Basic Electric Circuit
Professor Ankush Sharma
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module 4 Network Theorem 2
Lecture 17 Norton's Theorem

Namaskar. So, in the last class we discuss about the basics of Norton's theorem and we did some the examples with the help of the sources which were independent in those circuits. Now, in today's class we continue our discussion on Norton's theorem, but in this class we will discuss more about the cases where we have independent sources available in the circuit.

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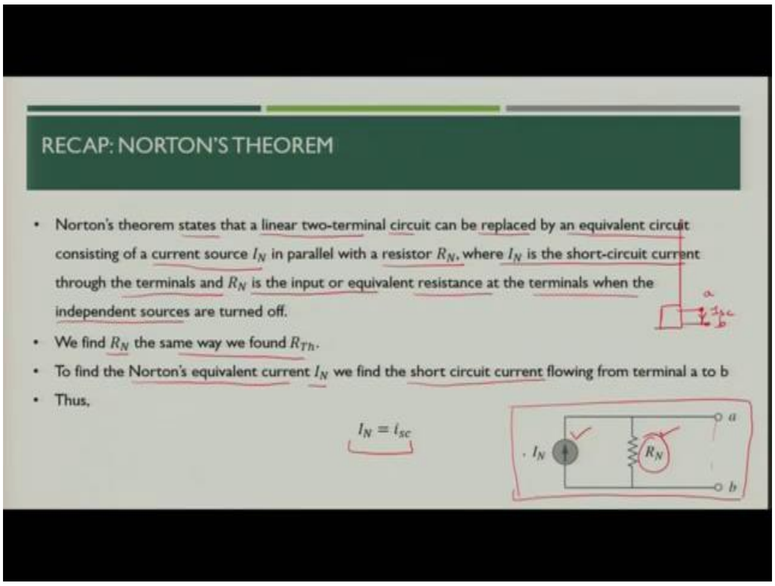

So, let us see the Norton's theorem which we discussed in the last class. Norton's theorem states that the linear 2 terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with resistor R_N where I_N is this short circuit current through the terminals and R_N is the input or equivalent resistance at the terminals, when independent sources are turned off.

Independent Sources turned off means, the voltage source would be short circuited and the current source would be open circuit. So, finally, we will get the Norton equivalent and as shown in this figure, where I_N is nothing but the short circuit current which is flowing through the terminal ab and the R_N is the input resistance which you will see when you see the circuit and you see through these 2 terminals, the input resistance the value of that would be nothing but Norton's resistance.

So, the independent or the unknown variables in this case are R_N and I_N and this is we have to find out. Now, R_N can be found in the same way we found R_{Th} that was the Thevenin's resistance, which we did in the Thevenin's theorem discussion. And for the Norton's equivalent current I_N what we have to do, we have to short circuit the current flowing from Terminal a to b. So, in the network suppose, this is your network and you have terminal ab like this you have to simply short circuit and find out the value of what is the short circuit current.

So, this network would be linear 2 terminal was you want 2 short circuit these 2 terminals and find out the value of circuit current. So, that circuit current would be nothing but the Norton's current.

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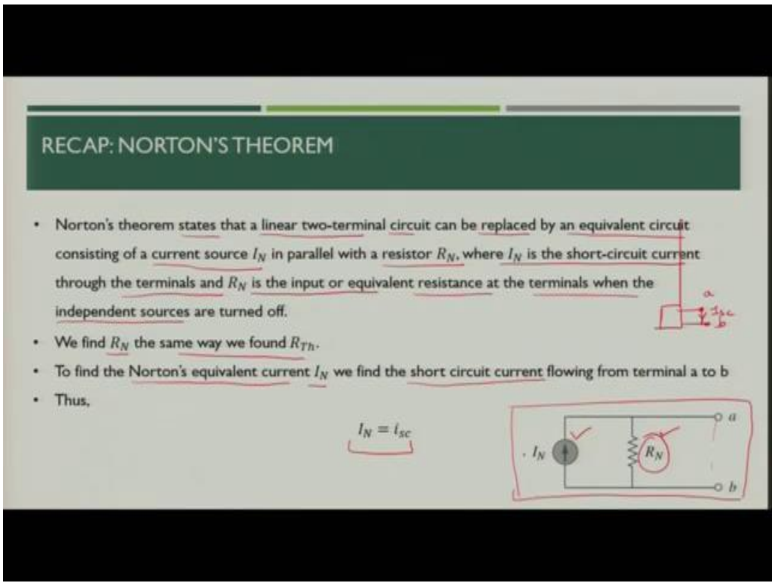

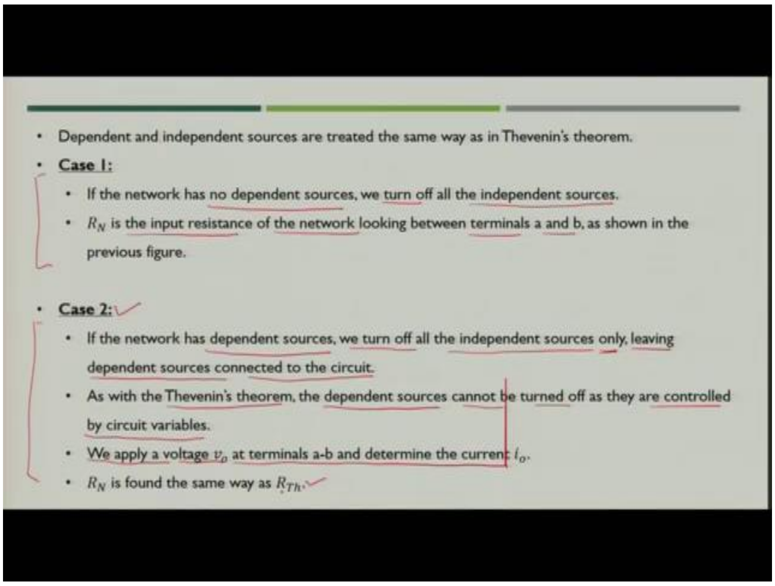

Now, in the last class, we discussed 2 cases. We discussed the first case in detail and we mentioned that there are 2 cases available there for finding out the Norton's resistance. So, now, in case 1, in the last class, we discussed that if the network has no dependent sources, then we have to turn off all the independent sources and, in that case, R_N would be the input resistance of the network looking between the terminals a and b.

So, as we saw in the previous figure, so, when you look through the terminal suppose, if these are the 2 terminals, and you want to find out the Norton's resistance, then you have to look through these 2 terminals and the resistance which you will see from these 2 terminals would be Norton's resistance. So, now, this was the case which we discussed in the last class. In today's class, we will discuss more about case 2, that means, if the network has dependent sources, so, in that case what we have to do, we have to turn off all the independent sources only and we leave the dependent sources connected to the circuit.

So, in that case, the as with the Thevenin's we discussed previously the dependent sources cannot be turned off as they are controlled by the circuit variables, since we have the dependency in the case of dependent sources, we cannot remove them from the circuit and we analyze the circuit for Norton's equivalent by keeping the dependent sources connected to the circuit. Now, how to solve it, we have to apply voltage v_0 at the terminals ab and determine the current i_0 and R_N is also found in the same way as we analyzed the Thevenin's equivalent and found the value of R_{Th} that is Thevenin’s resistance.

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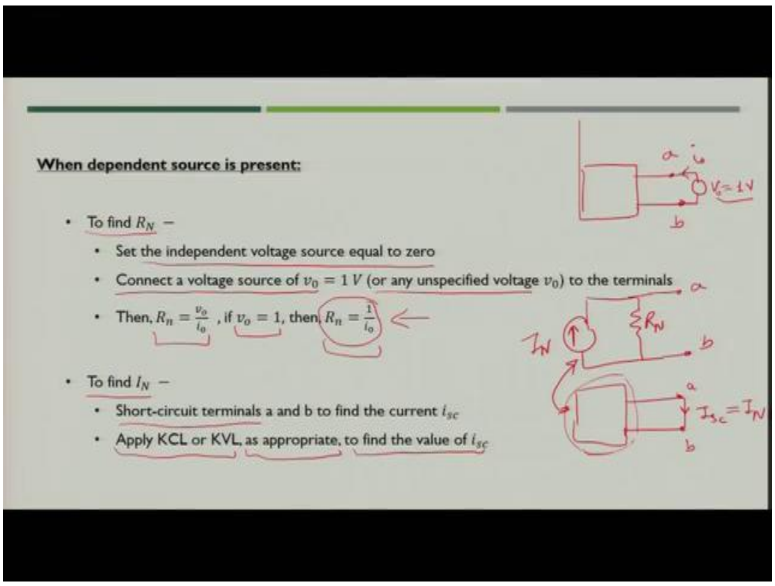

Now, what we have to do to find R_N, we have to first set the independent voltage sources equal to 0. So, suppose if you have the network like this, where you have 2 terminals ab now, what you have to do to find the value of R_N you have to connect a voltage source v_0 across terminal ab. So, you will connect a voltage source that is equal to maybe 1 volt or maybe any specified value. We usually keep 1 volt because it is easier to it simplifies the calculation for finding out the value of R_N.

So, it is not mandatory that you always keep v_0 is equal to 1 volt you can take any value and solve this circuit. We take as 1 volt because it is easier, now you can see that we have Norton's resistance as $\frac{v_0}{i_0}$. When we have $v_0 = 1$, R_N would be simply the reciprocal of the current which is flowing through the terminal a. So, using these equations, you can easily find out the value of Norton's resistance. Now, next we need to find out the value of Norton's current that means you have to short circuit the terminals a and b. So, you have the network, you have the terminals ab now you need to find out the value of short circuit current that means you have to first short circuit both of the terminals and find out the value of current flowing through short circuited terminal ab.

So, for this what you have to do, you have to just apply either KVL or KCL whatever is appropriate for that circuit, then find the value of short circuit current. So, this short circuit current will be nothing but the Norton's current and the final circuit which you will get would be the current source that is I_N and in parallel with you will have the Norton's resistance R_N and that is ab. So, you will simply replace the unknown that is linear to terminal network with the Norton's equivalent.

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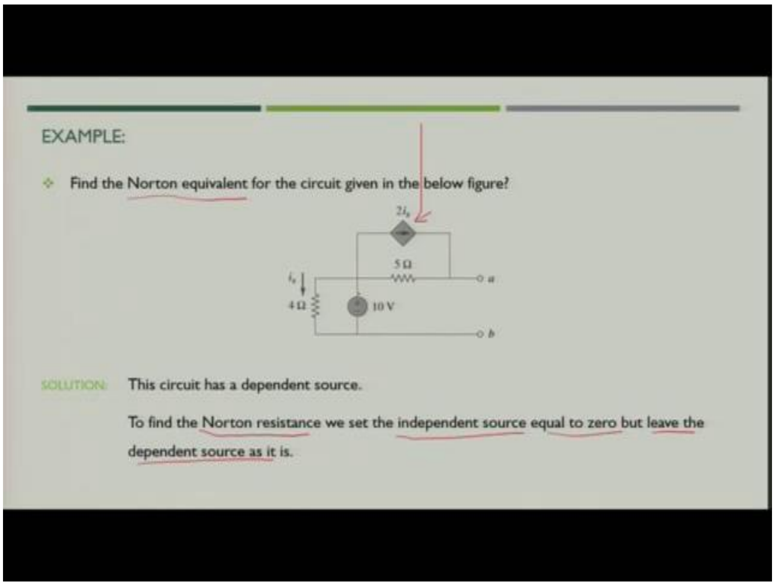

So, how we will do? let us take one example so that we can understand this concept clearly. Suppose, the circuit is as we see in the figure and we need to find out the Norton's equivalent if you see this circuit, you will see that there is one dependent current source and one independent voltage source, the dependent current source current source i_x the value of the current sources $2i_x$. i_x is dependent on the current which is flowing through the 4 ohm resistance. Now, what we have to do to find out the Norton's resistance, we have to first set the independent sources equal to 0, but leave the dependent sources as it is in the circuit.

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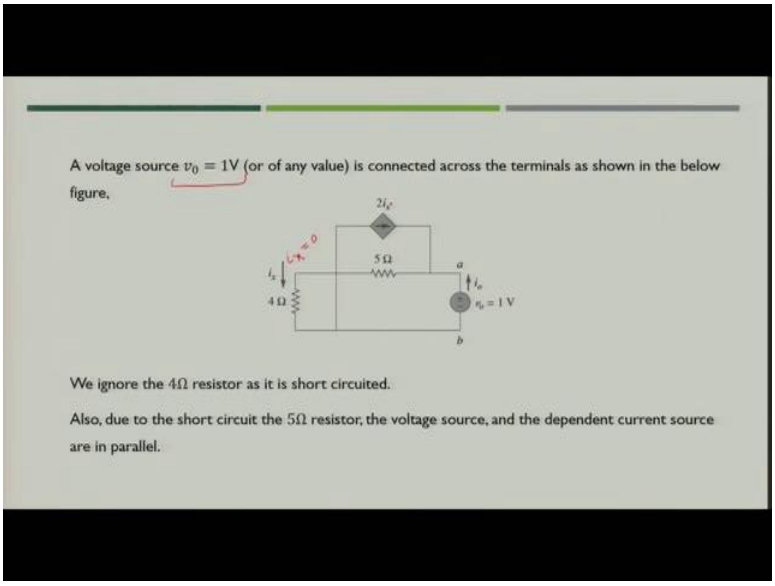

We short circuit the voltage source and leave the current source as it is. So, what will happen now the voltage source is short circuited, you are leaving current source as it is in the network, because it is a dependent current source and then you have 5 ohm resistance which is there in the circuit in parallel with dependent current source.

4 ohm resistance is now short circuited because the voltage was which was available here, we short circuited that voltage source. Now, to find out the value of R_N what we have to do, we have to apply external voltage supply $v_0 = 1$V and find out the value of i_0 which is flowing through this external voltage source. Now, if you see this circuit, this segment of the circuit what, what is there you will see this is the short circuit compartment because of the voltage source we short circuited.

So, the 4 ohm resistance which you see here is now short circuited. So, what happens when you short circuit resistance that means that the current flowing through resistance will be 0, because this will have the least resistance and whole current will pass through the short circuit wire that means that the value of i_x would be equal to 0. Now, what we are seeing here, since i_x is equal to 0, that means, this current source that is $2i_x$ will also be equal to 0.

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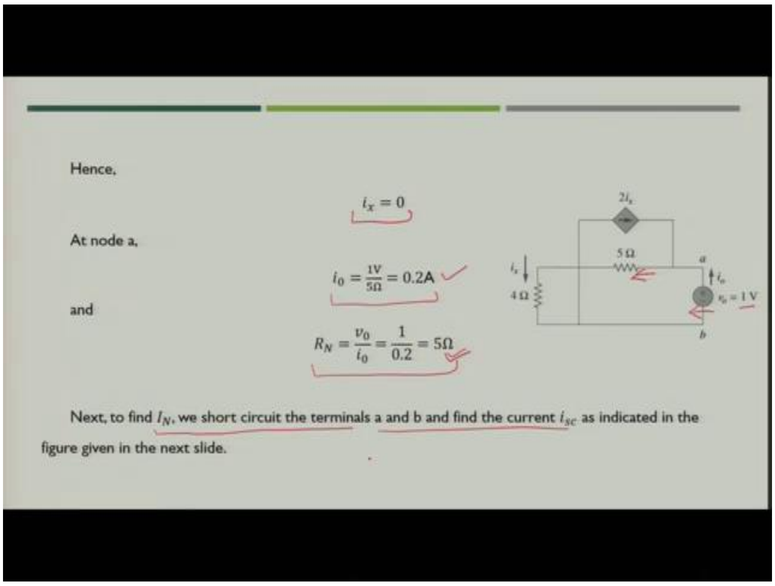

Now, what happens when i_x is equal to 0, you have this component 0, the current which is flowing from external voltage supply v_0 would be only through the 5 ohm resistance. So, what we get the current

$$i_0 = \frac{1V}{5\Omega} = 0.2A$$

$$R_N = \frac{v_0}{i_0} = \frac{1}{0.2} = 5\Omega$$

Now, if you see directly from this figure, if you do not apply voltage supply here then also you can calculate the Norton's resistance, why, how you can get it because this is any way short circuited. So, this compartment is out, this is 0 now, because the i_x value is 0. So, you are left with only this 5 ohm resistance which would be finally your input resistance when you see from terminal ab. So, you can straight away say that the Norton's resistance would be 5 ohm.

So, you can just cross verify that what you have got from $\frac{v_0}{i_0}$ is nothing but 5 ohm which you can find out through inspection also. Now, the next task is how to find out the value of I_N that means the shorts of current across the shorts of content flowing through the short-circuited terminal ab.

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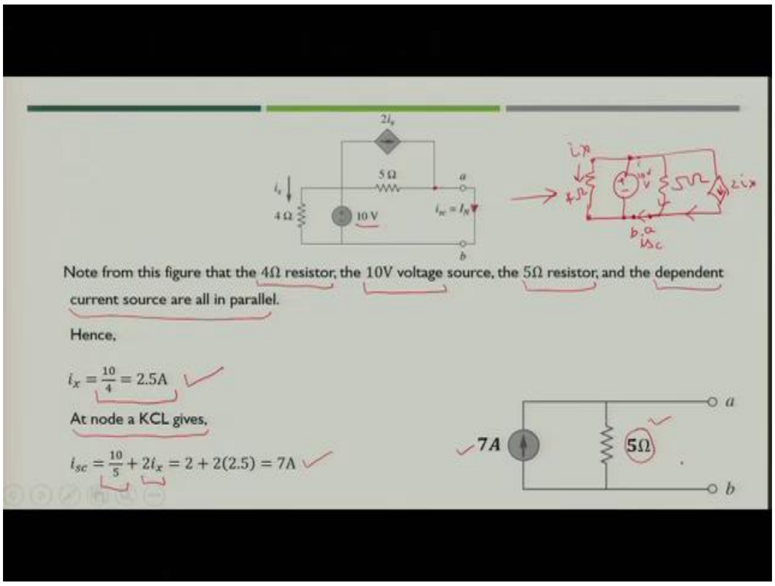

So, what we have to do we have to just short circuit the terminals ab. Now, we have assumed that there is a short circuit current i_{sc} which is nothing but the Norton's current and you have the circuit you need to find out the value of short circuit current flowing through short circuited terminal ab.

Now, if you see the figure that 4 ohm resistor is in parallel with 10 volt voltage source and 4 ohm resistor is also parallel with dependent current source. So, that means, if you see this particular segment ab short circuited. So, finally, what you are getting here is nothing but the voltage source in parallel with another 5 ohm resistance because this will be in parallel and then you will have dependent current source of $2i_x$.

So, this you get when you do the inspection of the circuit and where is your terminal ab, terminal ab would be here, ok. So, now, this is a simplified circuit which you will get from here. When you see this you get the value of current i_x which is flowing through the 4 ohm resistance, the 4 ohm resistance when you the current flowing through 4 ohm resistance is because, you see this voltage we which is across the node 1 and node 2 is nothing but voltage applied across 4 ohm resistance also.

So, with the help of this updated circuit, you can easily find out the value of i_x is nothing but 10 that is the value of the voltage source then we divided by 4 ohm resistance. So, now, you get you get the value of i_x, when you get the value of i_x you have to just run the KCL at node and find out the value of the short circuit current. So, if you see here the value of short circuit current is given by first 10 divided by 5 because if you see this figure this is again in parallel with 10 volt.

So, you get

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7A$$

This is your Norton's current and 5 ohm is the Norton's resistance which you calculated. So, in this way you can find out the Norton's equivalent of the circuit which was given in the example.

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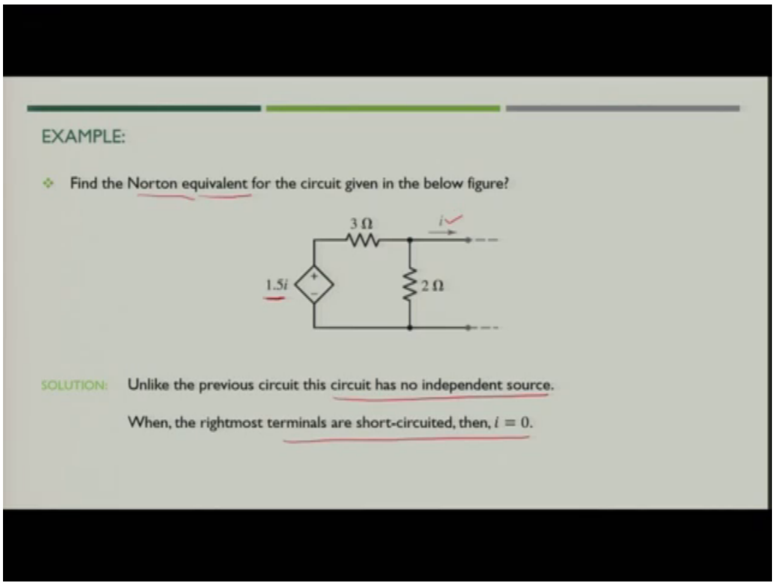

Now, let us see another case which is different from our previous example. In this case, we need to find out the Norton's equivalent, but the important thing which we see here that this circuit does not have any independent voltage or current source.

So, what will happen in that case? So, when you short circuit the right most terminal that is if you short circuits then current would be 0, why? Because there is no any other source available in the circuit, which can supply current i. So, if it is open circuited like in this case, the current i would be 0, even if it is short circuited current would still be 0 because the source which can supply this current is not available in the circuit and this dependent voltage source which we see here depends upon the value of current i.

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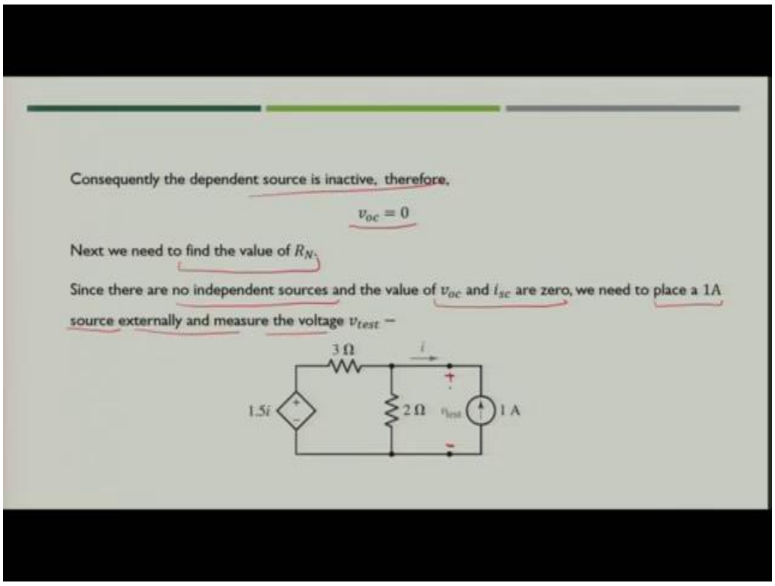

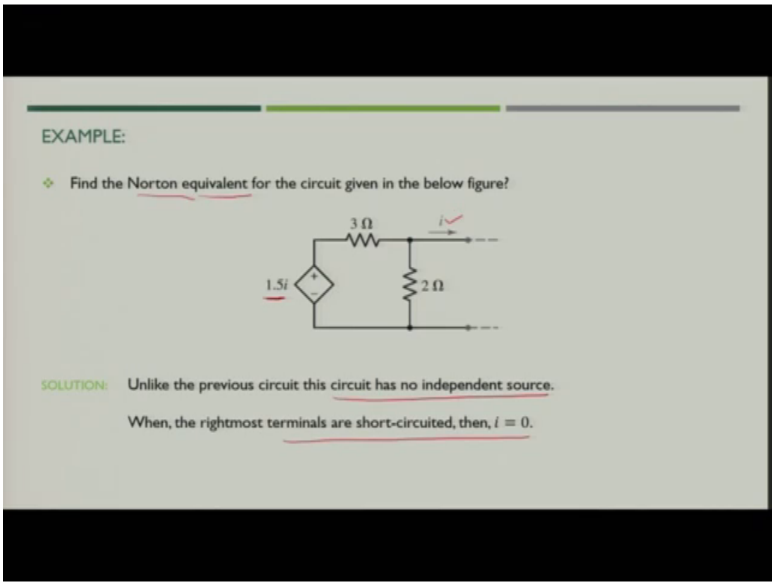

So, what you can say that you always have open circuit voltage 0, short circuit current also 0. So, it means that you need not to find out the value of Norton's current because you cannot find out Norton's current as we see there is no any depend independent voltage or current source. So, next what we need to do, we need to just find out the value of Norton's resistance. So, what we will do, since we do not have any independent source, the value of v_{oc} $and\, i_{sc}$ is 0. So, to find out the value of Norton's resistance, we just placed 1 ampere source externally and measure the voltage across terminal. So, let us say the voltage across terminal is v_{test} and we apply 1 ampere current source across these 2 terminals and find out the value of v_{test}.

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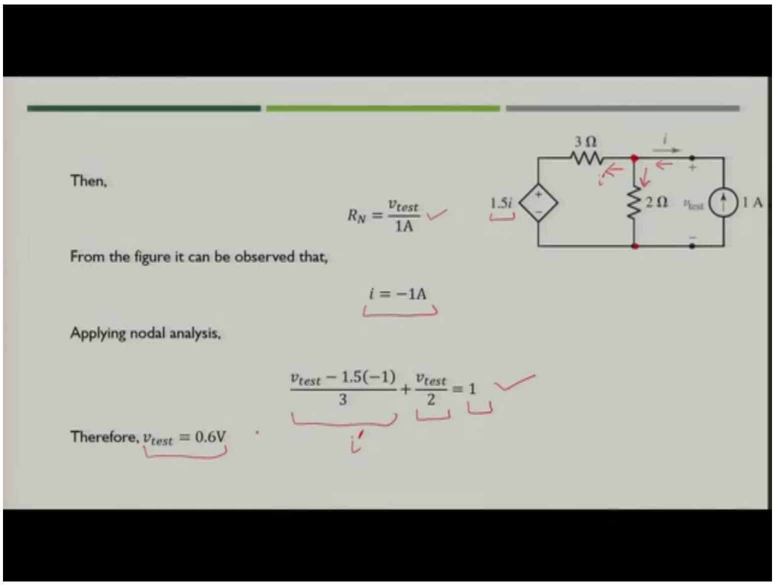

Now, the value of R_N that is Norton's resistance would be

$$R_N = \frac{v_{test}}{1\text{A}}$$

Now, if you see the figure what you can observe that the value $i = -1A$ because both are in opposite direction.

$$\frac{v_{test} - 1.5(-1)}{3} + \frac{v_{test}}{2} = 1$$

Therefore, $v_{test} = 0.6V$

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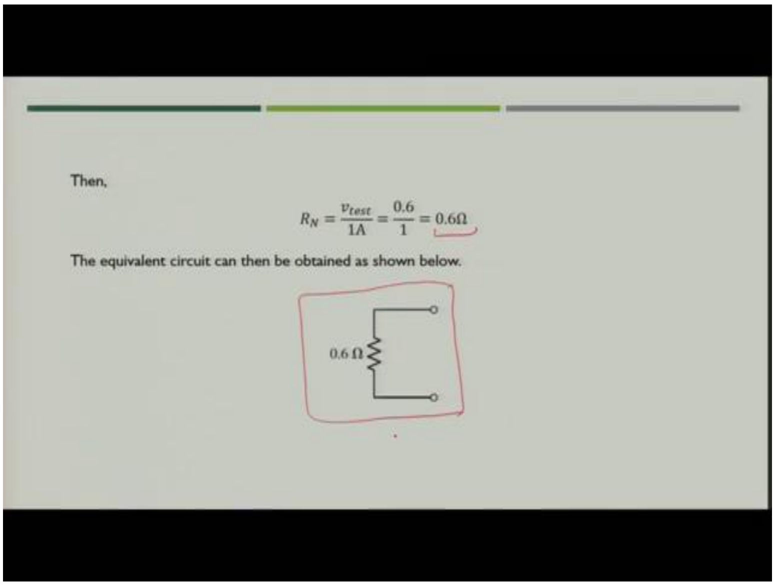

So, you get the value $v_{test} = 0.6V$.

Then,

$$R_N = \frac{v_{test}}{1\text{A}} = \frac{0.6}{1} = 0.6\Omega$$

The Norton equivalent of the circuit which is given in the example would be simply 0.6 ohm resistance without any current source. So, that means I_N will be 0 in this case.

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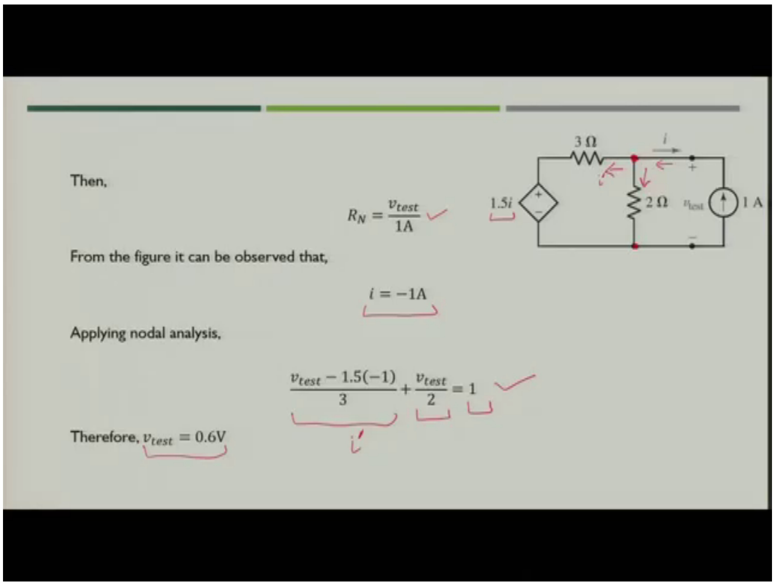

So, now, let us summarize what we discussed in case of Thevenin's and Norton's equivalent to determine the Thevenin's or Norton's equivalent the circuit can divided into 2 networks. As we see you have 2 sections of the circuit you can say this section is network A and where you see the load or the resistance which, which is of your interest to find out the value of either voltage or current that would be separated from the main circuit and it is called as network B.

So, you can divide the whole circuit into 2 parts. One is network A another is network B. So, what next is that network A simplified to evaluate either Thevenin's or Norton's equivalent. So, you will see terminal AB here the left of this would be having either Thevenin’s and Norton's equivalent so, that you can solve it by adding that equivalent to the resistance or that is the network B and find out the circuit variables across R_L.

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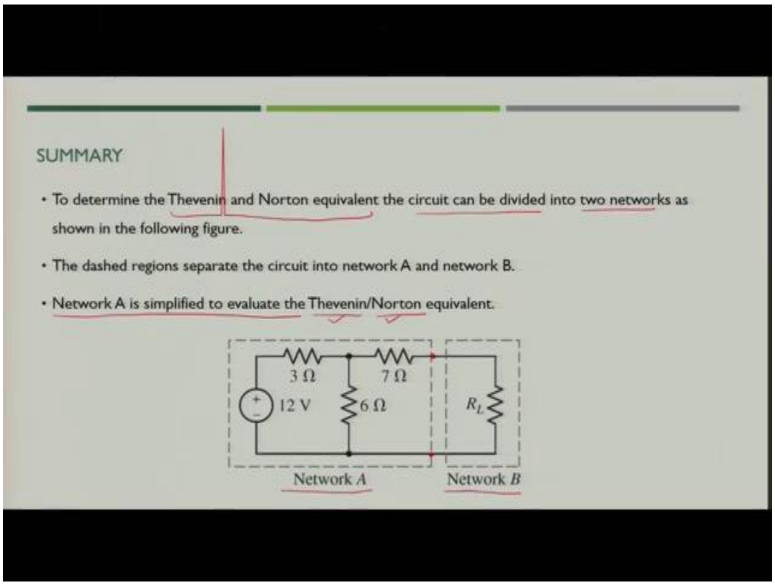

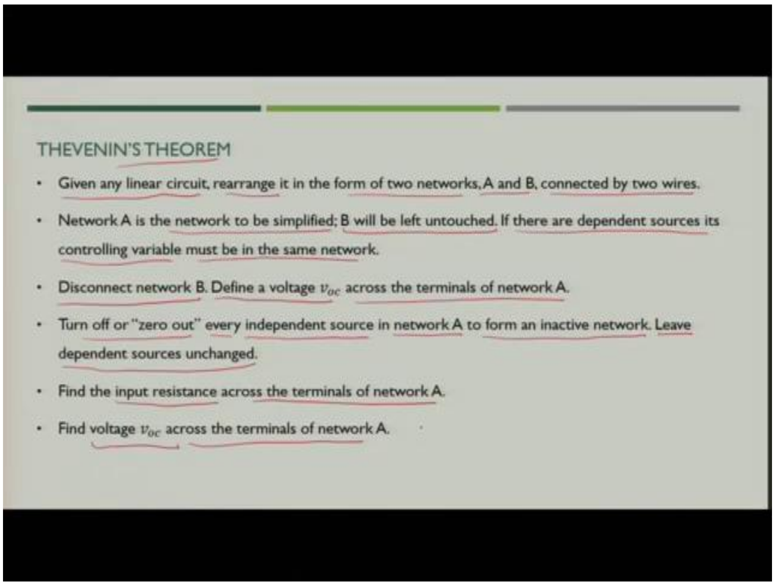

So, what we do in case of Thevenin's theorem. So, given any linear circuit, we rearrange it in the form of 2 networks A and B connected by 2 wires, network A is the network to be simplified and B will be left untouched. So, in this circuit, this would be simplified, then this would be untouched, if there are dependent sources, its controlling variable must be in the same network. That means if there is a dependent source in the circuit, the variable should also be in the same circuit.

So, both should be inside the network A for analysis. Now, disconnect the network be defined a voltage V open circuit across the terminal of network A. So, that means we will apply v_{oc} across these 2 terminals and turn off or zero out every independent source in the network A then what will happen we will form an inactive network. But, we have to leave the dependent sources unchanged. So, what will you get from this, we will get input resistance across the terminals of network A.

And when we get the input resistance, we again put all the Independent Sources back. That means when we found the input resistance of the circuit, we simply short circuited the voltage source and open circuit at the current source. So, we will again put the Independent Sources back to the circuit, and then we will find the voltage that is open circuit voltage across the terminal of network A.

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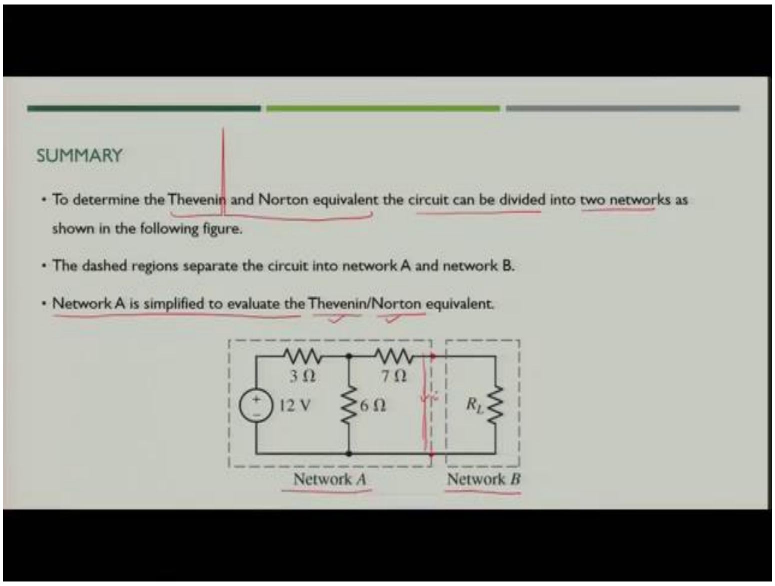

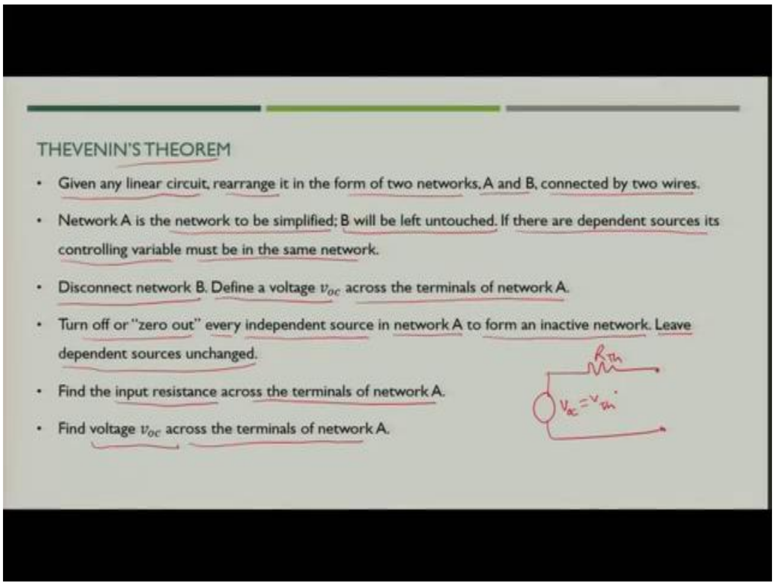

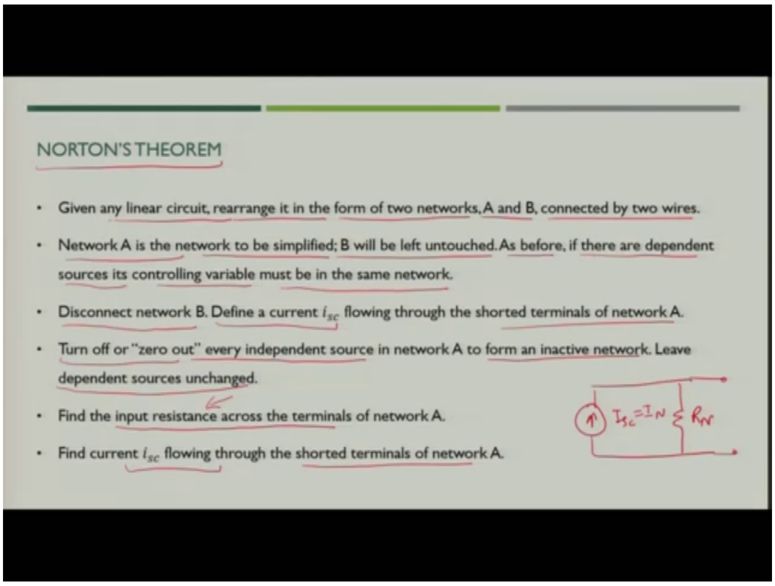

Opposite to the Thevenin's we have the Norton's theorem, because in this case, instead of open circuit voltage, we need to find out the circuit current. So, here also given any linear circuit, you have to rearrange it in the form of 2 networks that is A and B which are connected by 2 wires, network A is the network to be simplified and B will be left untouched as in the case of Thevenin's theorem in this case also the dependent sources and its controlling variable must be in the same network. Now, we disconnect the network B defined current i_{sc} that is the short circuit current so you will short circuit these 2 terminals right and you find out the value of circuit current flowing through the shorted terminal of network A turnoff or 0 out every independent source in the network to form an inactive network while keeping the dependent sources in the network.

Now, again, we have to find the input resistance. So, if you see Thevenin's and Norton's theorem, the input resistance is same in both of the cases the only change is the short circuit current which we need to find out in case of Norton's equivalent while in case of Thevenin's we need to find out the open circuit voltage. So, here if you see the opens the equivalent of the Thevenin's equivalent, this would be v_{oc} is nothing but V_{Th} in series with resistance R_{Th}.

So, this would be Thevenin's equivalent, while in case of Norton's you will have current that is short circuit current or you can say it is Norton's current and then in parallel you will have resistance R_{N} that is Norton's resistance, which will be, which would be found similar to what we did analysis for R_{Th}. So, in both of the cases finding out the resistance will be same, change would be in case of finding out either the short circuit current or open circuit voltage.

So, with this we close today's session. So, in this session we found the we analyzed the Norton's equivalent when the dependent sources were available. Thank you.