Basic Electric Circuits
Professor Ankush Sharma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Module 5: First Order and Second Order Circuits
Lecture 24: Step Response of R-C and R-L Circuit

Namaskar! In last class we were discussing about the step response of RC circuits. So, we will start our discussion from that point onwards and we will see the RC circuit response particularly the voltage and current value and then we will proceed for RL circuit and we will see what could be the step response for RL circuit.

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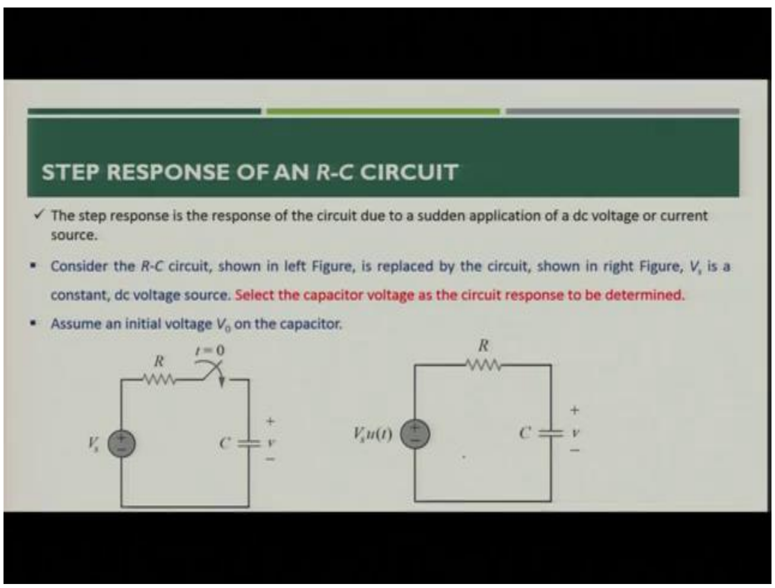

So, now let us recap what we discussed in the last class. We said that when time t is equal to 0 the circuit is closed then this particular phenomenon can be equivalently represented as a voltage source with a step function combined and then the RC circuit.

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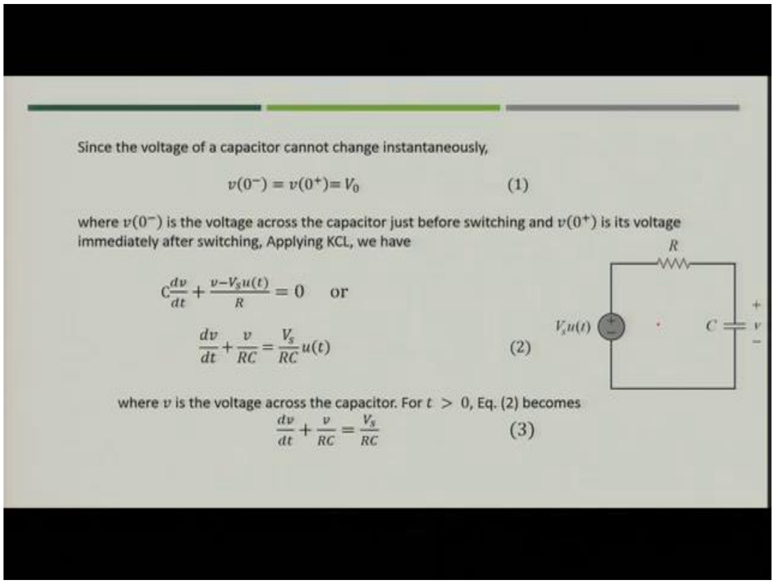

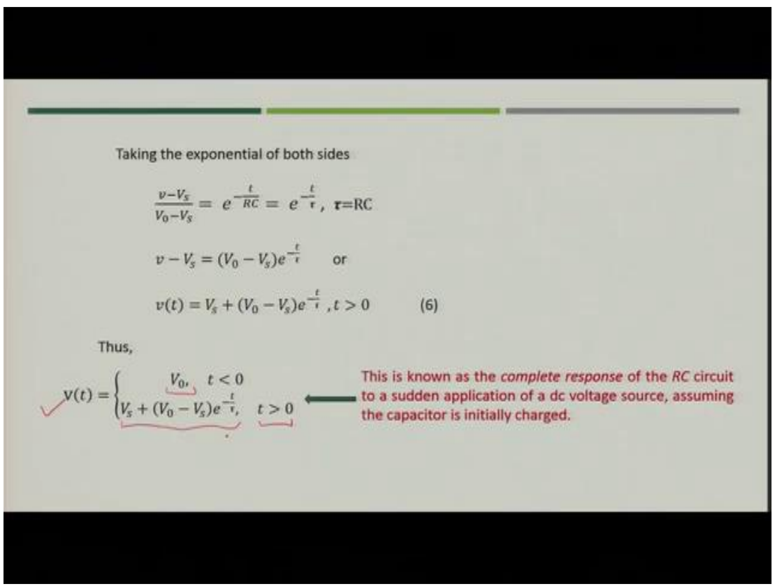

And we found that the equation for the RC circuit which we are seeing in the figure was derived with the help of mesh equations. Now, when we derived, we came to know V_0 will remain same when time t < 0 and when time t > 0 this value can be calculated with the help of this expression that is $V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}$. So, this is what we discussed in our previous lecture.

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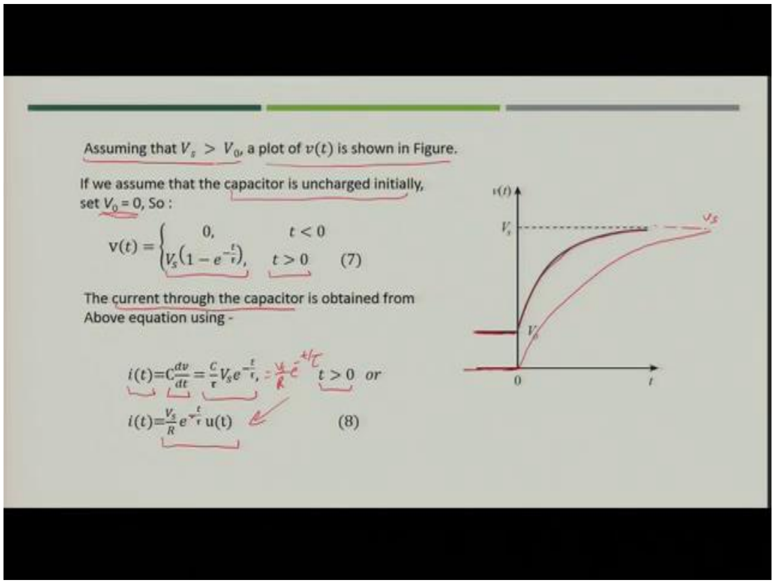

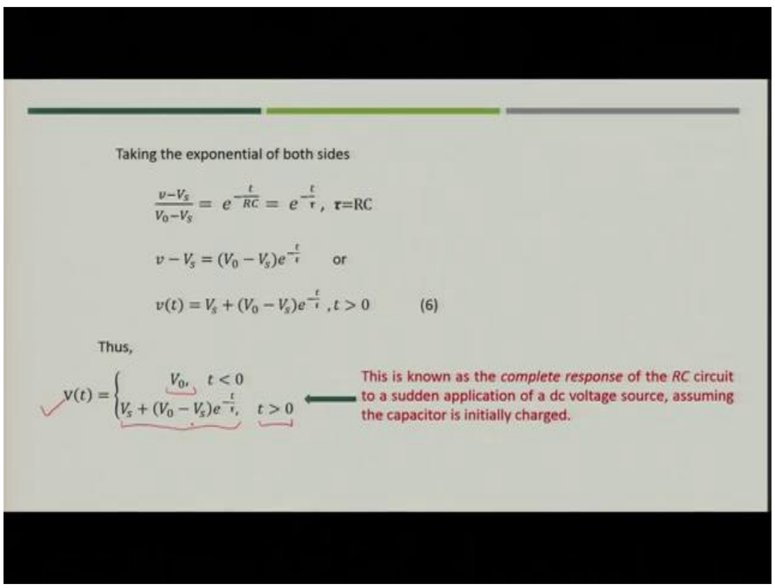

Now, let us see if the source voltage $V_S > V_0$. The plot would look like as shown in the figure. If you see this particular equation at time t is equal to 0 the value will start increasing. So, when we plot the curve for t less than just before 0 we will keep the value of voltage as V_0. So, what we have done? We have kept the voltage as V_0 and then after that it is $(V_0 - V_s)e^{-\frac{t}{\tau}}$.

So, it is now if you keep on putting the value of various t that is time as 1, 2, 3 you will get the curve which would like as shown in the figure. Now, the important thing which we have to remember is that at steady state value that typically occurs after 5-time constants. The value of voltage across capacitor would be equal to the voltage V_S that is the source voltage. Now, let us take another condition. If capacitor is initially uncharged that means that is $V_0 = 0$. So, what you have to do?

You must put the value of $V_0 = 0$ in this case. So, what you will get now is $V_S(1-e^{-\frac{t}{\tau}})$ for time t greater than 0. So, what will happen in that case? Your curve will start from 0 so you will see the curve like this where it will settle down again at voltage `V_S for time t greater than 0 and for time t less than 0 it will be 0. So, in that case it will the curve will look like as shown in the figure. Finally, it will settle down to value of V_S which is nothing but the source voltage.

Now, if you are asked to find out the current through the capacitor you must differentiate the voltage equation which we have got. So, current $i(t) = C\frac{dv}{dt}$. Now, if you differentiate you would get simply the value as it is nothing but $\frac{V_S}{R}e^{-\frac{t}{\tau}}$ and you will add *u(t)* just to make sure that you are referring to the time which is greater than 0 because this is a unit function. Its value is greater than 0 when time t is greater than 0.

So, this equation which is nothing but equal to $\frac{V_S}{R}e^{-\frac{t}{\tau}}$ for time t greater than 0 can be equivalently represented as $\frac{V_S}{R}e^{-\frac{t}{\tau}}u(t)$. Now, if you see these 2 equations this equation is increasing exponentially and when you see this equation this is decreasing exponentially which is obvious because when the voltage settles across the capacitor equals to the source voltage. The current will eventually die out.

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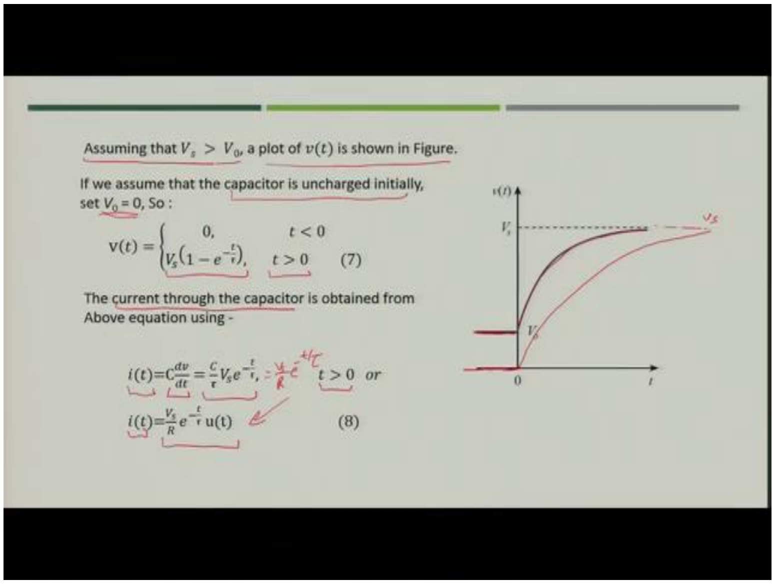

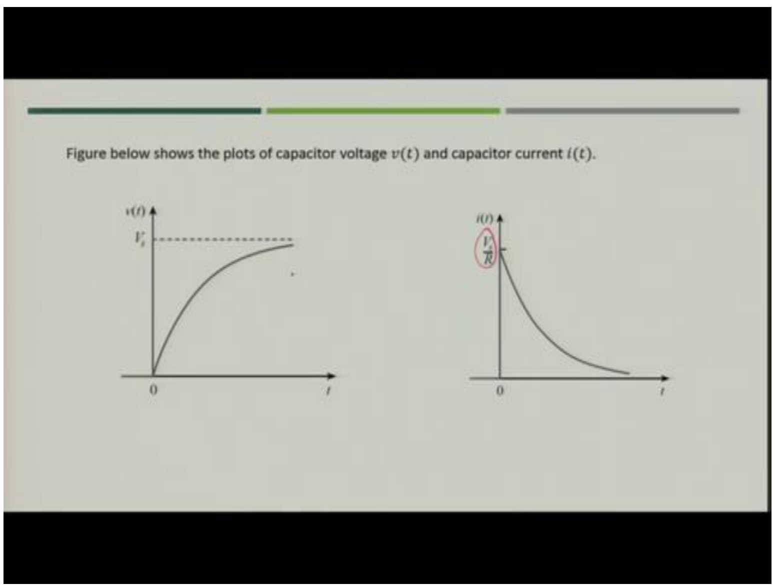

So, what will happen the responses this would be the response for voltage at across capacitor. It will settle down to some value to the value vs at some time t that tends to infinity and the current: current will start from $\frac{V_S}{R}$. So, if you put the value time t is equal to 0 here you will get it at time t is equal to 0, that is, I_0 is nothing but $\frac{V_S}{R}$ and then it is exponentially decaying. So, this will settle down after generally it settles down after 5 time constants because we assume that at 5 time constants the value what we get is less than 1 percent means it is almost settling to a 0 value. So, that is the approximation we take that after 5 time constants the value of current in this case or voltage in this case will settle down to a steady state value.

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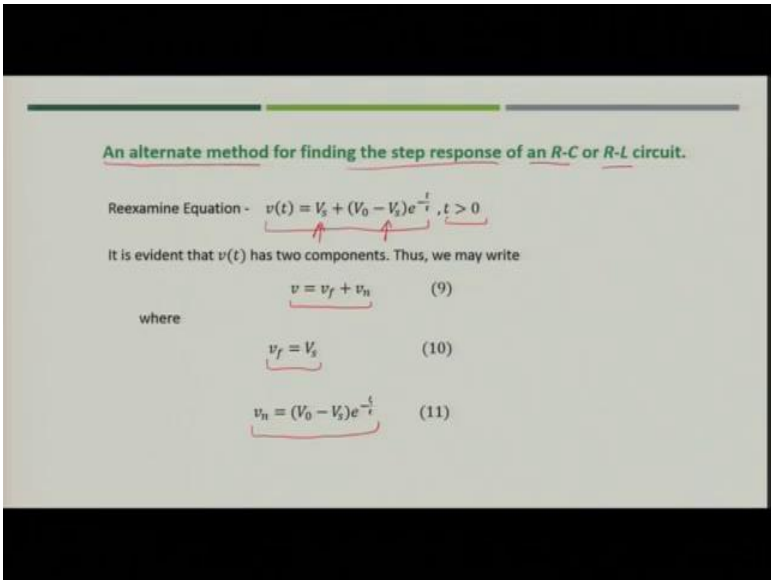

Now, there is an alternate method also for finding the step response of either RC or RL because it is applicable to both types of circuits. We derived in case of voltage across capacitor that was $V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}$ for any time t greater than 0. Now, if you see this equation you will come to know there are 2 important components in the equation. This can be written as $v = v_f + v_n$. Here, $v_f = V_s$ and $v_n = (V_0 - V_s)e^{-\frac{t}{\tau}}$. v_n is the natural response of the circuit because if you remember the source less RC circuit equation which we derived was similar to what is given in equation (11).

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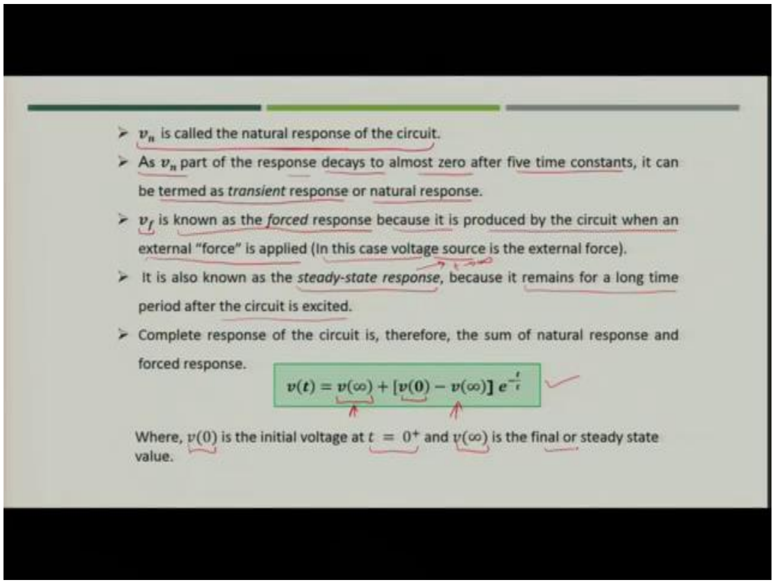

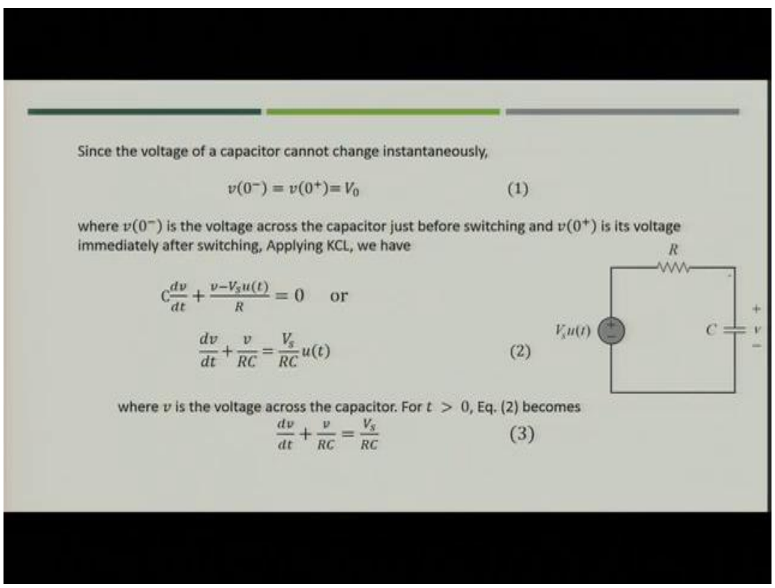

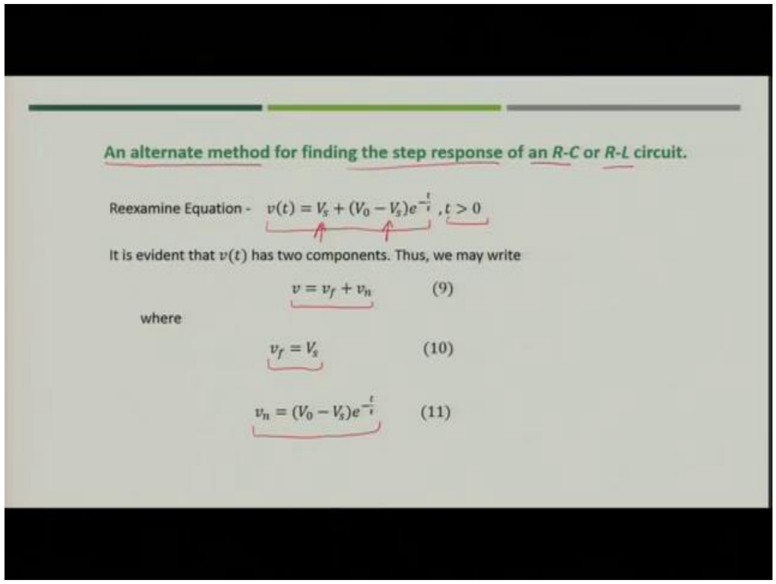

So, as v_n is part of the response which decays to almost 0 after 5 time constants it is generally termed as transient response or the natural response. Why is it called as natural response; because it is a source free or source less response of the circuit where no external source was applied. Now, v_f is known as the forced response because it is produced by the circuit when an external force was applied. So, that means when we apply any external source like voltage source or current source that acts as an external source for the circuit.

So, what happens in that case you term v_f is nothing but forced response. We also say this as a steady state response because it remains for a long time after the circuit is excited. In that case this would be nothing but steady state response of the circuit. So, now this steady state response is the response of the circuit at the time when time t tends to infinity.

So, you can now write that *v(t)* is equal to vs plus v naught minus vs into e to the power minus t by tau. V_s is the voltage across capacitor as time t tends to infinity. Similarly, what is V_0? V_0 is the value of voltage across capacitor at time t is equal to 0 minus. So, this equation which we see here can be equivalently represented as $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}$.

So, this is nothing but a combination of first response and natural response of the circuit. Here, V_0 is the initial voltage at time t is equal to 0 plus because what happens in the case of capacitor? The voltage across capacitor cannot change instantaneously. It means V_0 at time is equal to 0 minus and time t is equal to 0 plus will always remain same. And $v(\infty)$ is nothing but the steady state or the final value of the voltage across capacitor.

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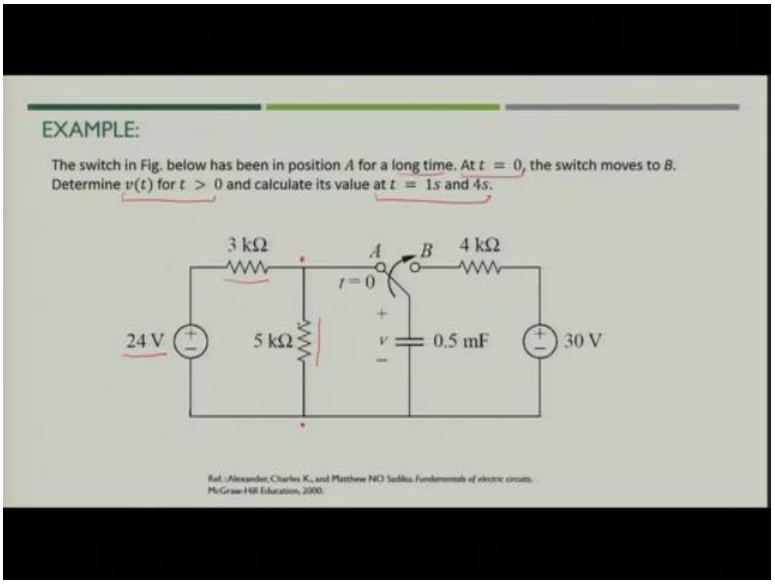

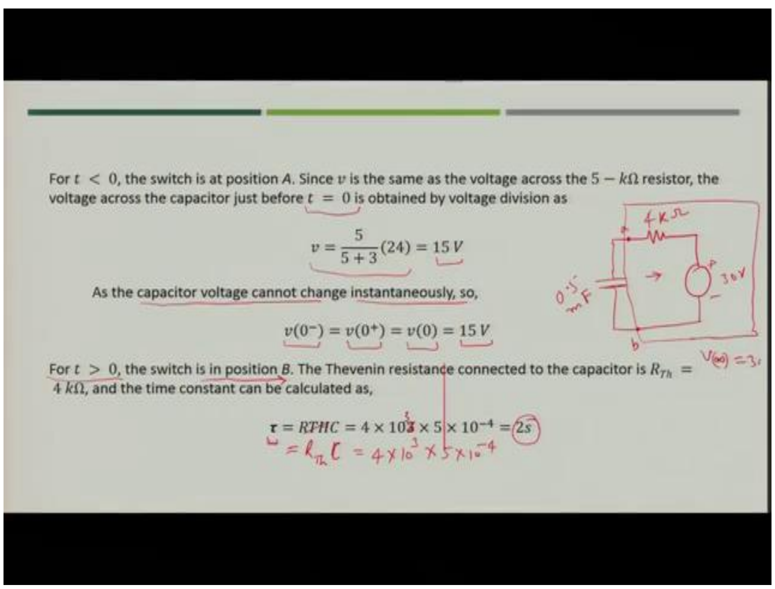

So now, let us take 1 example and let us what we discussed till now related to RC circuit. Now, in this figure position a is the position of the switch for a long time. Now, at t is equal to switch is moved from a to b. Then we have to find the voltage *v(0)* at time t greater than 0 and calculate the value of time voltage at time t is equal to 1 second and 4 second. Now, if you see this particular circuit, the value which you need to first find out is what is the value of the voltage across capacitor at time is equal to 0 minus means just before the switch was thrown from a to b.

Now, if you see the circuit you can easily find out what would be the voltage across capacitor. What would be the value? Because at time t is equal to 0 you see the voltage source value is 24 volt. Now, we have 3 kilo ohm and 5 kilo ohm resistances. The voltage across capacitor is nothing but voltage across the 5 kilo ohm resistance. So, what we can do? We can use by simply voltage division. Find out the voltage across capacitor that comes out to be 15 volts. Now, since the capacitor voltage cannot change instantaneously we can say $v(0^-) = v(0^+) = v(0) = 15$ volts.

So, we have found the value of $v(0)$. Now, for time t greater than 0 switch is closed. In that case if you close the switch the equivalent circuit will look like as shown in the figure where you have a capacitor connected then you have the resistance and again one voltage source. The value of capacitor is 0.5 milli farad then this value is 4 kilo ohm and voltage is 30 volts. This would be the condition when switch is thrown from a to b. so, if you see this circuit you will see if these are the 2 terminals of the circuit.

The right side of that is nothing but Thevenin equivalent which is applied across the capacitor. So, what you can say? The Thevenin equivalent for the circuit at time t greater than 0 would be looking like as shown in the figure. So, this section would be your Thevenin equivalent. So, you can simply say R_{Th} is nothing but 4 kilo ohm. So, what will happen in that case? τ is calculated as $R_{Th} * C$.

Then what is the value of capacitor because you need to find out the value of time constant Value of capacitance is .5 milli farads. When you simplify you get the value of time constant that is 2 second. Now, when you keep the circuit like this for a longer duration, the capacitor will be charged with the voltage v which is 30 volt. So, what will happen? You can simply say that v infinity is nothing but 30 volts.

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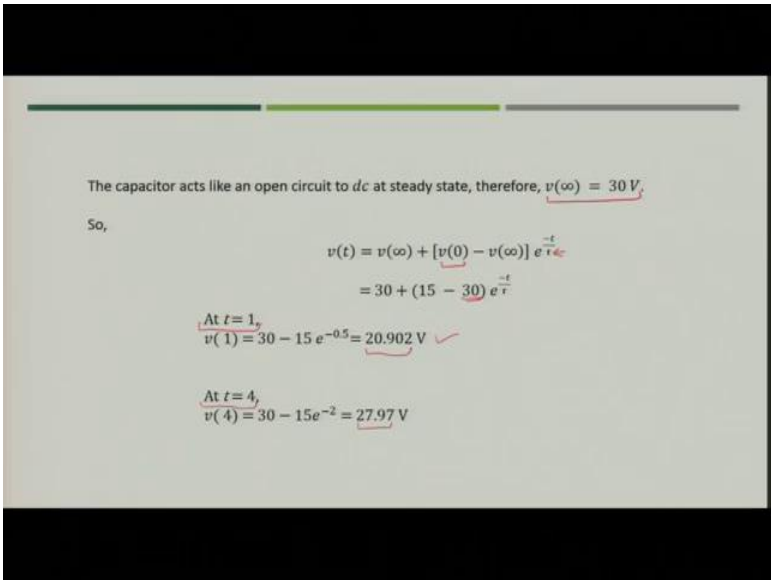

So, you got v at steady state value of voltage v is 30 volts. *v(0)* we have calculated already that is 15. So, you put the value of these 2 components and time constant value. You will get the value of voltage at any time t $30+(15-30)e^{-\frac{t}{2}}$. Now, time t is equal to 1 you put the value of t as 1 you will get the value of voltage v at t equal to 1 is 20 volts. 20.902 volts and at t is equal to 4 you simplify you will get 27.97 volts. You will see when time is changing from 1 to 4 the voltage across capacitor is rising and finally it will settle down to the steady state value that is 30 volts.

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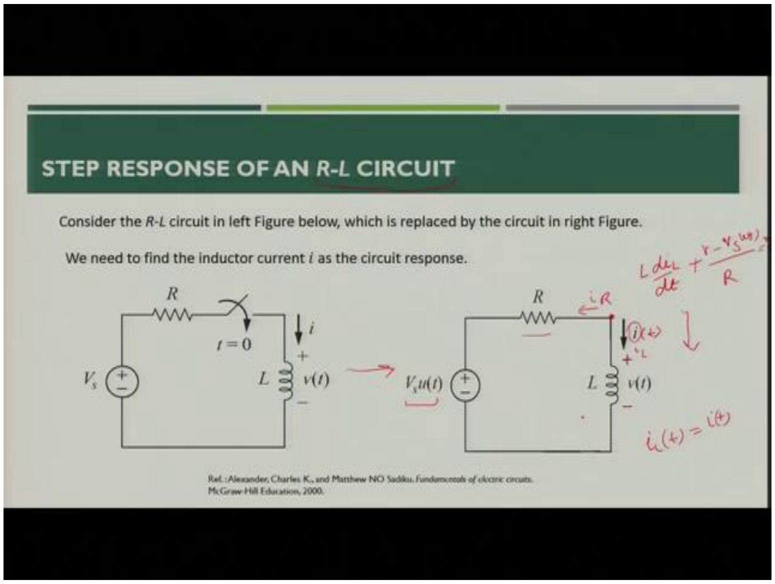

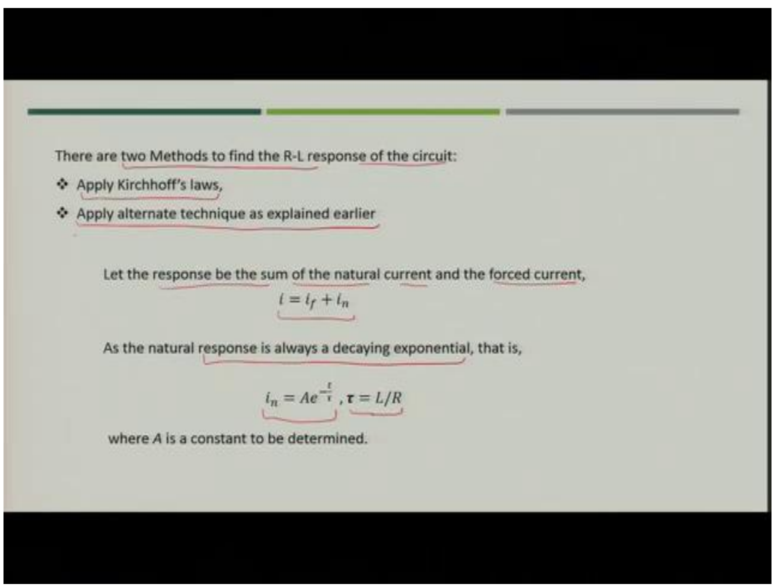

Now, let us talk about step response for a RL circuit. Now, if you see the figure on the left you have a resistance and inductor both are in series with voltage source V_s. If at time t equal to 0 switch is closed means voltage is applied to the circuit and finally you will see that the equivalent circuit including the switching phenomena can be represented as $V_s u(t)$. That would be the voltage source and then r and then inductor l. Suppose we need to find the value of current I at any time t for the inductor, voltage across inductor at any time t.

So, what we can do? We can use 2 methods to find the RL response of the circuit. First is that you can simply apply Kirchhoff's law. That means that let us say that at the node this is the current i_R. We know that this current is i_L which is nothing but time dependent current. Applying KCL we can write, $L\frac{di_L}{dt}+\frac{V-V_S u(t)}{R}=0$. This would be simply the Kirchhoff's current law application in the circuit and you can simplify and you can find the value of inductor current which would be nothing but function of time t. This can also be solved using an alternate technique which we just explained which is easier to derive and use.

So, now let us say that the response will be some of natural current some of forced current. Let us say natural response is $Ae^{-\frac{t}{\tau}}$ because you know that the natural response is always decaying because there is no external force applied. So, whatever is the current which is flowing through the inductor will eventually decay out to 0. So, it will always be exponentially decaying. So, natural response will be represented like this. *A* is a constant which we have to determine and τ is nothing but the time constant which we know that it is *L/R*.

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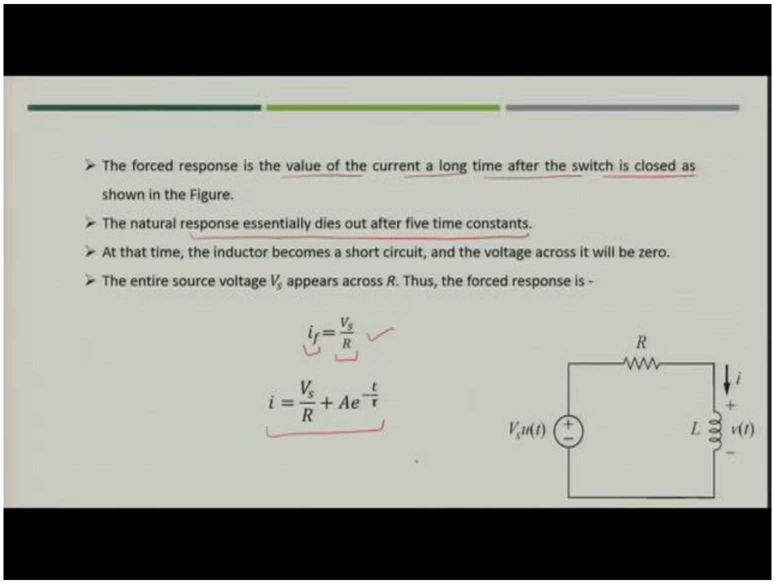

Now, what would be the forced response? Forced response is the value of the current a long time after the switch is closed. So, when you will see the circuit the natural response essentially dies out after 5 time constants. So, what will happen? At steady state condition say time t tends to infinity what will happen that the inductor will act as a short circuit.

So, in that case the if value that is forced response is nothing but V_s / R. Therefore,

$$i = \frac{V_s}{R} + Ae^{-\frac{t}{\tau}}$$

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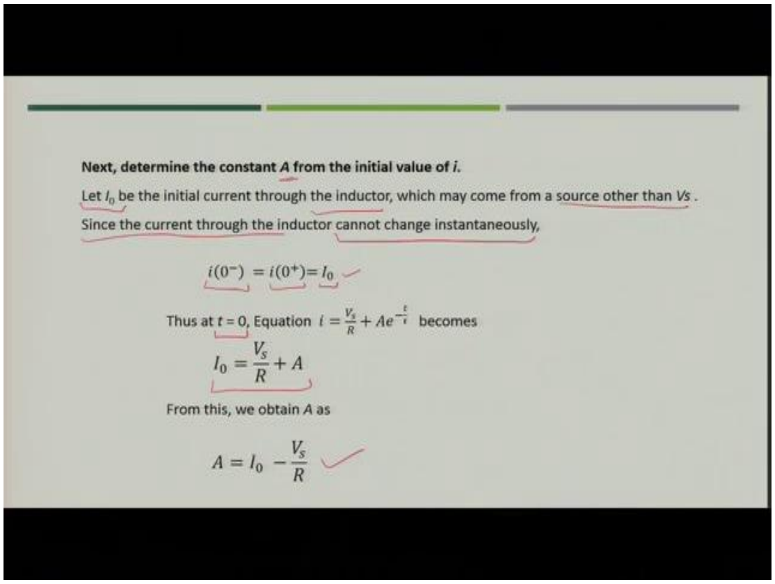

Now, next we have to find the value of constant *A*. Let us assume I_0 is the initial current which will be flowing through the conductor that means at time t is equal to 0 minus means the time just before the application of the switch means the circuit is switched on and voltage source is applied.

So, at that time we assume that the current flowing through inductor is I_0. I_0 will not come from voltage source because it is still not applied. It will come from some other source like initially applied and now is still in the circuit. We have the current flowing in the inductor because of some other phenomena and we can say that $i(0^-) = i(0^+) = I_0$ because we know that current flowing through inductor cannot change instantaneously.

So, because of some phenomena which had happened in the circuit we were having initially the current flowing I_0 at time equal to 0 and this will be maintained because the current through the inductor cannot change. So, what will happen at time t is equal to 0. If you put the value of time t equal to 0 in this, this will become 1 and I_0 would be nothing but $\frac{V_s}{R} + A$. So,

$$A = I_0 - \frac{V_s}{R}$$

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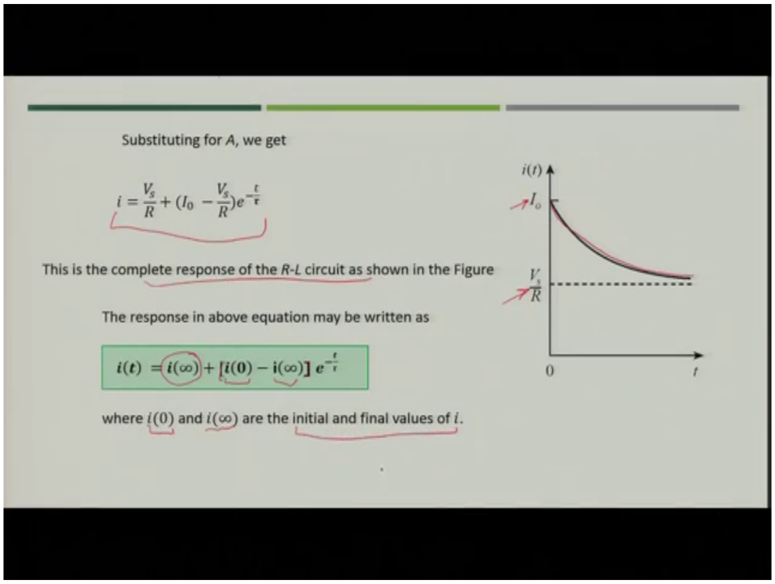

So, if you put the value of A you will get current as

$$i = \frac{V_s}{R} + [I_0 - \frac{V_s}{R}]e^{-\frac{t}{\tau}}$$

So, if you see this figure and suppose $\frac{V_s}{R}$ is less than I_0 so your response of the current across flowing through the inductor would be as shown in the figure. So, what we can say that the complete response of RL circuit. You can simply write that current has time t tends to infinity that is $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$. $i(0)$ is the value of current at t is equal to 0 and $i(\infty)$ is the current at time t that is tends to infinity. So, these are called initial and final values of current i.

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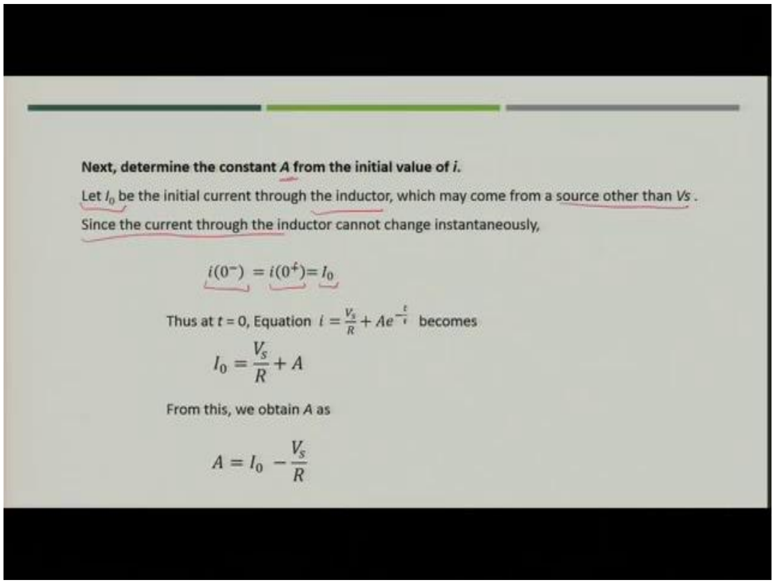

So, if you say that initially the there is no current flowing through the inductor that means $i(0)$ equals to 0. Your current i at any time t can be represented as i is 0 when time t less than 0. For time t greater than 0 it will be $i = \frac{V_s}{R}[1 - e^{-\frac{t}{\tau}}]$. So, if you see here since I naught is greater than $\frac{V_s}{R}$ so it is exponentially decaying but here it will be like this when it will be settling down to $\frac{V_s}{R}$ because this is exponentially increasing value. So, we can simply say if you apply unit step function it is nothing but $i = \frac{V_s}{R}[1 - e^{-\frac{t}{\tau}}]u(t)$. So, this would be called as a step response of the RL circuit when I_0 is equal to 0.

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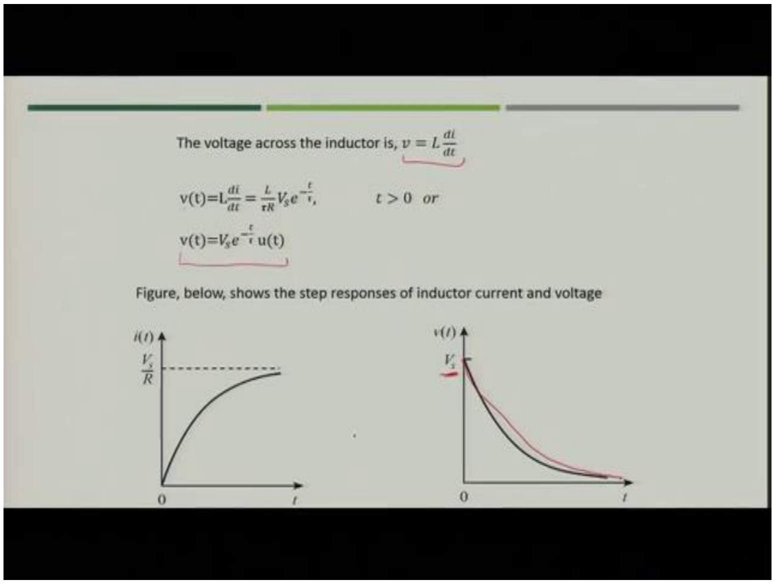

So, this would be the current which would be flowing through the inductor. Now, you know that voltage v is nothing but l di by dt so when you differentiate the current I which we got previously when we differentiate we get the value of vt which is nothing but vs into e to the power minus t by tau ut. So, now what will happen that at time t is equal to 0 the voltage across conductor would be vf and then it would exponentially decay to 0. So, this is just opposite to our response capacitor response.

When in case of capacitor voltage rises to steady state value while in case of inductor voltage settles down to 0 and similarly current in this case is settling to a steady state value while in case of capacitor the current will settle down to 0. So, in that way you can say that voltage current response for l and c are opposite to each other.

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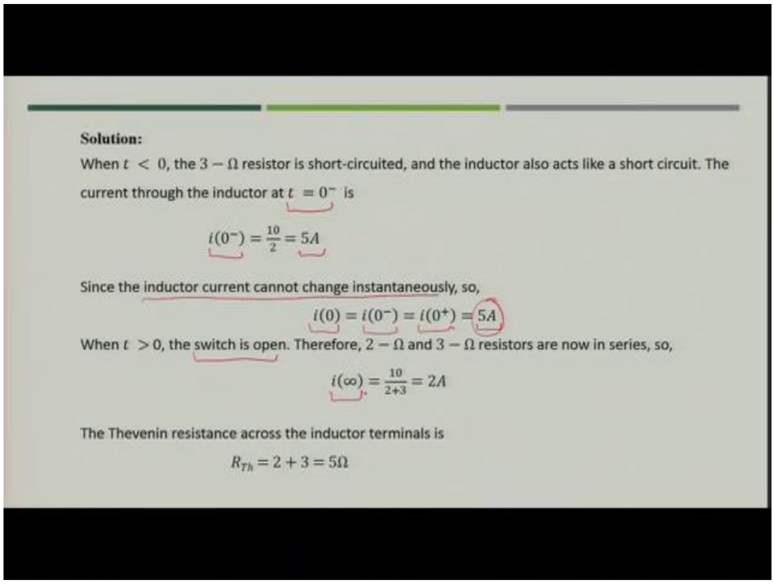

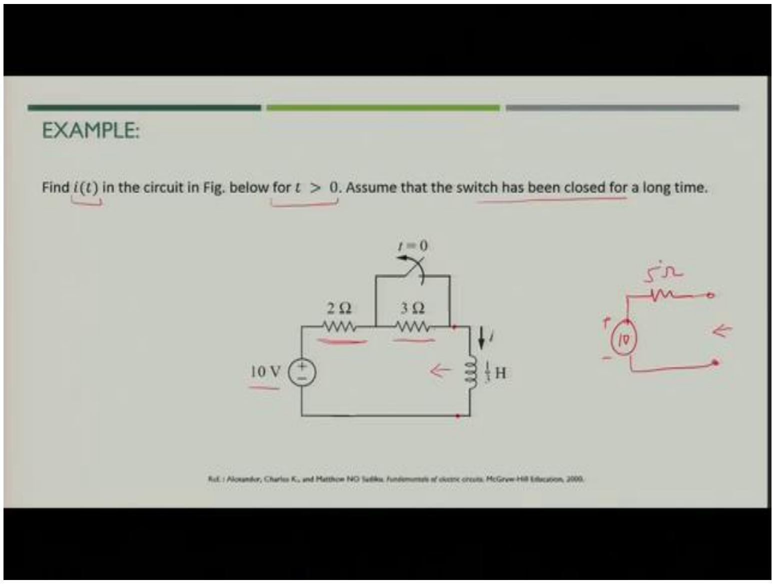

Now, let us take 1 example to understand the response of RL circuit. Let us say that we need to find the value of it in the figure for time t greater than 0. Now, we assume that switch has been closed for a long time and it was just opened at time t equal to 0. So, when switch is closed for time t less than 0 and it was switched on for sufficiently long time it means that the inductor was short circuited. And since switch was closed this 3 ohm resistance is also short circuited. So, what is the value of current I at time t is equal to 0? You can simply find out by applying the KVL that is 10 volt is applied across through the 2 ohm resistance and it will define the value of current in the circuit.

So, I_0 is nothing but 5 ampere and now we know that the inductor current cannot change instantaneously so even after switching off the circuit. When the circuit when the switched is off at time t is equal to 0 the current that is I_0 just before the switch is off of the circuit or just after the switch off phenomena in the circuit. The current cannot change instantaneously so the value of current I_0 will be 5 ampere. Now, when switch is off means it is opened the 2 ohm and 3 ohm resistances would be in series. So, what will happen in this case? The infinity at t is equal to infinity what would be the current $i(\infty)$?

If you see this figure and if you apply voltage this for very long duration with switch is open the inductor will again be short circuited and then the current flowing through the inductor will be driven by the series combination of 2 ohm and 3 ohm resistances. So, in that case your $i(\infty)$ would be nothing but 2 ampere. Now, if you see this particular figure if you see the Thevenin equivalent what will happen to the Thevenin equivalent will be 10 volt in series with 5 ohm resistance. So, this would be Thevenin equivalent in from the inductor side. So, R_{Th} is nothing but 5 ohm. So, you will simply write R_{Th}.

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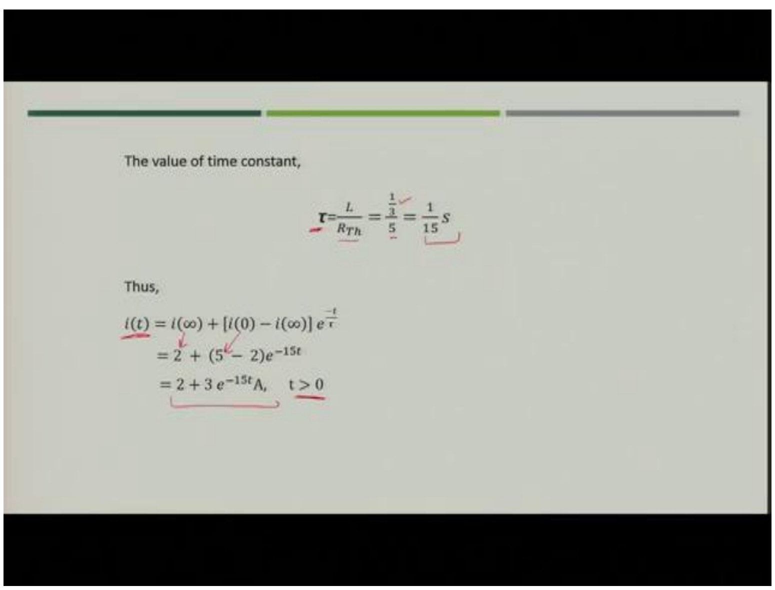

Now, you need to find the value of time constant tau which will be value of L. L is given by 1/3H and R_{Th} is 5, you will get tau as 1/15 seconds. Now, you have all the values in the hand. You can put the values in the equation and this expression for it. So, this will become $i(\infty)$ is 2 $i(0)$ is 5, τ is 1 by 15. So, finally you get the value of current I flowing through the inductor as $2+3e^{-15t}$ for time t greater than 0.

So, with this we close our today's session. In this session we discussed about the step response of RC as well as RL circuit. So, in the next lecture we will continue our lecture related to step response and we will also see the second order circuits also. Thank you!