Basic Electric Circuits
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Module 5: First Order and Second Order Circuits
Lecture 23: Singularity Function

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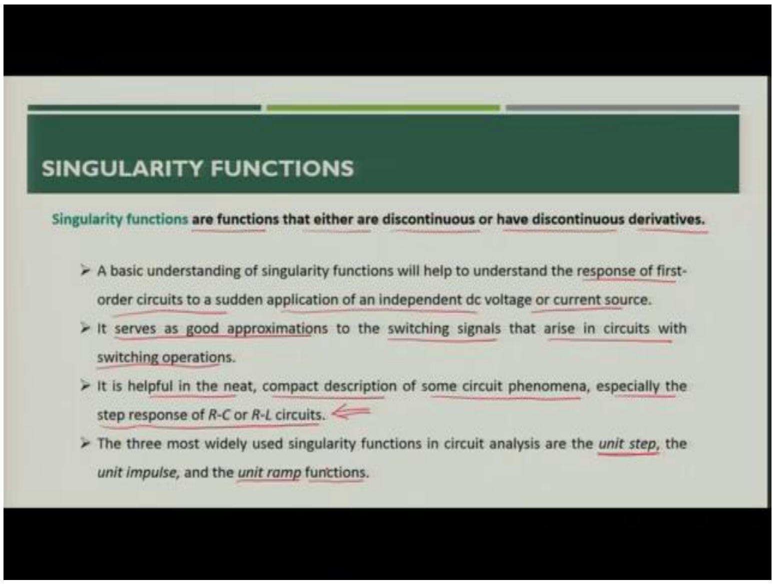

Namaskar! So, today in this lecture we will discuss about the singularity functions, what are those functions and we will see how we will use those functions in our circuit analysis. So, let us start. What are singularity functions? Singularity functions are those functions that are either discontinuous or have discontinuous derivatives. Now, the basic understanding of singularity function will help you to understand the response of first order circuit to a sudden application of independent voltage or current source.

So, in the circuit if you see, when you apply the voltage or current source it is applied suddenly. Suppose you are connecting the circuit with the voltage source through a switch and then you switch on the switch means you are basically adding the voltage source suddenly to the circuit. So, that is the sudden injection of current into the circuit. That would be something which you can represent with the help of a mathematical function. So, we will see how we will represent that phenomena with the help of a mathematical function.

Another phenomena is that whenever you apply any voltage there might be some sudden search coming up or maybe any event which is happening very for very small time period, you will have a surge kind of voltage appearing in the circuit. That is also another practical event which happens in the circuit. Then we will see how we can equivalently represent that event also into the mathematical function so that we can use them in our circuit analysis. So, let us see that the singularity functions which we use in the circuit theory are nothing but the functions which are like unit impulse, unit step and unit ramp.

When you suddenly apply the voltage source to the circuit it means that you are adding unit step to the circuit. Similarly, when there is a sudden surge coming into the circuit, then you will say this is the unit impulse being applied to the circuit and then if the voltage is building up gradually to the in the circuit then, you will say that is can be represented with the help of unit ramp function. Now, these basically the unit step serves as a good approximation to the switching signals that arise in the circuit with the switching operations and it is very helpful in the neat and compact description of some circuit phenomena especially the step response of RC and RL circuit.

So, in the later session of our discussion we will try to understand that what will happen to the circuit response when you apply step response to the circuit. So, this would be the important activity which we will try to understand. Before that, let us try to understand what are the various functions which we just mentioned here like unit step, unit impulse and unit ramp functions.

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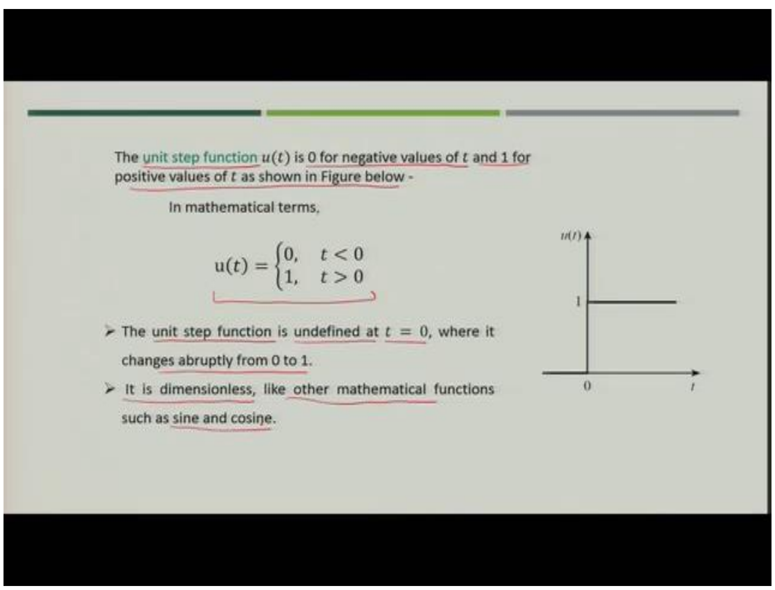

So, first let us see what unit step function is. Unit step function is 0 for negative value of time t and 1 for positive value of time t as shown in the figure. So, if you see the the function value is 0 up to time just less than 0 and suddenly it becomes 1 at time 0+. So, this kind of function is called unit step function. Mathematically, how we represent? We represent ut that is unit step function equal to 0 for t less than 0 and 1 when t greater than 0. Now, important thing to understand is that unit step function is undefined at time t is equal to 0 because it is changing abruptly from 0 to1 and that is why it is called as a singularity function. It is dimensionless like any other mathematical function such as sine and cosine.

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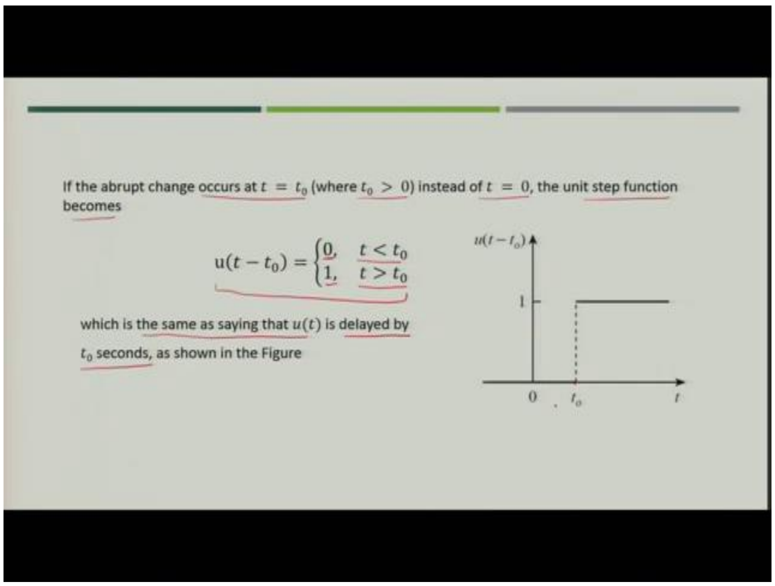

Now, if the abrupt change occurs at any time $t = t_0$ where t_0 is greater than 0 then instead of t is equal to 0 the unit step function will become 1 at time t_0 +. So, how the unit step function will look like? In that case, your value just up to t_0 - will be 0 and just after t_0 you will get the value as 1. At t_0 it is undefined so that is why it is shown as a dotted line.

So, how you will represent mathematically? You will say $u(t - t_0) = 0$ where $t < t_0$ and would be $u(t - t_0) = 1$ where $t > t_0$. So, in this case what we can say? We can say that unit step is delayed by t_0 seconds. So, it is delayed by t_0 seconds because it is starting not from 0 it is starting at some time t_0.

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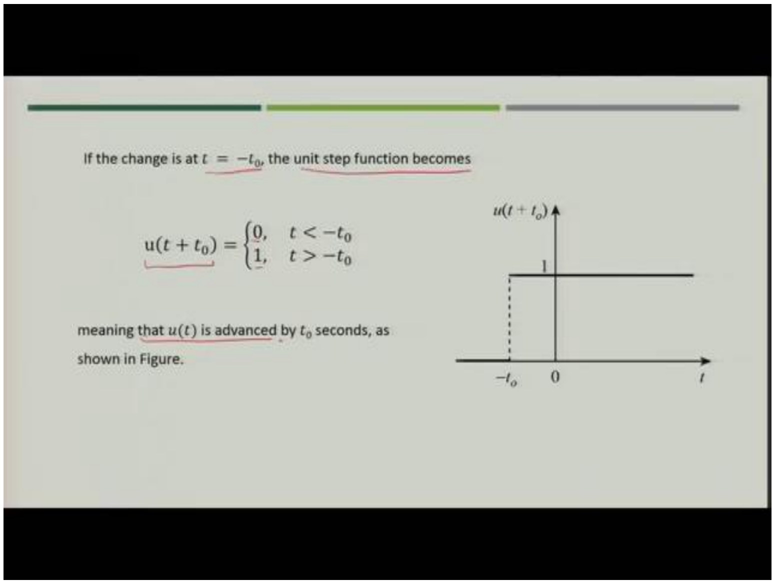

Now, if you change t to $-t_0$ the unit step function will now become $u(t+t_0)=0$ where $t<-t_0$ and would be $u(t+t_0)=1$ where $t>-t_0$. So, in this way you can say that the unit step function is advanced by t naught seconds.

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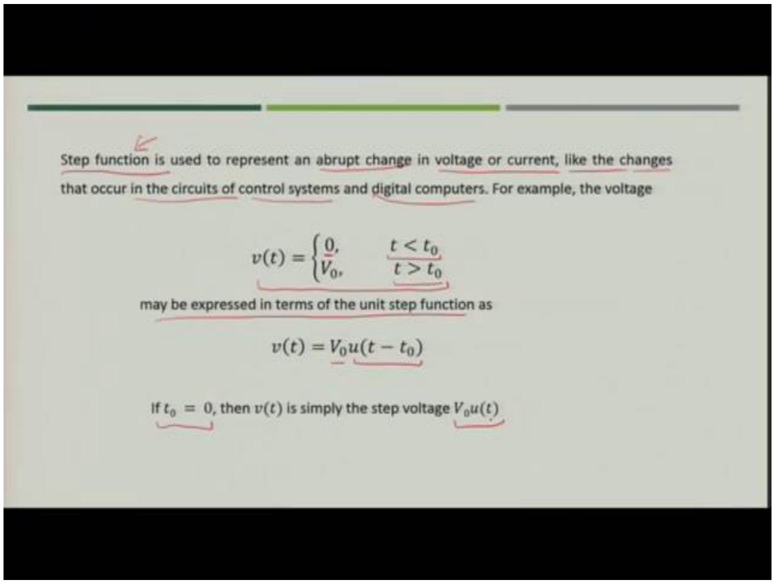

Now, step function is used to represent an abrupt change in voltage or current. So, whenever you apply voltage or current to the circuit through some switching arrangement it is nothing but you suddenly apply the source to the circuit and it is represented with the help of the unit step function because the value of voltage will not be 1 it will be some value.

So, you will simply say it is a step function. Now, it will occur in the circuit generally you will see in the control system and digital computers also. In the same manner you will see in electrical circuit also; the application of step function. Now, if you apply step voltage to the circuit; you can represent that phenomena with the help of this function. How will you represent? You will say $v(t) = 0$ where $t < t_0$ and would be $v(t) = 1$ where $t > t_0$. So, you can simply represent in terms of unit step function also as $v(t) = V_0 u(t - t_0)$. So, this is nothing but a unit step function. Then, if t is equal to t_0 is equal to 0, v(t) is simply the step voltage $V_0 u(t)$.

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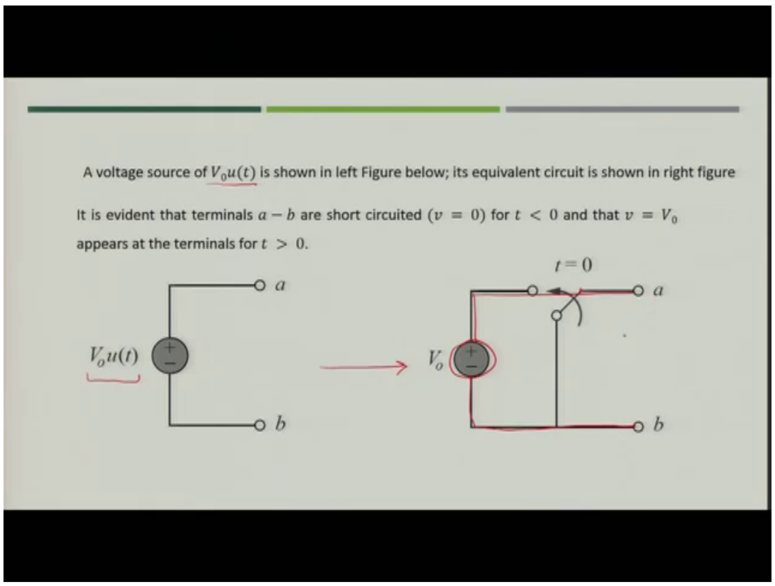

Now, the voltage source $V_0 u(t)$ which is shown in the figure is applied between terminals a and b. That is nothing but the condition which you represent with the help of switch in the circuit that the circuit is represented like this and this is the switch which you can initially put at this position. So, circuit would be like this and when at time t is equal to 0 when you apply the switch from this terminal to this terminal then the updated circuit would be different. So, if you apply this and this is the connection then it will be forced to connect to this point and your circuit would be like this. So, this is this the switch.

If you put this switch to this side then after that switching event the circuit will become this. So, this can be represented, the unit step response can be represented with the switching event which is represented with the help of switch in the circuit. So, we can say that when time t is equal to 0 the switch is thrown from this position to this position. So, it will at t 0 plus condition, the voltage across terminal ab will become v naught.

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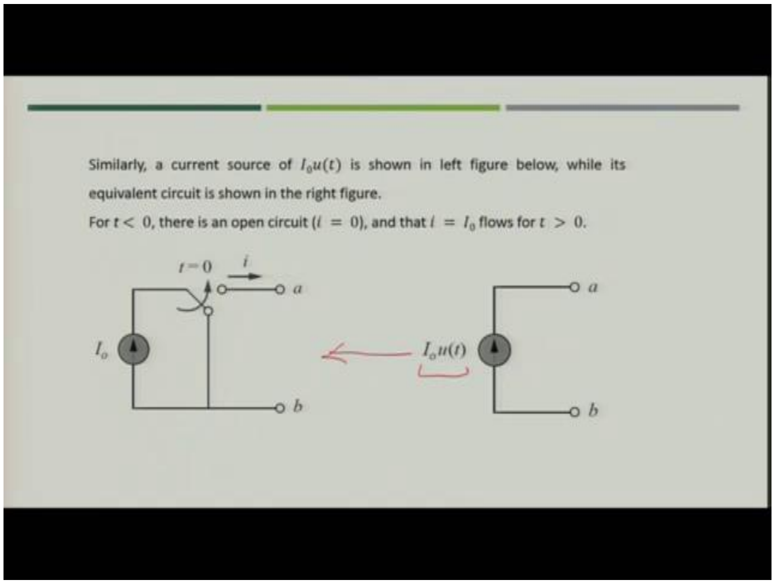

So, this is how you will represent the voltage. Similarly, for the current let us say the current source is $I_0u(t)$. So, $I_0u(t)$ is the current source which is connected between a and b. Now, if you want to represent through the switch this would be the circuit. Now, what we are doing? Initially it is like this, the circuit is connected. So, your current between a and b is 0 and when at time t you open the switch and connect from here to here the circuit will become like this. So, that also says that in the circuit the unit stop response can be represented for voltage as well as current source as shown in the figure.

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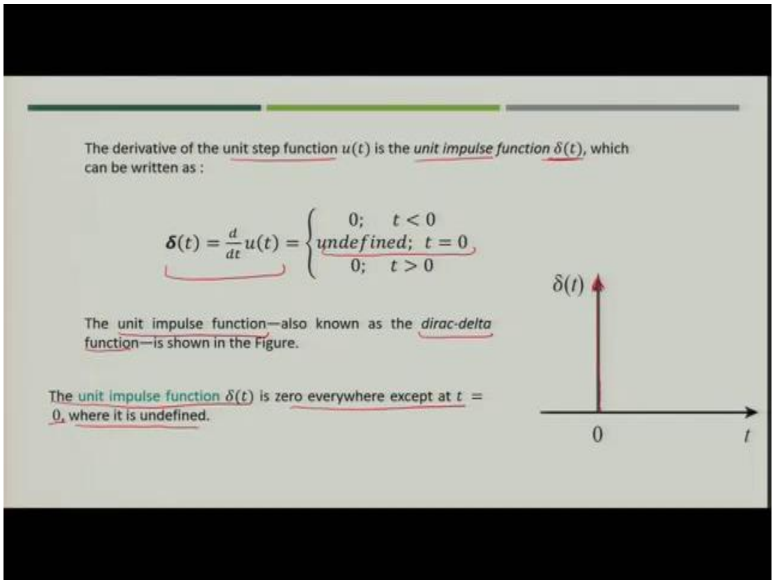

Now, derivative of the unit step function is the unit impulse function. You generally represent it as $\delta(t)$. So, how will you represent? You will say $\delta(t)$ is nothing but derivative of unit step function and it is represented as value 0 when t less than 0 and it is again 0 when t greater than 0 because the unit step function at time t greater than 0 is 1. So, derivative of that constant value becomes 0. So, for t less than 0 and t greater than 0 it will be 0. At time t is equal to 0 it will be undefined. So, because of this property this will become a singularity function.

So, generally the unit impulse function we also call as Dirac delta function and is shown in the figure. So, you will see that it have only 1 impulse kind of activity in the time scale at t is equal to 0 otherwise its value is 0. So, unit impulse function is 0 everywhere except at t is equal to 0 where it is undefined because it is it cannot be defined at time t equal to 0.

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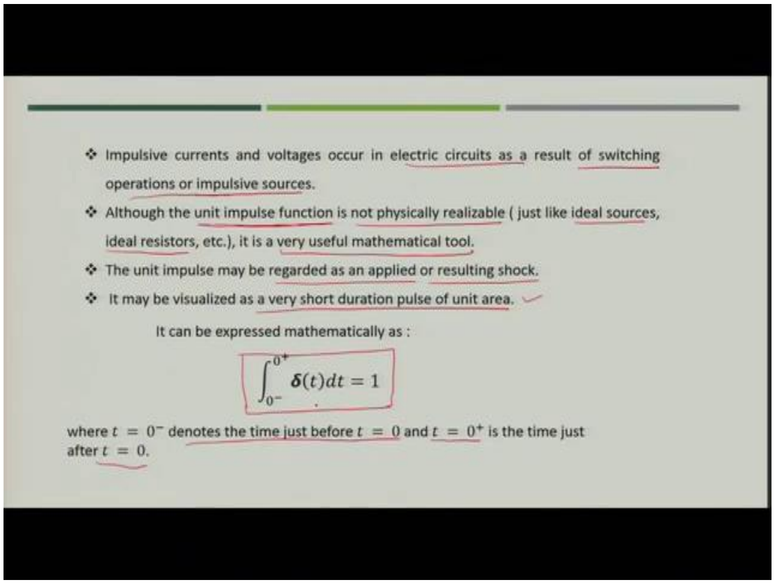

Now, impulse currents and voltage occur in an electrical circuit because of switching operation or an impulsive source. So, when you apply a voltage source to a circuit there would be an impulse which would be generated in the circuit. This phenomenon can also be represented with the help of this unit impulse function. Although the unit impulse function is not physically realizable like what we have seen in the case of ideal sources like ideal voltage source and ideal current source. Practically, we do not have something called ideal voltage or ideal current source. Similarly, ideal resistor does not exist but as these phenomena are very important and we used extensively in our calculations and circuit analysis.

Like that the unit impulse function can also be utilized for our circuit analysis and it is very important mathematical tool. Now, unit impulse can also be regarded as an applied or resulting shock into the circuit. It maybe visualized as a very short duration pulse of unit area. So, mathematically, how will you represent? You will represent that value from 0 minus to 0 plus $\delta(t)$ equal to 1. So, this is how you will represent the unit impulse or you will say direct delta function. So, t 0 minus denotes the time just before time t is equal to 0 and t 0 plus is the time just after t is equal to 0.

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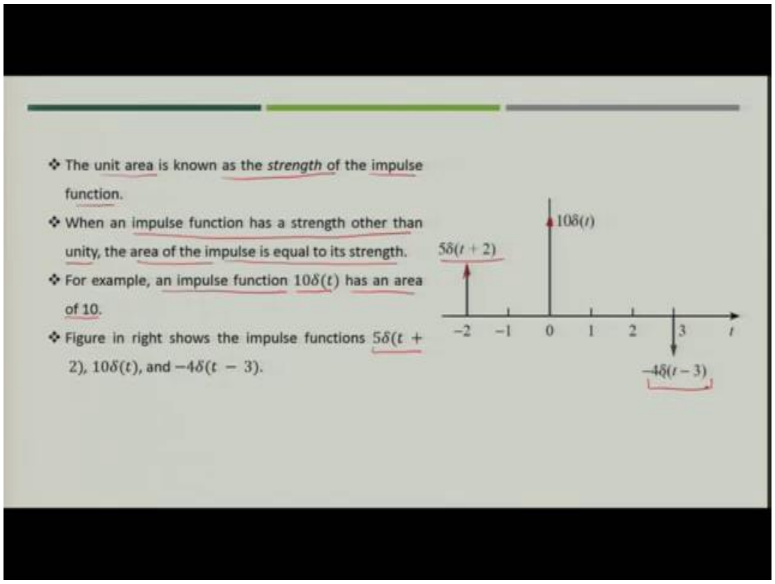

The value of unit area is called as a strength of the impulse function. So, when the impulse function has a strength other than unity, the area of the impulse is equal to its strength. Say for example if the impulse function is 10 $\delta(t)$ means it has an area equal to 10. So, how you will represent them? You will represent with some scale and say in this figure you have you can see easily that this is the value of $10\,\delta(t)$. This value is $5\delta(t+2)$ because it is, if you compare with respect to unit step which we discussed, that it is advanced by some time. So similarly, here also it is advanced by -2 seconds that is why the value of the unit the impulse function is $5\delta(t+2)$. The third function is delayed by 3 seconds. So, that is why it is minus. The value is also negative, and the value of this function is $-4\delta(t-3)$.

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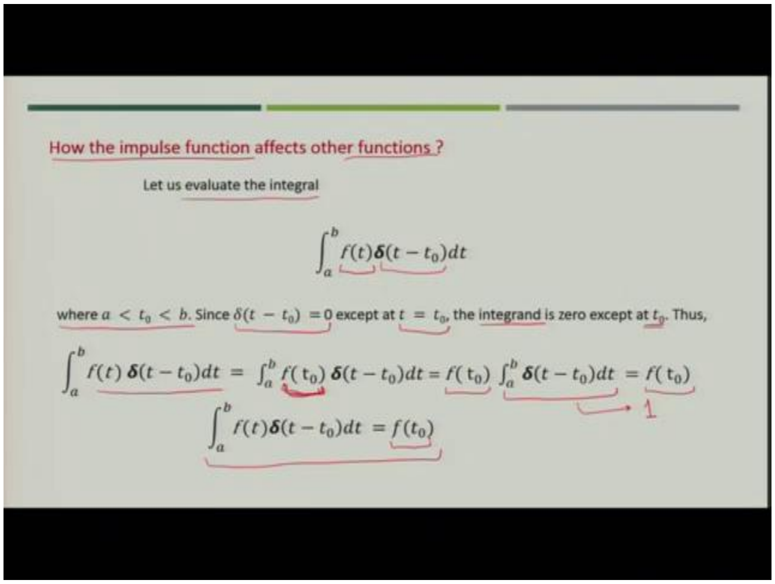

Now, how does this impulse function affect the other functions. Let us see what happens when you club the impulse function with any other function. So, let us say that a function $f(t)$ is associated with a delta function. We want to integrate between a to b for $f(t)\delta(t-t_0)$. So, if you see this expression $\delta(t-t_0)$ is 0 except at $t=t_0$. So, it means that your integrand is 0 except at time $t=t_0$. Therefore, you can simply this function as $f(t_0)$. So, what you get finally is the value of f at time t_0. So, whenever you associate unit impulse function with any other function the value of that combined function will become the function value at time $t=t_0$.

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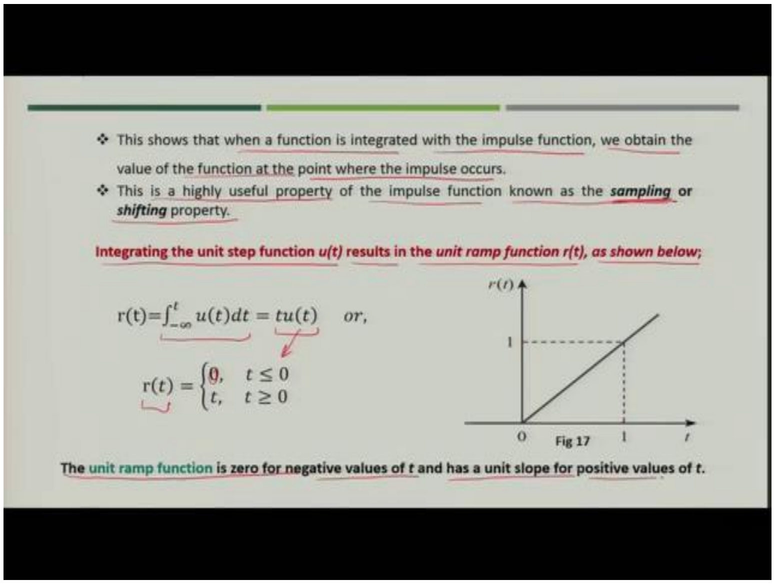

So, this shows that when the function is integrated with the impulse function we obtain the value of the function at the point where impulse occurs and this is highly useful property of the impulse function which is known as sampling or shifting property.

So, when you want to sample a function, you will associate the impulse function with that function at multiple intervals then you get the sample of that function at various time intervals. So, this is very important property and we use extensively in the signal processing. Now, let us understand the third function which is unit ramp function. Now, if you integrate the unit step function, so, in case of unit impulse function, we differentiated the unit step function. Here, we are integrating the unit step function.

When you integrate you will get a unit ramp function as shown in the figure. Now, if you see the unit ramp function you can define it $r(t) = \int_{-\infty}^{t} u(t)dt = tu(t)$. This is because *u(t)* is 0 for time less than 0 and 1 for time t > 0. So, you can define the unit ramp function as 0 at time t less than or equal to 0 and t equal to the value t, when time t greater than 0. So, because of this t you will see a ramp at time t is equal to 1 you will get the value of ramp as 1.

So, this function will be represented by this graph as you are seeing in the figure. So, unit ramp function is 0 for negative values of t and has a unit slope for positive value of t.

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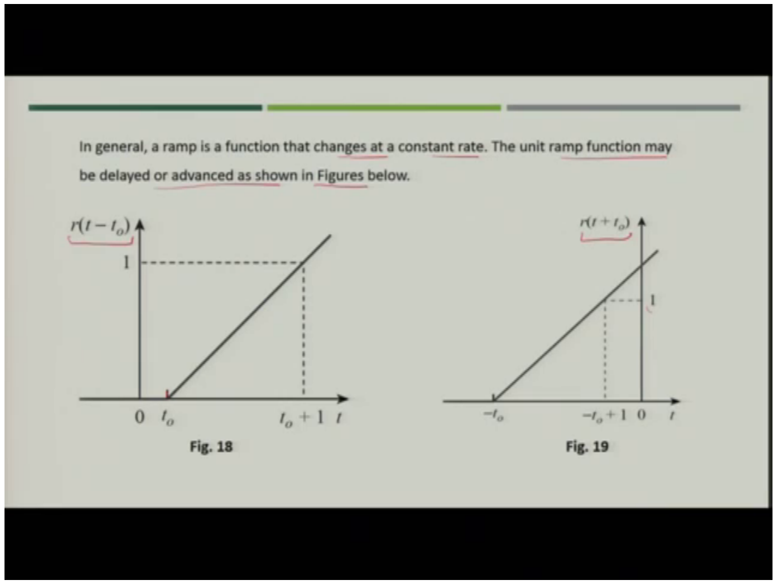

Now, in general the ramp is a function that changes at a constant rate. The unit ramp function maybe delayed or advanced as we saw in case of unit step and unit impulse also. So, here also you can advance or delay. When you delay with some time t_0 you get ramp function $t-t_0$ like as shown in the figure. When you advance, then your ramp will start from $-t_0$ and ramp function will be $r(t+t_0)$.

So, important thing which you can see from here is that at time t is equal to minus 1 you will get the value of unit ramp.

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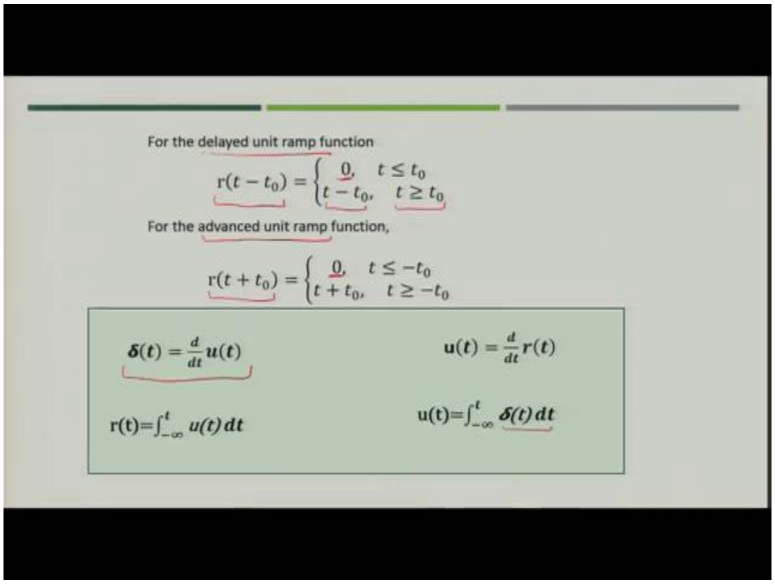

Let us try to summarize what we have discussed till now. For delayed ramp you will say that for time $t \le t_0$ the value would be 0. For time $t \ge t_0$ it will be $t - t_0$ when the delayed ramp function is defined. When you define the advanced unit ramp function $r(t + t_0)$ you can simply write the value is 0 when $t \le -t_0$ and it is $t + t_0$ when time is $t \ge t_0$.

So, now you can summarize that the unit impulse. You can simply write that the unit impulse is nothing but derivative of unit step function and the unit step function is derivative of unit ramp function. Similarly, unit ramp is integration of unit step function and unit step function is integration of unit impulse function. So, you can simply say that these 3 functions are either the derivative or integration of the functions which we discussed.

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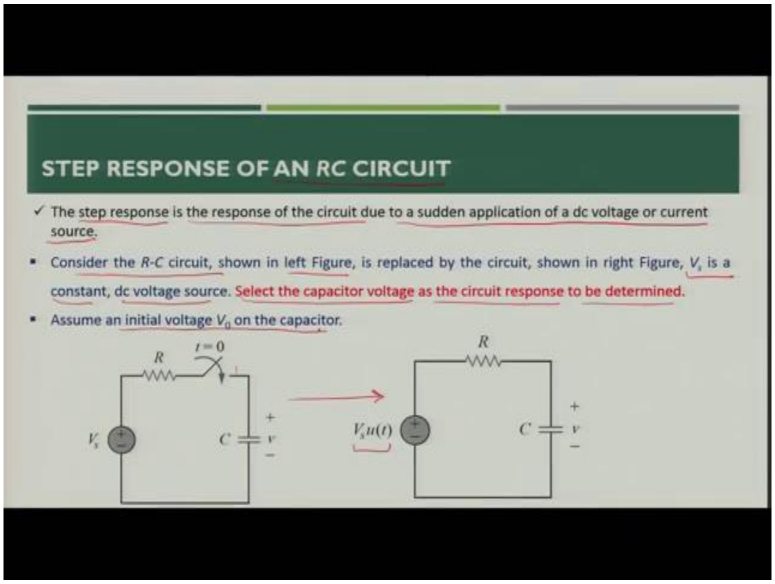

So, now let us start the step response of a circuit because we have understood the basic functions which are called as singularity functions. The unit step function is very important because when we talk about the step response of an RC circuit, we will say that when we apply the voltage that means, we are talking about the step input to the source step input to the circuit.

So, step response is the response of the circuit due to sudden application of dc voltage or current source. Now, let us see the circuit here. Now, if you see the circuit at the left you will simply switch on the circuit at time t = 0. When you switch on the circuit it will be the circuit like shown in the figure. So, you can simply say that V_s is nothing but the unit step because this unit step is being defined by the switch. Here V_s is constant because you are applying a constant dc voltage source. We then select the capacitor voltage as a circuit response which we have to determine. So, for determination let us assume that the capacitor is initially charged with some voltage V_0.

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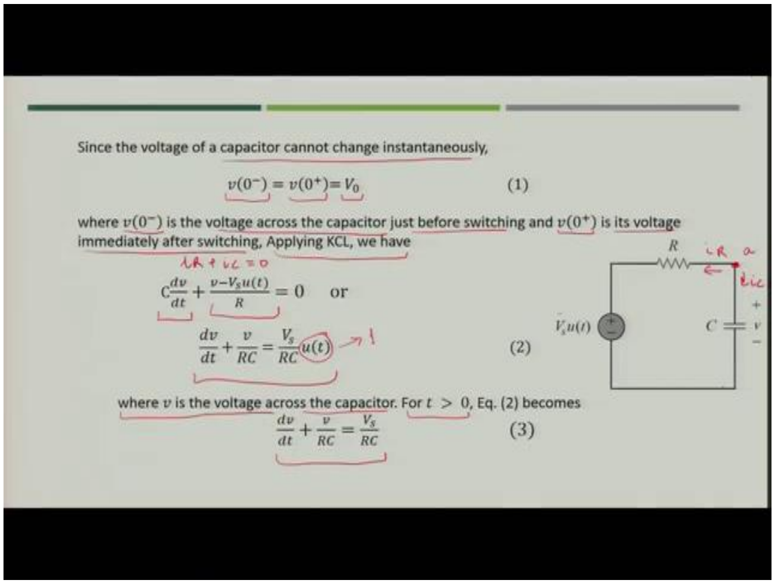

So, when you will say that it is charged with some voltage V_0 we can say that at time t = 0$^-$ or at time t = 0+, voltage will remain same as V_0 because capacitor cannot change the voltage instantaneously. So, at t = 0- the voltage across capacitor just before the switching happens and 0 plus is the voltage immediately after the switching happened. Now, if you apply KCL in this circuit what do you get?

Let us take this as a node a and you say that this is the current i_R this current i_C so your you can simply say $i_R + i_C = 0$. What is the value of i_C ? i_C is nothing but $C\frac{dv}{dt}$ because this is the voltage across the node. Then, i_R can be represented as $\frac{v - V_s u(t)}{R}$. This can be rewritten as shown in the slides.

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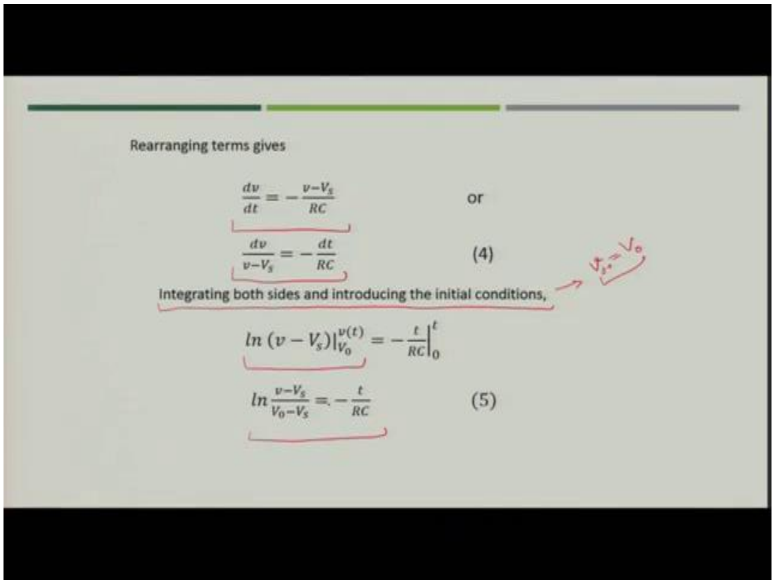

So, now you have the value $\frac{dv}{dt} = \frac{v - V_s}{RC}$. This can be simplified as $\frac{dv}{v - V_s} = \frac{dt}{RC}$. Then integrate at both sides and use the initial condition. What was the initial condition? The voltage across the capacitor was 0 at time 0 plus. So, we apply that. We use that initial condition and take integration at both sides. We get the expression as given in the slide.

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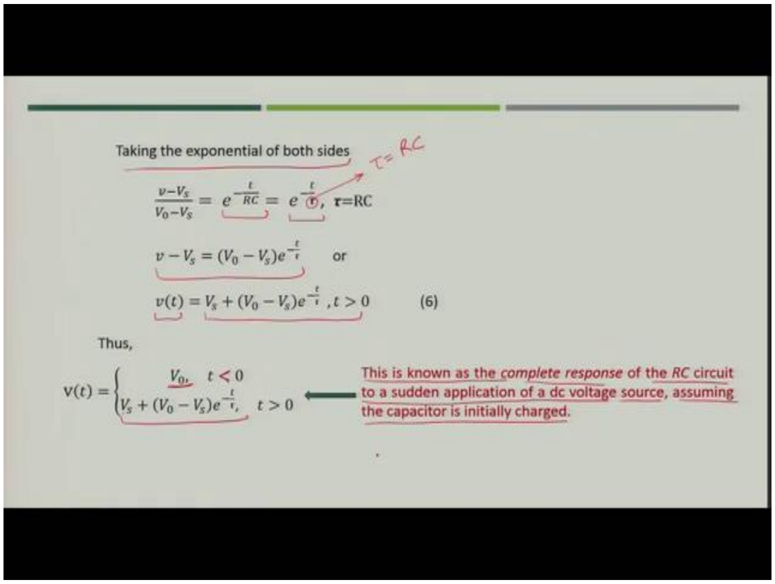

And when you take exponential at both sides you will get $\frac{v-V_s}{V_0-V_s} = e^{-\frac{t}{RC}} = e^{-\frac{t}{\tau}}, \tau = RC$. So, when we were discussing about the source free response of RC circuit we termed this τ as time constant which was nothing but RC. So, this will be simplified as shown in the slides. Now, this condition is valid only for time t greater than 0. For time t less than 0 the value is equal to the initial voltage across the capacitor that is v naught.

So, what we can say; that when we apply a step input to the RC circuit the capacitor which was initially charged with voltage V_0 will remain same when time t < 0 and when time t > 0 this value can be calculated with the help of this expression that is $V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}$ and this is known as the complete response of the RC circuit towards the sudden application of dc voltage source with the assumption that capacitor is initially charged. So, with this we close our today's session. We continue this journey in the next lecture where we will discuss few more concepts related to RC circuits. We will see some examples and then we will move onto RL circuit also. Thank you!