**Basic Electric Circuit**
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**Module 4 Network Theorem 2**
**Lecture 19 Maximum Power Transfer Theorem**

Namaskar. So, in yesterday's class we discussed about the maximum power transfer theorem and we discussed few of the examples when independent sources were available in the circuit. So, now, let us start the today's lecture. In this lecture, we will more focused about the circuit where the dependent sources are available, and we will try to solve the circuit for maximum power transfer.

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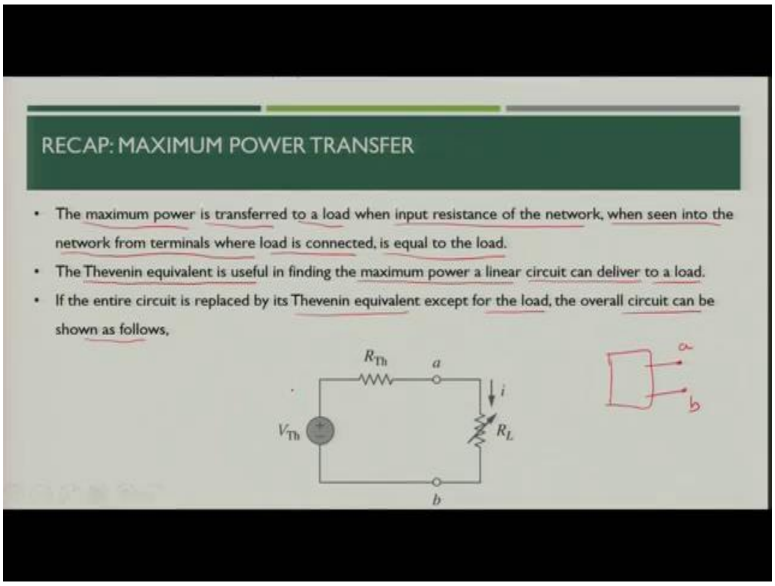


So, before going to the dependent source condition let us try to understand what we discussed in the previous class. We discussed about the maximum power transfer theorem, we said that the maximum power is transferred to a load when input resistance of the network, when seen from the terminal where the load is connected, is equal to load.

To do this we use Thevenin equivalent, because, if you see the network, which may be having multiple sources and multiple elements, what we have to do we have to convert it into the Thevenin equivalent which is there in this in the figure and then we can analyze the maximum power transfer. So, now, Thevenin equality is very useful in finding the maximum power a linear circuit can deliver to the load and we also discussed that entire circuit is replaced by Thevenin equivalent except for the load the overall circuit can be shown as given in the figure.

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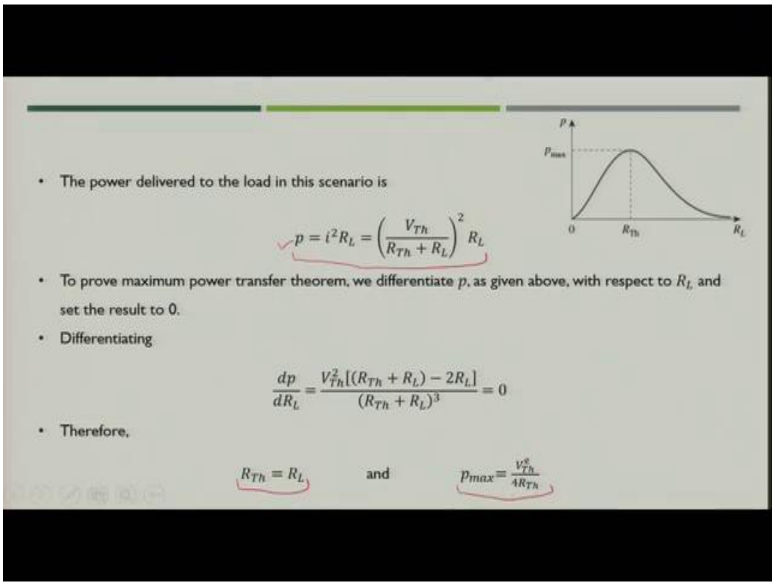


Now, we also discuss that, if you keep on changing the load, from 0 to infinity, there would be one point at which your power being absorbed by the load is maximum and that is the power being given to the load by the sources is maximum. So, that condition comes when $R_L$ that is the load resistance is equal to Thevenin resistance and we stabilize that the power is nothing but $i^2 R_L$ and we derived the maximum power transfer condition.

How did we derive this? We used $\frac{dp}{dR_L}$ that is differentiation of power $p$ with respect to variable that is load and we established that the derivative of p with respect to $R_L$ is 0 when $R_{Th} = R_L$. Therefore, at $R_{Th} = R_L$, $R_L$ the maximum power which will be transferred to the load is $\frac{V_{Th}^2}{4R_{Th}}$.

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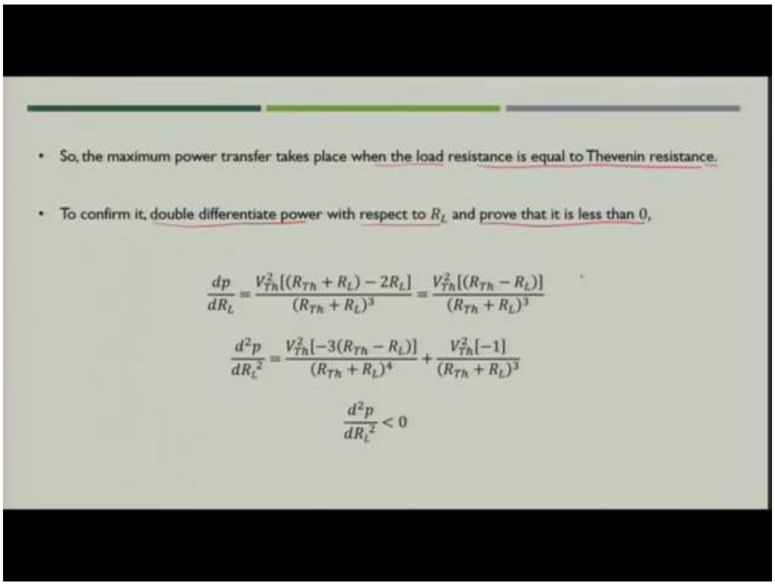


Now, as we discussed in the previous class, the maximum power transfer takes place when the load resistance is equal to Thevenin resistance. This we derived when we took the derivative of power $p$ with respect to $R_L$ which is variable and now, as we know that at the when the derivative $\frac{dp}{dR_L}$ is equal to 0 at that condition. The $R_L$ value what we get if you supply that value $R_L$ into the power p power might be maximum or minimum.

Now, we have to find out whether the condition which we have stabilized with the help of taking the derivative $\frac{dp}{dR_L}$ is really a maximum or not. So, what we have to do we have to double differentiate the power with respect to $R_L$ and will prove that it is less than 0. So, when it is less than 0 that means that the $R_L$ value which we have found when we took the derivative that is $R_{Th}$ would be maximum.

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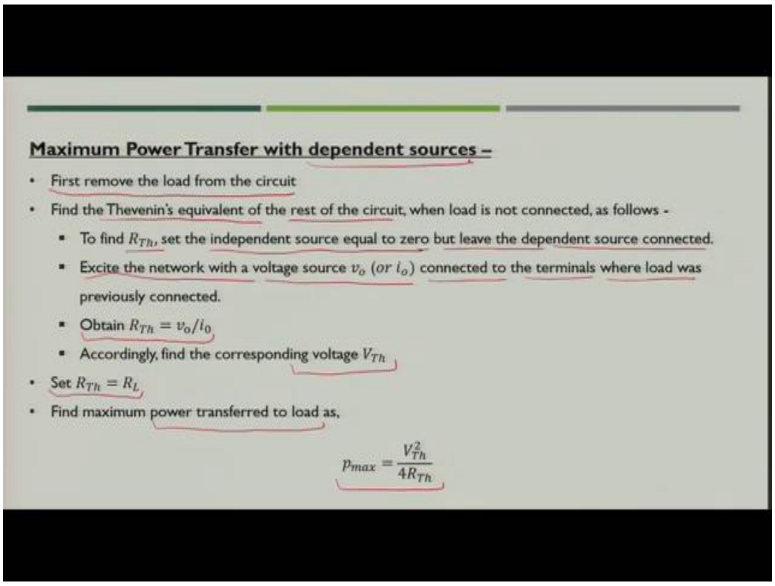


So, how we will find out the values of various elements like $V_{Th}$ $R_{Th}$ and $R_L$ when the dependent source is available in the circuit. First you have to do is first remove the load from the circuit. Now, the circuit which is left without load, what you have to do you have to find the Thevenin equivalent of the rest of the network. Now, when you find the Thevenin equivalent first thing you have to do you have to find the $R_{Th}$ where you will set the Independent Sources equal to 0. It means that if you have an independent voltage source, you will keep it as a short circuited and if you have independent current source, you will keep the open circuit and you leave all the dependent Sources connected to the network.

Now, you will excite the network with the voltage source $v_0$, you can use either $v_0$ or $i_0$ depending upon your convenience of simplifying the circuit. You can use $v_0$ as a 1-volt source or $i_0$ as 1 ampere connected to the terminal were load was previously connected. Then you obtain the value of $R_{Th} = v_0/i_0$. So, $v_0$ is the value which is across the terminal where you have connected either the voltage source or current source and then divided by current supplied by the source that is $i_0$ and now, after that you will find the corresponding Thevenin voltage.

Finally, what you have to do you have to set $R_{Th}$ is equal to $R_L$ because as per maximum power transfer theorem, your Thevenin resistance should be equal to the load resistance and then the maximum power being supplied to the load would be

$$p = \frac{V_{Th}^2}{4R_{Th}}$$

So, we will utilize this process to find out the maximum power being transferred to the load when dependent sources are available.

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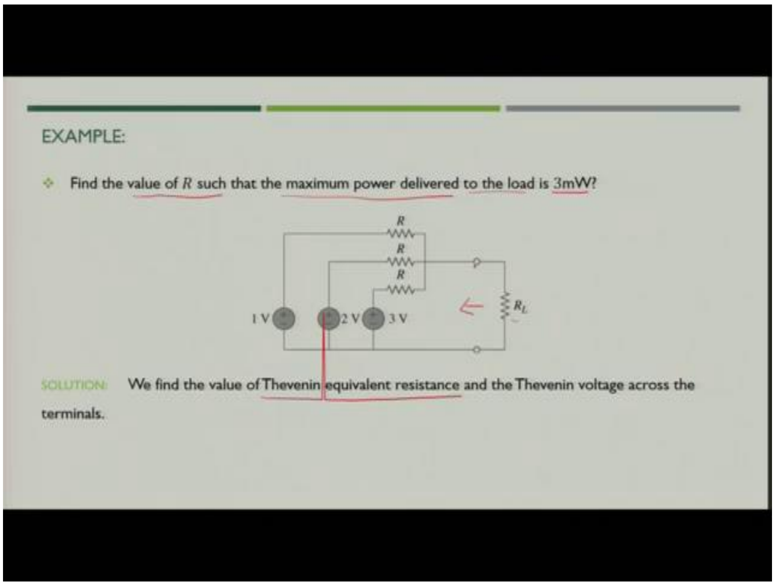


So, let us take one example of maximum power transfer theorem then we will go to the dependent sources condition suppose if this is the circuit and you have to find out the value of R such that the maximum power is being delivered to the load is, 3 milli watt. So, now, we are not asking for load $R_L$. We are asked to find out the value of the unknown resistance R so that the power maximum power being delivered to the load 3 milli watt.

We have to first find the Thevenin equivalent. So, for Thevenin equivalent the equivalent circuit for $R_{Th}$, that is, Thevenin resistance would be, you will short circuit all the voltage sources because they are you have 3 independent voltage sources. Then you will remove the load $R_L$ and you will see what would be the input resistance across these 2 terminals.

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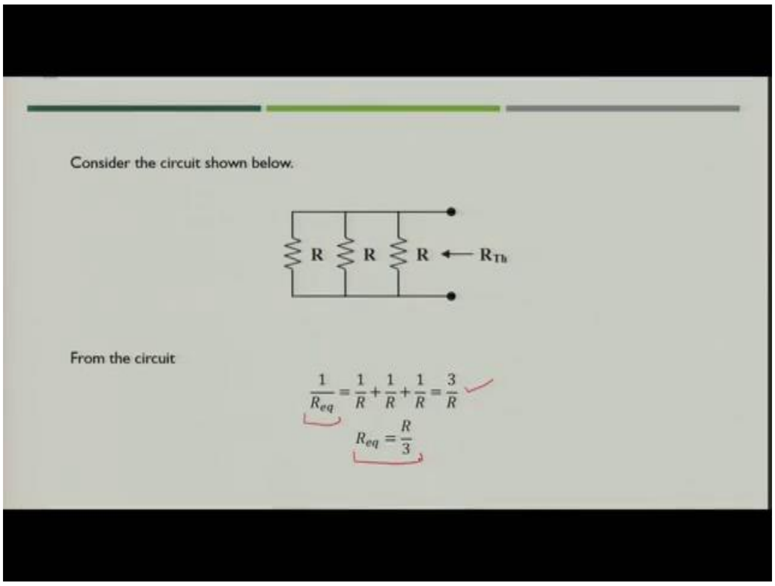


So, you will get this circuit as an equivalent circuit for after short circuiting all the voltage sources and then you need to find out the value of $R_{Th}$. So, here you will see all the 3 resistances are in parallel. So, what you can say if it is $R_{Th}$ is nothing but $R_{eq}$ across these 2 terminals. Then you can write

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

$$R_{eq} = \frac{R}{3}$$

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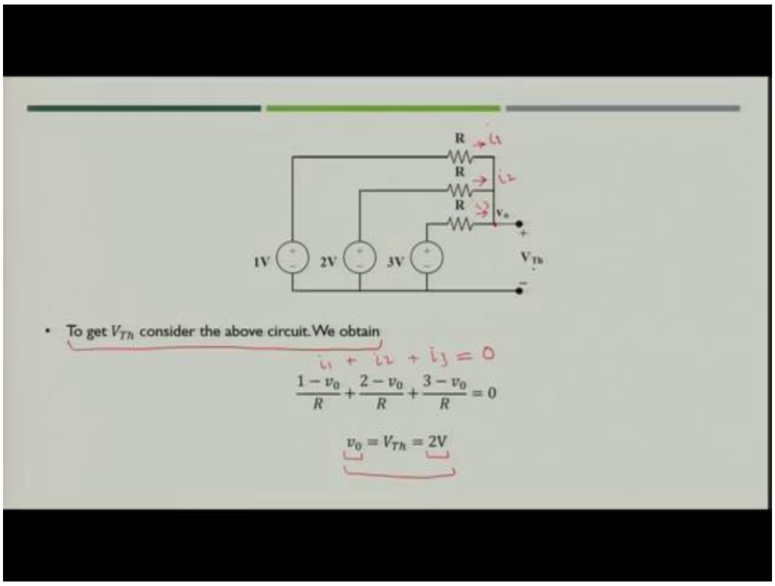

Now, next task is to find out the value of Thevenin voltage. Now, let us assume that the voltage at this particular node is $v_0$. So, what we can write we can use Kirchhoff's current law at this particular node and we will try to find out the value of $V_{Th}$. So, what you can right at this point for 1 volt what would be the current.

$$\frac{1-v_0}{R}+\frac{2-v_0}{R}+\frac{3-v_0}{R}=0$$

$$v_0=V_{Th}=2V$$

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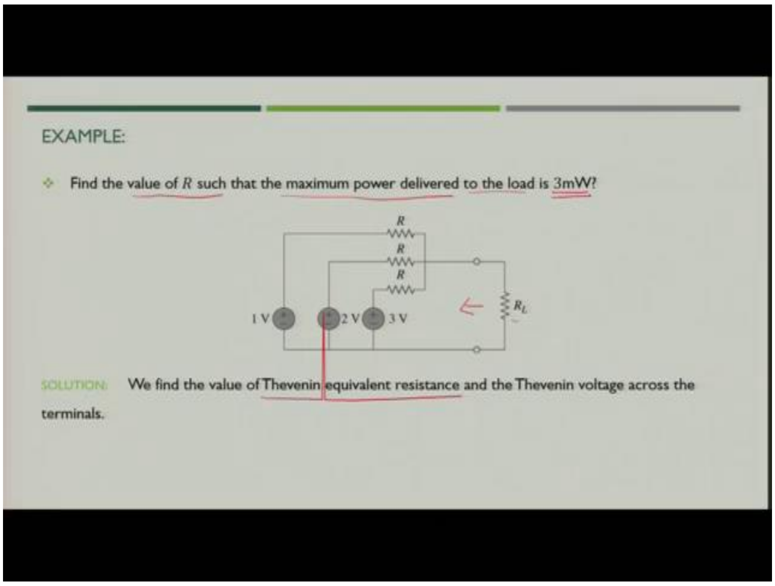


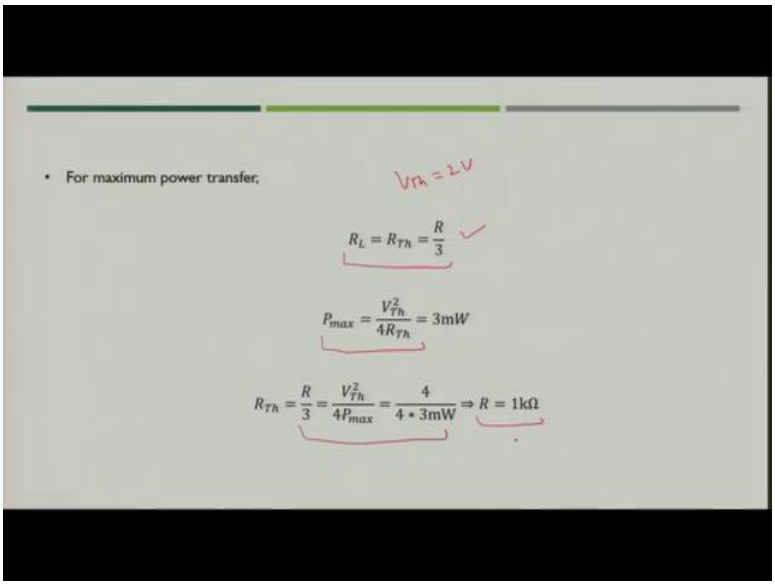


So, maximum power transfer condition is $P_{max}=\frac{V_{Th}^2}{4R_{Th}}=3\text{m}W$

$$R_{Th} = \frac{R}{3} = \frac{V_{Th}^2}{4P_{max}} = \frac{4}{4 * 3\text{mW}} \Rightarrow R = 1k\Omega$$

So, this will help you to find out the value of unknown resistance R in the circuit, which will make sure that the load which is connected to that circuit is getting the 3 milli watts of power.

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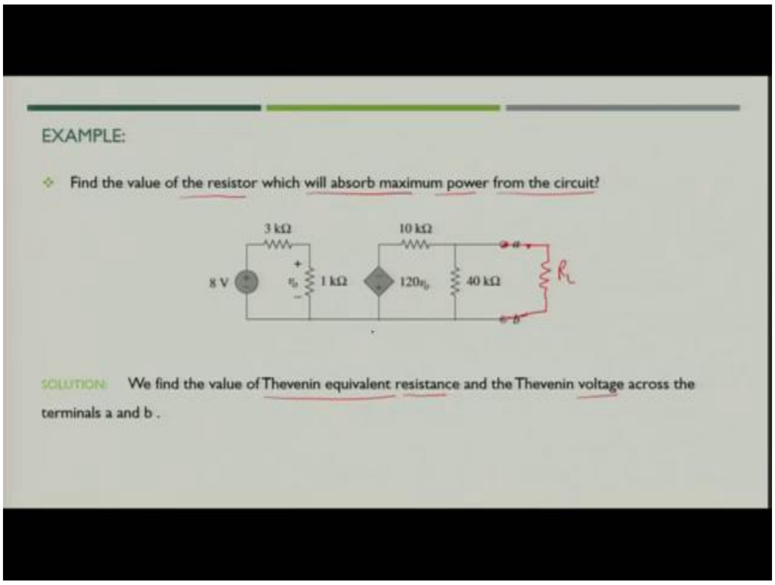


So, now, let us come to another example, where you will see there is one dependent voltage source available. Now, we have to find the value of resistance suppose if there is a load say $R_L$ connected across the terminal ab. Now, we need to find out the value of $R_L$ which will absorb maximum power from the circuit, that is the circuit which was given in the question. So, the load which will absorb maximum power from the circuit.

So, what we have to do we have to first find the value of Thevenin equivalent that is Thevenin equivalent resistance as well as Thevenin voltage across the terminal ab. So, what we will do first, first we will short circuit the independent voltage source, leave the dependent source as it is in the circuit and then we connect the voltage or current source to solve the circuit to find the value of $R_{Th}$.

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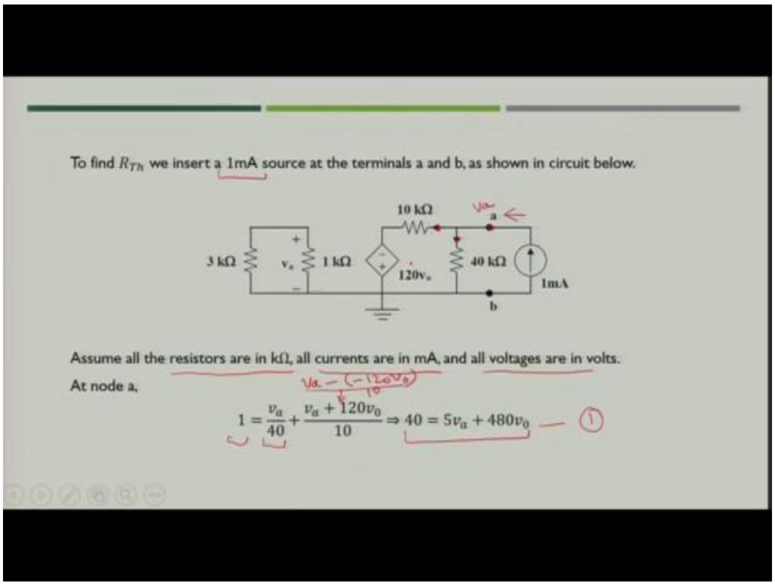


So, here what we are going to do we are adding 1 milli ampere of current source across the terminal ab. Why 1 milli ampere because the resistance R is in kilo ohm. Hence, we can connect 1 milli ampere so that the circuit can be simplified. This ensures that voltage would remain in volts. So, now, you have got this circuit, you see in this circuit, there is no voltage or current source because the connected voltage source in this particular segment is short circuited; we are left with only the dependent voltage source and then we are connecting 1 milli ampere as a source external to the circuit across terminal ab.

Now, what we have to do? Let us apply the Kirchhoff current law at node a. So, these current sources supplying current through node a from this side and let us assume that the direction of current in other 2 segment is has shown in the figure. You can say 1 milli ampere is nothing but the sum of current going outside the node. So, the value of current going outside the node, this current would be the value of node voltage. Suppose, the node voltage is $v_a$.At node a,

$$1 = \frac{v_a}{40} + \frac{v_a + 120v_0}{10} \Rightarrow 40 = 5v_a + 480v_0$$

So, this is the equation which we get from this particular segment of the circuit.

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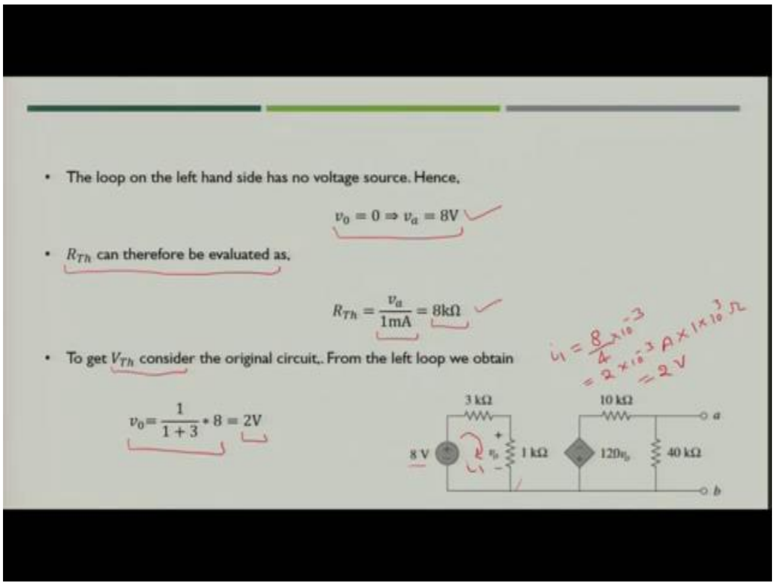


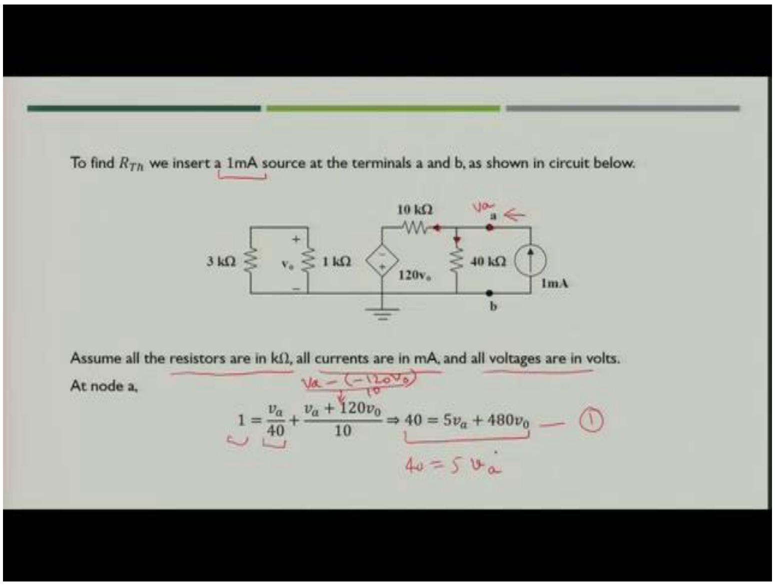


Next if you see the circuit, if you see the circuit here you will say there is no source available in this segment. So, what does it mean the value of $v_0$ would be 0, right. So, since $v_0$ value is 0 and this value $v_0$ which is across 1 kilo ohm is the variable which defines the value of dependent voltage source. So, what will happen in that case? Since $v_0$ here is 0 the value of dependent voltage source that is $120v_0$ will be 0.

Finally, if you put the value of $v_0$ as 0 here, you will, you will finally get $40 = 5v_a$, if you simplify you get $v_a = 8V$. So, then Thevenin resistance

$$R_{Th} = \frac{v_a}{1\text{mA}} = 8k\Omega$$

Now, the next task would be the finding out the value of Thevenin voltage. So, what we have to do now, you have to take the original circuit. So, you will retain the sources now. So, will the voltage source, which we short circuited in case of $R_{Th}$ will come back in the circuit again and now, we have to find out the value of the voltage which is across 1 kilo ohm that is $v_0$. So, from this segment, if you see this segment the 8 volt is supplying current to both of the resistors is that is 3 kilo ohm, 1 kilo ohm, both, so, this is nothing but voltage divided circuit.

So, finally, voltage $v_0$ which is across 1 kilo ohm would be

$$v_0 = \frac{1}{1+3} * 8 = 2V$$

So, you can use either of the technique either voltage dividers technique, which is like you can simply you find out the value of $v_0$ or you can write the loop equation and find out the value have $v_0$. So, now, you got the value of $v_0$ which is the dependent variable for this dependent voltage source.

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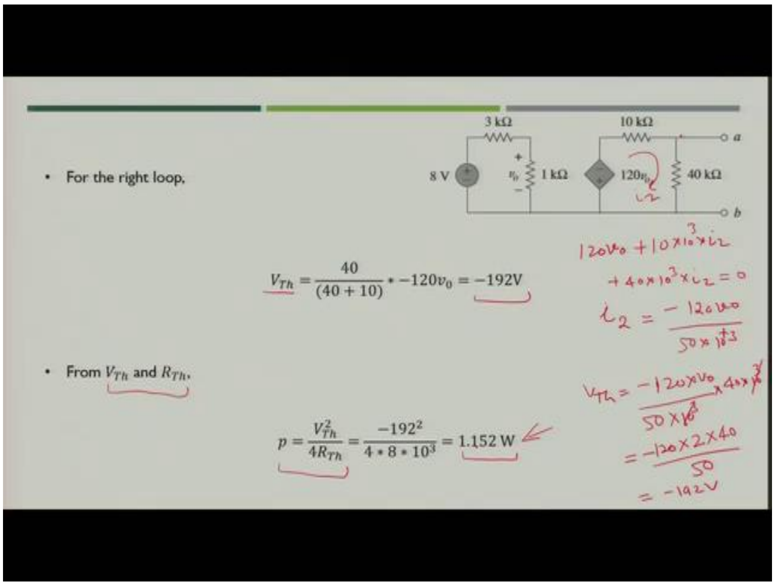


So, next what we have to do we have to find out the value of $V_{Th}$. So, here if you find out the value of $V_{Th}$ what you get you will get again the voltage divider circuit or alternatively you can write the loop equation say $i_2$. This would be

$$120v_0 + 10 * 10^3 i_2 + 40 * 10^3 i_2 = 0$$

$$i_2 = -120v_0/(50 * 10^3)$$

Therefore,

$$V_{Th} = \frac{40}{(40 + 10)} * -120v_0 = -192V$$

So, now you have got $V_{Th}$ and $R_{Th}$. So, you will find out the value of power $p$.

$$p = \frac{V_{Th}^2}{4R_{Th}} = \frac{-192^2}{4 * 8 * 10^3} = 1.1.52W$$

This would be the maximum power which is being transferred to the load, which would be connected across the terminal ab. So, in this way, you can find out the value of maximum power being transferred to the load when any dependent sources available.

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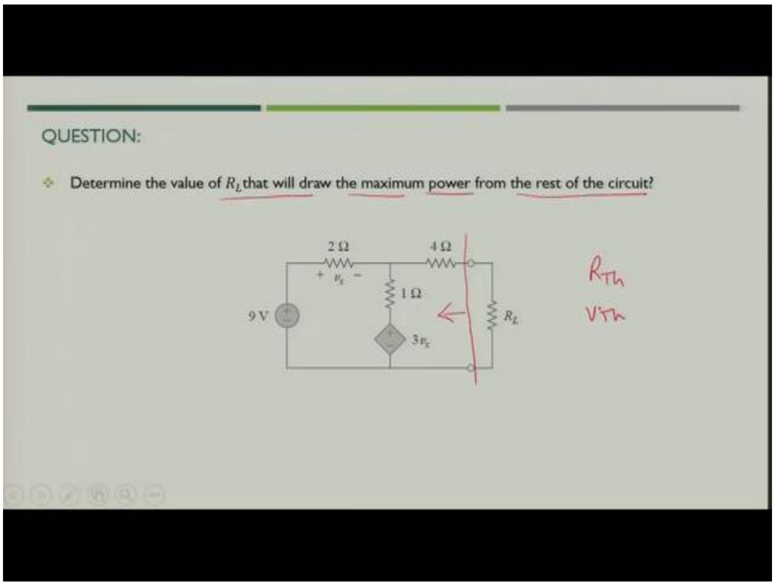


Now, next, we have another circuit where the dependent source is available. I will leave this particular circuit for you to do it at your home, so that you get the better idea about how to solve the circuit when you have the dependent sources available. So, I will just give you the clue that how you will solve it. So, we have to find out the value of $R_L$ which will draw the maximum power from rest of the circuit. So, this would be rest of the circuit, what you have to do you have to find out the value of $R_{th}$ and $V_{th}$.

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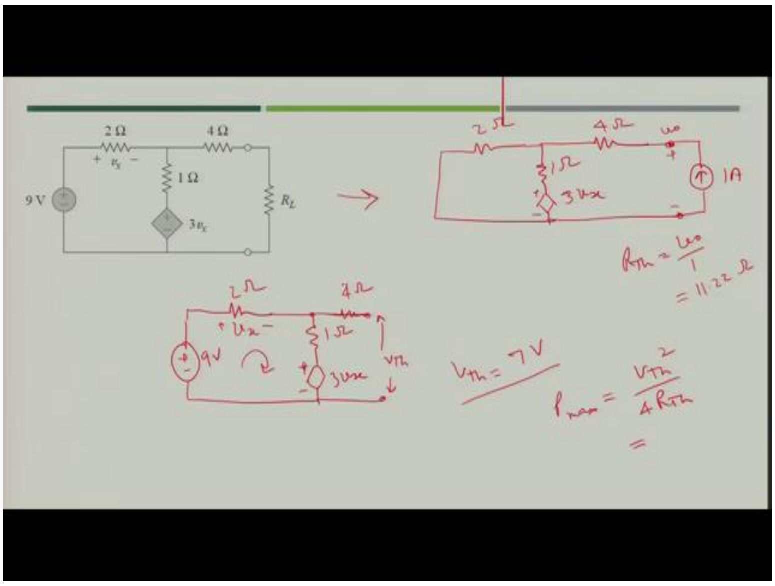


To find $R_{th}$ you will simply short circuit this. You will get circuit like this in this case. So, this is dependent voltage source. This is $3v_x$, this is 1 ohm, this is 2 ohm and then you have 4 ohm and you need to find out the Thevenin resistance.

So, what you will do, you will apply 1 ampere current source across the terminal and you find out the value of $v_0$ across the terminals and the value of $R_{th}$ would be $v_0/1$ ampere. So, just solve it and when you will solve you will get the value of $R_{th}$ as 11.22 ohm. Similarly, for $V_{th}$ what you have to do you have to just solve this circuit where you have 9 volt supply 2 ohm and you have resistance then you have dependent voltage source and here you have 4 ohm resistance which does not have any impact because the terminal is open circuited.

So, here you can just try to find out the value of $V_{th}$ we are these $3v_x$ is there in 1 ohm is the resistance along the $3v_x$ dependent voltage source and the dependent variable for the depending source the dependent voltage sources the voltage across 2 ohm resistance. So, what you can do, you can just simply write the loop equation and simplify to get the value of v. So when you will simplify you will get the value of $V_{th}$ as 7 volts.

So, now you have got $R_{th}$ and $V_{th}$ and you can simply find out the value p_{max}. So, you apply this particular equation and find out the value of p_{max}. So, I will leave it for you to do it at home so, that you get the value of the Thevenin equivalents. With this, we close our today's session. Namaskar.