Basic Electric Circuits
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Module 6: First Order and Second Order Circuits
Lecture 26: Step Response of Second Order Circuits

Namaskar, so in the last class we were discussing about the series RLC circuit and its response when the source is not available, so, we will continue our journey from there in this week also and let us see when you apply step input then what will happen.

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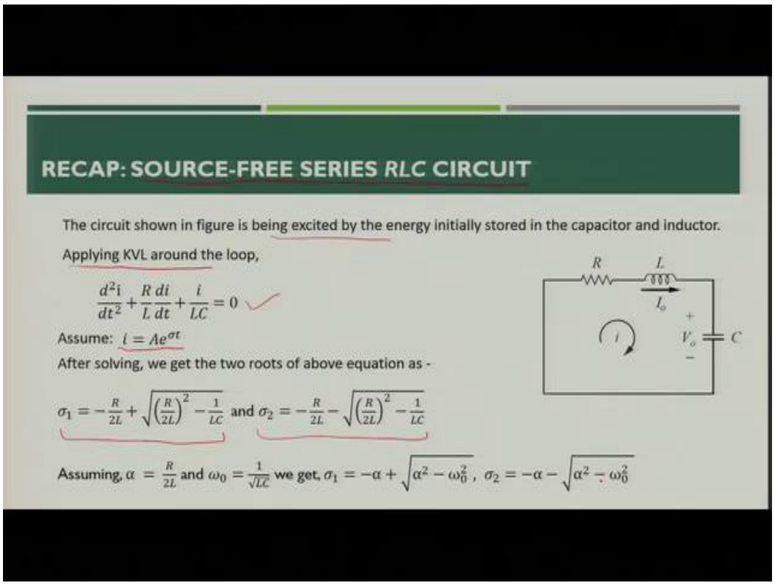

So let us first understand what we discussed in the last class, we discuss about the source free series RLC circuit and when we were discussing we mentioned that the characteristic equation which we have or we get from Kirchhoff's voltage law is of second order in nature, that why it is called second order circuit. Now if you see this figure the energy is initially stored in inductor as well as capacitor, so the capacitor is initially charged with voltage V_0 and inductor is having some initial current as I_0, Since this two elements are storing the energy that is why the circuit is having some current flowing, now we stabilized that the equation using KVL we will get is second order equation that is $\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}=0$.

Now we assumed that the current is $Ae^{\sigma t}$, now if you put the value in the above equation and we solve it we found that there are 2 roots possible for the equation and we saw that σ_1 and

σ_2 and we assumed that $\alpha = R/2L$ and $\omega_0 = 1/\sqrt{LC}$. When we simplified, we got σ_1 and σ_2 as shown in the slides.

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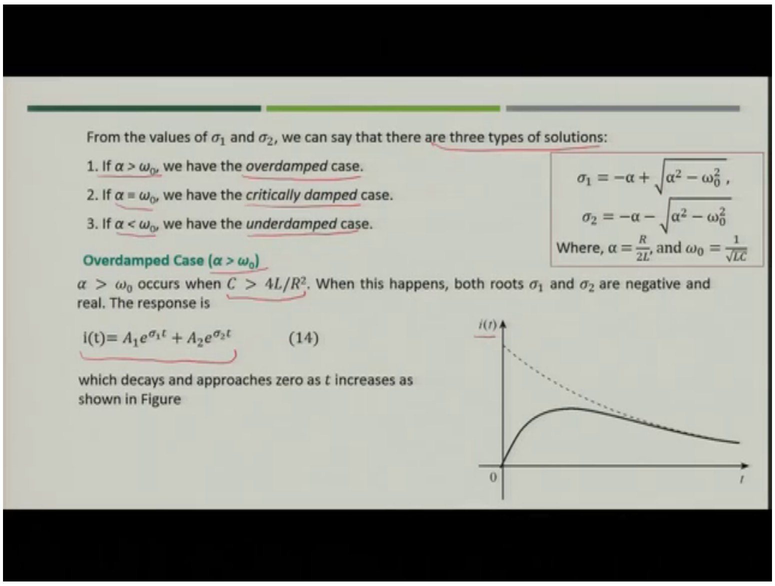

Now based on the values of σ_1 and σ_2 we found that there are 3 types of solution available for the equation, that is first case when $\alpha > \omega_0$ it means the circuit is overdamped, when $\alpha = \omega_0$ we have critically damped case, and when $\alpha < \omega_0$ we have under damped case.

Now when we say that it is over damped and $\alpha > \omega_0$ the condition which we got was $C > \dfrac{4L}{R^2}$. Now based on that condition the current value what we found was $A_1 e^{\sigma_1 t} + A_2 e^{\sigma_2 t}$.

On plotting the curve that is the variation of current *i* with respect to time we saw that the curve will look like this which will start from 0 and eventually it will settle down at some point because it is a source free then it will come back to 0 as time t tends to infinity.

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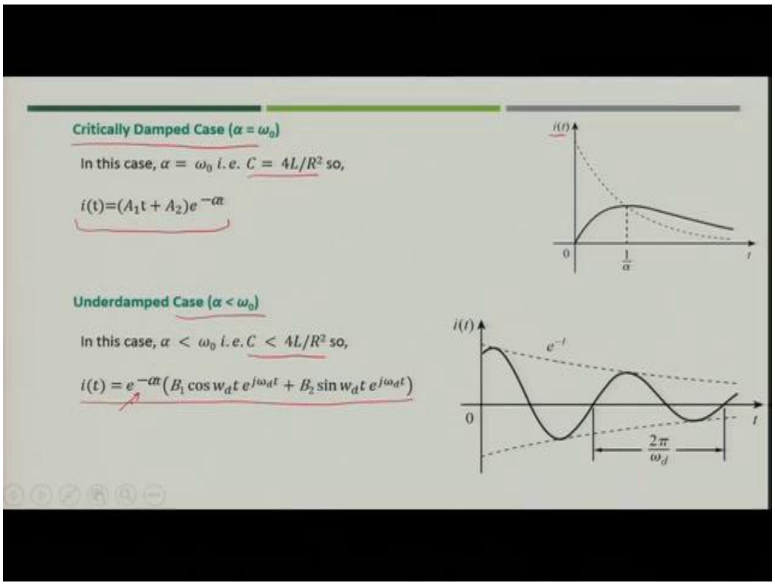

Now second case what we discussed was $\alpha = \omega_0$ where the condition is $C = \frac{4L}{R^2}$ and we found that current in that case is *i* at time t is equal to $A_1 e^{-\alpha t} + A_2 e^{-\alpha t} = A_3 e^{-\alpha t}$ and when we plot this the value of current i with respect to time t we saw that the characteristic of the current is as shown in the figure and we saw that at time t is equal to 1/α the current will be at its maximum value. Now the third case what we discussed was $\alpha < \omega_0$, in that case the $C < \frac{4L}{R^2}$ and we saw the current equation which we got was having exponential term as well as sin and cosine terms, so when we plot the variation of the current i with respect to time we saw that it was it was exponentially decaying as well as it was oscillatory that is because of the sinusoidal component available and the time period was $2\Pi / \omega_d$ and it was exponentially decaying because of the exponential component available, so this is what we discuss in the last class.

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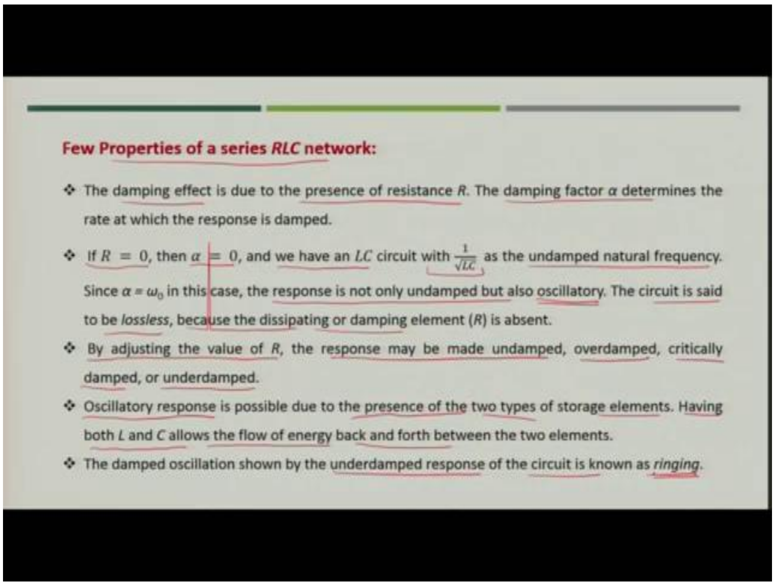

Now let us see what are the various properties of series RLC network, now we can say that damping effect is due to the presence of resistance R because the damping factor α depends upon resistance R, so we can tune the value of R on and get the various kind of damping effect, when the condition is α is equal to 0 means R is equal to 0, it means that we have a pure LC circuit and in that case we will have only $\omega_0 = 1/\sqrt{LC}$. This is nothing but your undamped natural frequency. In that case that response is only oscillatory and the circuit is said to be lossless and because of no dissipating component the energy will keep on oscillating.

By adjusting the value of R the response can be made undamped, overdamped or critically damped or may be under damped, so the R is the important component in the series RLC circuit, oscillatory response is possible due to the presence of two types of the storage elements, so here we have inductor as well as capacitor which act as a storage element, so having both L and C allow the flow of energy back in forth because of that we get the oscillatory response. Now the damped oscillation show by the underdamped response of the circuit is also known as ringing.

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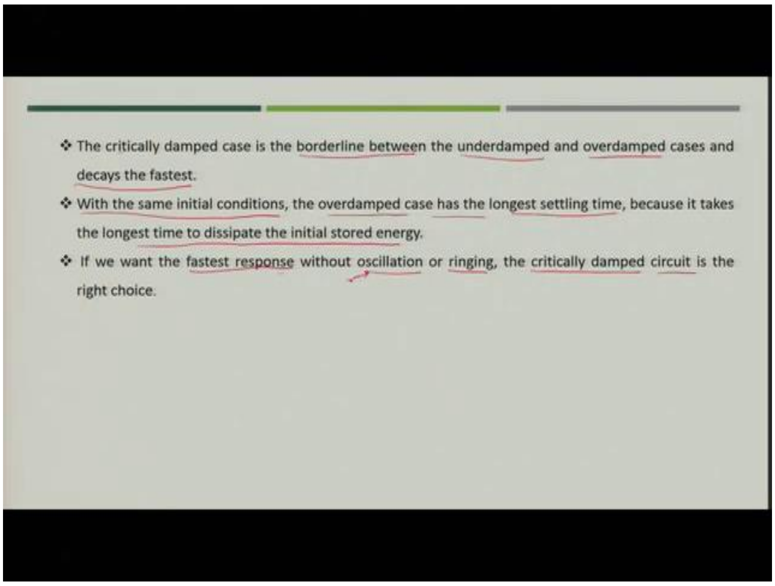

Now the critically damped case is the border line between the underdamped and overdamped cases and decays the fastest, so if you have a system which is critically damped it means it will decay the fastest, with the same initial conditions the overdamped case has the longest settling time, because it takes the longest time to dissipate the initially store energy in the storage elements, now if you want the fastest response without oscillation or ringing, what we need to do? We have to make the system critically damped circuit, so in that case we will get the fastest response without any oscillations.

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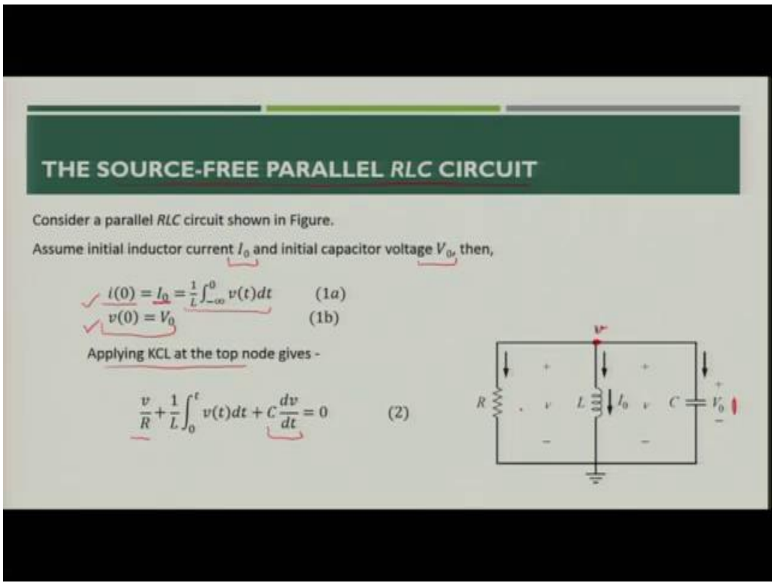

Now let us talk about the source free parallel RLC circuit, so what will happen in this case you will have R,L and C all this component in parallel, so what you need to do, you have to first figure out what is the initial condition. Let us assume that initial current flowing through the inductor is I_0 and initial capacitor voltage is V_0. Hence, current at time is equal to 0 is represented using equation (1a). Integration ranges between minus infinity to 0 because this is the time where the system is at rest initially, so what is the current flowing at time is equal to 0 will continue to flow in the circuit at time before t equal to 0, similarly for time t is equal to 0 we are assuming that capacitor is initially charged with voltage V_0.

Now what will happen, if you assume this two initial conditions and the circuit is in parallel, then the best thing is that you can apply Kirchhoff current law at the node where all the 3 elements are connected. Now sum of the resistor, inductor, and capacitor currents will be 0 at this node as given in equation (2).

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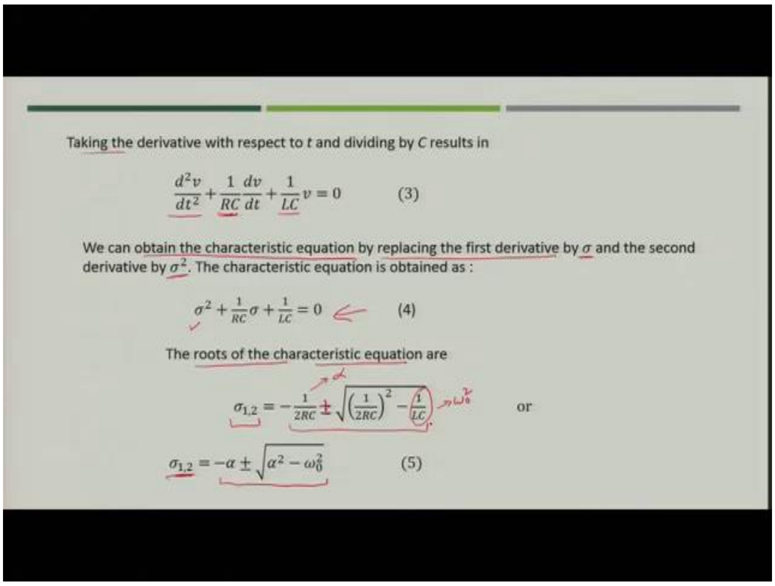

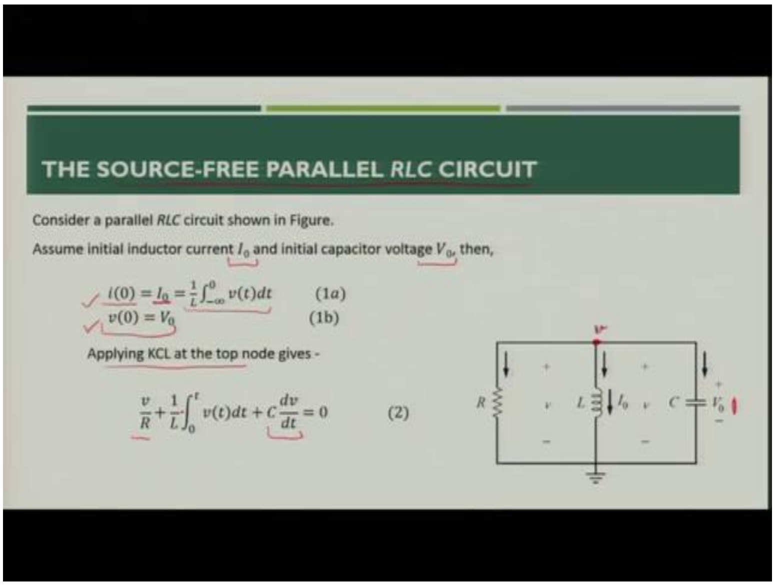

Now differentiating equation (2) and dividing by C we get equation (3). Now we can obtain the characteristic equation by replacing the first derivative by σ and the second derivative by σ^2 you get $\sigma^2 + \frac{1}{RC}\sigma + \frac{1}{LC} = 0$. Since it is a second order equation you will get two roots of this equation, what would be the roots? The roots would be σ_1 and σ_2, let us assume the values are the characteristic equations, roots are σ_1 and σ_2. The value of σ_1 and σ_2 is obtained as given in equation (5), so you will get simply two roots.

Now let us assume that $\alpha = \frac{1}{2RC}$ and ${\omega_0}^2 = \frac{1}{LC}$. Using this the roots can be written as, $\sigma_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$. Now if you see this roots, this are similar to what we got when we were analyzing the series RLC circuit.

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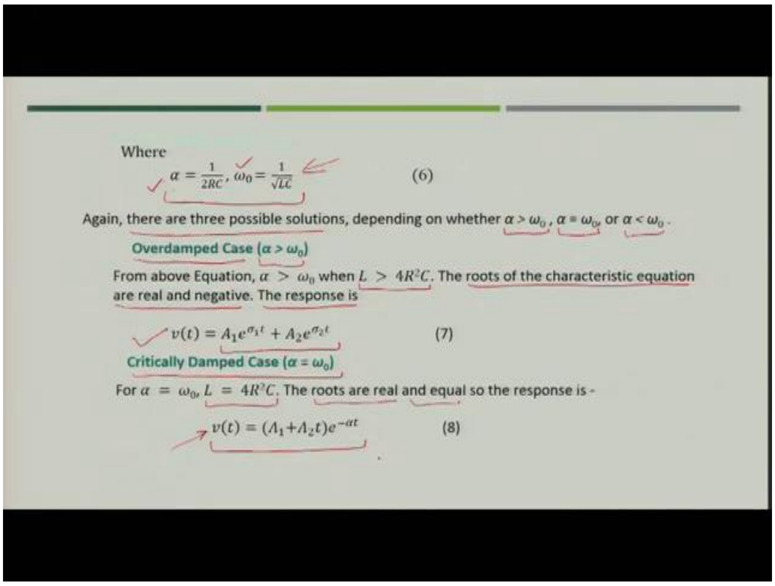

Based on the values of α and ω_0 you can again have the three possible solutions. If you compare with the series RLC circuit in this case also you will get three possible solutions. First condition is $\alpha > \omega_0$, second is $\alpha = \omega_0$ and third is $\alpha < \omega_0$.

Now when $\alpha > \omega_0$ you call it as overdamped case, so what will happen if you put the conditions if you replace α and ω_0 with this value which you assumed previously, you will get the condition $L > 4R^2C$ the circuit will be overdamped. In that case the roots of the characteristic equations are real and negative, this is similar to what we got in case of series RLC circuit also, so similarly the response which you will get in this case is as given in equation (7). This response would be similar to what we got in case of series RLC circuit.

Now in case of critically damped case where $\alpha = \omega_0$ the system will be critically damped when $L = 4R^2C$, so in that case also the roots are real and equal, so you can say the response is given by equation (8), so this is again similar to what we got in case of series RLC circuit.

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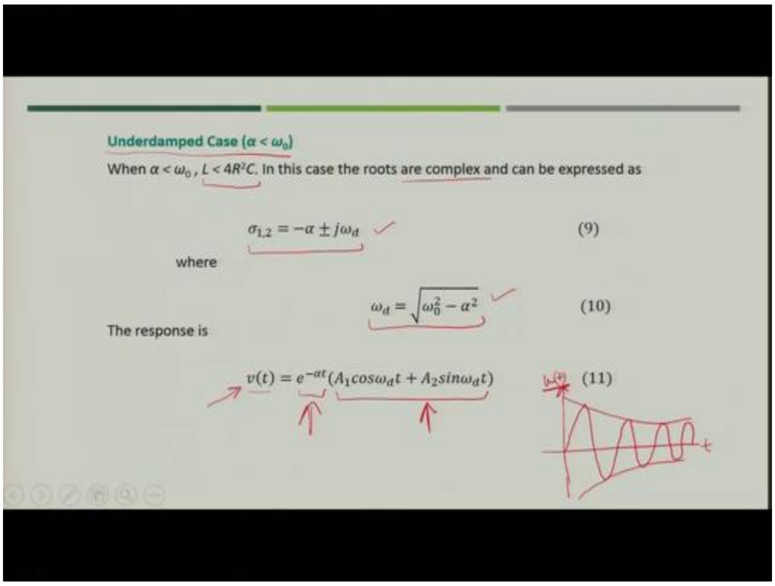

Now the third case is underdamped case when $\alpha < \omega_0$ that means the value of $L < 4R^2C$, so in this case the roots would be complex and can be expressed as in equation (9), where $\omega_d = \sqrt{\omega_0{}^2 - \alpha^2}$. So you can compare both of the cases like series RLC and parallel RLC circuits, you will come to know that the roots are same, the only change would be the condition because here the elements are in parallel. This is why the here relation between L ,R and C would be different in both of the cases, but finally the response would be similar to what we got in case of series RLC circuit. In this case the expression is given by (11) will give you the exponentially decaying response together with the oscillatory response.

So in both of the cases like in case of series RLC circuit, we were comparing the current i with respect to time, in this case since it is a parallel RLC circuit we are interested in finding out the voltage with respect to time, so here you will again get the oscillatory response with exponentially decaying, so exponentially decaying because of this term oscillatory response would be because of this term.

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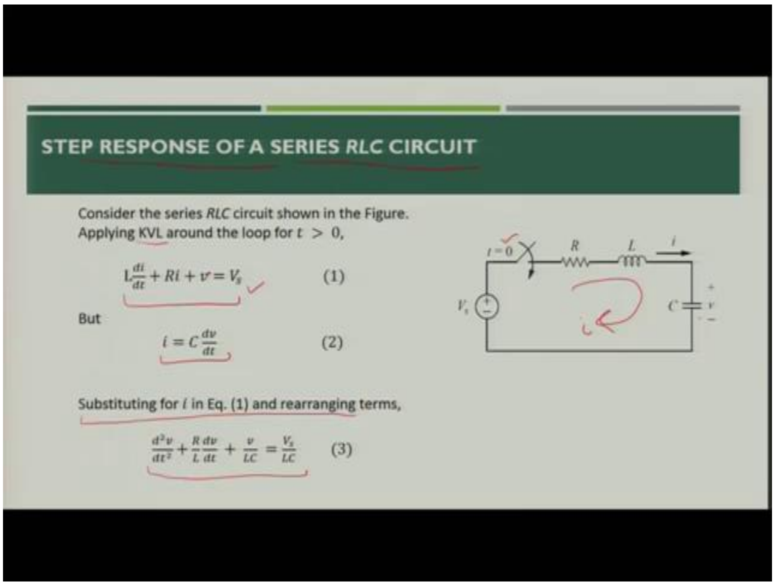

Now let us talk about the case when we apply a step input to a series RLC circuit. By closing the switch at time t is equal to 0 there would be a current flowing in the mesh, let us say this is *i*. Now what you need to do you have to write the mesh equation for this circuit? We will try to simplify the loop or mesh equation which we get from the KVL, so when you apply KVL at time t greater than 0 the moment when you close the switch you will get,

$$L\frac{di}{dt} + Ri + v = V_s$$

Now you know that, $i = C\frac{dv}{dt}$. Using this in the mesh equation you will get the equation in the second order as,

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

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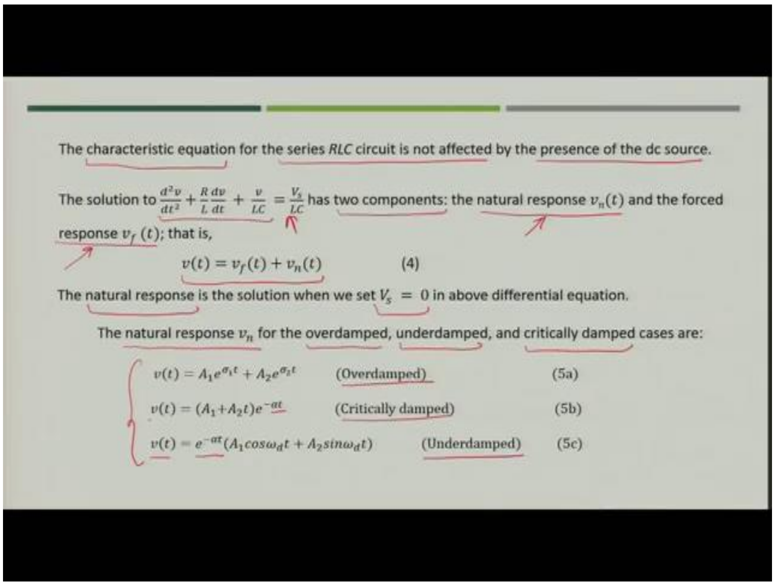

Now when you compare the characteristic equation, the characteristic equation will not be affected by the presence of the DC source. You can see the solution of this equation can be divided into two parts. It will have two components; one is natural response and other is forced response. So, this will be like what we discussed in case of series RL or series RC circuits. In case when we apply the step input to those circuits, so similarly here also we can divide the total response of the circuit into two parts, one is natural response and other is force response.

Now you can see that the total response can be written as the forced response plus natural response as in equation (4). Now to find the natural response, you need to set the value Vs equal to 0. So when you put the Vs equal to 0 in this equation (4), so you will get only the left side component and this would be equal to 0 and this is same equation which we get when we analyze the series RLC circuit without any external source, so based on the previous discussion in case of source free RLC circuit we can write simply the natural response of the circuit and we can also say that there are three conditions which are underdamped, overdamped and critically damped cases. These are described using equations (5a), (5b), and (5c).

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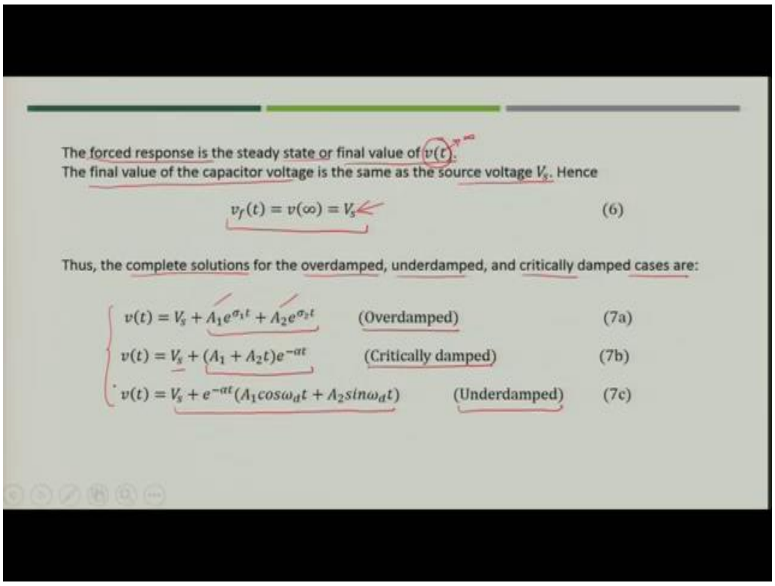

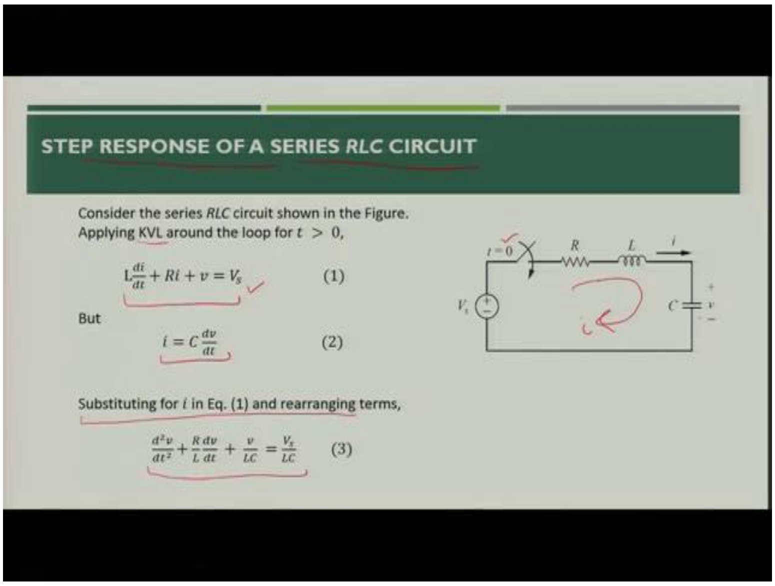

Now to find the force response you need to find out what would be the steady state or final value of voltage t, so that is t means the time which tends to infinity, the final value of capacitor voltage will be the same voltage that is the source voltage, if you see the figure when you keep on this particular switch on for a very long period what will happen, in that case your inductor will act as a short circuit, so the total voltage would be applied across the capacitor will be equal to the source voltage, because after the capacitor is charged to its full value your current i will also be 0, so you will get finally the capacitor voltage equal to Vs.

So the same value we will use and we will say that the forced response is nothing but the source voltage, so this we have got, so what we can write now the complete solution for the overdamped, underdamped and critically damped cases can be written as given in equations (7a), (7b), and (7c).

Now what is left is the value of parameters A1 and A2, so this values we can calculate with the help of initial conditions which we assumed at time t is equal to 0, we have initial voltage and initial current which is flowing through inductor. So, we can use that value and find out the value of those constants which are unknown in this three equations, so with this we can close our today's session where we discussed about the step response of a series RLC circuit. We will continue our discussion in the next class where we will discuss about the step response in case of parallel RLC circuit, thank you.