Basic Electric Circuit
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Module 4 Network Theorem 2
Lecture 18 Maximum Power Transfer

Namaskar. So, till now, we have discussed 3 important circuit theorems those were Norton's theorem, Thevenin's theorem and superposition theorem. So, now, today we will discuss about the maximum power transfer theorem.

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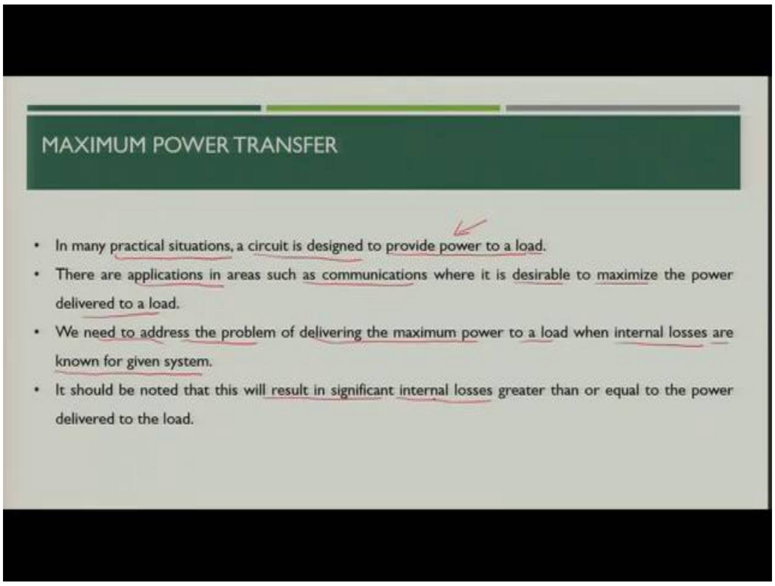

So, in many situations the circuit is designed to provide the power to the load. So, rather than finding out the voltage and current we are more concerned about the power which is being supplied to the load. So, there are such applications such as those in communication system, where it is desirable to supply maximum power to the load. So, what we have, we have to do we have to address this problem of delivering the maximum power to the load when internal losses are known for the given system. Now, one important thing which you have to keep in mind is, when you see the internal losses present in the system, and if you apply maximum power transfer theorem, it means that there would be significant internal losses, how it will happen, when we will discuss the maximum power transfer theorem in detail, we will understand this particular phenomena.

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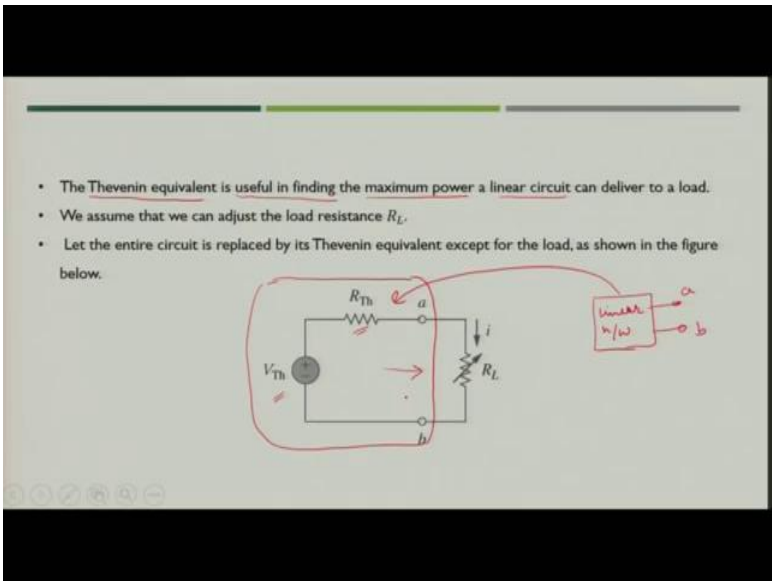

Now, the Thevenin equivalent which we discussed in the previous classes, will be used for finding out the maximum power in linear circuit. So, whatever the network you have suppose if you have a 2-terminal network and you see this is linear network. So, this network can be replaced by the Thevenin's equivalent this we have already seen when we discussed the Thevenin's equivalent.

So, now, whatever the network we have, we can represent with its equivalent Thevenin voltage and Thevenin resistance. Now, to the terminals across which we have just found using the Thevenin equivalent we have to add the load. Then we have to find out that under what condition the maximum power would be supplied by this network to the load. So, what we are assuming here is that the load is variable, so, that we can tune the load in accordance with the maximum power transfer condition.

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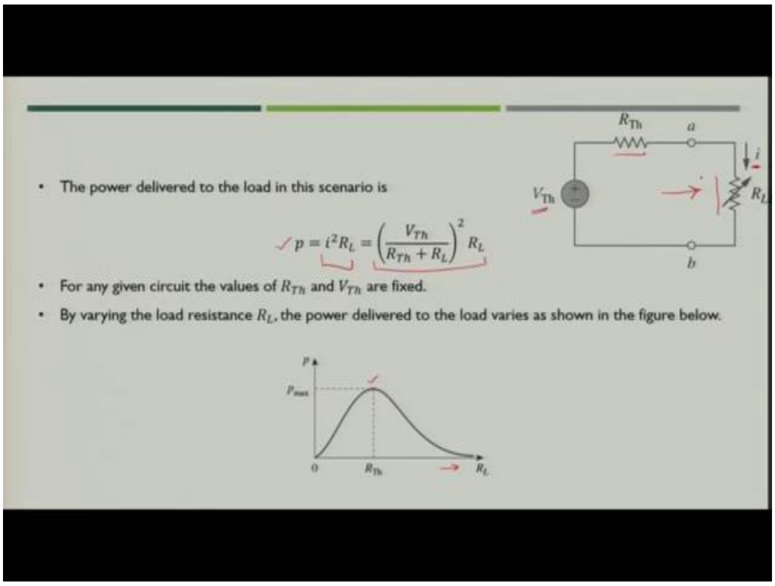

So, now, if you see the circuit which is having the Thevenin equivalent of the network and then the variable load that is R_L connected. So, what would be the power in that case, power would be i^2R. In this case resistance R is R_L because we are to find out the power dissipated by this resistance R_L. So, the value of current would be

$$i = \frac{V_{Th}}{R_{Th} + R_L}$$

Then the square of the current multiplied by R_L would be the total value of power which would be delivered to the load. Now, if you see this particular equation V_{Th} and R_{Th} is fixed because this is network equivalent at a particular point of time. So, for that particular instance the value of V_{Th} and R_{Th} will remain fixed, the only thing which is variable in this circuit is R_L.

So, now, if you vary the value of R_L from 0 to say infinite and you measure the value of power supplied by the network to the load then you can draw a curve which will look like as shown in this figure. So, when you start increasing the value of R_L, you will see that power which is required for this particular load is increasing and now, at one particular instant the power would be maximum and after that the value of power will again start reducing even if the value of R_L is increasing.

So, from this you can understand that there is one instance at which the maximum power would be supplied to the load. And now, what we have to do we have to find out under which condition or what would be the value of this R_L at which the maximum power would be delivered by the network to the load.

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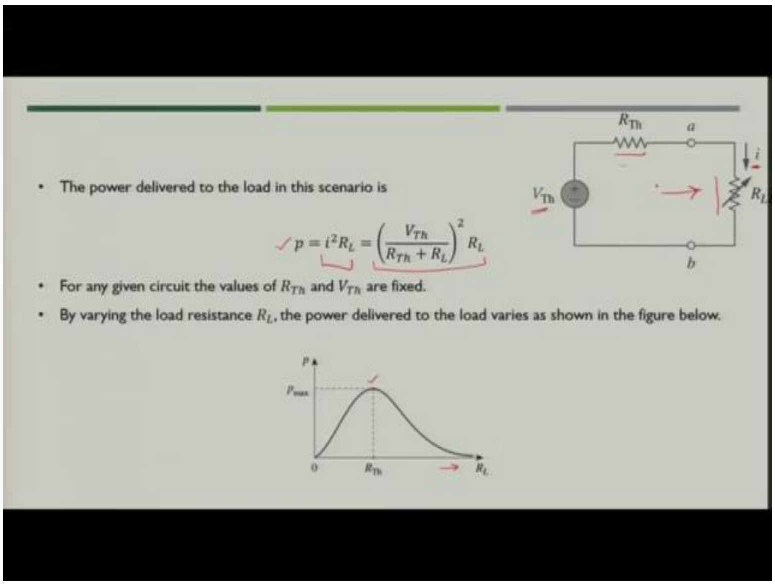

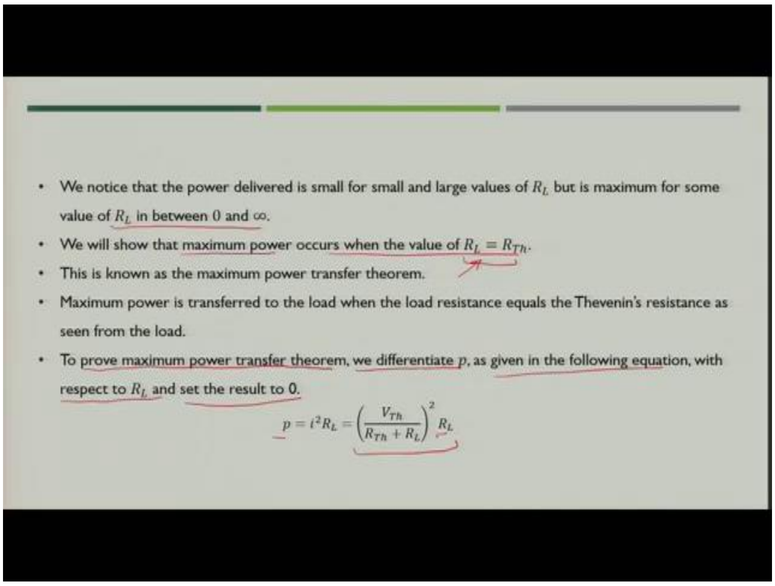

So, now, let us try to find out the values we have seen that the R_L is changing between 0 to infinity, the value which at which the maximum power occurs is $R_L = R_{Th}$. If you see this figure, the maximum power transfer to the load happens at the value of R_L which is equal to R_{Th}. So, in this particular network, when the value of R_L is equal to Thevenin resistance, maximum power transfer happens. We find this with the help of the power p, which we just calculated. Now, let us see how we will say that R_L is equal to R_{Th}. So, we need to to find out the optimality condition, what you have to do, you have to differentiate the function which is given here as p.

Now, power is given by

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

So, to prove the maximum power transfer theorem, what we have to do, we have to differentiate p which is given in this equation with respect to R_L and then for finding out whether we have reached the maximum value or not we have to set

$$\frac{dp}{dR_L} = 0$$

So, we have to differentiate the power p with respect to R_L which is the only variable in this particular equation.

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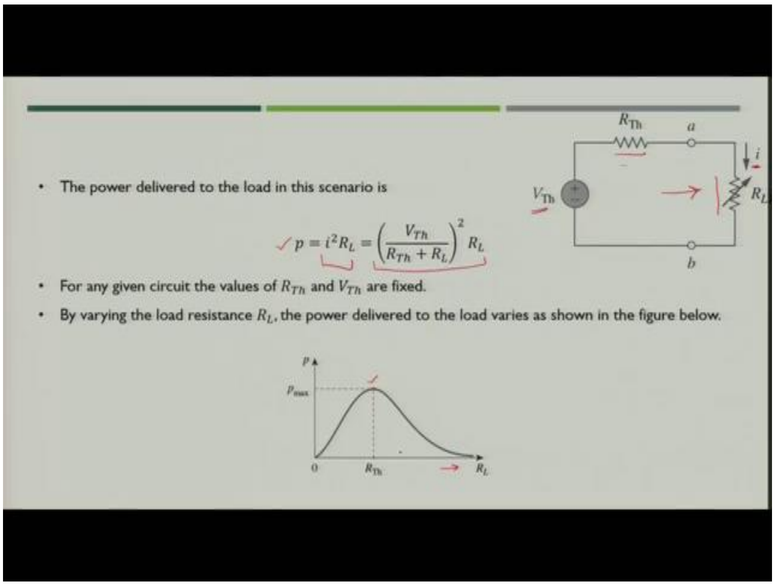

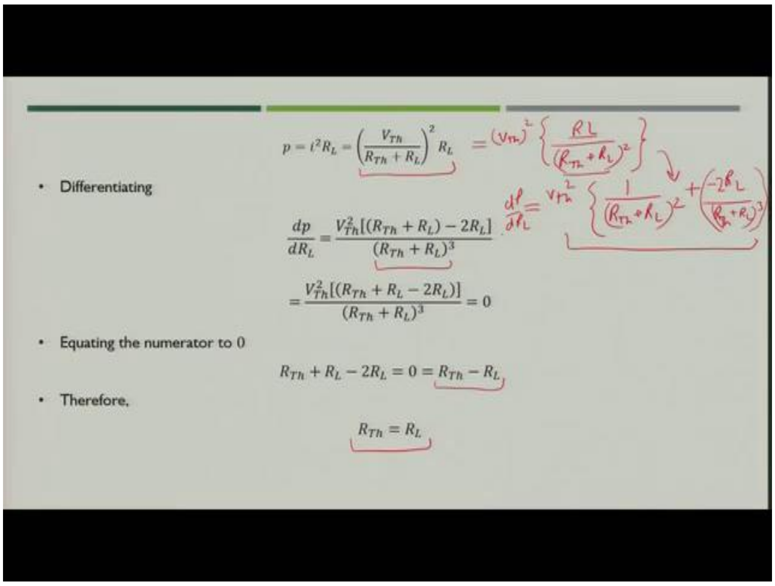

So, now, if we differentiate the power equation we

$$\frac{dp}{dR_L} = \frac{V_{Th}^2[(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)]}{(R_{Th} + R_L)^4}$$

$$= \frac{V_{Th}^2[(R_{Th} + R_L - 2R_L)]}{(R_{Th} + R_L)^3}$$

So, when you set this equal to 0, you get the numerator terms that is $R_{Th} + R_L - 2R_L = 0$. Hence, $R_{Th} - R_L = 0$ and finally, you get $R_{Th} = R_L$.

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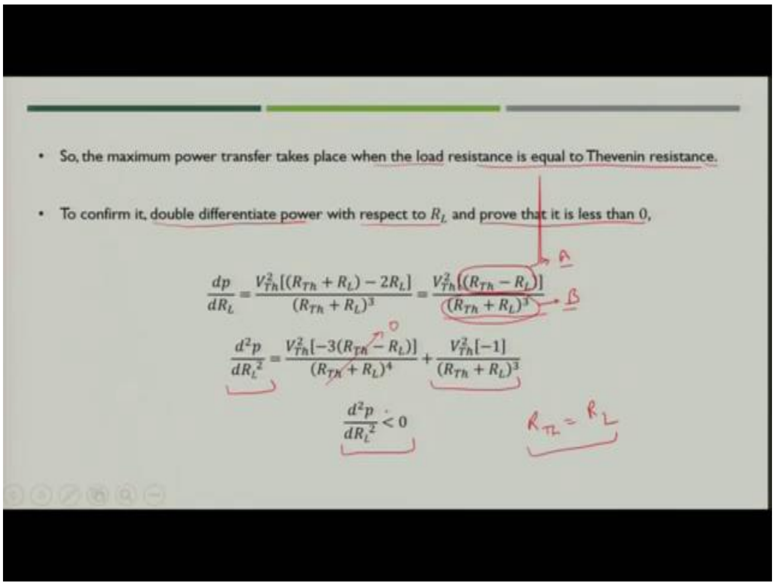

The maximum power transfer takes place when the load resistance is equal to Thevenin resistance this we derived when we took the derivative of power p with respect to R_L which is variable. And now, as we know that when the derivative $\frac{dp}{dR_L} = 0$ at that condition, the R_L value what we get if you supply that value R_L into the power p, power might be maximum or minimum. Now, we have to find out whether the condition which we have is derived with the help of taking the derivative is really a maximum or not. So, what we have to do we have to double differentiate the power with respect to R_L and will prove that it is less than 0. Solving this as shown in the slide you get the double differential to be less than 0 and hence, when the load resistance is equal to the Thevenin's resistance maximum power is transferred to the load.

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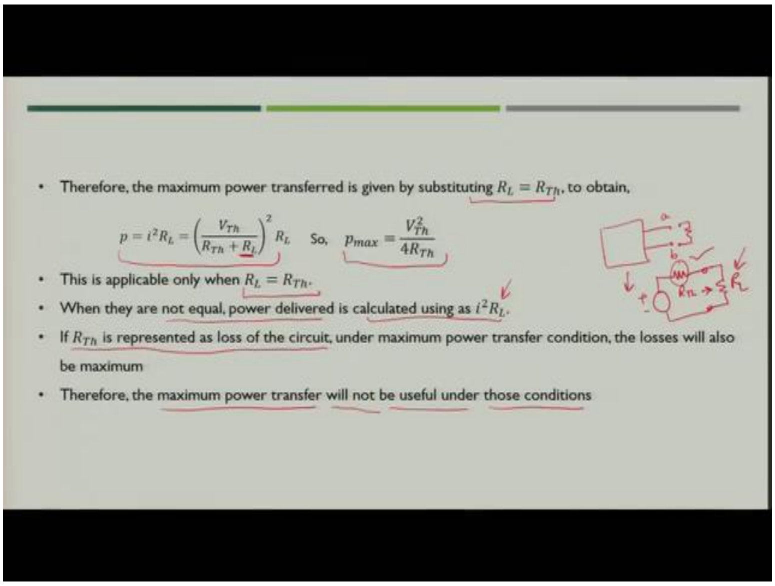

So, therefore, when we substitute the value of $R_L = R_{Th}$ in the power equation which we just calculated, that is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L,$$

the maximum power which the network will transfer to the load would be

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

And important thing to consider is that this value will hold valid only when $R_L = R_{Th}$. So, when the value of R_L value is either more than R_{Th} or less than R_{Th} we have to use the conventional formula of $i^2 R_L$ to find out the power being transferred to the load.

Now, if R_{Th} is represented as loss of the circuit, so, what happens if you have the network like this and you are connecting load to this external terminal ab if this network is represented as Thevenin equivalent, but, the value of R_{Th} which we are seeing here is nothing but total loss which is there in the network. And then if you connect the load R_{L} at maximum power condition, although the load will receive maximum power from the source, but losses will also be maximum.

So, that is why the maximum power transfer condition will not be useful when the losses are present in the system. So, in that case loss, whatever we get in the system would be equal to whatever power is being absorbed by the load.

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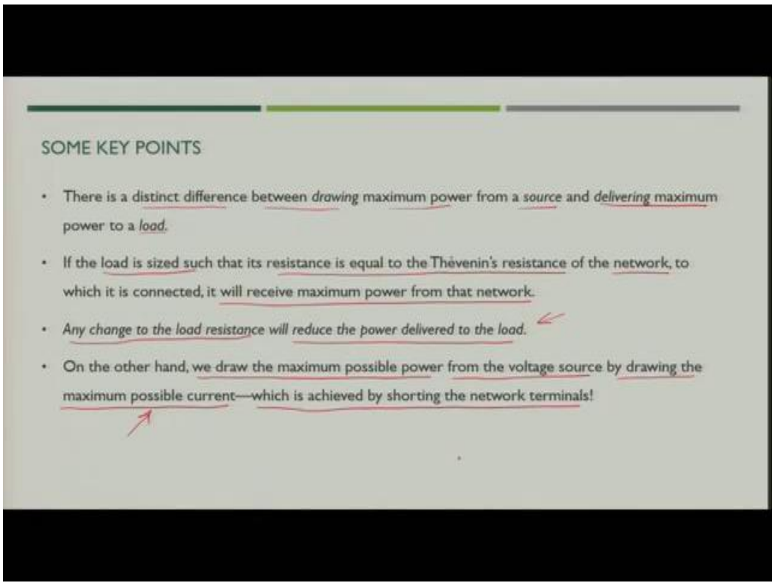

So, what we can say now, that, there is a distinct difference between drawing maximum power from the source and delivering maximum power to the load. Now, when the load is sized, such that the resistance is equal to Thevenin's resistance of the network it will receive maximum power from that network. Any change in the load resistance whether it is increasing or decreasing with respect to Thevenin resistance, it will reduce the power which would be delivered to the load.

So, on the other end, we can now say that, we draw the maximum power possible power from the voltage source by drawing the maximum possible current. So, this was the condition when we say that the source is delivering maximum power to the load. Now, let us see what is the meaning of drawing maximum power from the source. So, when you want to draw maximum power from the source means, you want to draw maximum possible current from the source how it will be achieved?

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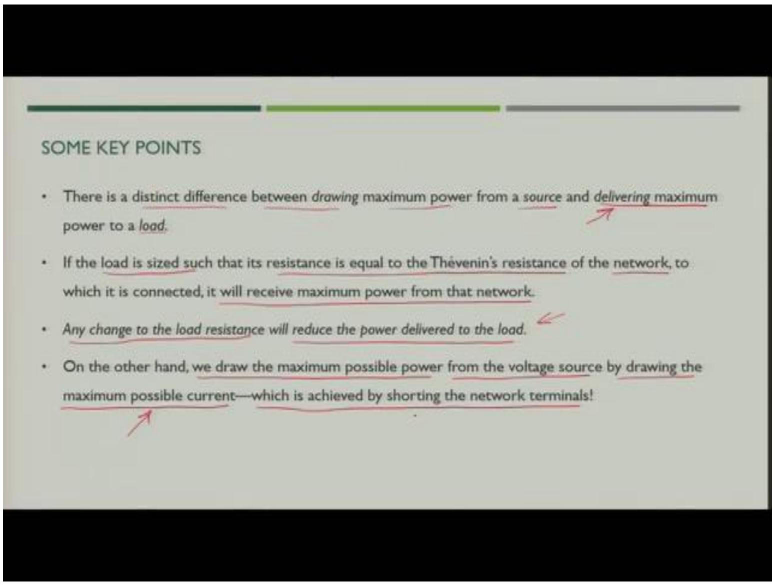

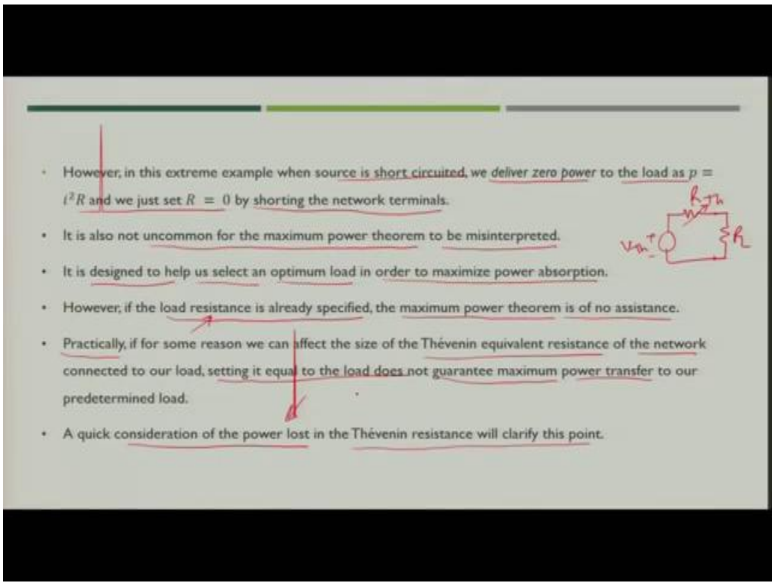

It will be achieved simply by shorting the terminals because only at that condition the maximum current would be delivered by the source. So, in that case what will happen, you will effectively deliver 0 power to the load because in that case the R value is 0 because you have short circuited the terminals.

So, you can now understand that there is a distinct difference between drawing maximum power and delivering maximum power to the load. So, maximum power transfer theorem is applicable when you say that source is delivering maximum power to the load, when we say drawing maximum power it means that at that condition, you are simply sorting the source terminals and maximizing the current output from the source.

Now, another important thing which you have to keep in mind that maximum power theorem is sometimes misinterpreted. What happens that generally we designed the maximum power transfer theorem to select the optimal load in order to maximize the power absorption, but, if the load is fixed, that means, if the load resistance is already specified, then maximum power theorem will not be able to help why because, since you have fixed the value of load that is your R_L is now fixed rather than the variable component.

So, your R_{Th} is anywhere driven by the network components. So, your V_{Th} and R_{Th} is already fixed, if R_L is also fixed, you cannot find the maximum power transfer condition. So, in that way, if the load resistance is already specified, the maximum power transfer theorem will not hold. Now, suppose by any chance if you make R_{Th} as a variable, in that case the Thevenin equivalent resistance of the network when you want to set equal to load, it will not always guarantee the maximum power transfer condition, because what happens in practical situation the network will have some loss components also.

So, because of the available loss components, we have the power loss in those components. And in that case the circuit which you see as a Thevenin equivalent will not be able to help in identifying the maximum power condition why because there would be one component which is always be a loss component and that component will not give you the condition under which you have the $R_{Th} = R_L$. So, that is something which you have to keep in mind that in practical scenario, you always have to see whether there is a power loss in the circuit are not.

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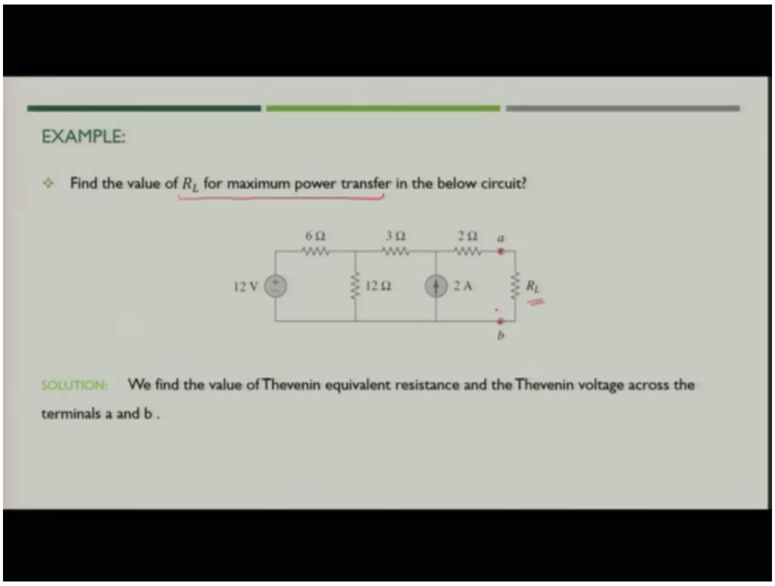

Now, let us understand the maximum power transfer theorem with the help of an example. So, we have the circuit given in the figure. Now, we have to find the maximum power transfer at what value of R_L we get the maximum power transfer. So, what is the first step? the first step would be to find the Thevenin equivalent of this circuit which is left to the terminals ab.

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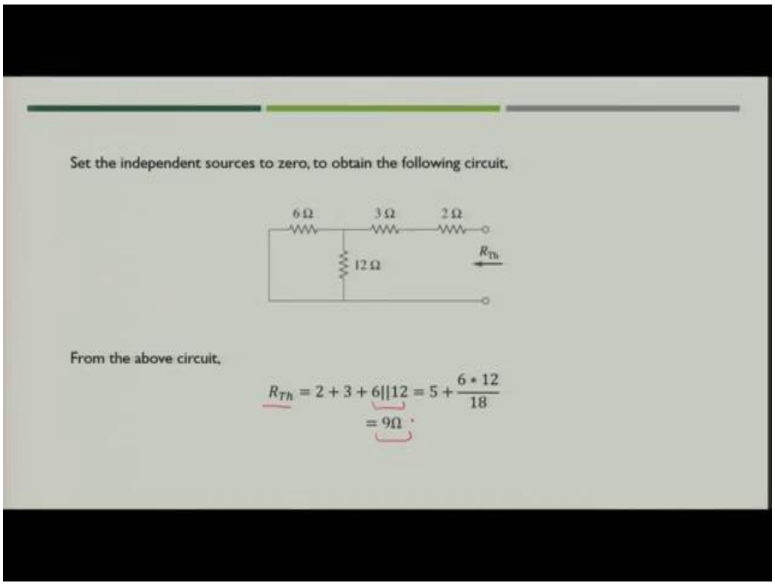

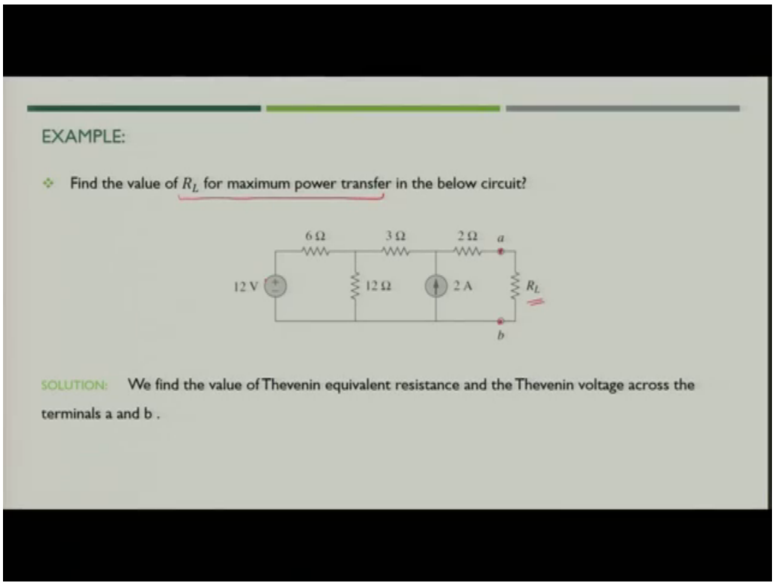

So, for that what you have to do, you have to first short circuit the voltage source and open circuit the current source so, that you can find out the value of Thevenin resistance. So, now, if you see the circuit, when you short circuit the voltage source, open circuit the current source 6 ohm resistance would be in parallel with 12 ohm and these 3 ohm and 2 ohm resistance would be in series. So, what would be the value of R_{Th}?

$$R_{Th} = 2 + 3 + 6||12 = 5 + \frac{6 * 12}{18}$$

$$= 9\Omega$$

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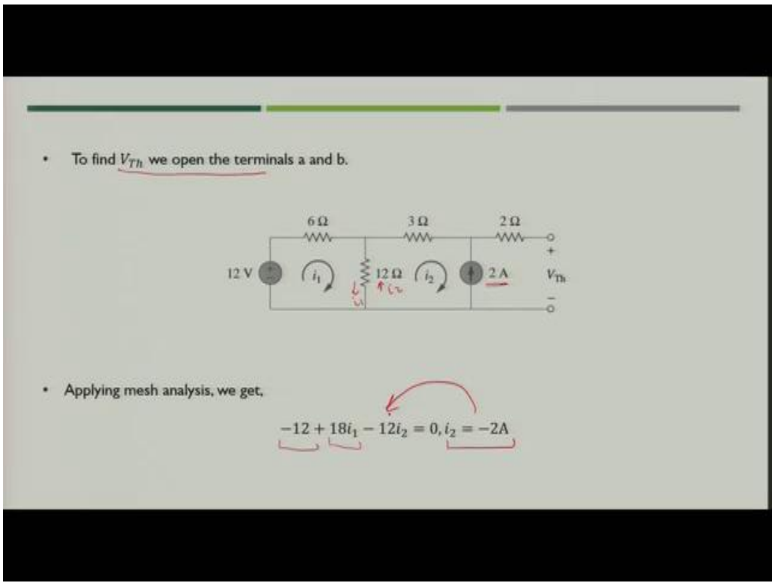

Now, next task is to find the value of V_{Th} that is Thevenin voltage. Now, when you see the circuit, why by keeping the sources back to the circuit, you will see there would be 2 meshes in the circuit. So, let us say i_1 is the mesh current in mesh 1, i_2 is the mesh current in mesh 2 and voltage across terminals ab is V_{Th}. Now, in that case, if you see the circuit, $i_2 = -2A$. Now, if you write the KVL for this particular mesh, what you will write? You will write

$$-12 + 18i_1 - 12i_2 = 0$$

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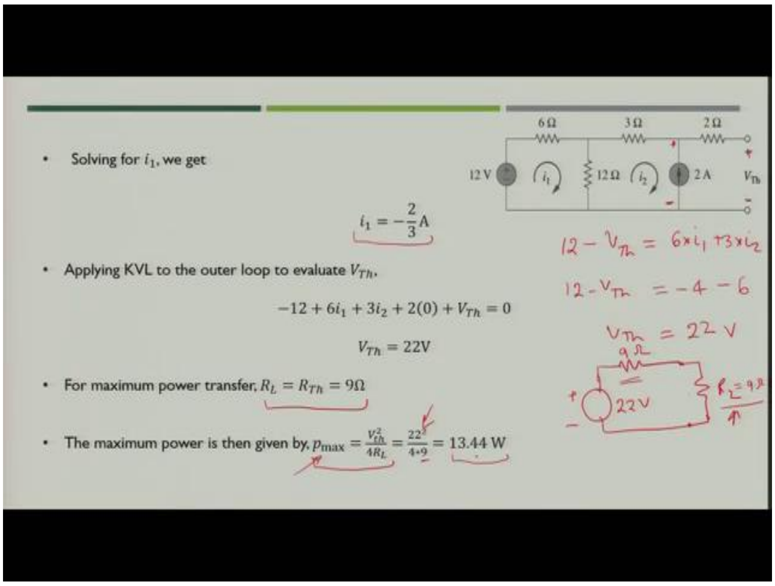

If you put the value of i_2 into first equation and simplify you get

$$i_1 = -\frac{2}{3}A$$

Next, you have to just see from the figure that V_{Th} is nothing but the voltage across the current source because there would be no current flowing in 2ohm resistor because it is open circuit. So, voltage V_{Th} would be across the current source. So, what you can write in this case, if you apply KVL to this particular segment, you get

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$V_{Th} = 22V$$

So, now, you have got the value of V_{Th} and you have the value of R_{Th} as 9 ohm. So, your equivalent, Thevenin equivalent of the circuit would be the 22 volt in series with 9 ohm resistance and then you have connected and unknown the load that is R_L.

Now, using maximum power transfer theorem, what you can say that the maximum power will be transferred only when the 9 ohm resistance is equal to the load that is R_L. So, under maximum power transfer condition 9 ohm would be

$$p_{max} = \frac{V_{th}^2}{4R_L} = \frac{22^2}{4 * 9} = 13.44\text{W}$$

So, with this let us conclude the today's session. We will continue our discussion on maximum power transfer theorem in next class also, where we will discuss that in case the dependent sources available in the circuit. How you will calculate the maximum power which would be transferred to the load. Thank you.