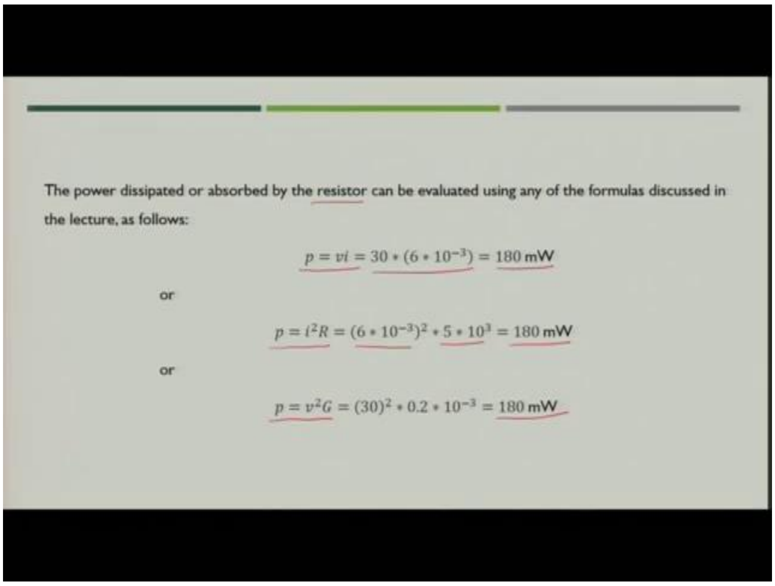

Now, let us try to find the power dissipated or absorb by the resistor. You can use any formula which we have discussed till now. Let us take $p = vi$, so when you put value v and i you directly get the value of power dissipated that is 180 milliwatt. Similarly, $p = i^2R$, so put the value of current i. So, i^2R you substitute the value again you will get the same value of power dissipated that is 180 milliwatt.

Now third option is you can calculate with the help of conductance. That is $p = v^2G$, put the value you will again get the value of 180 milliwatt. So, you can use any formula which you want to use for calculation of power, you will get the same value eventually.

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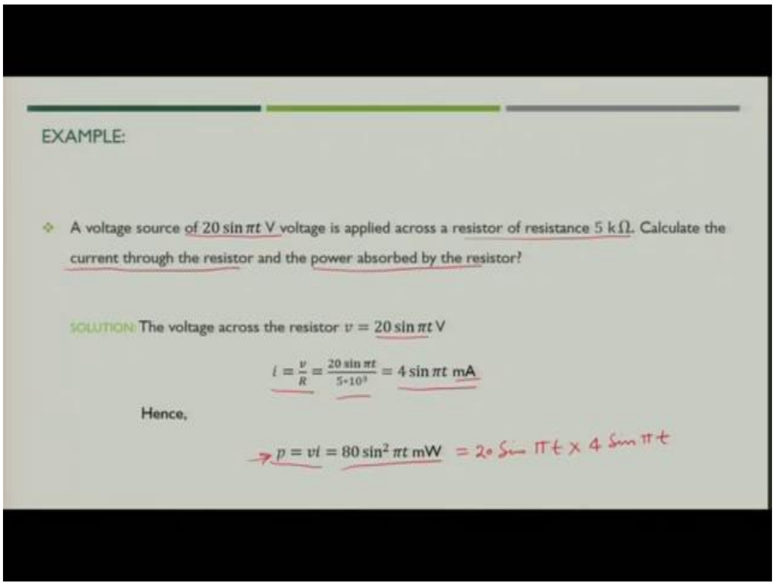

Now, let us take another example. Suppose if the voltage source of $20\sin\pi t$ V is applied across the resistor of resistance 5 k Ω. How will you calculate the current through resistor and power absorb by the resistor?

Simply use the formula

$$i = \frac{v}{R} = \frac{20\sin\pi t}{5*10^{3}} = 4\sin\pi t\text{mA}$$

Now, to calculate the value of power p,

$$p = vi = 80\sin^{2}\pi t \text{ mW}$$

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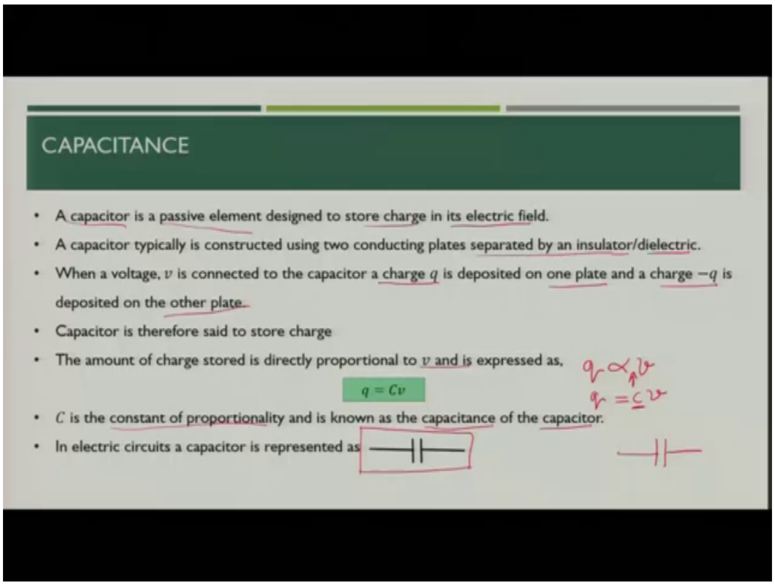

Now, let us consider another element called capacitance. Let us try to understand what is capacitor. So, capacitor is a passive element design to store energy in its electric field. So, capacitor is typically constructed using two conducting plates, separated by an insulator or dielectric. Now, when you apply voltage v across the plates of the capacitor a charge q is deposited on one plate of the capacitor and charge - q is deposited on the other plate of the capacitor.

So, therefore you can say that capacitor is an element which stores charges. Now, what would be the amount of stored charge? That would be directly proportional to voltage v. So, can say that q that is stored charge is directly proportional to v. Now, you can introduce another proportionality constant that is called C. So, here what is C? C is the constant of proportionality which is known as capacitance of that capacitor and $q = Cv$.

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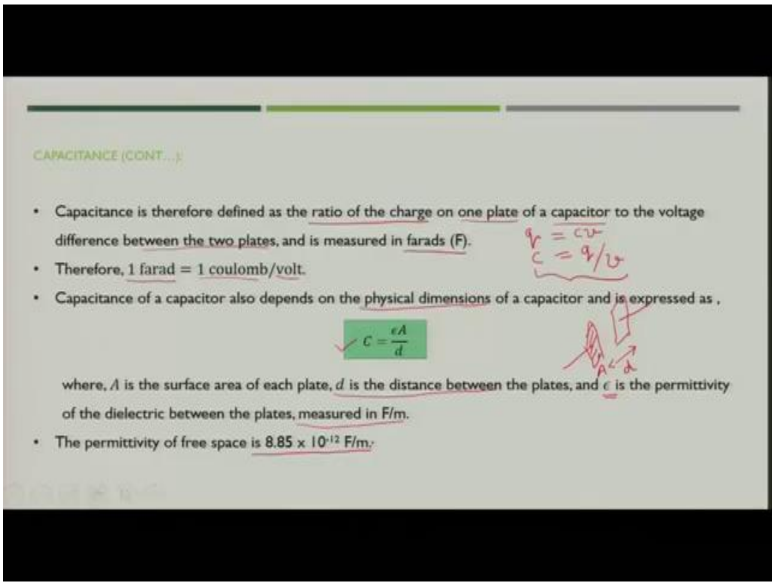

So, how you will define capacitance now? Capacitance can be defined as the ratio of charge on one plate of the capacitor to the voltage difference between the two plates. So, earlier we saw $q = Cv$. So, C would be q by V which is the ratio of charge on 1 plate of the capacitor to the voltage difference between two plates. Now, it is measured in farads. What is the value of 1 farad? 1 farad is nothing but, 1 Coulomb per volt.

Now, capacitance of a capacitor also depends upon its physical dimension. So, if you see the capacitor plates, like if it is two plate capacitor, the area A and the distance between the plates define the value of capacitance of that capacitor. So, how you will define capacitance C?

$$C = \frac{\epsilon A}{d}$$

where, A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric between the plates, measured in F/m. Permittivity of free space is given as 8.85 x 10^{-12} F/m.

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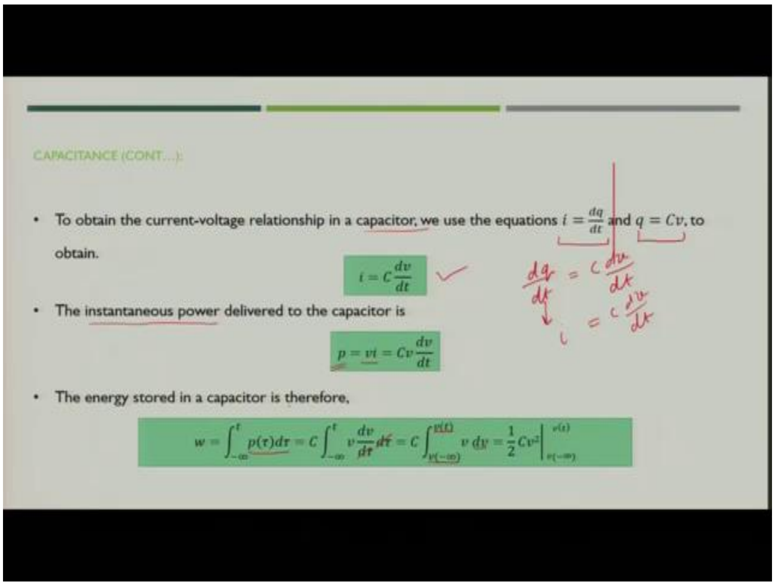

Now, to obtain current voltage relationship in a capacitor, we can use the equation $i = \frac{dq}{dt}$, which we calculated in our first lecture and then $q = Cv$ which we just now saw. So, if you use these 2 equations you can find the value of current I. So, what you have to do? You have to simply take the derivative of both side of the equation with respect of time.

So, this will become $\frac{dq}{dt} = C\frac{dv}{dt}$. C is a constant, so it cannot be differentiated, if you differentiate it will become zero. So, $\frac{dq}{dt} = C\frac{dv}{dt}$. What is $\frac{dq}{dt}$, it is current i.? So, you get $i = C\frac{dv}{dt}$. So, you can easily calculate the value of current i for capacitor.

Now, if you are asked to find the instantaneous power. Then, you have to simply multiply the current which you calculate previously with v, you will get

$$p = vi = Cv\frac{dv}{dt}$$

Now, to calculate the energy stored in the capacitor? You have to just integrate over time this power, you will get the value of total energy stored. So, what would be the value? You integrate from minus infinity to time say t, at particular instant where you want to find out the value of energy stored.

$$w = \int_{-\infty}^{t} p(\tau)d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

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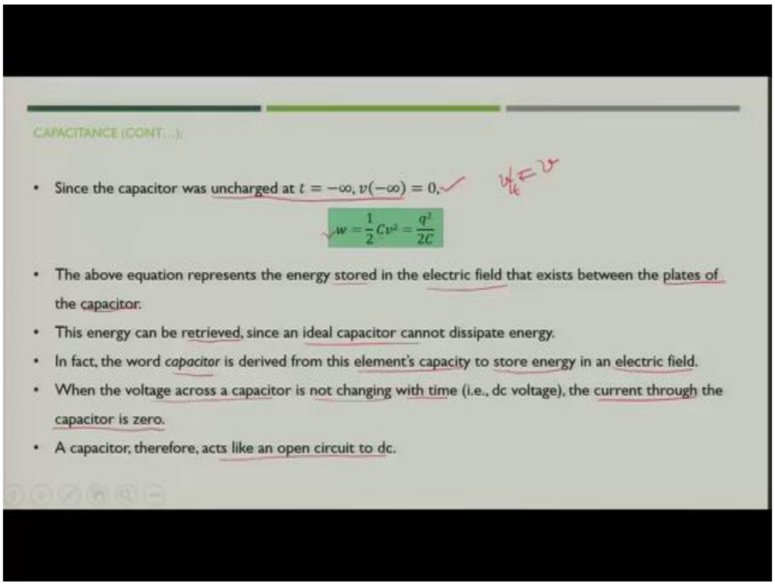

So, it means that your value $v(-\infty) = 0$. And if you take value $v(\mathrm{t})$, v at time at particular time t, then you will say simply as v. So, your total value of energy

$$w = \frac{1}{2}Cv^2 = \frac{q^2}{2C}$$

So, using that particular formula also can find out the value of energy W in terms of q and C. So, now this equation represents energy stored in the electric field that exists between the two plates of the capacitor.

So, this energy can be retrieved because it is an ideal capacitor, so it will not dissipate energy by itself. So that is why you can say that capacitor is element having the capacity to store the energy in its electric field. Now, when the voltage across the capacitor is not changing with respect to time the current through the capacitor would be zero. That means the capacitor acts like an open circuit for DC voltage.

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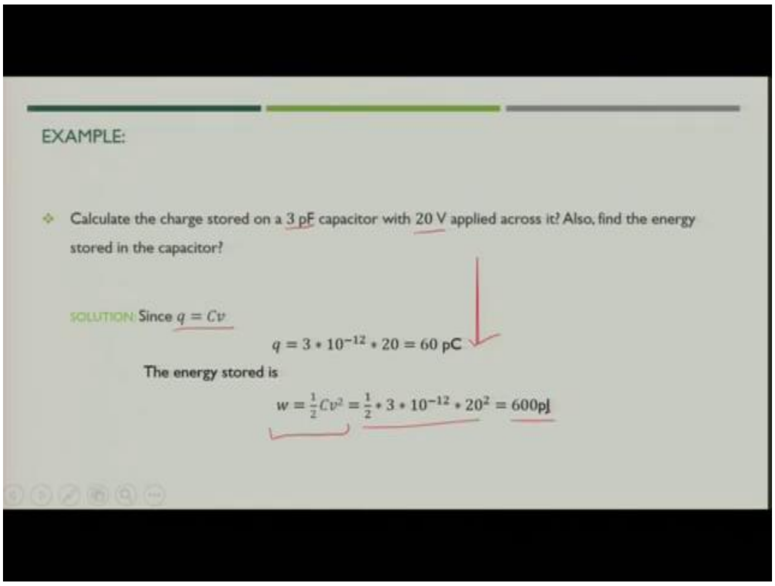

Now, take 1 example, let us calculate the charge stored on 3 pF capacitor with 20 volt applied across it. You just simply use the formula $q = Cv$ and you will calculate the value of

$$q = 3 * 10^{-12} * 20 = 60 \text{ pC}.$$

Now, for energy stored you will use

$$w = \frac{1}{2}Cv2 = \frac{1}{2} * 3 * 10^{-12} * 20^2 = 600\text{pJ}$$

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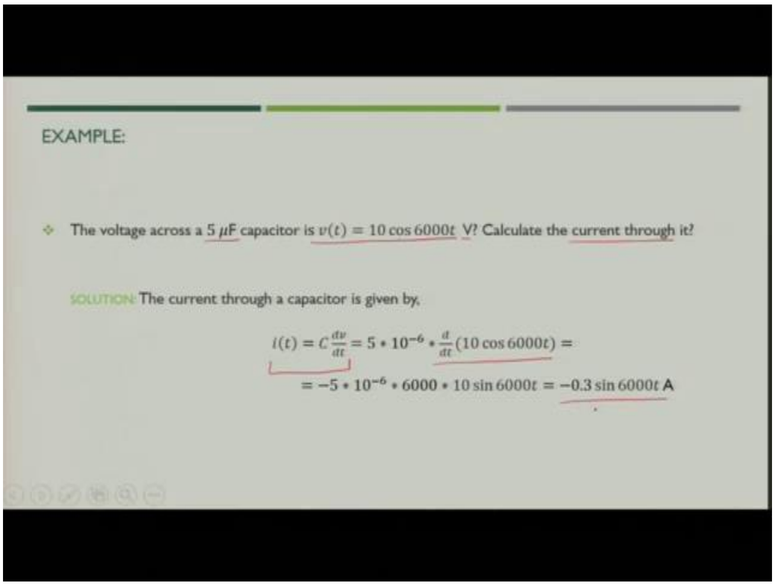

Now, if the voltage across a 5 μF capacitor is $v(t) = 10\cos 6000t$ V. How you will calculate the current passing through it?

You have to use the formula

$$i(t) = C\frac{dv}{dt} = 5 * 10^{-6} * \frac{d}{dt}(10\cos 6000t) = -5 * 10^{-6} * 6000 * 10 \sin 6000t =$$

$$= -0.3 \sin 6000t \text{ A}$$

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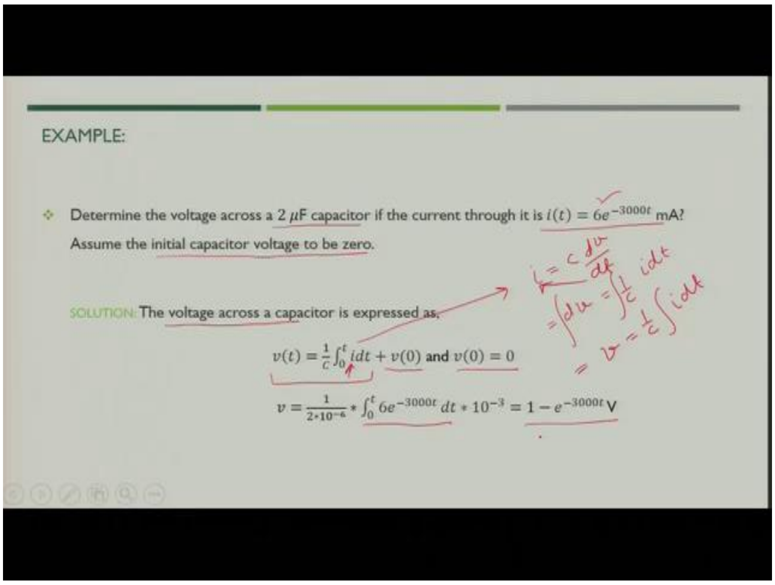

Now, if you are asked to determine the voltage across 2 μF capacitor if the current through it is $i(t) = 6e^{-3000t}$ mA. Assume initial capacitor voltage to be zero, so what will happen? You have to

$$v(t) = \frac{1}{C}\int_0^t idt + v(0) \text{ and } v(0) = 0$$

$$v = \frac{1}{2*10^{-6}} * \int_0^t 6e^{-3000t}\, dt * 10^{-3} = 1 - e^{-3000t} \text{ V}$$

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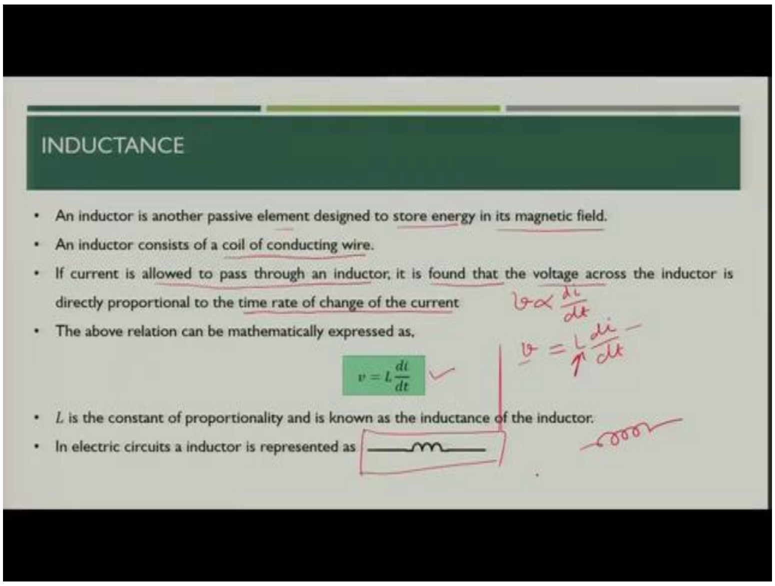

Now, let us come to the third element that is inductance. Now, inductor is another passive element which is design to stored energy. But here the stored energy is in the form of magnetic field. Inductor consists of coil of conducting wire and when current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to time rate of change of current. So that means $v \infty \frac{di}{dt}$. So now if you convert it into equality,

$$v = L\frac{di}{dt}$$

So, this proportionality constant is nothing but, the inductance of that element through which the voltage is measured and current is changing with respect to time. So, $v = L\frac{di}{dt}$ is the property of the inductor and how you will represent this? This is represented in the form of coil. So, the symbol you will see in the literature like this, which is nothing but the symbol for inductor.

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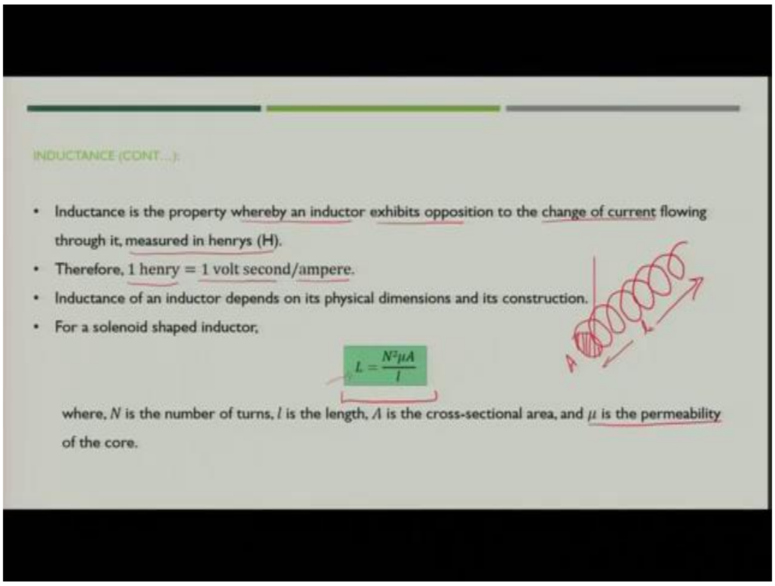

Now, what is inductance? Inductance is the property where by an inductor exhibit opposition to the change in current flowing through it and is measured in henrys. So, what is 1 henry? 1 henry is nothing but 1 volt second per ampere.

Inductance of inductor also depends upon the physical dimension. So, suppose if you have a coil and you are asked to find out the value of inductance. See coil will have cross sectional area say it is A, and length is l. So, for this type of inductor your inductance value

$$L = \frac{N^2\mu A}{l}$$

N is the number of turns, A is the cross sectional area, l is the length of the inductor, and μ, what is μ? μ is the permeability of the core of the inductor.

So, with this you can calculate the value of

$$L = \frac{N^2\mu A}{l}$$

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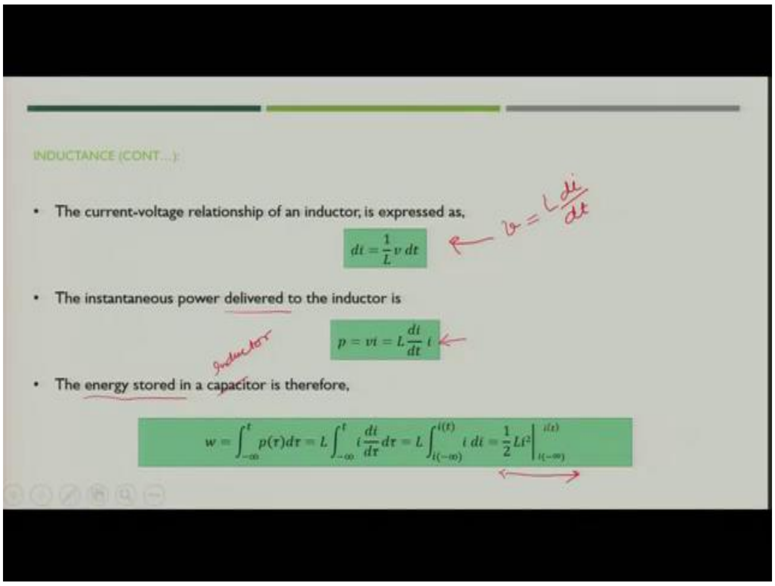

So, now what can you write? You can write

$$di = \frac{1}{L} v\, dt$$

So, to find out the power delivered to the inductor, you will write

$$p = vi = L\frac{di}{dt}\, i$$

Now, if you are asked to find out how much energy has been stored in a

$$w = \int_{-\infty}^{t} p(\tau) d\tau = L\int_{-\infty}^{t} i\frac{di}{d\tau} d\tau = L\int_{i(-\infty)}^{i(t)} i\, di = \frac{1}{2}Li^2\Big|_{i(-\infty)}^{i(t)}$$

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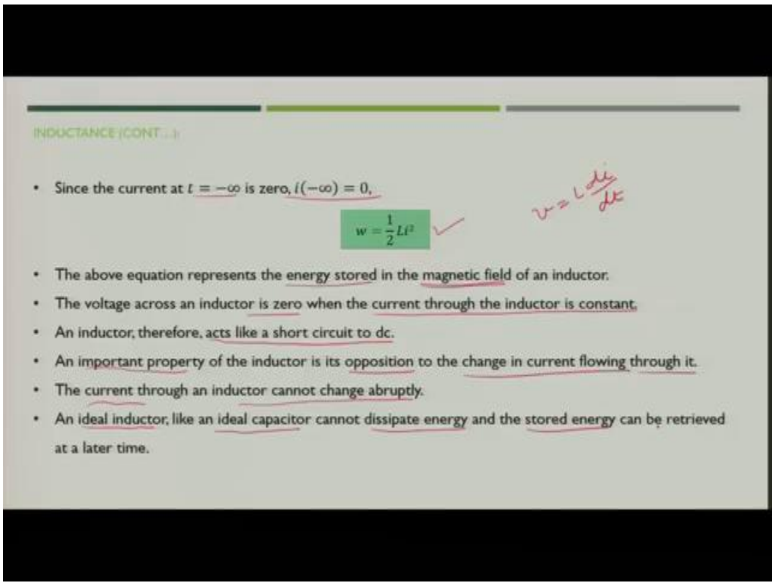

Now, at the initial condition we assume that $t = -\infty$ is zero, $i(-\infty) = 0$,So, finally what we get? We get the total energy stored is

$$w = \frac{1}{2}Li^2$$

So, above equation represents the total energy stored in the magnetic field of an inductor. So, opposite to what we saw in the capacitor. Capacitor energy is stored in the electric field. But the inductor stores energy in the magnetic field. Now voltage across inductor is zero, it means that current through inductor is constant.

Because,

$$v = L\frac{di}{dt}$$

v is zero means rate of change of current is zero. So, voltage across inductor would be zero. In that case the inductor would act like a short circuit to DC. Now, important property of this inductor is that it will oppose to the change of flow of current through that particular element. So, it means that you can say that current through an inductor cannot change abruptly. Therefore, the ideal inductor, like an ideal capacitor cannot dissipate energy and it will store energy which can be retrieved at later time.

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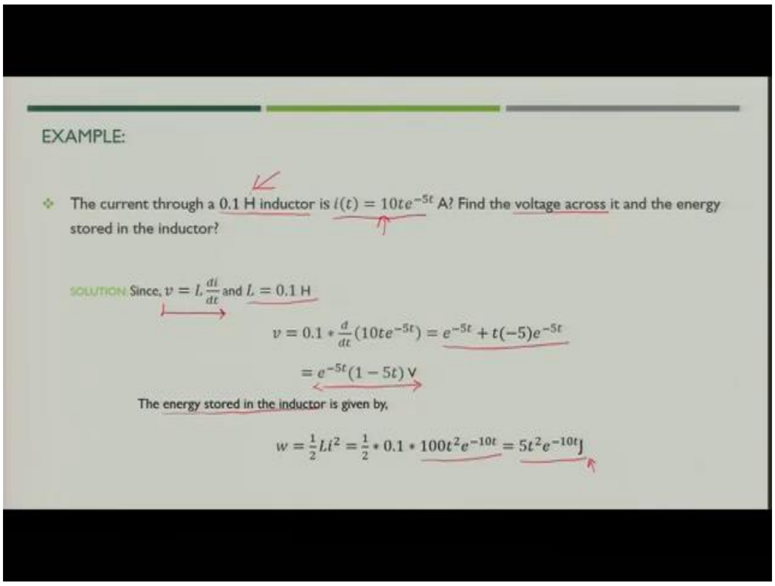

So now let us take 1 example. Suppose if the current through 0.1 H inductor is

$$i(t) = 10te^{-5t} \text{ A.}$$

We need to find the voltage across that inductor. So, what will we do?

We will use the formula that is

$$v = L\frac{di}{dt} \text{ and } L = 0.1 \text{ H.}$$

We then differentiate

$$\frac{d}{dt}(10te^{-5t}) = 10*(e^{-5t} + t(-5)e^{-5t})$$

So, finally what you will get voltage

$$v = 0.1 * \frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t}$$

$$= e^{-5t}(1 - 5t) \text{ V}$$

Now, if you are asked to find the value of energy stored in an inductor. You simply use formula

$$w = \frac{1}{2}Li^2 = \frac{1}{2} * 0.1 * 100t^2e^{-10t} = 5t^2e^{-10t}\text{J}$$

So, today we saw 3 basic elements of our electrical circuit, that are resistor, inductor, and capacitor. So, we also saw that the property of these 3 elements and also saw what is the ohms law. So, in next lecture we will try to find out, how you will utilize this knowledge in analyzing the circuit. So, thank you very much.