Basic Electric Circuits
Module 01
Basic Circuit Elements and Waveforms
Lecture-02
Sinusoids and Phasors
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So welcome to the second lecture on sinusoid and phasors. So before going into this particular topic let me give you one good historical event example. Basically, this is related to a war that happened in late 19th century. So, this war was not where fought with the weapons but it was a war of words. What were those words? Those words were AC and DC. So, at that time lot of debates were going on that was around 1890 period, which is superior whether DC is superior or AC is superior. Eventually AC won. Why? Because AC was able to transfer the power over longer distances, losses were less, and it was easier to generate for a higher voltage. So that is why finally in around 1895 people accepted AC as a preferred voltage source. So now let us try to understand why the AC is important in our circuit analysis and why we want to study AC.

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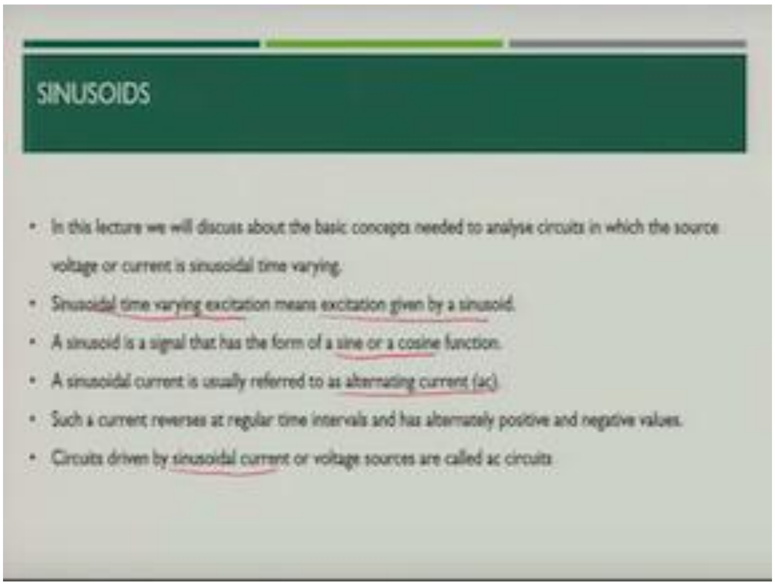

So let us first understand sinusoid. In this particular lecture will try to understand the basic concepts which are required to analyze those circuits which are connected with some time varying sinusoidal voltage or current source. This sinusoidal time varying excitation means that is excitation given by a sinusoid.

Now the question would be, what is a sinusoid? Sinusoid is a signal that has the form of sine or cosine functions. Now sinusoidal current is usually referred to as alternating current. Whenever you say that this is alternating current that means that you are talking about sinusoidal current that would essentially be either in the form of sine or cosine function. Such current reverses at every regular time interval. That means it will alternatively become positive and negative. So, the circuit driven by these sinusoidal current or the voltage source are called AC circuits.

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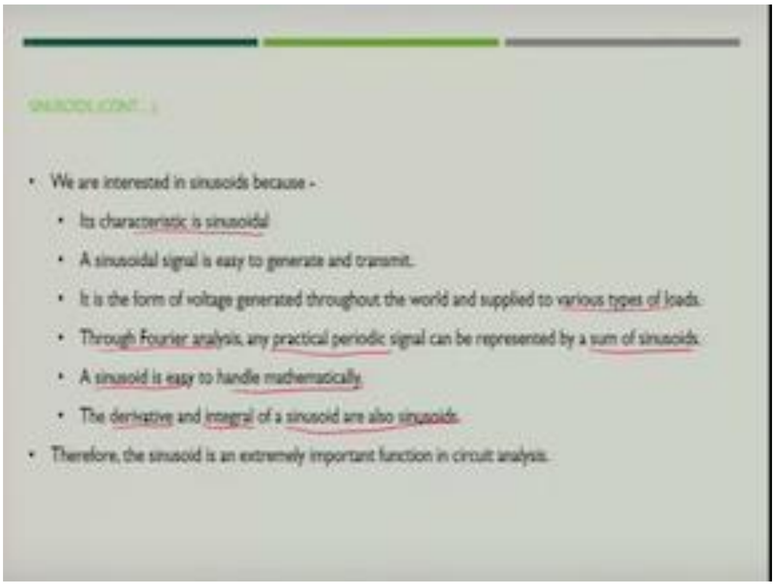

Now why are we interested in sinusoids? Firstly, the characteristic is sinusoidal it is alternatively going positive and negative. Sinusoidal signal is very easy to generate and transmit and right now almost all across the world the voltage, which is generated is AC and it is being supplied to various loads that may be residential, industrial or commercial.

So, since AC is very prominent and it is highly utilized across the globe we need to understand the AC circuits, which are connected through AC current or voltage source.

Now another important aspect is that using Fourier analysis we can transform any practical periodic signal into sum of sinusoids, and these sinusoids are very easy to handle mathematically, because the derivatives, that is, derivative of sin is cos, derivative of cos is sin. So, it means that the derivative and integral itself would be sinusoids. Therefore, the sinusoids are very important for our circuit analysis.

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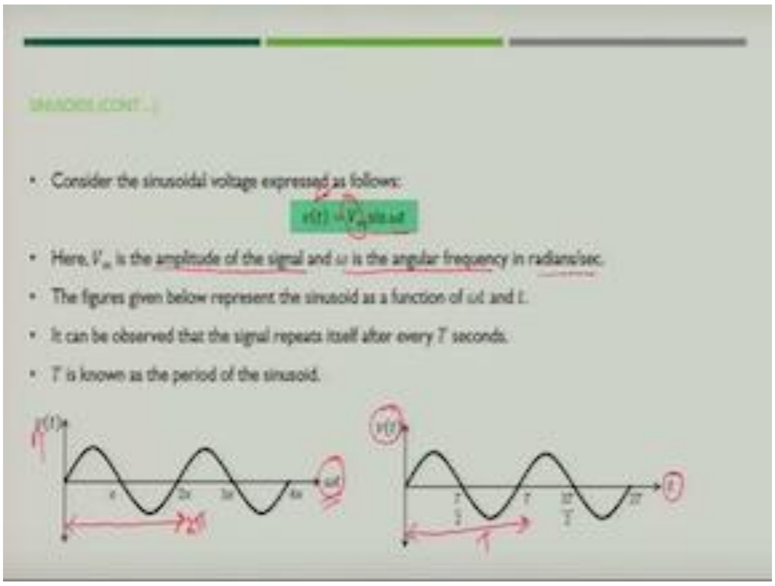

Now let us consider one sinusoid and try to find out what are the various components of the sinusoids. So how do we represent the sinusoid, we represent sinusoid like $v(t) = Vm \sin \omega t$. Now t is the time variation, Vm is the amplitude of the signal and then you have another component $\sin \omega t$ which tells us that this particular signal is varying alternatively from positive to negative, as shown in the figure. So now if you see there are two figures given in the slide.

In the first figure the variation of the signal is with respect to ωt. Now here what is ω? ω is called as angular frequency, which is measured is radiance per second. Now what are we doing in this figure? We are trying to find out the amplitude of voltage signal with respect to angle, because this is ωt; that is nothing but angle.

In the second figure what are we trying to do? We are trying to find out the voltage amplitude with respect to time. So, if you see both of the figures they are going to be alternatively positive and negative. So here you see the signal is positive and here it is negative, and again it is repeating after the angle of 2π.

Similarly, if you see the time variation you will see that the signal is repeating again after the time T. So, what is T? T is called the time period of the sinusoid and after every time period the sinusoid is repeating itself.

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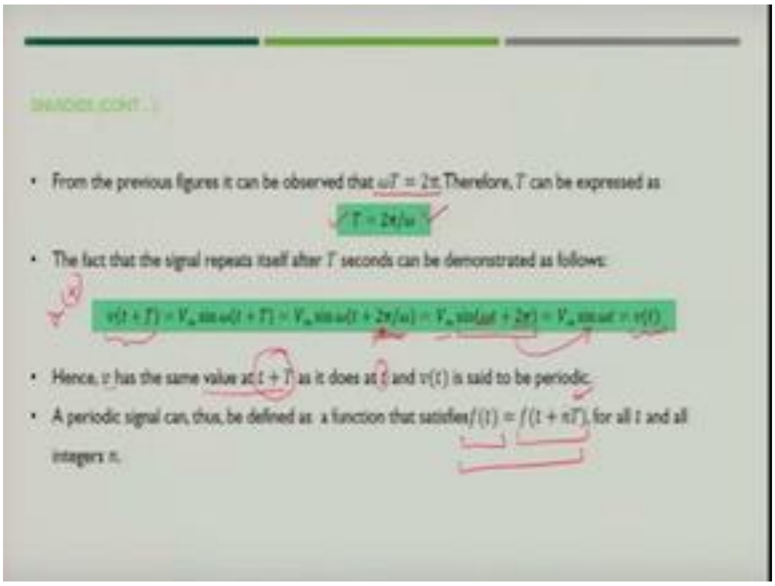

So what we can get from these two figures? If you compare both of the figures we see that the $\omega T = 2\pi$. So, what can we say? We can say $\omega T = 2\pi$ and it can be shown that the time period $T = 2\pi/\omega$.

How can you say that it is repeating after every time period capital T? Let us try to find out. If the our original signal is $v(t)$, then $v(t + T)$ is,

$$v(t+T) = Vm \sin \omega(t+T) = \ Vm\, sin\, \omega(t + 2\pi/\omega)$$

So now the output is $Vm \sin(\omega t + 2\pi)$, because if you take ω inside the first term become ωt and second term become 2π. Now what is the value of $\sin(\omega t + 2\pi)$? It again becomes ωt. So finally, what do we get? We get $Vm \sin \omega t = v(t)$.

In this way we can say that if we add T in the function it will eventually become the original signal. It means the signal is repeating after every time period that is T. So, we can say that v has the same value at $t + T$ as it does at t, and that is why it is periodic. So, in general we can say that if any function $f(t) = f(t + nT)$, that is the time period multiplied by any integer value then the signal is periodic.

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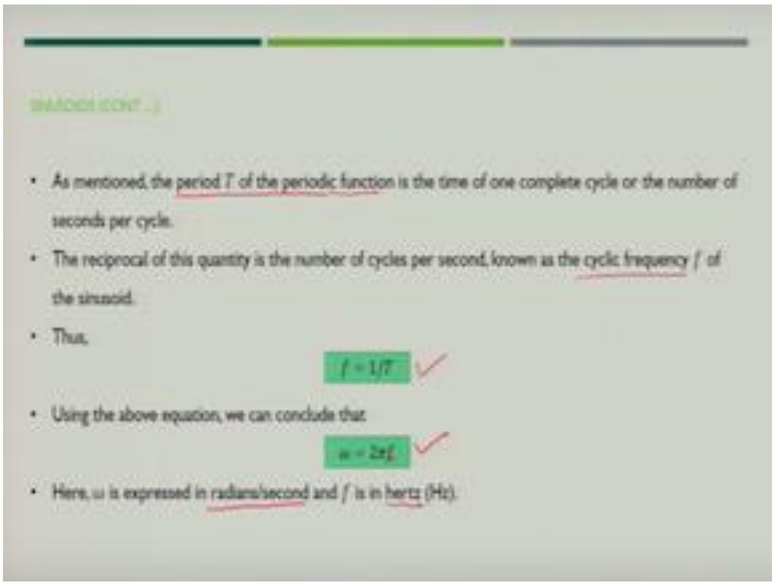

Now as we have seen that T is nothing but time period and using this we can say that function is periodic function. Now let us try to find out another quantity, which is called cyclic frequency. In general terms we do not say cyclic frequency, we simply say as frequency. So, what is frequency f, f is nothing but the value of $1/T$, that is time period. Now if you put this value in the previous equation, which we derived that is $T = 2\pi/\omega$, what do we get? We get $\omega = 2\pi f$. So, this is basically the relationship between angular frequency that is measured in radiance per second and your cyclic frequency that is measured in hertz.

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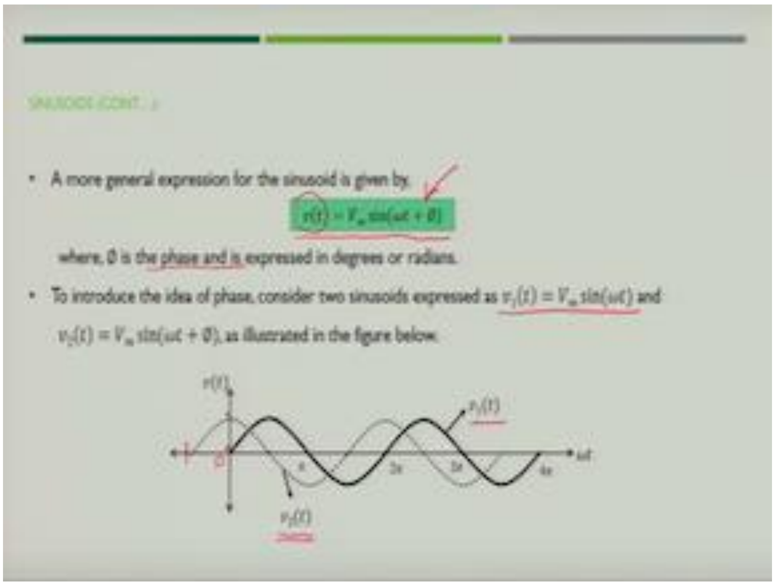

Another generic term in the literature you commonly see is $\emptyset$ which gives $v(t) = Vm\, sin(\omega t + \emptyset)$. So we add another term $\emptyset$, which is nothing but the phase of that particular wave form. What does it mean? If you see the figure and you see the voltage $v2(t)$, its wave form is not starting from 0, it is starting from somewhere before 0.

So, if you put value of t equal to 0, you will get some value $Vm\sin(\emptyset)$. It means that at time t equal to 0 this signal has some value. So that is why it is having some positive value at time t equal to 0. So this means that it is starting before the 0 value.

So now if you compare with another signal, which is $v1(t)$, then $v1(t)$ is nothing but our original signal, which we considered previously that is $v1(t)$ is equal to $Vm\sin(\omega t)$. If we plot both of them, so this will become $v1(t)$ is nothing but Vm sin omega t, which starts from origin while $v2(t) = Vm\sin(\omega t + \emptyset)$, which starts before origin. So, we can say that the angle $\emptyset$ is giving some leading component to the voltage $v2(t)$ and that is called our phase angle.

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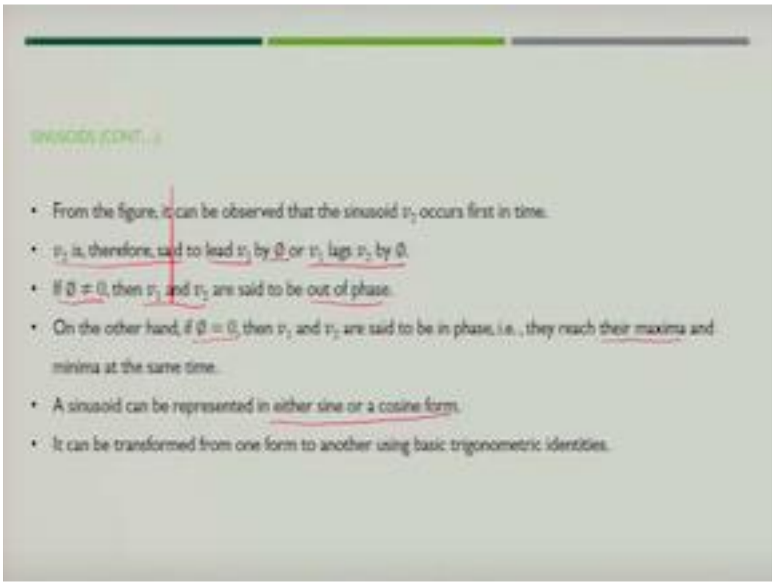

So what we can observe $v2(t)$ is leading $v1(t)$ by angle $\emptyset$ or we can say $v1(t)$ lags $v2(t)$ by $\emptyset$. Now if angle $\emptyset$ is not equal to 0 that means voltage v1 and v2 are out of phase or if we make $\emptyset$ is equal to 0 then voltage v1 and voltage v2 both wave form will coincide and they will reach at their maxima and minima at the same time. So, this phase difference will define that how two waveforms are out of phase with each other. Now sinusoid can be represented either in sin or cosine form and it can be transformed from one form to another form using trigonometric identities.

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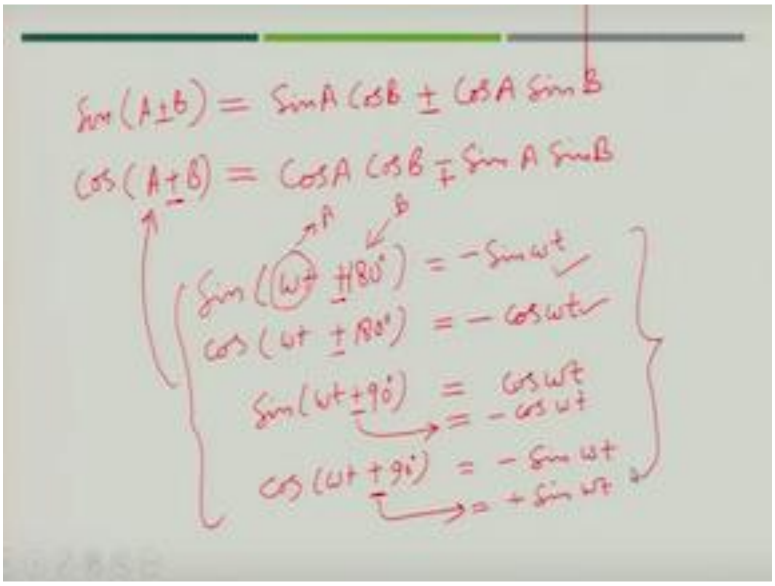

So before going into the details let's see few of the trigonometric identities, which you will keep on using while you work on sinusoids as well as phasor. So let us the first one, what is first one? First one is sin A plus B. So we will consider $\sin(A + B)$. You have $\sin(A + B) = \sin A\, cosB + cosA\; sin\; B$. Now instead of $\sin(A + B)$ if you are asked to find $\sin(A\; -\; B)$, what do you have to do? You have to replace plus with minus. That is $\sin(A\; -\; B) = \sin A\; cosB\; -\; cosA\; sin\; B$.

Now for $\cos(A + B)$, you have $\cos A\; cosB\; -\; sinA\; sin\; B$. So opposite to that if you are asked to find out the value of $\cos(A - B)$, you have $\cos A\; cosB\; +\; sinA\; sin\; B$.

Now using these you can find out few values, which are important in our sinusoid as well as phasor calculation. What are these values? One is sin omega t plus 180 degree. What is the value of $\sin(\omega t + 180)$, which is equal to $-\sin \omega t$. Similarly, if you are asked to find out the value of $\sin(\omega t - 180)$, still the output will remain same; that is $-\sin \omega t$.

Now, what is $\cos(\omega t + 180)$? That is again minus of $-\cos \omega t$. So here also if you are asked to find out $\cos(\omega t - 180)$ again the output will remain same, i.e., $-\cos \omega t$.

Similarly, if you are asked to find out the value of $\sin(\omega t + 90)$ what would be the output? It would be $\cos \omega t$. Now instead of plus if we say that find out the value of

$\sin(\omega t - 90)$, now this will become minus of $-\cos \omega t$, right. So, if it is minus then your output will be $-\cos \omega t$.

Now for $\cos(\omega t + 90)$ this will become $-\sin \omega t$. Similarly, $\cos(\omega t - 90)$ will become $\sin \omega t$. So, what you can do, you can verify these values by putting values in this particular original formula.

So, what you can do, you can put ωt, you can replace A with ωt and B with 180 degree or 90 degree, whatever the case would be and you will find out these expressions. And these are very important because you will keep on using them when you try to find out the value of sinusoids, like if you want to change sinusoids from sin to cosine or if you want to find out value of phasor, which we will discuss in next few slides.

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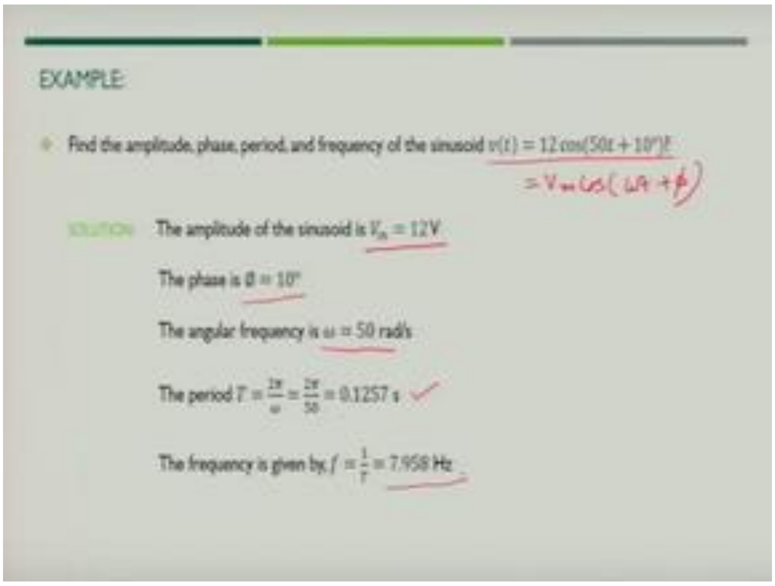

Now let us take few examples so that you can better understand the concept of sinusoid. Let us take the example of one sinusoid that is $v(t) = 12\cos(50t + 10°)$. Now you can easily compare this with $Vm\cos(\omega t + \emptyset)$. So, when you compare the two expressions you will come to know that Vm is nothing but 12 volts, because V is measured in volts, then what is the phase angle, phase angle would be 10 degree positive. Now angular frequency that is ω, you will get 50 radians per second. So, time period would be $2\pi/\omega$, that will become 0.1257 second and frequency will be opposite to the time period that is you will get 7.958 hertz.

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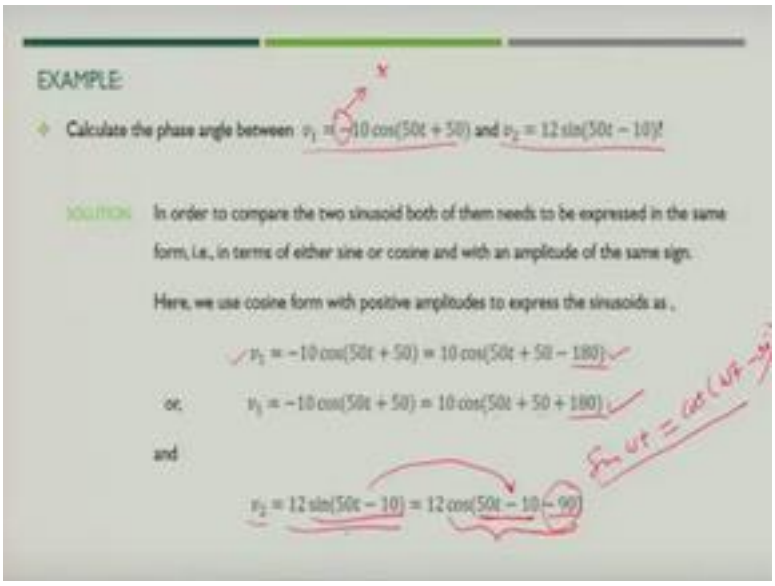

Now if you are asked to calculate the phase angle between two voltage signals that is $v_1 = -10\cos(50t + 50)$ and $v_2 = 12\sin(50t - 10)$. Now in order to compare these two both them either have to be sin or cosine. So, what we are doing, we are trying to find out the phase angle by converting sin into cosine. And we have to convert the minus sign into the positive sign with the help of some trigonometric operations we just saw in the previous slide.

So, we can say, $v_1 = -10\cos(50t + 50)$ can be converted as $10\cos(50t + 50 - 180)$ because $\cos(\omega t - 180)$ will become $-\cos\omega t$ as well as $\cos(\omega t + 180)$ will also become $-\cos\omega t$. So, in this case there are two possible options.

One is v_1 is equal to $10\cos(50t + 50 - 180)$, as well as $10\cos(50t + 50 + 180)$, because whether it is plus or minus your output would be minus of cos of that angle. So, we are taking both of the cases and we will justify that in both of the cases we will get the same output.

Now v_2 is $12\sin(50t - 10)$ that means $12\cos(50t - 10 - 90)$, that we are trying to find out the value of $\sin\omega t$ in terms of cos so what we can say $\sin\omega t$ is nothing but $\cos(\omega t - 90)$, right. This is what we saw in the previous slide. Now if you use this value this particular expression will become $12\cos(50t - 10 - 90)$. So, in this way you are converting the second voltage signal from sin to cosine.

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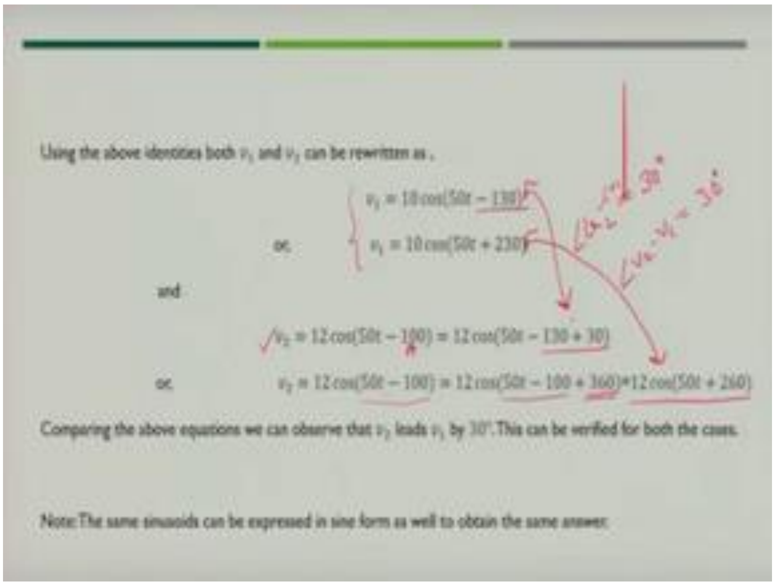

So v_1 we got, is $10\cos(50t - 130)$ because the value was +50-180, so it will finally become -130. It can also be +230. So, these are the two signals, which we have computed. Now v_2 you can simply write $12\cos(50t - 100)$ because that is -10-90, so it will become -100. You can change this value as -130 + 30. So that you can easily compare both of them.

Similarly, you can also write 50t -100 as 50t-100+360, 360 degree is nothing but 2π, being a periodic signal, you will eventually get the same signal. So, we are taking that particular establishment into the consideration and we are adding 360 degree and finally we get $12\cos(50t + 260)$.

So now if you compare these two you will come to know that v_2 leads v_1 by 30. So, angle of v_2 minus angle of v_1 will be 30 degree. Similarly, for these two cases if you compare both of them again you will come to know the angle of v_2 minus v_1 is equal to 30 degree. So whether it was plus 180 degree or minus 180 degree in case of v_1 we found the same answers.

Similar exercise can be done with the help of converting cos into sin and them comparing with the voltage v_2, so that is something, which you can do it as an exercise.

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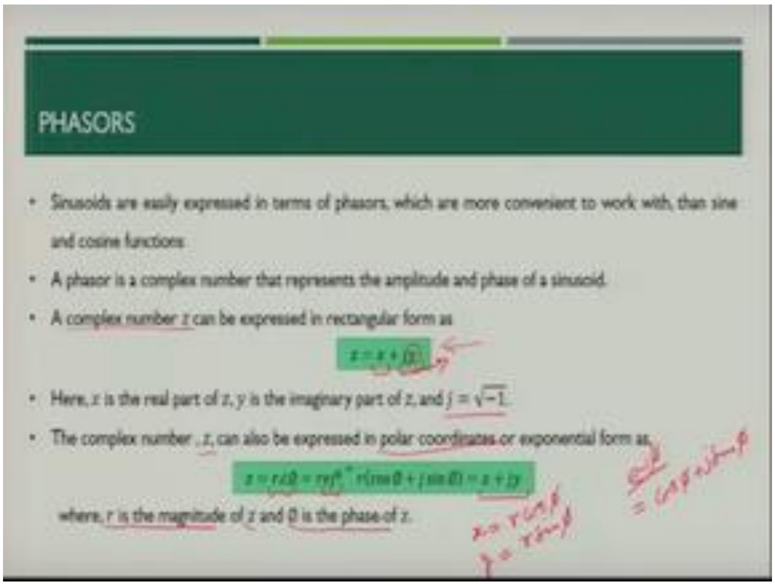

Now let us come to another concept called phasor, because sinusoids are easily expressed in terms of phasor, which are more convenient to work with than the sin and cosine functions. So how will we define phasor? Phasor is a complex number that represents the amplitude and phase of sinusoid. So, if you are asked to define the complex number what you will write? You will write complex number $z = x + jy$.

So, j is the operator, which have the value $\sqrt{-1}$, which says that when it is attached to one particular value then this particular value will become imaginary. So, this is basically the rectangular form of the complex number z.

The same z can also be expressed in polar coordinates or in exponential form, that is, z can be simply expressed as $r\angle\emptyset$ that is nothing but $re^{j\emptyset}$. Now what is $e^{j\emptyset}$ that is called Euler formulation, $e^{j\emptyset} = (\cos\emptyset + j\sin\emptyset)$.

This Euler formula is very important for phasor definition, so we will use it most frequently in our various phasor calculations. Hence, $z = r\angle\emptyset = re^{j\emptyset} = r(\cos\emptyset + j\sin\emptyset)$. So, if you decompose it will again will become $x + jy$ where x is nothing but

$r \cos \emptyset$ and y is nothing but $r \sin \emptyset$. So in this form r is considered to be the magnitude of z and $\emptyset$ is the phase of the complex number z.

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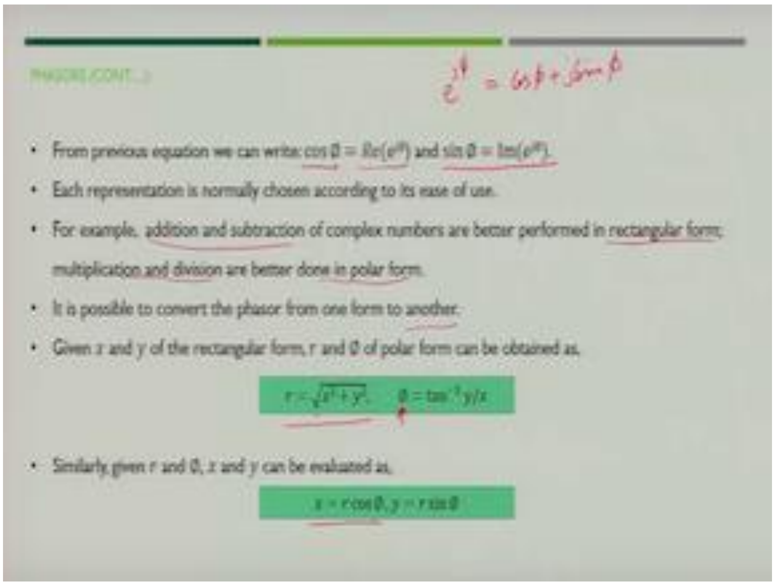

Now $e^{j\emptyset} = (\cos \emptyset + j \sin \emptyset)$, $\cos \emptyset$ can be represented as real terms of $e^{j\emptyset}$ and $\sin \emptyset$ can be represented as imaginary term of $e^{j\emptyset}$. So, each representation is normally chosen according to the ease of use, that is either you will use the rectangular form of the complex number or the polar form that is exponential form of the complex number representation.

Now, if you want to add or subtract the two complex numbers it is better to go with rectangular form or if you want to do the multiplication and division it's better to go in the polar form. So, it is possible to convert the phasor from one form to other form. If you are given the rectangular form, that is, $x + jy$, then $r = \sqrt{x^2 + y^2}$ and phase angle of the complex number would be $\emptyset = \tan^{-1} y/x$. Similarly, if you want to convert polar into rectangular then $x = r \cos \emptyset , y = r \sin \emptyset$

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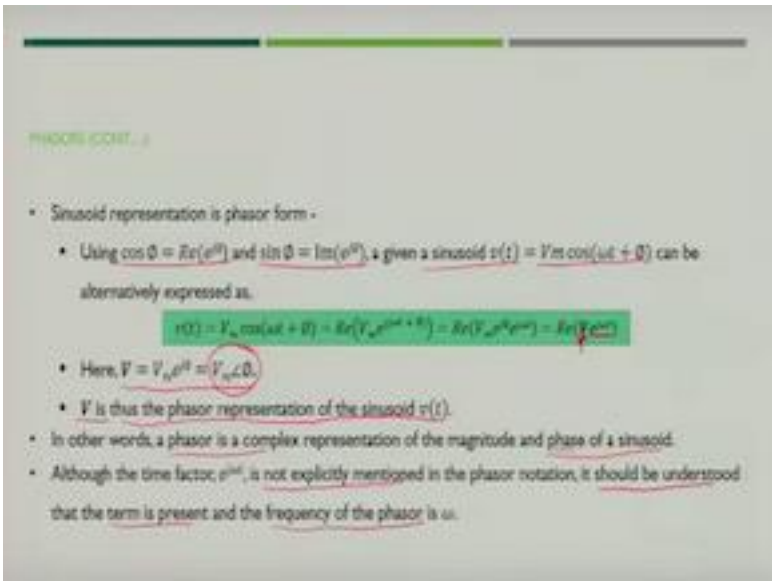

Now how you will represent sinusoid in the phasor form, because this is very important task in circuit analysis. $\cos \emptyset$ is nothing but real of $e^{j\emptyset}$, $\sin \emptyset$ is equal to the imaginary of $e^{j\emptyset}$, so we will use them for a given sinusoid that is $v(t) = Vm \cos(\omega t + \emptyset)$ and convert them into phasor form.

So, what will happen if you write $v(t) = Vm \cos(\omega t + \emptyset)$, you know that $\cos \emptyset$ is equal to the real of $e^{j\emptyset}$. We can write the same expression as $Re\left(V_m e^{j(\omega t+\emptyset)}\right)$. So, this will become $Re\left(V_m e^{j\emptyset} e^{j\omega t}\right)$.

So, we write this expression as $Re(\boldsymbol{V} e^{j\omega t})$. what is capital $\boldsymbol{V}$? $\boldsymbol{V}$ is nothing but $V_m e^{j\emptyset} = V_m \angle \emptyset$. So that is nothing but the phasor representation of the sinusoid. So, in short form we write as $V_m \angle \emptyset$, so in this way $\boldsymbol{V}$ which you see in the slide is the phasor representation of sinusoid.

What you can say phasor is a complex representation of your magnitude and phase of a sinusoid. So important thing to see here is that although $e^{j\omega t}$ is an important term but we do not explicitly mention it because in phasor notation we always see this particular term as a time varying component, so we drop this particular component for simplicity but while you do the phasor notation you should always keep in mind that this term will always be present and the frequency of phasor will be ω.

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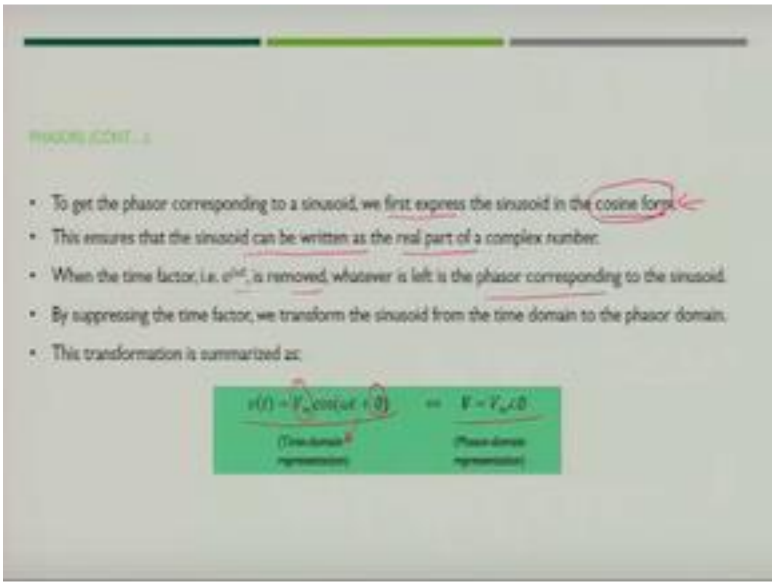

Now the to get the phasor corresponding to sinusoid, we first express the sinusoid in the cosine form. So, this is also very important point because whenever you want to present sinusoid in phasor terms it has to be first converted into cosine form. Why cosine form? Because this is the real part of the Euler expression. This will ensure that sinusoid can be written as a real part of the complex number when time factor is removed whatever is left is the phasor corresponding to that sinusoid.

So, in short what we can say that the signal $V_m \cos(\omega t + \emptyset)$ which is written in time domain can be converted into phasor domain with taking V_m and angle $\emptyset$. So simply you can see $\boldsymbol{V} = V_m \angle \emptyset$ to convert sinusoid into phasor domain.

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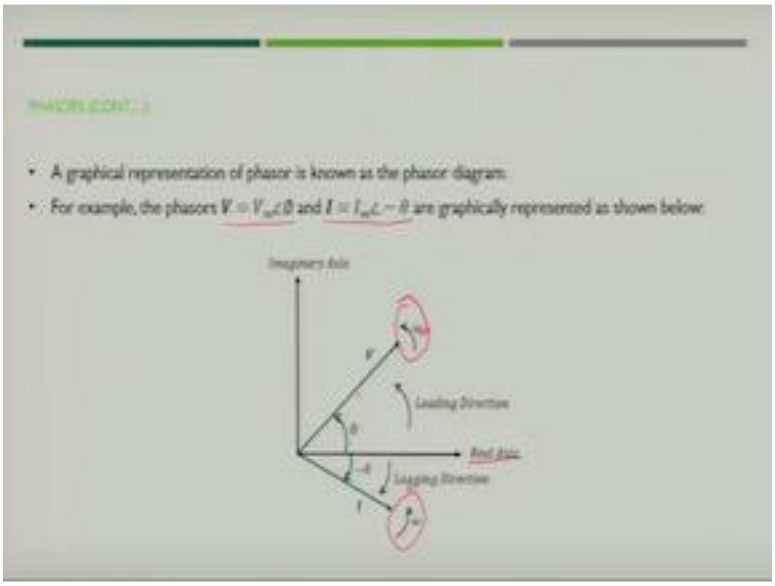

Now if you represent graphically, then if you take two phasors like, $\boldsymbol{V} = V_m\angle\emptyset$ and $\boldsymbol{I} = I_m\angle-\theta$, so how you will represent graphically, you will take real axis, imaginary axis, you plot $\boldsymbol{V}$, V has angle $\emptyset$ with respect to real axis and current $\boldsymbol{I}$ is $-\theta$ with respect to real axis. So here the voltage $\boldsymbol{V}$ phasor is leading while the current $\boldsymbol{I}$ is lagging.

Now if you see with respect to time both would be rotating with respect to the stationary plane, that is the real and imaginary axis plane, because we dropped $e^{j\omega t}$ term but eventually the both of the phasor would be rotating in the frame and having the constant angular frequency. They are always constant with respect to each other but rotating in a absolute term.

So, for simplicity we do not represent omega for the phasor because that is seems to be understood and we simply represent voltage and current as a phasor.

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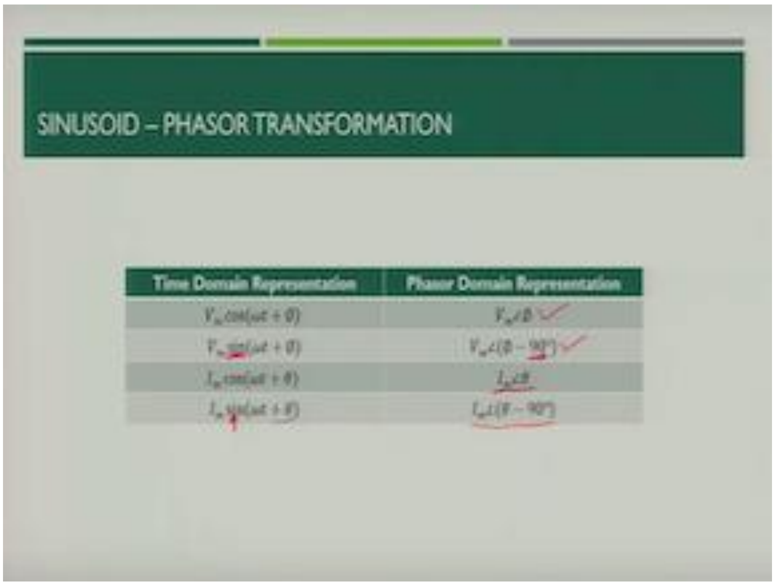

Now if you compare for both of the domains like time domain and phasor domain, $Vm\cos(\omega t+\emptyset)$ can be written as $Vm\angle\emptyset$. $Vm\sin(\omega t+\emptyset)$ $Vm\angle(\emptyset-90°)$, why 90 degree is coming, because this is in sin terms so you have to convert them into cosine term and then you will get $Vm\angle(\emptyset-90°)$.

Similarly, $Im\cos(\omega t+\theta)$ would be $Im\angle\theta$ while $Im\sin(\omega t+\theta)$ would be $Im\angle(\theta-90°)$ because sin you need to convert into cosine and before converting into phasor domain.

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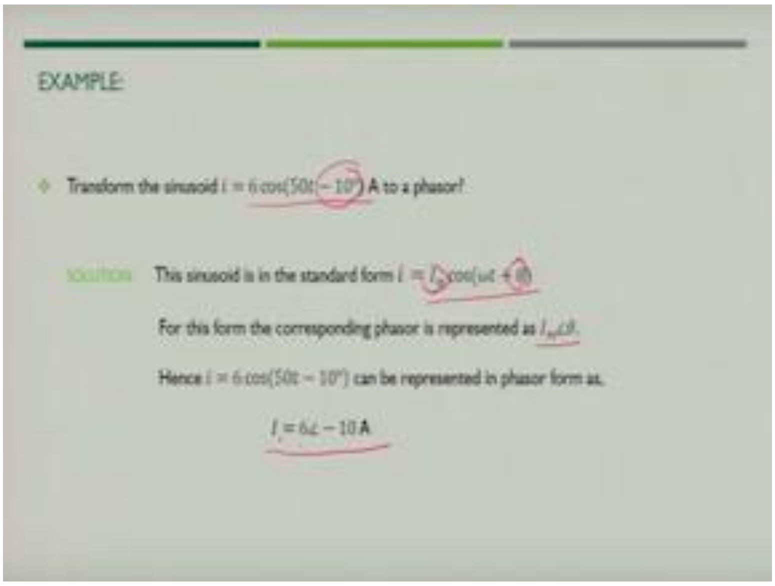

Now before closing let's take a couple of examples, if you are asked to transform the sinusoid $i = 6\cos(50t - 10°)$ to a phasor, what you will do? You will simply compare it with the standard sinusoid then you will come to know that in phasor term you have to just represent as the amplitude and the phase angle. So, what will happen in this case, amplitude is 6, phase angle is minus 10 degree, so we will write $6\angle - 10$ ampere as a phasor representation of the given sinusoid.

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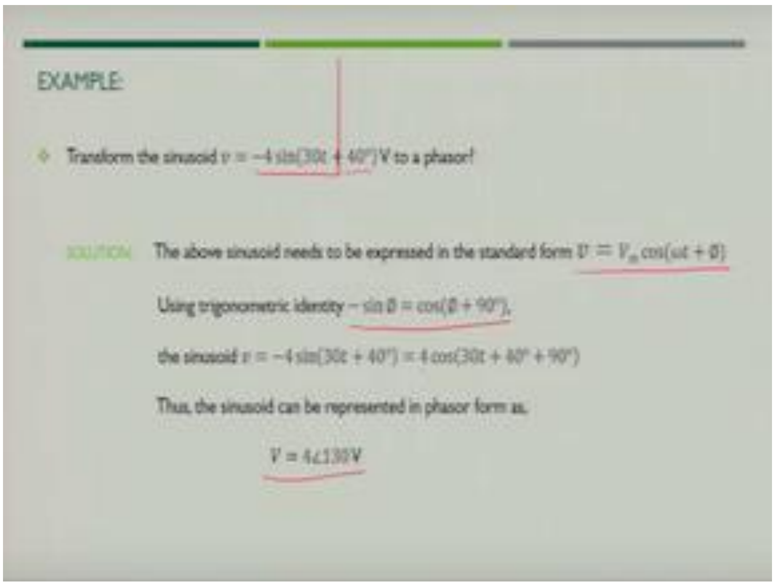

Now if you are asked to find out the phasor value of $-4\sin(30t + 40°)$, first what you have to do, you have to convert it into a standard cosine in term. So that is $Vm\cos(\omega t + \emptyset)$. Now using trigonometric identity, you know that $-\sin\emptyset = \cos(\emptyset + 90°)$, you will use this formula here and you will convert this into cosine terms and then it will become $4\cos(30t + 40° + 90°)$. So now that you will write in phasor terms, the phasor terms the value of this sinusoid is $4\angle 130$ degree volt.

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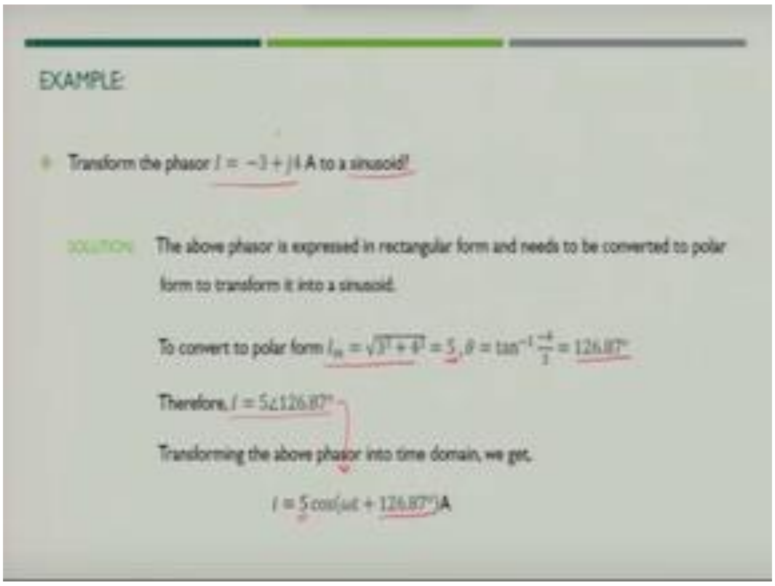

Now if you are asked to transform a phasor into a sinusoid what you will do, you have to first see what is the (rectangular) since this is given in rectangular form you can simply convert it into Euler form or polar form. So, what will happen the magnitude I_m that will become $\sqrt{x^2 + y^2}$. So, x, y these two terms are given, so I_m will become 5, then angle theta is nothing but y/x. So, this will become $\tan^{-1}\frac{4}{3}$ that comes out to be $126.87°$.

Therefore, you can simply write in phasor form, current $I = 5\angle 126.87°$. Now it is very easy to transfer the phasor into a sinusoid. You have to just simply put this value in place of amplitude and this as a phase angle. So finally, what will become $i = 5\cos(\omega t + 126.87°)$A.

So, with this we close our today's session. Thank you very much.