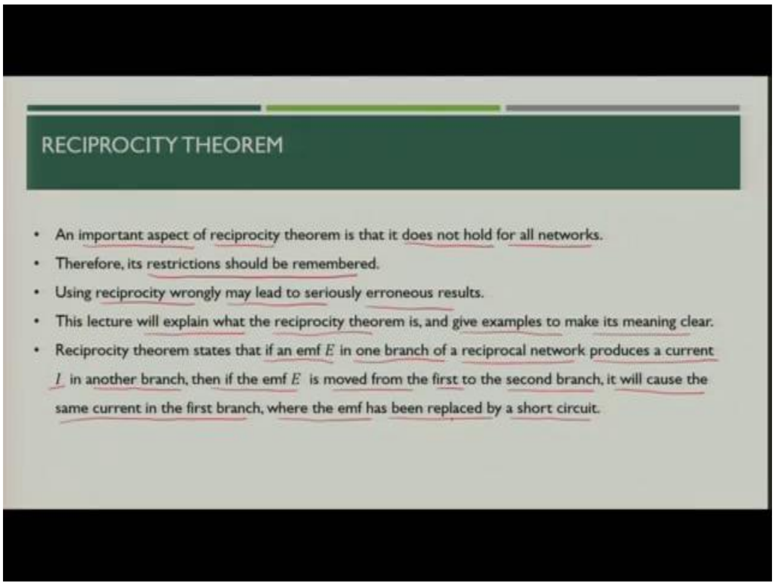
**Basic Electric Circuit**
**Professor Ankush Sharma**
**Department of Electrical Engineering**
**Indian Institute of Technology Kanpur**
**Module 4 Network Theorem 2**
**Lecture 20 Reciprocity and Compensation Theorem**

Namaskar. So, in today's lecture will discuss about 2 theorems, those are reciprocity theorem and compensation theorem. So, let us start with the reciprocity theorem.

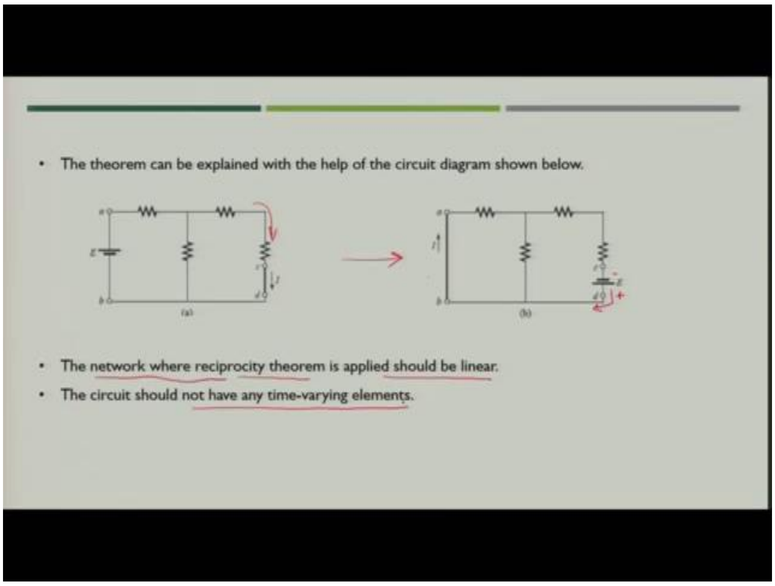
(Refer Slide Time: 0:33)




So, what does reciprocity theorem says basically. It is a very important aspect of reciprocity theorem that it does not hold for all network. So, we have to be very careful when we deal with the reciprocity theorem and we have to keep remembering the restrictions which are imposed on the reciprocity theorem because if you do not follow then the reciprocity theorem you will use wrongly and that may lead to a serious erroneous result in the circuit.

So, in this lecture we will explain what do you mean by reciprocity theorem. And we will also take 1 example to make the things clear. Now, let us see what do you mean by reciprocity theorem the theorem says that if an emf E, emf is nothing but electro motive force E in 1 branch of reciprocal network produces a current that is I in another branch, then, if the emf is moved from first to the second branch, it will cause the same current in the first branch where emf has been replaced by a short circuit.

(Refer Slide Time: 2:06)



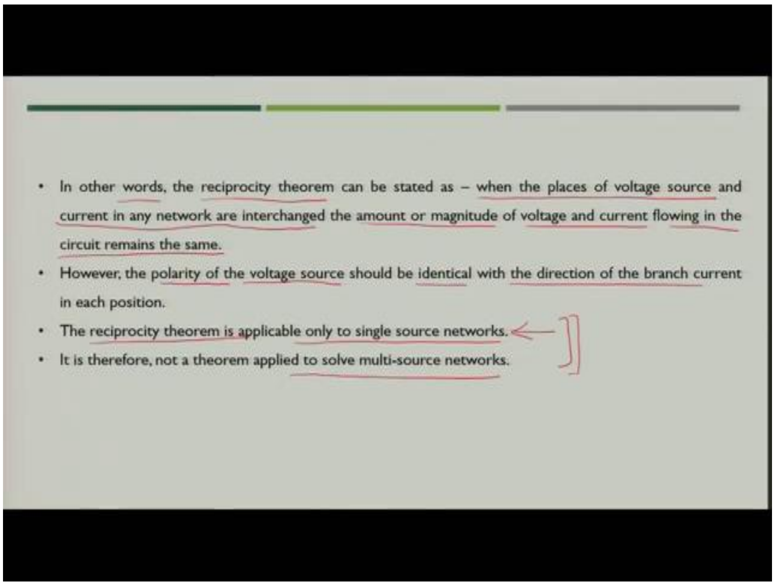

So, let us understand this definition with one circuit. Suppose, we have originally a circuit as shown in this figure, where a voltage source E is present between node a and b and it is causing a current I to flow in the resistance and along the node a and b. So, when voltage source is energized, the circuit will be energized, the current will flow in this particular branch. So, we are saying that suppose this current is I. So, what reciprocity theorem says that if you replace or if you exchange the locations of this current and voltage source, E, then the other values are not going to change in the circuit.

So, what does it mean? It means that we are replacing the value of E. So, how you will replace you will just switch the locations of both of the, the parameters 1 is E that is voltage source and second is current flowing in this particular branch. So, if you place the voltage source at current location, you have to make sure that the polarity of voltage source would be in such a way that it will give power to the circuit in the same direction as the previous direction of the current was.

So, that is why if you see here, it between cd the polarity of E is of minus is on the top and plus is on the bottom. So, this will make sure that the current which is flowing from the source would be in the same direction as if it was there even the original circuit was energized. Now, next is that, when you will place the current in that branch where voltage source was connected. So, what you have to do you have to simply short circuit the nodes a and b because now voltage source is not available.

So, this would be short circuited and the same current will flow in the branch keeping other parameters same. So, that means that you can exchange the locations of voltage source and the current. Now, important thing is that the reciprocity theorem can be applied only when the circuit is linear and there is no any time wearing element.

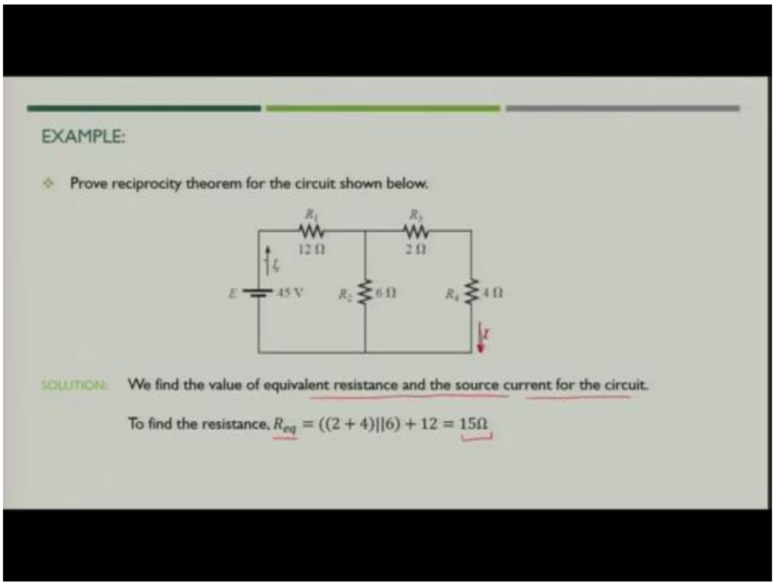
(Refer Slide Time: 4:57)




Now, in other words, you can also say that reciprocity theorem can be interpreted as when the places of voltage source and current in any network are interchanged the amount of magnitude of voltage and current flowing in the circuit remains same.

Now, as we discuss that polarity of voltage source should be identical with the direction of branch current in each position. So, important factor to keep in mind is that the reciprocity theorem is applicable only to a single source network. So, if there is a network were multiple sources are connected you cannot utilize the reciprocity theorem over there. So, this is the limitation which we have to always keep in mind otherwise, we will use reciprocity theorem wrongly.

(Refer Slide Time: 5:55)



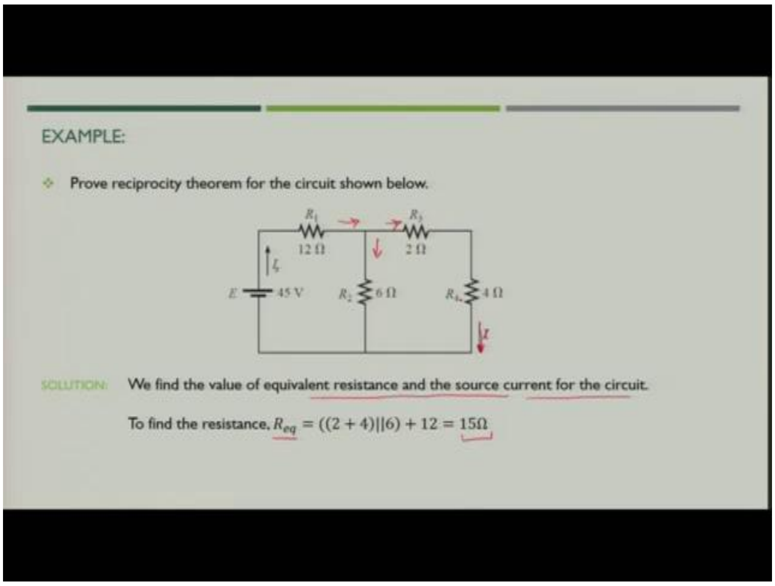

Now, let us understand this aspect with the help of an example. So, that you are clearer about the particular theorem. Let us see a particular circuit which is shown in this figure, we have a voltage source E that is 45 volt giving current as $I_s$ to the circuit and the 4 resistance as you will see in the circuit are shown. Now, we have to prove that if you exchange the locations of I and E the values of current I is not going to change.

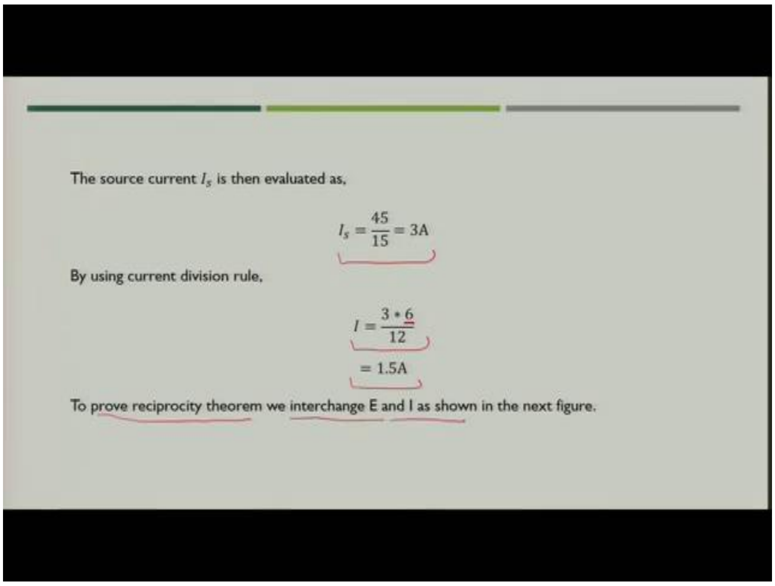
So, what we will do will first find out what would be the value of I in this particular arrangement. So, we will first try to find out the equivalent resistance of the circuit. So, the equivalent resistance the circuit would be

$$R_{eq} = ((2 + 4)||6) + 12 = 15\Omega$$

So, now, we have got 15 ohm as a total equivalent resistance of the circuit. So, you can find out the value of current $I_s$.

(Refer Slide Time: 7:37)




So, the current $I_s$ would be

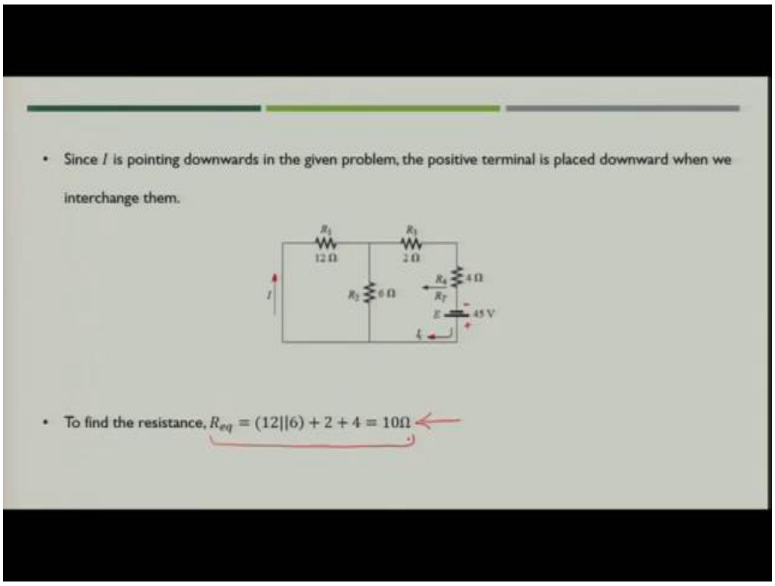
$$I_s = \frac{45}{15} = 3A$$

Now, you can use current division rule, because here the current is being divided into 2 parts. So, the current flowing in this would be the value of resistances divided by the sum of the resistances. So, you will see here

$$I = \frac{3*6}{12} = 1.5A$$

Now, to prove the reciprocity theorem, what we will do we will interchange the locations of E and I.

(Refer Slide Time: 8:44)

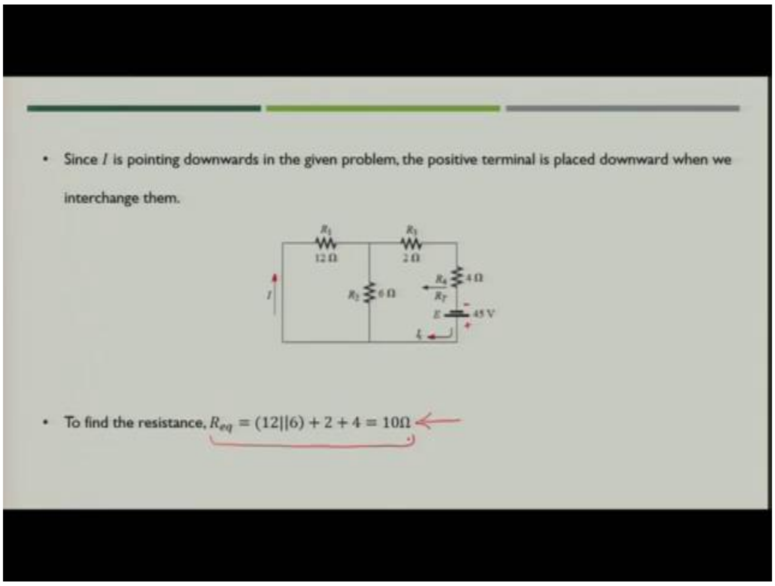


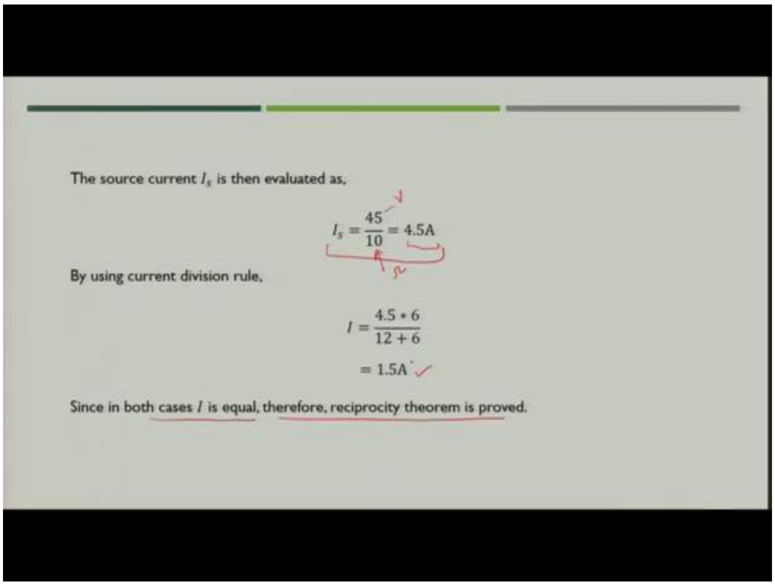

So, what would happen in that case, the circuit will be modified as shown in this figure, where you will see the direction of ease downward. So, the minus sign is on the top, plus sign would be on the bottom and $I_s$ current would be flowing in this direction which is same as the direction of I was in the previous figure and the location where the voltage source was connected we have short circuited and we assume that the current I which is same as it was in the previous case is flowing in the short circuit leg segment of the circuit.

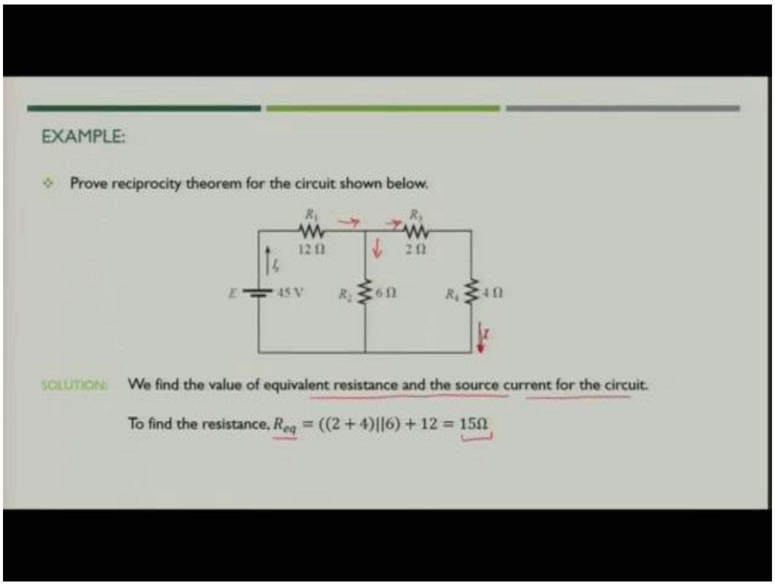
So, now, we have to prove this value of I again would be 1.5 ampere when the circuit is modified as per reciprocity theorem. So, now again if you see what would be the value of equivalent resistance,

$$R_{eq} = (12||6) + 2 + 4 = 10\Omega$$

(Refer Slide Time: 10:08)



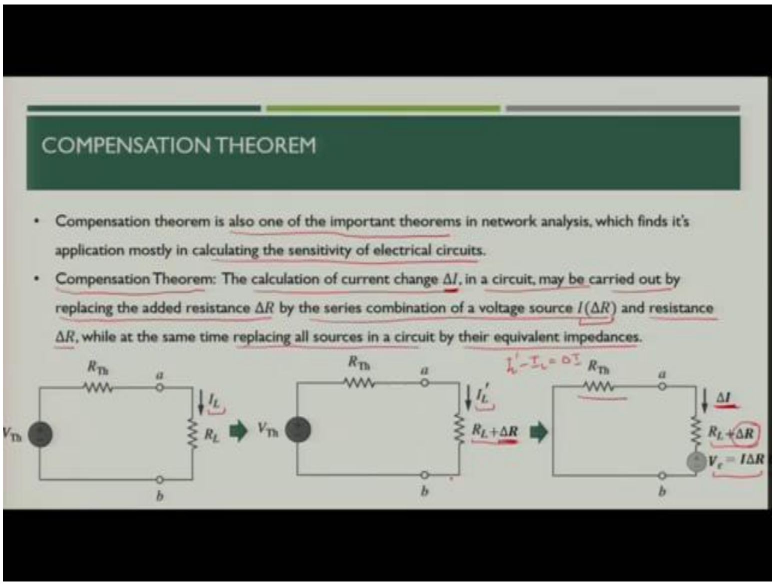



Now, in this case what value of $I_S$ that is source current you have got 45 is the value of voltage source and the resistance which you have got seen by the source from this site that is equivalent resistance of the circuit. So, when you calculate the value of source current it will become 4.5 ampere. Now, by current division rule, you can again find out what would be the value of current I as,

$$I = \frac{4.5 * 6}{12 + 6} = 1.5A$$

So, we can see now, in both of the cases the current I is same. So, that means that we can say that the reciprocity theorem is justified in this particular circuit. So, the circuit which is modified and we place the voltage source in such a way that it is the current coming out of the voltage sources in the same direction and we short circuit the current I these segments where the voltage source was connected and we found that the value of I is same as it was there in the previous case. So, this justifies that the reciprocity theorem is applicable in this particular case.

(Refer Slide Time: 12:21)




Now, let us talk about another theorem, which is called as a compensation theorem. Compensation theorem is also 1 of the important theorems in the network analysis, which finds its application mostly in calculating the sensitivity of the electrical circuit. Sensitivity means, when you change the value of voltage a little bit, say, $\Delta V$, how much the current will change in that particular segment, where you are trying to find out the sensitivity.

So, that will give you the value of sensitivity of that particular circuit element or the particular segment in which you are trying to find out now, particularly, what does compensation theorem means, now, the calculation of current change $\Delta I$ in a circuit may be carried out by replacing the added resistance $\Delta R$ by a series combination of voltage source that is a combination of original current $I * \Delta R$ and resistance $\Delta R$ while at the same time replacing all sources in the circuit by their equivalent impedances.

What does it mean? Suppose, you have a network and the 2 terminals of the network are having the load connected that is the value of load is $R_L$. So, what you can do you can replace the whole circuit as a Thevenin equivalent that that is what we discuss when we discuss the Thevenin theorem. So, linear 2 terminal network can be replaced by its Thevenin equivalent. So, here we have created Thevenin equivalent of the network.

Now, suppose, if there is a change in value of load $R_L$ by $\Delta R$. So, what will happen in that case, the value of, the updated value of the resistance that is $R_L + \Delta R$ will cause a change in the load current. So, now, earlier it was the load current $I_L$, it has become $I_L^{'}$. So, $I_L^{'} - I_L$ is nothing but equal to $\Delta I$. So, if you want to see the impact of only $\Delta I$, you can equally represent the circuit as the new voltage source we see which opposes the direction of the flow of the current and the value of $V_C$= $I(\Delta R)$ which would be connected in series with $R_L + \Delta R$.

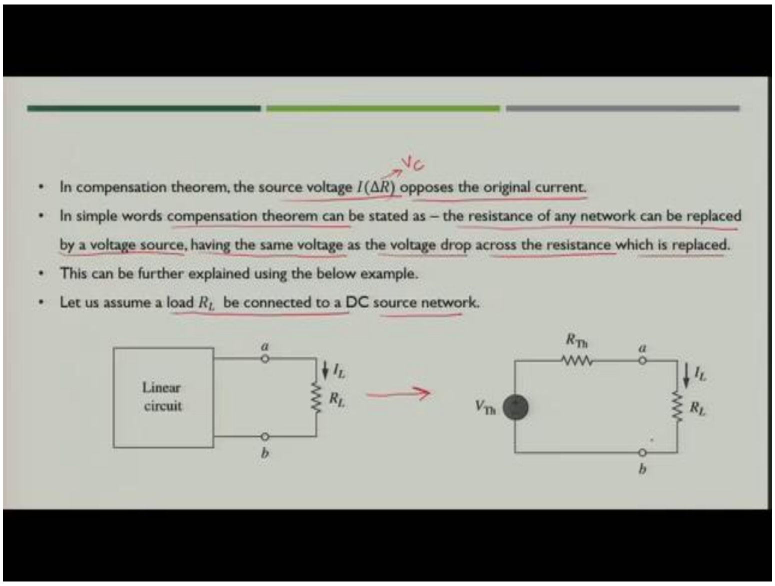
So, now, the value of current flowing $\Delta I$ would be the value of $V_c$ which you will see in the voltage source would be $I\Delta R$. So, now, you again see the compensation theorem the calculation of current $\Delta I$. So, when you will place the arrangement like this, you can easily find out the value of $\Delta I$. So, value of $\Delta I$ would be

$$\Delta I = \frac{-V_c}{R_{Th} + R_L + \Delta R}$$

So, this would be the value you can simply calculate with the help of the updated circuit. So, by using the compensation theorem you can directly see what would be the impact of $\Delta I$. because of the change in resistance $\Delta R$ and this is what the compensation theorem says. So, it says that calculation of current change $\Delta I$ in this circuit may be carried out by replacing the added resistance $\Delta R$, that is the added resistance $\Delta R$ by a series combination of voltage source $I\Delta R$.

So, this is a voltage source $I\Delta R$ and resistance $\Delta R$. So, the resistance will be intact the original resistance $R_L$. So, $R_L + \Delta R$ will be the updated resistance while at the same time replacing all sources in the circuit by their equally impedances. So, that means, that the original Thevenin voltage we are replacing with the short circuit, if we have a current source then it will be open circuited while keeping all resistance value in the circuit. So, equivalent resistance value of the circuit is $R_{Th}$. So, that is kept in the circuit and with this the arrangement you can easily find out the value of $\Delta I$ which is flowing into the circuit because of the value of the resistance changes from $R_L$ to $R_L + \Delta R$. So, this is what compensation theorem says.

(Refer Slide Time: 17:58)

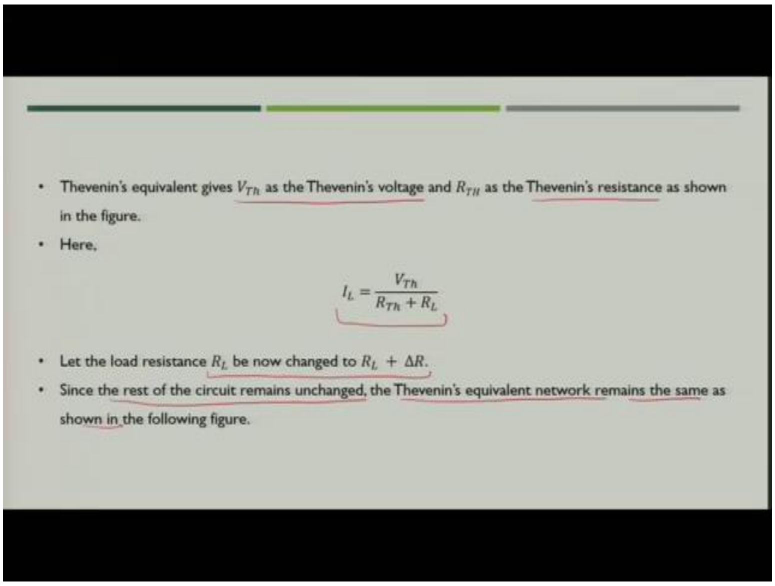



Now, let us understand and justify our understanding what we have discussed till now, the compensation theorem will give you the value of voltage source $I\Delta R$ that is nothing but the value $V_c$ which we have seen and the look the orientation would be in such a way that it will oppose the original current.

So, alternatively we can also say that the resistance of any network can be replaced by a voltage source having the same voltage as the voltage drop across the resistance which is replaced. So, if you see the change in resistance is $\Delta R$ voltage drop across $\Delta R$ is nothing but $I\Delta R$. So, that is why it is replaced with the equivalent voltage source $V_c$. So, let us now try to understand and derive whether the updated circuit which we have just calculated justifies the claim or not.

So, let us assume that the load $R_L$ is connected to a dc source network that is, we have a source here and load $R_L$ is connected to that. So, when we discuss the Thevenin's theorem we discuss that this particular circuit can be replaced by its equivalent voltage in series with Thevenin resistance. So, this circuit can be can be replaced by the equivalent circuit as shown in the figure and the load $R_L$ is connected across the terminal ab.

(Refer Slide Time: 19:48)

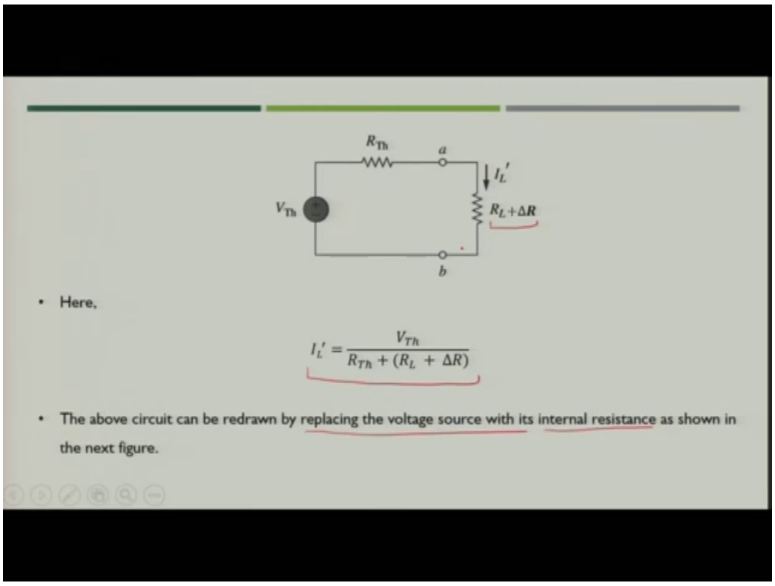



Now, if you are calculating the value of $I_L$ would be

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

So, this is we have got. $R_{Th}$ is nothing but Thevenin's resistance. Now, let us see, if we change the value of load resistance from $R_L$ to $R_L + \Delta R$, then what will happen. Since the rest of the circuit remains unchanged, the Thevenin's equivalent network will remain same as shown in the next slide.

(Refer Slide Time: 20:35)

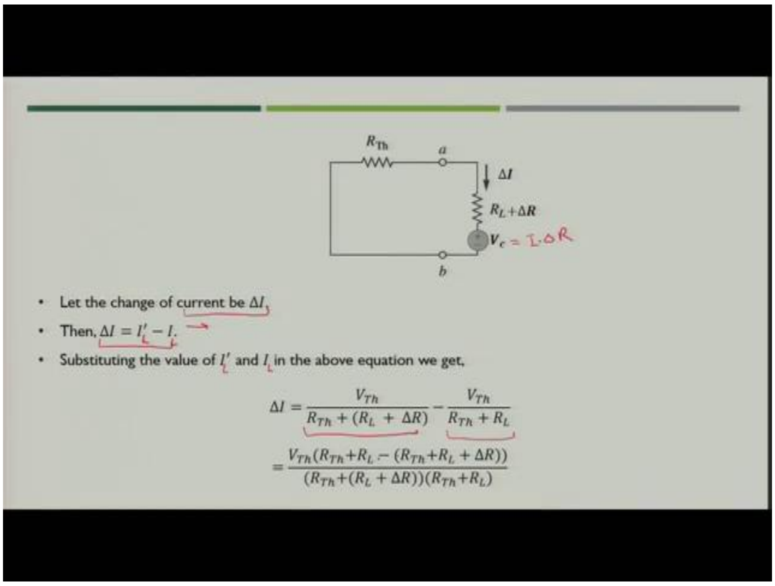


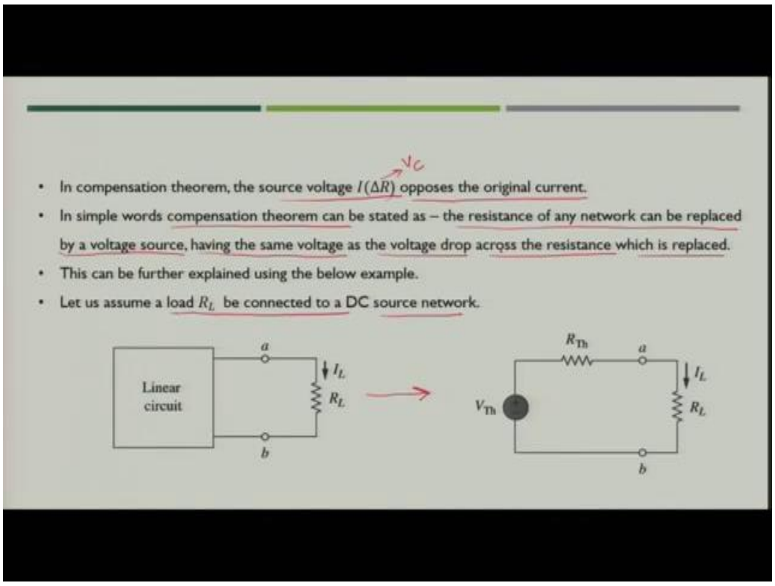

So, what happens now, the other parameters of the circuits are same. So, there is no change in the Thevenin equivalent, Thevenin equivalent will remain same, the only change would be the load resistance that is now $R_L + \Delta R$. Current now, flowing through the updated load resistors is now $I_L^{'}$. So, what will happen, what would be the value of $I_L^{'}$ now,

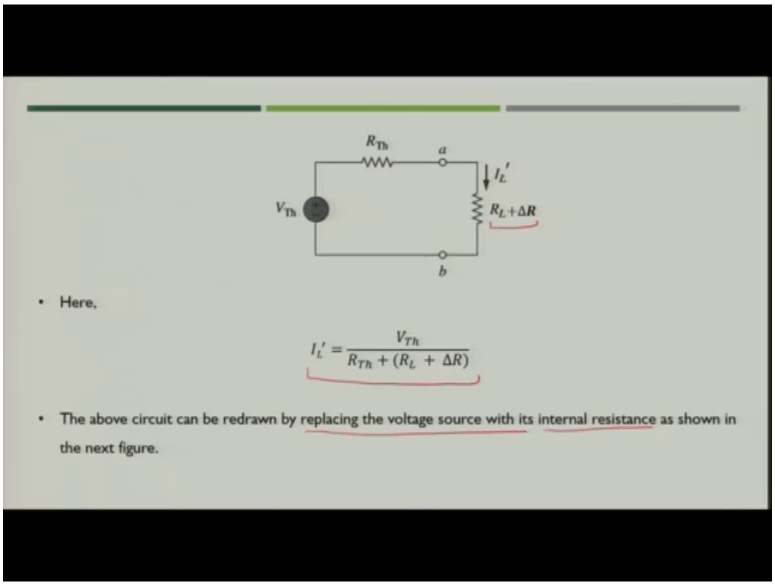
$$I_L^{'} = \frac{V_0}{R_{Th} + (R_L + \Delta R)}$$

Now, we apply the compensation theorem and we replace the voltage source with its internal resistance and put 1 another voltage source that is $V_c$.

(Refer Slide Time: 21:40)






So, now, as per compensation theorem our circuit would be like this, where we are short circuiting the voltage source which is there in the network in this case it was $V_{Th}$. So, we have short circuited the voltage source Vth. Now, we have connected the voltage source $V_c$ the value of $V_c = I\Delta R$.

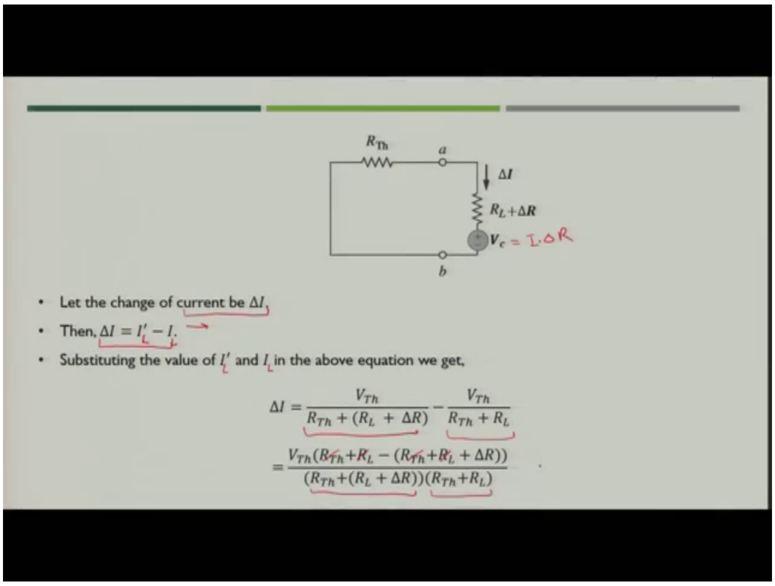
So, here the magnitude would be like this, but, since it is in the opposite direction it will become negative. So, that we will see when we will finally derive the condition. So, now, let us assume that the value of $\Delta I$ is the change. So, $\Delta I = I_L' - I_L$, $I_L'$ is the value of the updated current when $\Delta R$ was added and $I_L$ is the original current.

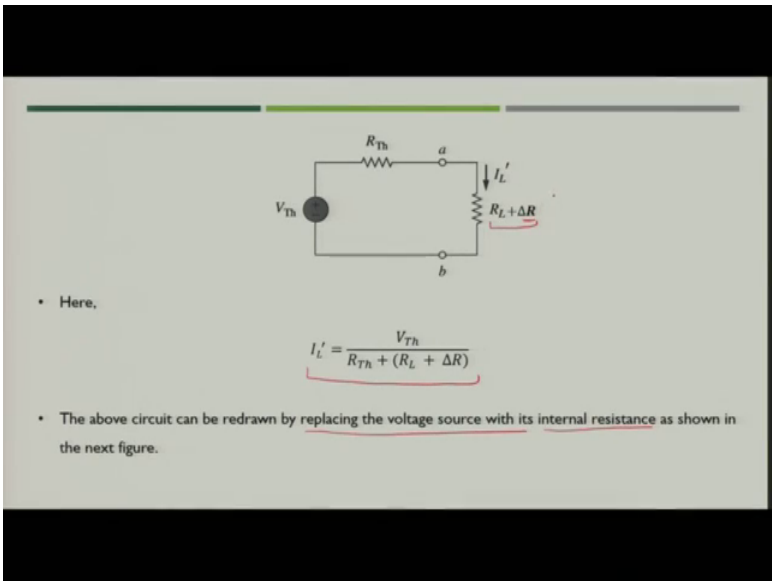
Now, substituting the value of $I_L'$ and $I_L$ in the above equation we get $\Delta I$.

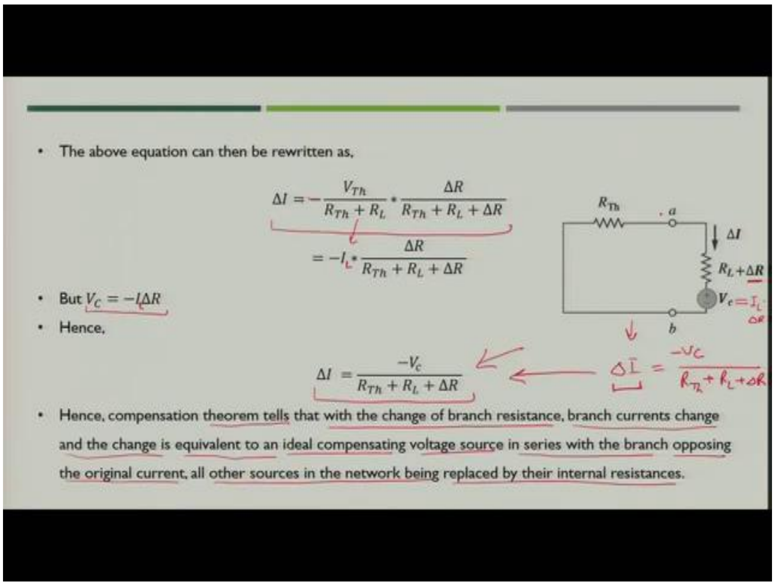
$$\Delta I = \frac{V_{Th}}{R_{Th} + (R_L\ +\ \Delta R)} - \frac{V_0}{R_{Th} + R_L}$$

Now, we simplify, so, we just take the product of both the denominator and use the, you place this at here and this at this particular location and simplify.

(Refer Slide Time: 23:40)






$$\Delta I = \frac{V_{Th}}{R_{Th} + (R_L + \Delta R)} - \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{V_{Th}(R_{Th} + R_L - (R_{Th} + R_L + \Delta R))}{(R_{Th} + (R_L + \Delta R))(R_{Th} + R_L)}$$

$$= -\frac{V_{Th}}{R_{Th} + R_L} * \frac{\Delta R}{R_{Th} + R_L + \Delta R}$$

$$= -I_L * \frac{\Delta R}{R_{Th} + R_L + \Delta R}$$

Now, what you know $V_C = I_L \Delta R$.

Hence,

$$\Delta I = \frac{-V_c}{R_{Th} + R_L + \Delta R}$$

So, with this we calculated that what would be the value of $\Delta I$ when there is a change in value of $\Delta R$.

So, this you can see directly from this circuit and this is what we have derived while taking the value of change of current that is $I_L' - I_L$. So, this justifies that, when you follow the compensation theorem, you can directly replace the voltage source, directly placed voltage source in series with the at the resistance where the change in resistance happened in this case it was $\Delta R$. So, the value of placed to voltage source would be equal to the original current multiplied by the updated the change in resistance that is $\Delta R$ and at the same time, we have to keep all the other voltage sources in the network of that means that we will short circuit the voltage source which are there in the circuit.

So, without going into the derivation, you can simply use the change in the circuit that is the change in $\Delta R$ and you can replace directly with the updated voltage source and find out the change in current because of change in resistance $\Delta R$. So, what you can say the compensation theorem can basically summarized as follows that with the change of the branch resistance, branch current changes and the changes equivalent to an ideal compensating voltage source that is in series with the branch opposing the original current and all other sources in the network being replaced by their internal resistance.

So, that means, that the internal resistance of the network is nothing but $R_{Th}$. So, other sources are replaced by only the internal resistance. So, this you can justify that the compensation theorem can be directly applied for a linear circuit by replacing updated voltage source in series with the resistance where the change of $\Delta$ is happening and find out the value directly the change in the value of current directly with the help of updated circuit.

So, with this we close our today's session. So, in this particular session, we discussed about 2 theorems. So, in the next week, we will start with some new topic on the course that is our basic electrical circuit. Thank you.