**Basic Electric Circuit**
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**Module 6**
**First Order and Second Order Circuit**
**Lecture - 27**
**Step Response of Parallel RLC Circuit**

Namaskar, In the last class we were discussing about the Step Response of Series RLC Circuit. In this class we will continue a discussion on Series RLC Circuit and we will also see the condition when your RLC are connected in parallel and then you provide the step response to that circuit. So, let us recap what we were discussing in the last class.

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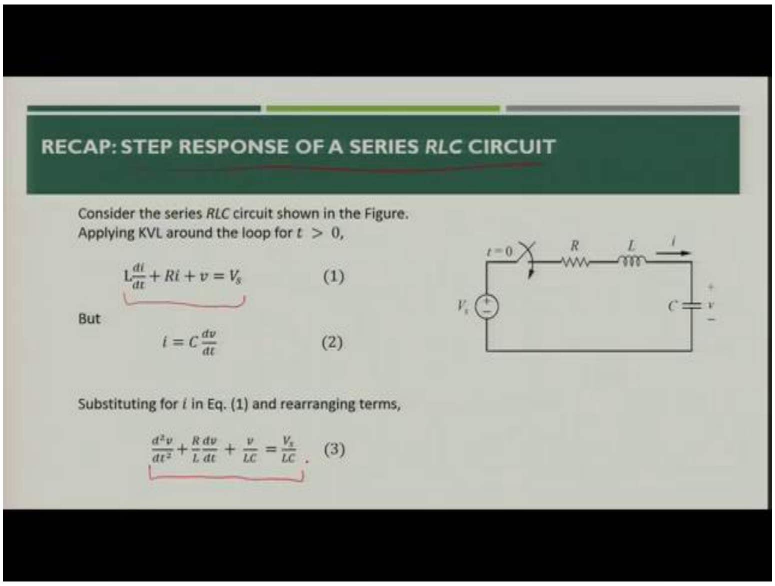


In the last class we discussed about the step response of a Series RLC Circuit and we discussed that at time t is equal to 0 when the switch is closed, the equation for the particular circuit is $L\frac{di}{dt} + Ri + v = V_s$, where the *Vs* is the voltage source. Now when we rearrange then we got the second order differential equation.,

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

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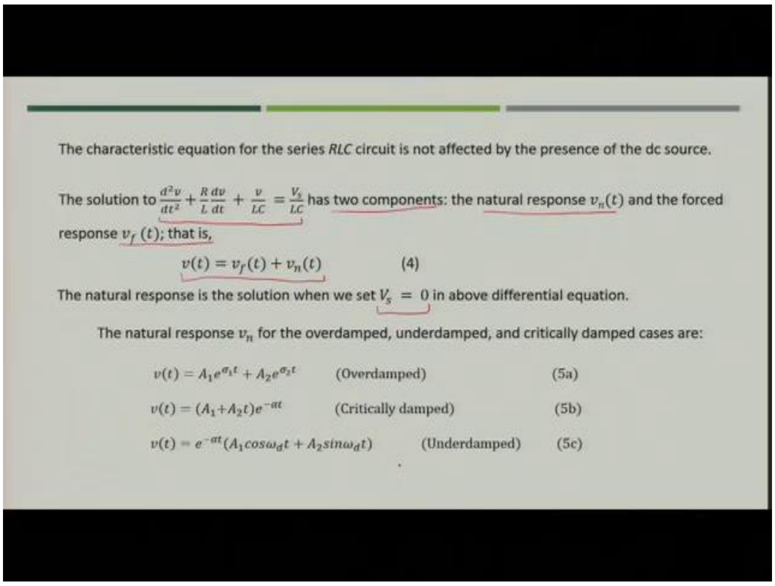


And then we discuss that the solution of the equation which we collected based on the mesh analysis of Series RLC Circuit has 2 components. 1 is natural response and another is forced response and we saw that the total response is the sum of force response as well as natural response. To get the natural response we set *Vs* equal to 0 and we found the 3 conditions like overdamped, critically damped and underdamped situation and we got the 3 voltage value under this 3 condition.

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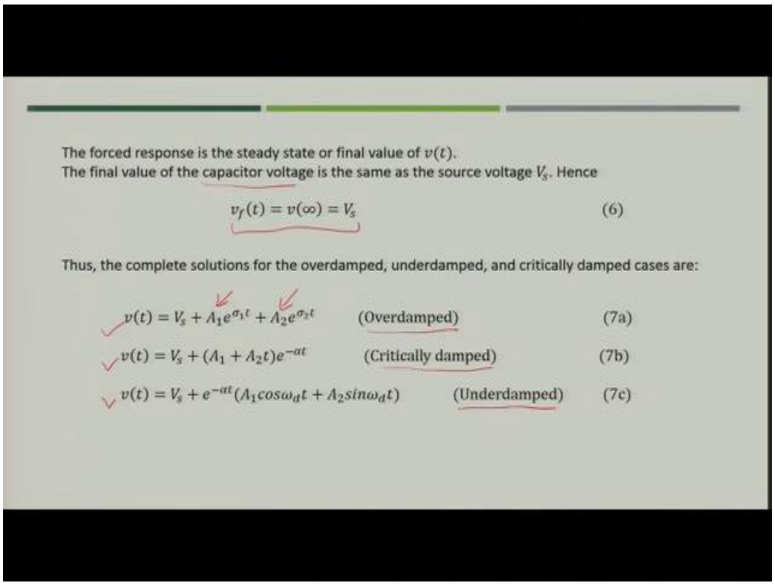


And then we discussed that the force response is nothing but the steady state value across the capacitor that is nothing but the voltage Vs and finally we got the 3 conditions that is overdamped case, critically damped case and underdamped case and what we were discussing in the last class is that, we have got this 3 equations in terms of 2 unknowns those are A1 and A2 which we can the value of this A1 and A2 can be found with the help of seeing the initial and final conditions, like initial voltage or voltage across capacitor, initial current through inductor and similarly the final values so on. So what we will do?

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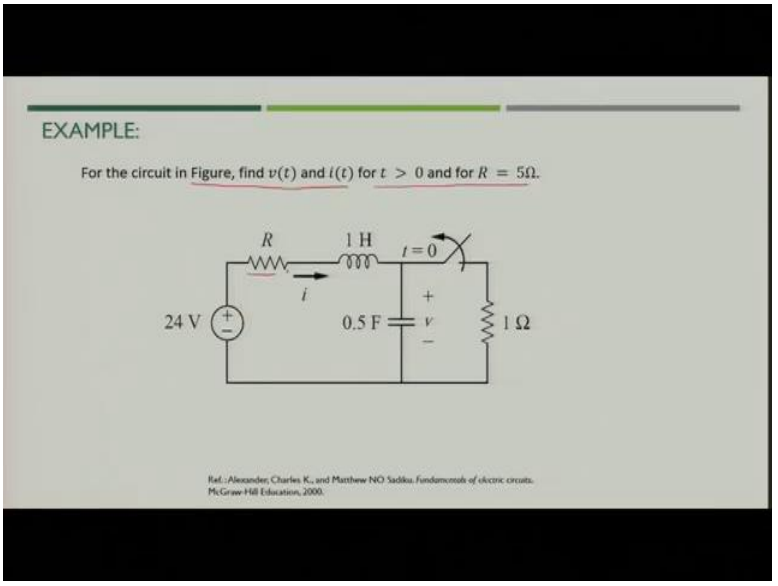


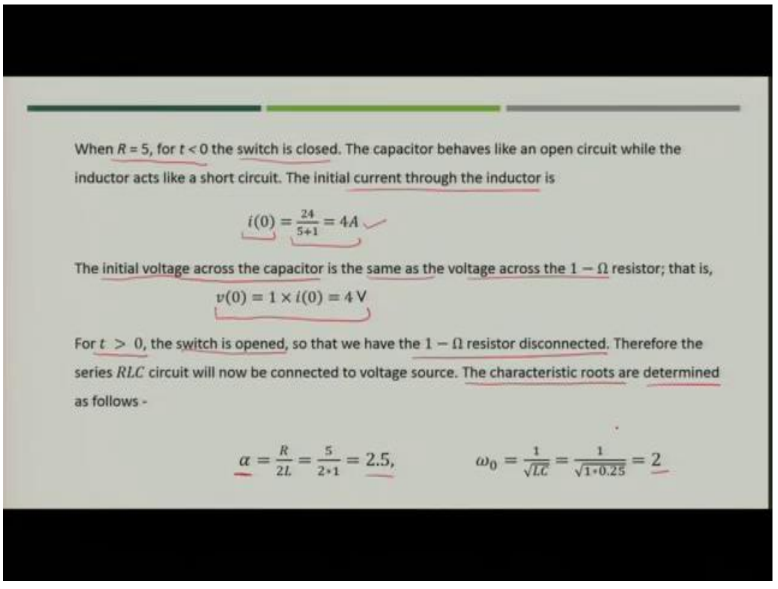


We will try to understand the fact with the help of an example, so what we will do? We will take this example. In this example the switch is open at time t is equal to 0 and the resistance R which is given in the figure is 5 ohm, so what happens the switch is initially closed and when it is opened the circuit is converted into Series RLC Circuit. So, now let us try to find out the value of current i at any time t and value of voltage at any time t.

If you see this particular figure when this switch is initially closed for time $t < 0$, what happens? This particular circuit will give inductor as a short circuit because it is switch is connected for very long period and the capacitor will be open circuit because it will be charge to certain value and it will act as an open circuit. So, what is left in the circuit is only these 2 resistances.

So based on these 2 resistances we can find what will be the value of voltage across capacitor at time t is equal to 0 and what will be the value of current which is flowing through the inductor at time t is equal to 0.

So, that would be our starting point. Now given in the question as a 5 ohm, now switch is closed at time is equal to 0, so particularly at that time that is at time t is equal to 0 we need to first find out the value of current flowing through the inductor so what we will get since we can see from the figure that only these 2 resistances are in the circuit and current flowing through the inductor is nothing but current flowing through the resistances.

So what we get is that 24 volt is the voltage source and value of R is 5, value of 1 ohm resistance is added in series with 5 ohm resistance so total resistance of the circuit is 6 ohm and which is applied across 24 volt. The initial current that is current at time t is equal to 0 is 4 ampere. Now initial voltage across the capacitor would be same as the voltage across 1 ohm resistor.

So, you can see from the figure, we need to find out the value of the voltage across the 1 ohm resistor. Since, we know that the current which is flowing in the circuit is 4 ampere which we just calculated. So the voltage across the capacitor will be 1 ohm multiply by 4 ampere that is 4 volt, so we get the initial voltage across capacitor is nothing but 4 volt, now when time t greater than 0 the switch is open that means the 1 ohm resistor is now disconnected so when you the switch at time t is equal to 0 this particular register will be disconnected from the circuit.

Now in the circuit we will have only R of 5 ohm, 1 henry inductor and 0.5 farad capacitor. So we see the typical case of Series RLC Circuit, now in this case what we have to do we have to first find out the characteristic roots because it is series RLC circuit we discussed what will be the value of the characteristic roots. So let us find out the value of α, $\alpha = R/2L$ and we get the value of alpha as 2.5.

$\omega_0 = 1/\sqrt{LC}$ and its value is 2, now you see that $\alpha > \omega_0$ it means that we have the overdamped condition.

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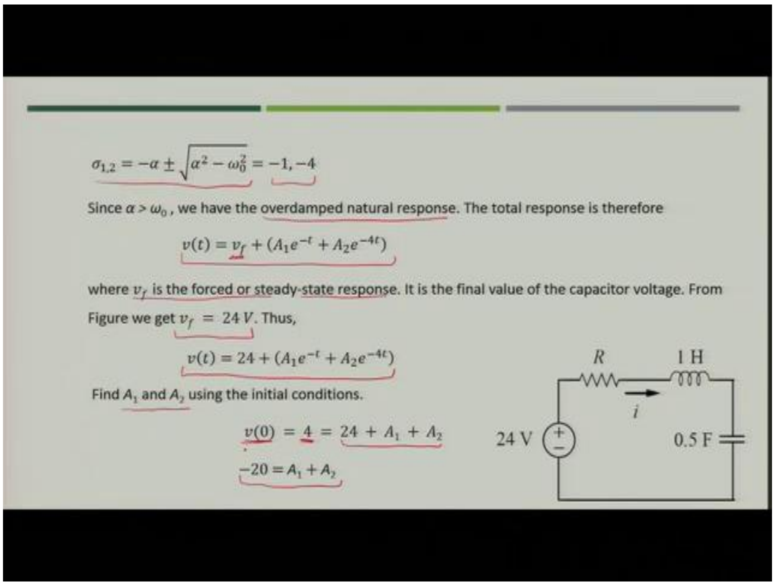


So, in this case the overdamped means your value of the characteristic roots would be -1 and -4 which you can find from the formula that is $\sigma_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}}$ .

So, since it is an overdamped natural response the value of voltage across capacitor at any time t can be given as $v(t) = v_f + (A_1 e^{-t} + A_2 e^{-4t})$. Now, $v_f$ is the forced or steady state response. So, if you see from the figure which is the modified circuit when the switch is opened so the *Vs* the final value that is value of $v_f$ is nothing but *Vs* because when the capacitor is charged to its value the current will be 0.

So at steady state the value of $v_f$ would be 24 volt, so we can now write that $v(t) = 24 + (A_1 e^{-t} + A_2 e^{-4t})$. Now next task is to find out the value of A1 and A2 now from initial condition that is v at time is equal to 0 we know that the value of voltage was 4 volt which we just calculated so if we put t is equal to 0 in this equation we get

$$v(0) = 4 = 24 + A_1 + A_2$$
$$-20 = A_1 + A_2$$

So this is what we get from voltage across the capacitor at time t is equal to 0.

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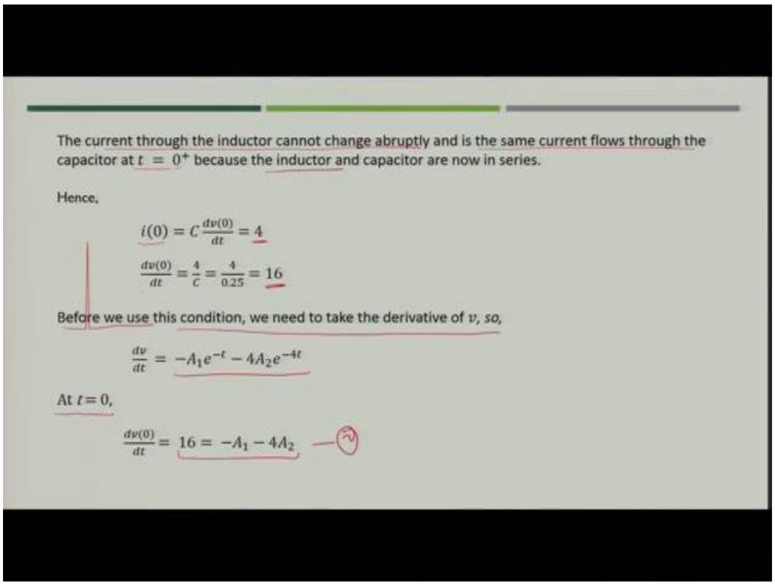


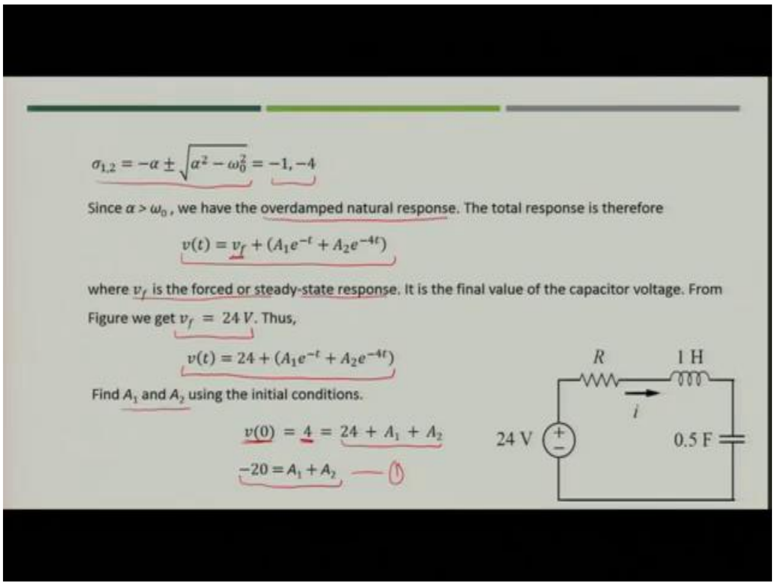


Next we know that the current through inductor cannot change abruptly and it is the same current flows through at time t is equal to 0 plus means even when we open the switch the current across the current through the inductor cannot change abruptly, since it is the series circuit so the current which is flowing through inductor at time t is equal to 0 will continue to flow through the capacitor also, so we can say $i_0 = C\frac{dv_0}{dt} = 4$, which we calculated previously.

So, $\frac{dv_0}{dt}$ at time t is equal to 0 is nothing but 16, so this you can calculate from the equation. Now before using this condition that is the $\frac{dv_0}{dt}$ at time t is equal to 0. We have to first take the derivate of $v$. So we know that this is the value of $v$ at time t. So, let us take the derivative of this. When we take the derivate we get the value of $\frac{dv}{dt} = -A_1e^{-t} - 4A_2e^{-4t}$ , now you will put t is equal to 0 so you get another equation $16 = -A_1 - 4A_2$ .

So, now you have 2 equations for A1 and A2 first you got from here and second you got from here, so you can use these 2 equations and find out the value of A1 and A2.

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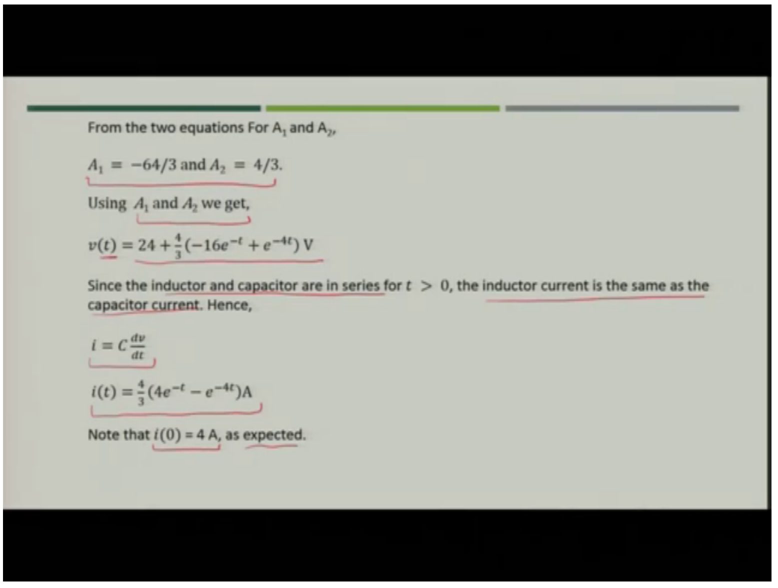


So when you get the value of A1 and A2 you can put the values in the value for voltage at any time t and you get the expression for voltage at any time t, as shown in the equation. Now, since the inductor and capacitor are in series the inductor current is the same as the capacitor current, so what we can say? We can say inductor current i can be given as C dv by dt, so when you differentiate this and multiply by C that is the capacitor you will get the value of current i at any time t also. Now if you put the value of t as 0 you will get the same expression which we derive for current i at any time at time t is equal to 0 which is our expected result. So, in this way you can calculate the value of unknowns that is A1 and A2 with the help of initial values that is inductor current and capacitor voltage.

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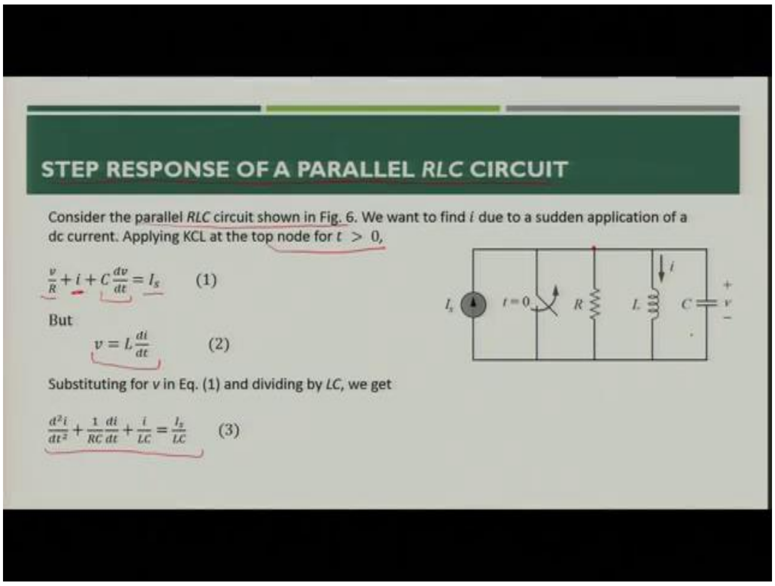


Now, let us move on to the second case that is step response for a parallel RLC circuit. Now in this case suppose the parallel RLC circuit is shown is in this figure here, what we are doing? We are initially keeping the switch closed at time t is equal to 0 we are opening up the switch so that we get the parallel RLC circuit at time t less than 0 the current source will be short circuit so when we open it then it will be converted into Parallel RLC Circuit.

Now, at time t is equal to greater than 0, let's apply KCL at the top node, so what will happen? The value of current I s will be divided into 3 circuit elements that is R, L and C so what would be the value of current flowing through resistance R that is V by R, V is nothing but voltage at any t across the capacitor plus i that is the initial we assume that is current i which is flowing through the inductor at any time t plus the capacitor current, capacitor current can be given as C dv by dt and the sum of this 3 currents will be equal to the value of source current that is $I_s$.

Now, we know that $v = L\frac{di}{dt}$. We put the value of v in this equation and when we rearrange to get the final equation as $\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$ So this is what you get from the parallel combination of the circuit.

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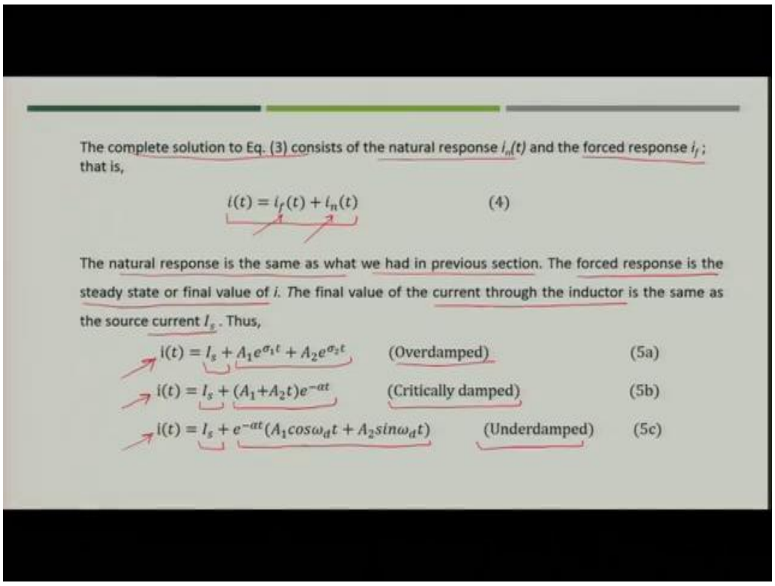


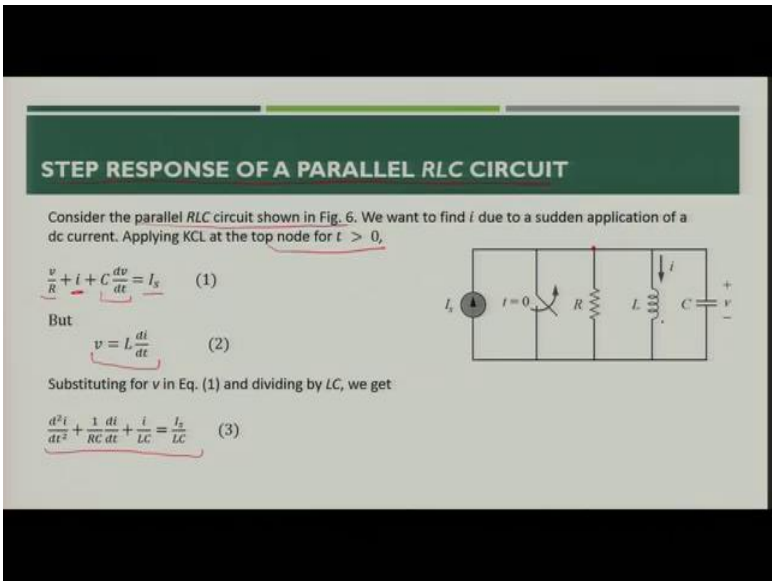


Now, the complete solution will be consisting a natural response as well as forced response. So, you can write the total current i is nothing but forced response and natural response of the current. Now natural response of the current is same as we got in previous section when we discuss source free parallel RLC circuit, so we can write those equations, so what we got? That natural response in case of overdamped is, $A_1e^{\sigma_1 t} + A_2e^{\sigma_2 t}$, where $\sigma_1$ and $\sigma_2$ are the characteristics roots of the governing equation.

And in case of critically damped the natural response was $A_1e^{-\alpha t} + A_2e^{-\alpha t} = A_3e^{-\alpha t}$, α was the damping coefficient and in case of underdamped we got $e^{-\alpha t}(A_1\cos\omega_d t + A_2\sin\omega_d t)$ where,

$\omega_d$ was the damping frequency . Now these 3 we got when we discussed about the source free response that is natural response of the circuit.

Now forced response is the steady state or the final value of i. So if you see the circuit when the switch is open for very long period this inductor well be short circuit, so whole current that is $I_s$ will flow through the inductor and you can say that the final value of current which is flowing through inductor is same as source current that is $I_s$. So, you will add the value of $I_s$ in place of the $i_f(t)$ that is the force response.

So you will get the total value of current is nothing but $I_s$ plus natural response of the circuit in case of overdamped, again $I_s$ plus critically damped response of the circuit when source is not available and plus in case of underdamped we get $I_s$ plus the natural response of underdamped circuit.

So, these 3 we get based on the various conditions with respect to your damping the coefficients and the natural frequency of the circuit, so that is $\alpha$ and $\omega_0$. So under various condition you will get these 3 values for current. In case of overdamped your $\alpha > \omega_0$, in case of critically damped $\alpha = \omega_0$ and in case of underdamped $\alpha < \omega_0$.

So, now in this set of current equations you know that the A1 and A2 are the 2 things which are unknown, so how we can find? We can again use the initial values of current and voltage that is current flowing through the inductor and voltage across the capacitor to find out the value of A1 and A2. Now how we will do?

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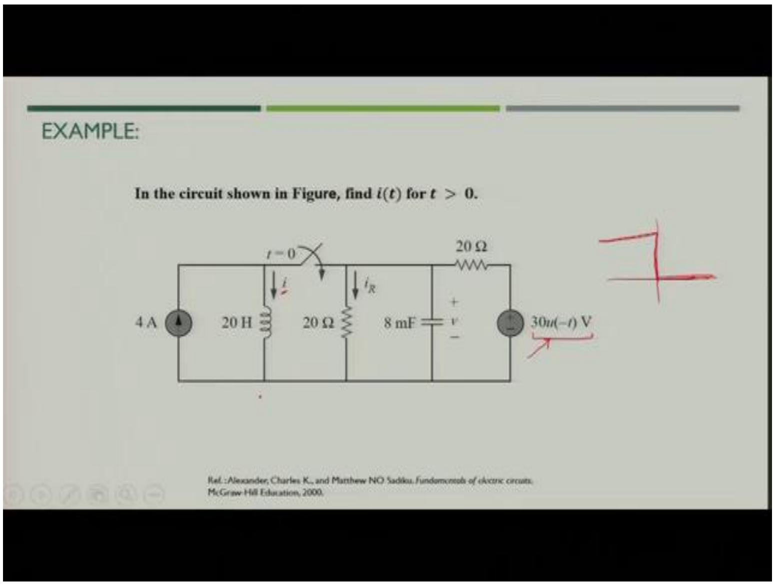

Let us take 1 example so that the things are more clear, here if you see the circuit the 4 ampere current source is connected across the inductor so at time t less than 0 when the switch is opened, the current is flowing only through the inductor so you can say that at time t less than 0 the circuit is divided into 2 parts. One part is where the current source is available second part were voltage source is available, so at time t less than 0 when the switch is open the current will be flowing through the 20 henry inductor.

It means that you can simply say current $i$ at time is equal to 0 is nothing but 4 ampere. This you can get easily from the circuit, now you see that the voltage which is connected across the second part of the circuit is $30u(-t)$ volt. So, what does it mean? It means that the time t less than 0 the value of voltage is 30 volt, at time t greater than 0 these value became 0 volt, means when the switch is closed this voltage source is short circuit.

So, this is something which you have to keep in mind because here the unit is step function is $u(-t)$ it means that the value of unit step function is like this were you have a value at time t is equal to, less than t is equal to 0 but it is 0, when t greater than 0. So with these 2 things let us proceed to solve the circuit.

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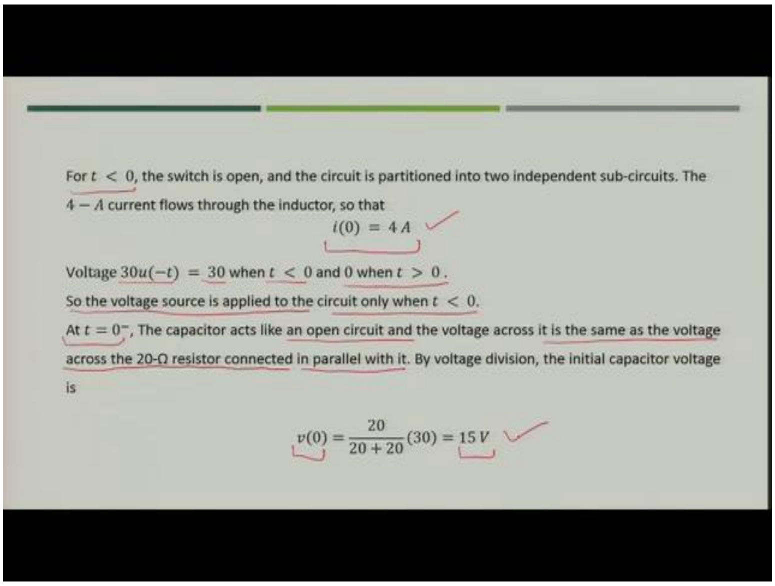

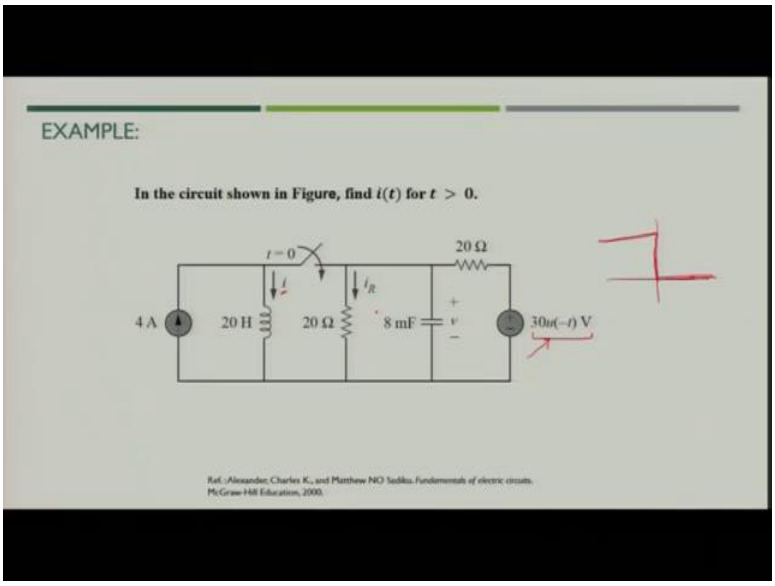


For t less than 0 this switch is open, so we found that the initial current which is flowing through inductor is 4 ampere. Now $u(-t)$ is equal to 30 when t less than 0 and 0 when t greater than 0. So, the voltage source is applied to the circuit only when t less than 0. Now at t is equal to 0 minus that means just before the switch is closed the capacitor acts like a open circuit and voltage across it is the same as the voltage across 20 ohm resistor connected in parallel with it.

So if you see this segment of the circuit the voltage because the switch is open for very long period this will the capacitor will act as an open circuit, so the voltage across capacitor will be same as the voltage across 20 ohm resistance. Now, at time t less than 0 the value of voltage source is 30 volt and these 2 are in series because the current will flow through this direction. So, what will be the value of voltage across 20 ohm resistor which is in parallel with capacitor.

You can simply use the voltage division and you will come to know that the value of this voltage at time t is equal to 0 is nothing but 15 volt. This is our initial voltage across the capacitor when the switch is closed. So, we got 2 initial conditions current which is flowing through the inductor at time t is equal to 0 and voltage across capacitor at time t is equal to 0.

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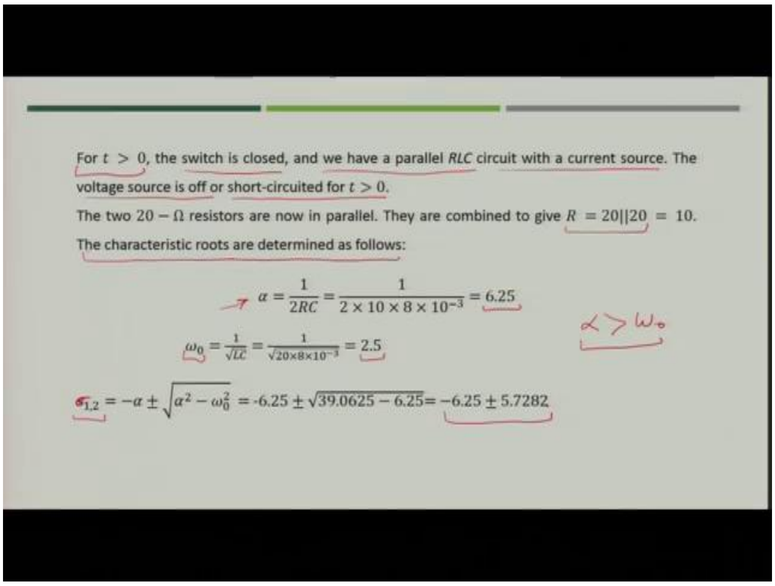


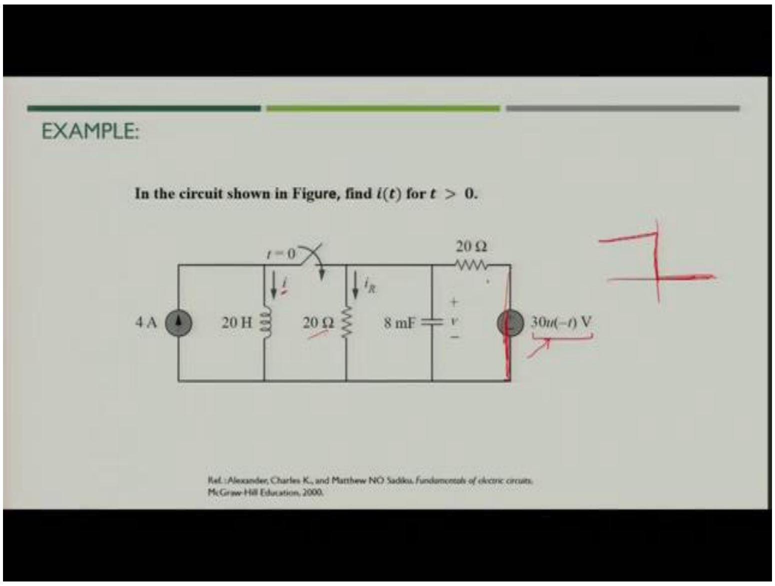


Now for time t greater than 0 what will happen the switch is now closed and we have Parallel RLC Circuit with a current source now voltage source is off because for time t greater than 0 this will be 0 so it means that it will be short circuit. So, now 20 ohm and this 20 ohm both would be in parallel, so what we can say that the value of effective R which is across the circuit will be 10 ohm because 20 ohm and 20 ohm are in parallel now. This will be short circuited.

So, now we have a case of parallel RLC circuits, so we have inductor, we have capacitor and we will have another equivalent resistance of 10 ohm which is again in parallel. So, what we can do now? We can apply the equations which we got in case of Parallel RLC Circuit, so characteristic roots we can simply determine with the help of the values which we calculated previously.

So α is nothing but $1/2RC$, so if you put the value of R and C, R is 10, C is 8 milli farad so it will became 8 into 10 to the power 3 and we get the value of α as 6.25. Similarly, $\omega_0 = 1/\sqrt{LC}$ and you get it as 2.5. Here, you will see that $\alpha > \omega_0$ that means that this is a case of overdamped circuit. We can say, the value of the roots, i.e., $\sigma_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}}$ and we get 2 characteristic roots.

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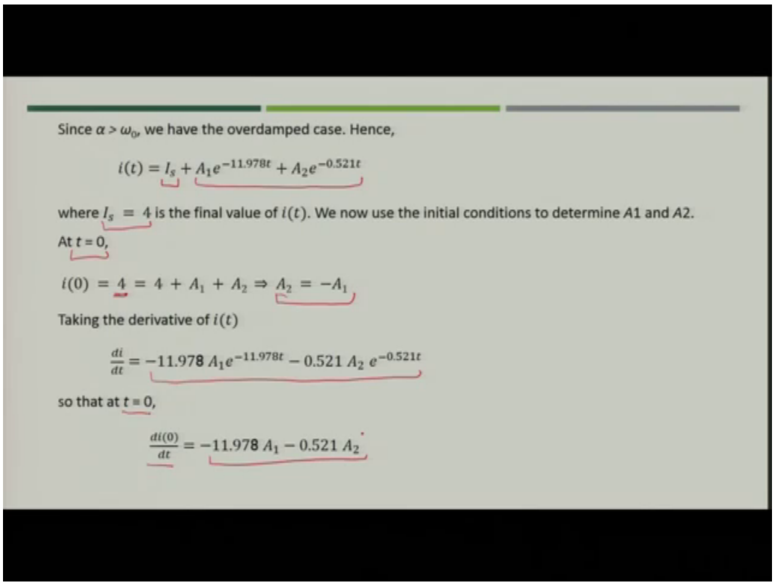


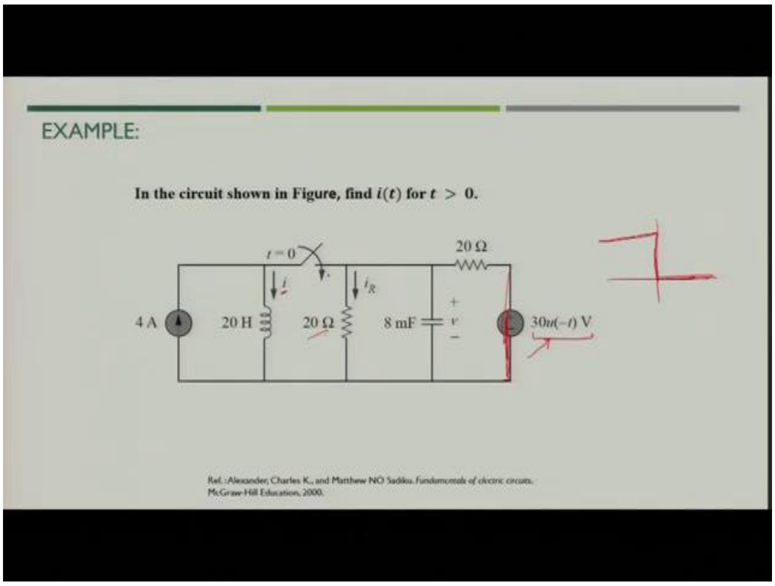


We can write the value of current $i(t)$ that is nothing but equal to the force response that is $i_f(t)$ and here the value of $i_f(t)$ is $I_s$, which we just calculated in the case of parallel RLC circuit plus the natural response, i.e., $A_1e^{\sigma_1 t} + A_2e^{\sigma_2 t}$. Here $I_s$ is equal to 4 this is what we can see from the figure when the circuit is the switch is closed for very long period the capacitor will be acting as a open circuit and the whole current will flow through 20 henry inductor.

So, it means that the $I_s$ is 4 ampere in the circuit, so this is what we got that $I_s$ is equal to 4 ampere. Now if you put the value of t is equal to 0 in the above equation you know that *i* at time t is equal to 0 is 4 which you just calculated so you get $A_2 = -A_1$, now if you take the derivative of the above equation $\frac{di}{dt}$ you will get this expression at time t is equal to 0, $\frac{di(0)}{dt} = -11.978A_1 - 0.521A_2$.

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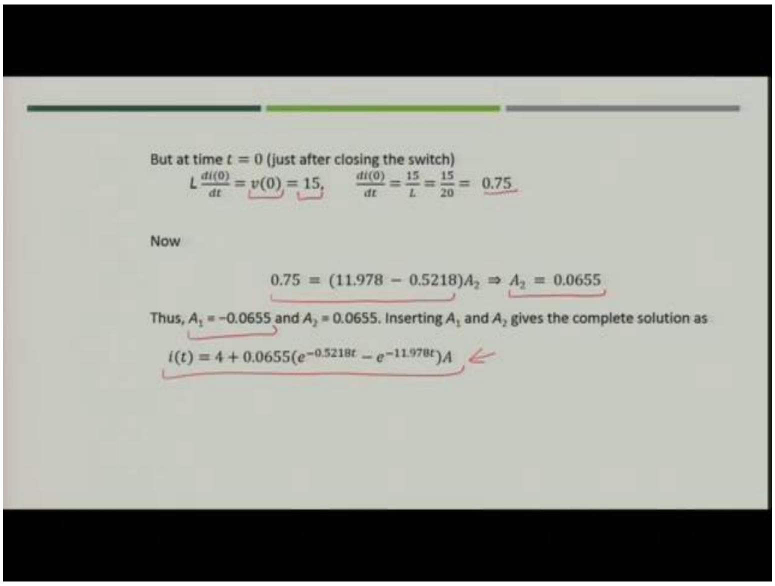


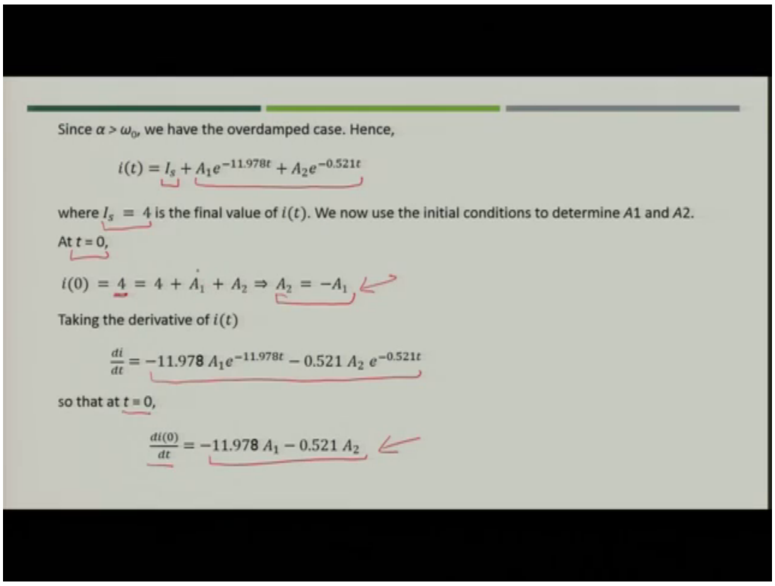

Now, you know that the voltage which is across the capacitor cannot change abruptly, so it will remain as 15 volt and since capacitor is in parallel with inductor the same voltage will be applied across the inductor also, so you can simply write the voltage across the inductor is nothing but $L\frac{di(0)}{dt} = 0.75$.

Now you put the value in this equation so you get the value of A2 you can use the expression which we calculated when we were putting the value of time t is equal to 0 and we were finding out the equation A1 plus relation if between A1 and A2 when i is at time t is equal to 0 equal to 4 ampere.

So now we got 2 equations and we can solved it and we can get the value of A2 and similarly we can get the value of A1. So now we have A1 and A2 we can simply say the value of current *i(t)* is nothing but 4 plus you can put the value of the expressions, value of the components so you get this particular equation.

So, this is the final expression of current in terms of t, so with this you can calculate the value of A1 and A2 when you have parallel RLC circuit. so with this we can close our today's session, so in this session we discussed the step response for Series RLC Circuit as well as the Parallel RLC Circuit and now we will proceed forward to solve this first order second order equations in more easier format, because in this type of equations till know what we discussed was based on the differential equations those were first order and second order differential equations.

It is sometimes difficult to solve, so we will use another technic called laplace transform and we will try to understand first the laplace transform and then we will apply the technic to solve this first order and second order equations and first order and second order circuits again. Thank you.