Basic Electric Circuits
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Module 5:
First Order and Second Order Circuits
Lecture 22:
First Order RL Circuits

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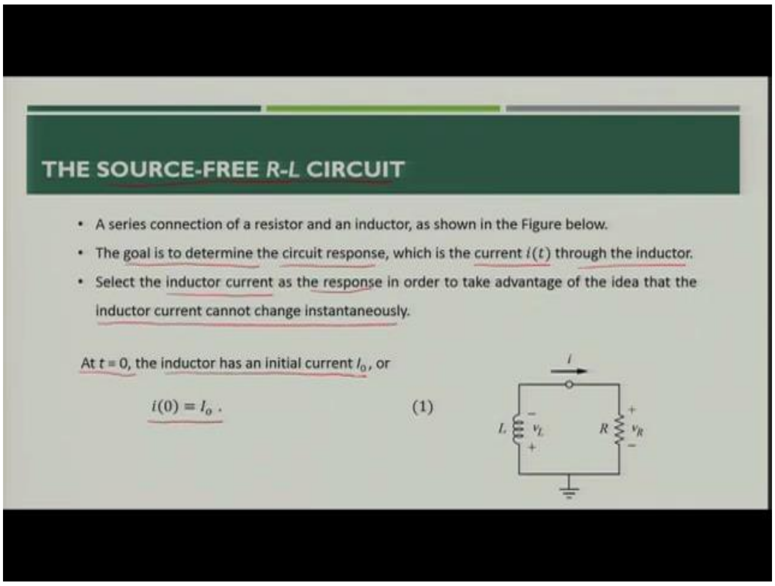

Namashkar so, in last class we discus about source free RC circuit response, in this class we will discuss about the RL circuit, and we will particularly see, when the source in not available, how the circuit will respond. So now, let us starts the second lecture of this week we will discuss about the source free RL circuit now, if you see the RL circuit which we will consider for today's discussion is RL circuit you can say that L is connected in parallel or you can say that the elements are connected in series in a loop.

So now, the goal of the particular circuit analysis is that we will try to find out the current which would be flowing through the inductor. Here, we have selected inductor current as the response because we know that, the current through inductor cannot change instantaneously. So, like your RC circuit where capacitor voltage cannot change abruptly similarly, here the inductor current cannot change instantaneously. Now, let us assume that at time t is equal to 0, the inductor has an initial current I_0.

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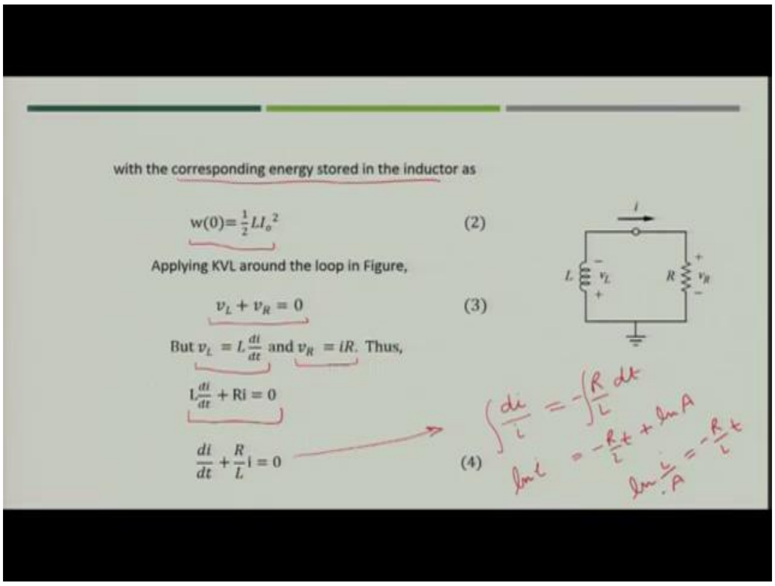

So, let us analyze the circuit. The energy which would be stored in the inductor initially is

$$w(0) = \frac{1}{2} L I_0^{\ 2}$$

Now, if apply KVL around this particular loop you will see that

$$v_L + v_R = 0$$

Where $v_L = L\frac{di}{dt}$ and $v_R = iR$

So, we can simply write

$$L\frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

In this equation you can use the integration technique to solve it as we did in the case of RC circuit. Hence you get,

$$\frac{di}{i} = -\frac{R}{L} dt$$

Integrating, you get

$$\ln i = -\frac{R}{L}t + \ln A$$
$$\ln \frac{i}{A} = -\frac{R}{L}t$$

So let us assume that constant is ln A similar to what we assumed when we were analyzing the RC circuit.

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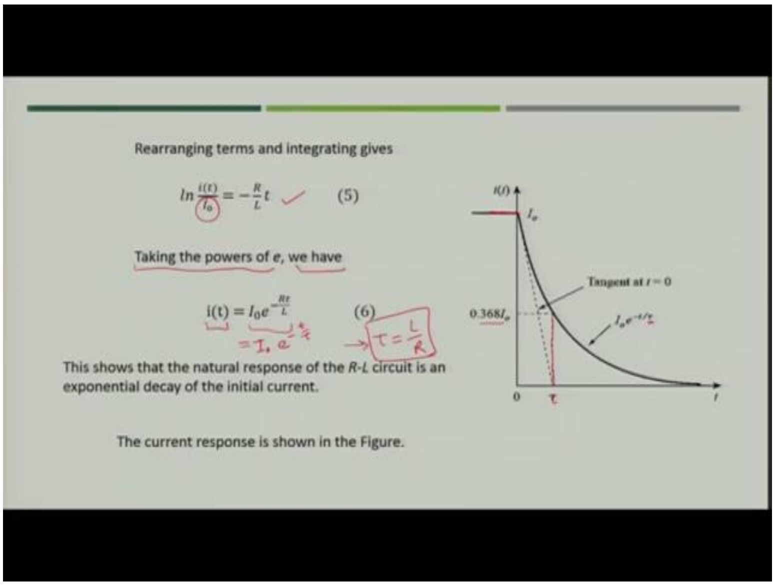

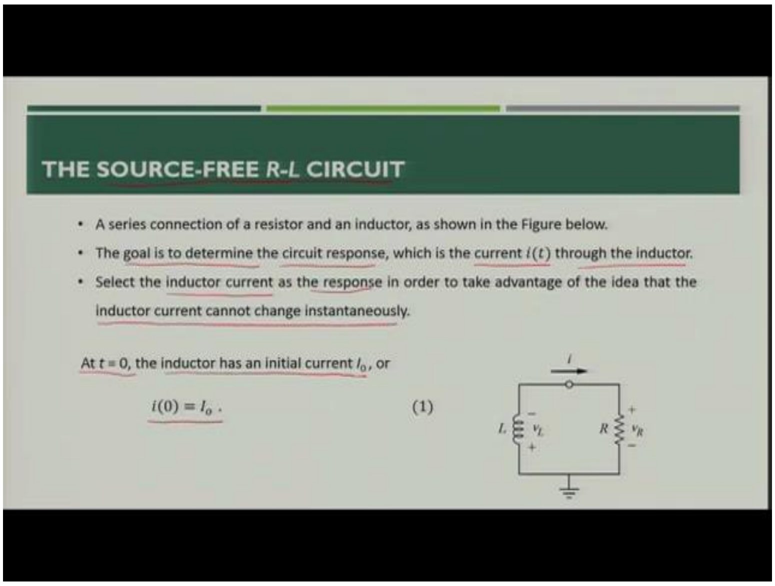

The initial value that we assume I at time t is equal to 0, is I_0. Hence, the above equation can be written as

$$i(t) = Ae^{-\frac{R}{L}t} = I_0 e^{-\frac{R}{L}t}$$

So, if you compare it with the RC circuit where we calculated $v(t) = V_0 e^{-t/RC}$ here, the current $i(t) = I_0 e^{-\frac{R}{L}t}$. The function would be exponentially decaying so it will start with the initial value that is I naught and then it will decay with the function $e^{-\frac{R}{L}t}$.

So, if you see particularly this figure you will see that at time t is equal to tau this will become 36.8 percent of its initial value now, what is tau, if you correlate with the RC circuit tau was the time constant here also the tau is nothing but the time constant but the value of tau would be if you compare this with the I naught e to the power minus t by tau, then you can simply, say tau is nothing but L by R.

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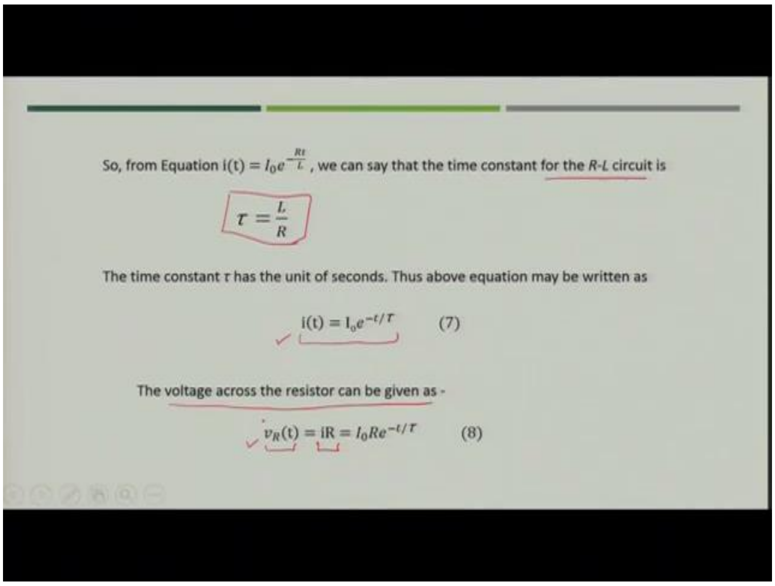

So, we got τ that is our time constant for RL circuit is nothing but L by R. so instead of writing in terms of L and R the current I can be written as

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

Hence, what would be the voltage across the resistor? Voltage across the resistor, would be

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

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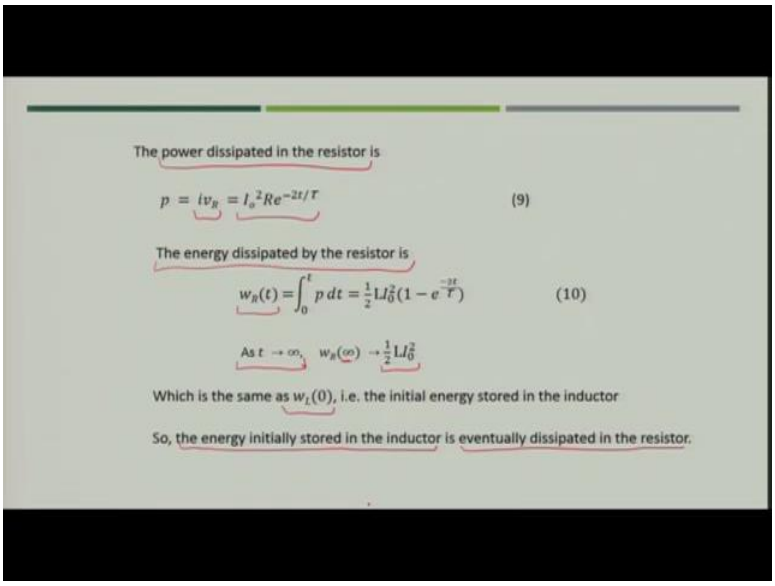

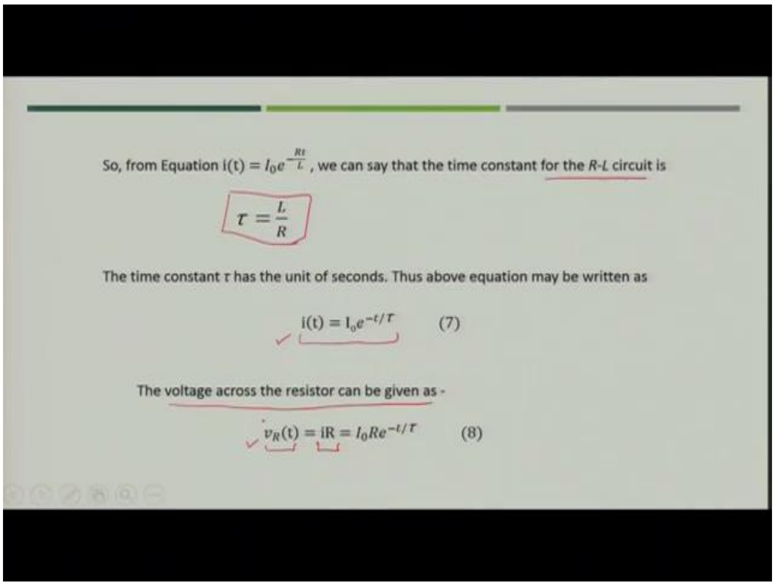

Now, instantaneous power dissipated by the resistor will be p,

$$p(t) = v_R * i = I_0{}^2 R e^{-2t/\tau}$$

The energy absorbed by the resistor till time t is,

$$w_R(t) = \int_0^t p dt = \frac{1}{2} L I_0{}^2 (1 - e^{-2t/\tau})$$

From this equation when, t times to infinity the

$$w_R(\infty) = 1/2 L I_0{}^2$$

And this is what we assume that the initial value stored in the inductor, so that means that whatever the initial energy was stored in the inductor, is eventually dissipated in the resistor.

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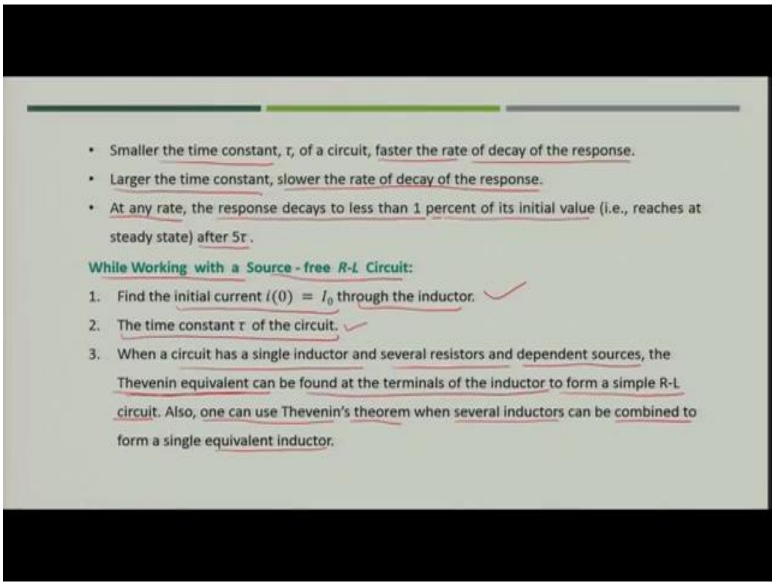

Now, let see some important facts about the time constant, smaller the time constant of the circuit faster would be rate of decay of the response. So, this is similar to what we saw in case of RC circuit so similar to that in RL circuit also the small time constant means faster the rate of decay of response.

Larger the time constant slower would be the rate of decay of response. Now, at any rate the response decays to less than 1 percent of its initial value after 5 time constants so, the same we saw in case of RC circuit also, when the time is 5 tau that is 5 times of the time constant the value of voltage was left with less than 1 percent. So, here in case of RL circuit the response will decay that means the value of current that is flowing through the inductor, will reach to less than 1 percent of its original value, after 5 time constants.

Now, while working with the source free RL circuit, we have to keep 3 things in mind and we have to solve the circuit accordingly. Firstly, we have to find the initial current which is flowing through the inductor then, find the time constant tau of the circuit and then thirdly when the circuit has a single inductor and there are several resistors connected or there may be some dependent sources also the Thevenin equivalent would be a good option to analyze the circuit.

So, we will use Thevenin equivalent at the terminals of the inductor to form a simple RL circuit or we can use Thevenin theorem when several inductors can be combine to form a single equivalent inductor, so in summery we can say that if we have circuit where we have multiple inductors and resistors we can utilize the Thevenin theorem to simplify the circuit. And find out the response of the circuit.

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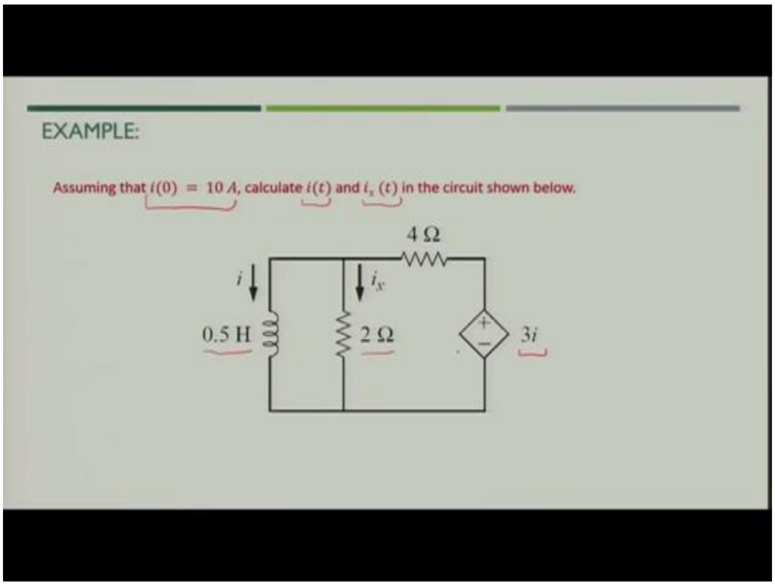

Now, let us take one example so that we can clear the concept, let say we have one circuit as shown in the figure, here one inductor of 0.5 H is carrying some current i and in parallel with a 2 ohm resistance where the current i_x is flowing an then we have 4 ohm in series with the parallel combination of inductor and resistor and which is connected in series with one dependent voltage source, whose current component that is if the voltage source value given in the circuit is 3i the current component is depending upon the current flowing through the inductor.

Now, initial value which is given here, is I at time t is equal to 0, is 10 ampere we need to find out, the value of I at time t through the inductor. And find out the value of current I which is flowing through the 2 ohm resistor.

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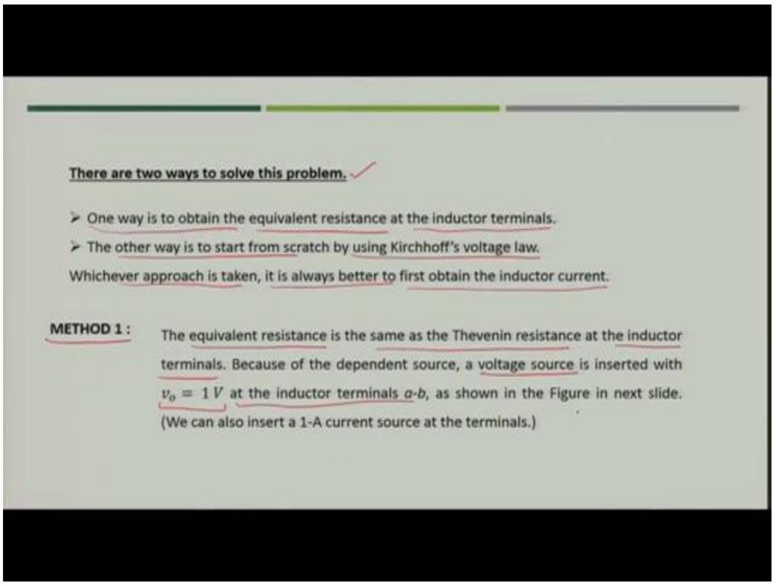

Now, here we have two ways to solve this problem, one way is that we can obtain the equivalent resistance of the inductor terminal, or other way is that, we start from scratch using Kirchhoff voltage law.

So, when we have to find out the equivalent resistance we can utilize over knowledge of Thevenins theorem and simplify the circuit to find out the equivalent resistance second when we have the dependent voltage source, we can also use the simply Kirchhoff voltage law and simplify the circuit as we do in case of mesh analysis so, whatever the approach we will take the value will remain same but, it is always better to obtain first the inductor current.

Now, in first method the equivalent resistance is the same as Thevenin resistance at the inductor terminals now, since, we have a dependent voltage source available in the circuit, what we can do, we can use one voltage source that we ley say that the value of that voltage source is 1 volt. At the inductor terminal AB which is shown in the next slide.

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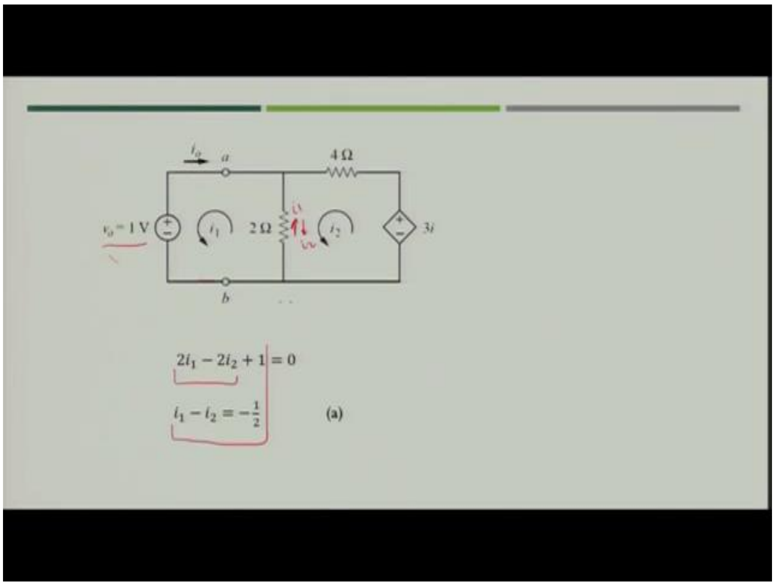

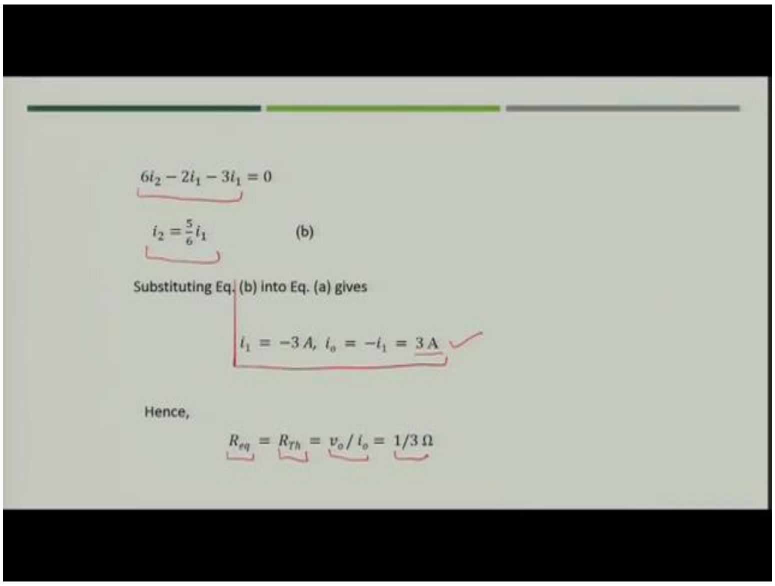

So, what we have done we have remove the inductor from terminal AB and we added one voltage source equal to 1 volt now, we have two meshes in the circuit, let say the current i1 and i2 is flowing in this direction both are in the same direction so, let us try to write the KVL equation for both of the meshes in this case, you can write,

$$2i_1 - 2i_2 + 1 = 0$$

Now, for second loop what we can write, we can write

$$6i_2 - 2i_1 - 3i_1 = 0$$

So, here the value of dependent current source would be $3i_1$. So this we have got now when we solve $i_2 = 5/6i_1$

Now, we have two equations in term of i1 and i2. If we solve, we get the value of $i_1 = -3$, now i_1 is opposite to the direction of i_0 which is the value of current flowing through the 1 volt voltage source so, the value of $i_0 = -i_1$. So, we get $i_0 = 3$ampere. Now, the R equivalent is nothing but the Thevenin resistance and its value is v_0 / i_0.

v_0 value is 1 volt, and i_1 we have just calculated as 3 ampere we get, 1 by 3 ohm as a Thevenin resistance.

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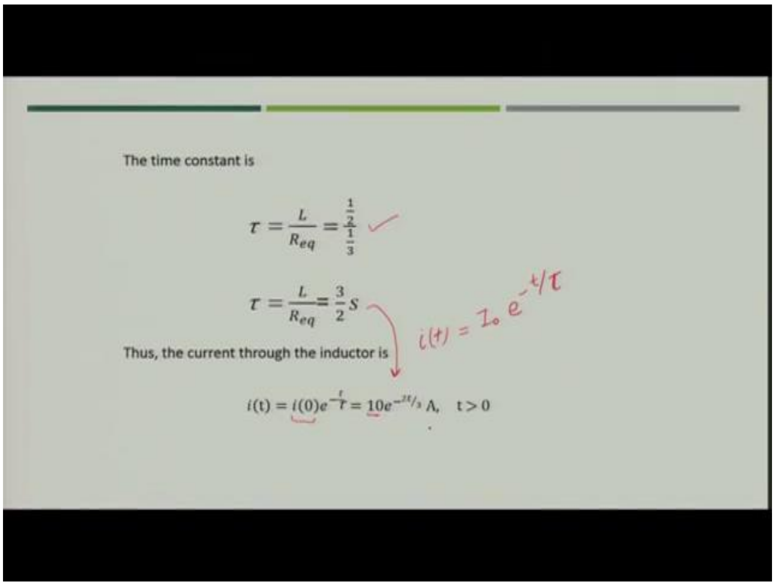

Now, what would be the value of time constant, time constant value will be L upon R equivalent value of L is given as 0.5 henry so that was given in the question so, we will put that value and the R equivalent we have just now calculated as 1 by 3 ohm so, value of tau will be 3 by 2 second, this value we will put in the original equation of current

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$

So, $i(0) = 10$ is there and we put the value of tau in the equation we get, the current flowing through the inductor, at any time t is

$$i(t) = 10e^{-\frac{2t}{3}}$$

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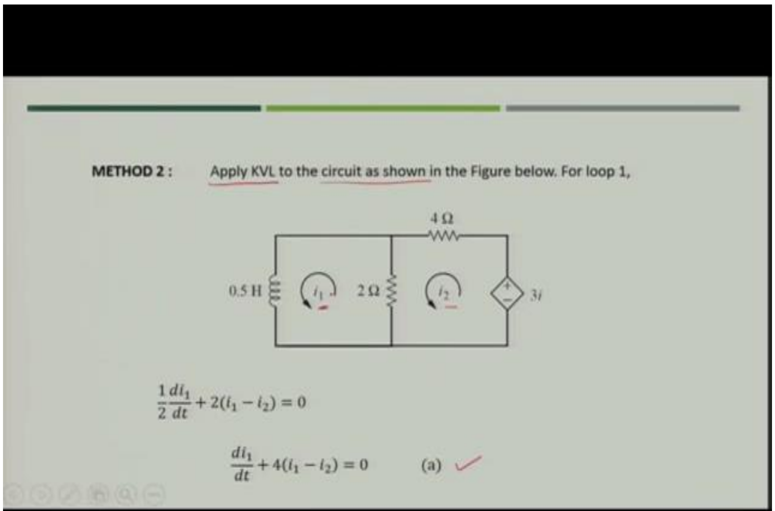

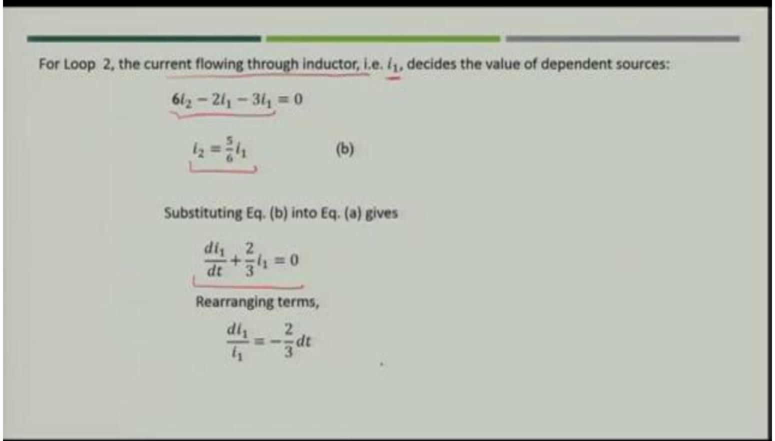

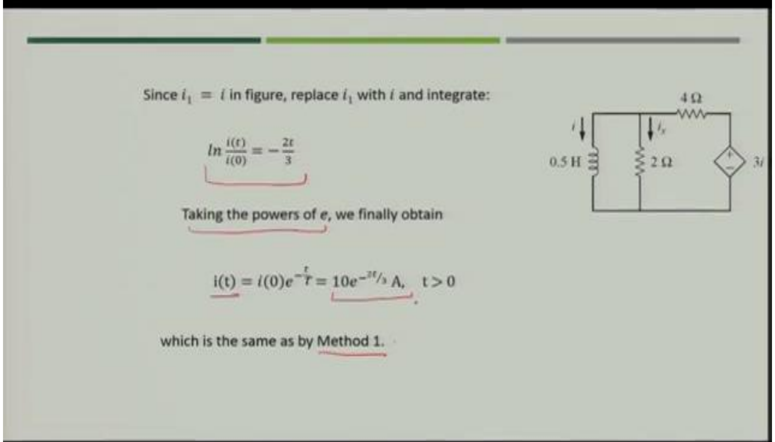

Now, if you see second method, we apply KVL to the circuit so, if you apply KVL you simply, put the inductor back to the circuit then the circuit will again have two meshes let us take the value of those currents as i1 and i2. The calculations are as shown in the above slides and we get the same answer as in method 1. Hence, either you use the Kirchhoff voltage law directly to the circuit or you will use Thevenin theorem you will get the same result so, now we have got the value of i(t).

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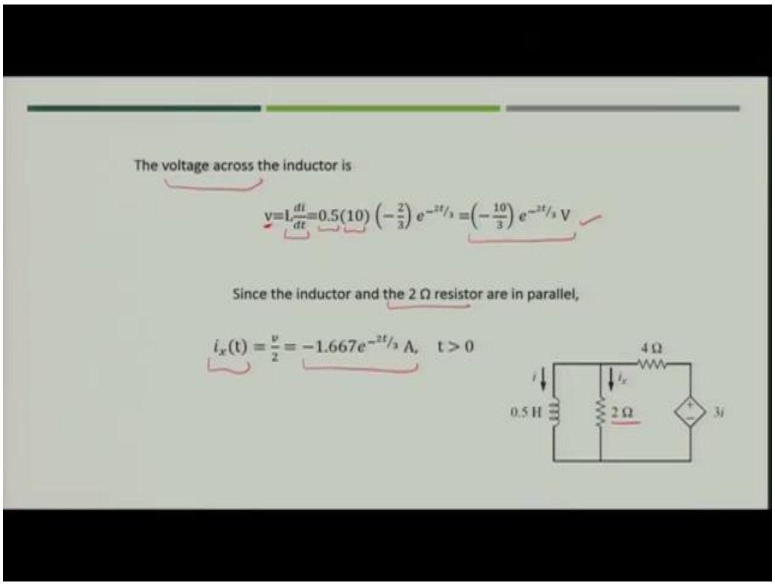

Next task is we need to find out the value of v which is nothing but the voltage across the inductor. so voltage across the inductor is, L di by dt we can put the value of inductor L then, the initial value of current I naught because di by dt is when you differentiate It you will get the value of di by dt so, this will become, 0.5 into the value of I naught into the minus 2 by 3 term which is coming from here, into e to the power minus 2t by 3. So you will get value of voltage V is nothing but minus 10 by 3 into e to the power minus 2t by 3 volts.

Now, since the inductor and the 2 ohm resistors are in parallel that means whatever is the voltage coming across the inductor will be the same voltage which is coming across the 2 ohm resistor. So, we can calculate the value of current $i_x = \frac{v}{2} = 1.667e^{-\frac{2t}{3}}$. So in this you can use either Thevenin equivalent or the mesh analysis method, using Kirchhoff voltage law and you will get eventually the same answer so, we close our RL circuit session today from this point in the next lecture we will discuss about the various sources specially those sources which are required for the analysis of the circuit so, if you see the particular electrical circuit we apply either voltage or current source so, when you apply voltage or current source those are suddenly apply to the circuit.

So, we will get one step response of the source so, that is one important type of source which we will study plus when we have the surges coming in the electrical circuit we get another type of pulse in the circuit, that is called the pulse response so, that also we will see how we will interpret that particular event with the help of mathematical function because these two are required for analysis of our electrical circuit so we will see how we will represent both of them in a mathematical term.

And then we will apply both of them into the various RC and RL circuits and see how they will behave when we will apply either stepper impulse type of sources into the circuit, Thank You.