Basic Electric Circuits
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Module 4
Network Theorem 2
Lecture – 16
Norton's Theorem

Namaskar. So, in the last week we discussed about two very important theorems those were superposition theorem as well as Thevenin's theorem. So, we will continue in this week with few other theorems which are very important for the circuit analysis purpose and we will start with Norton's Theorem today.

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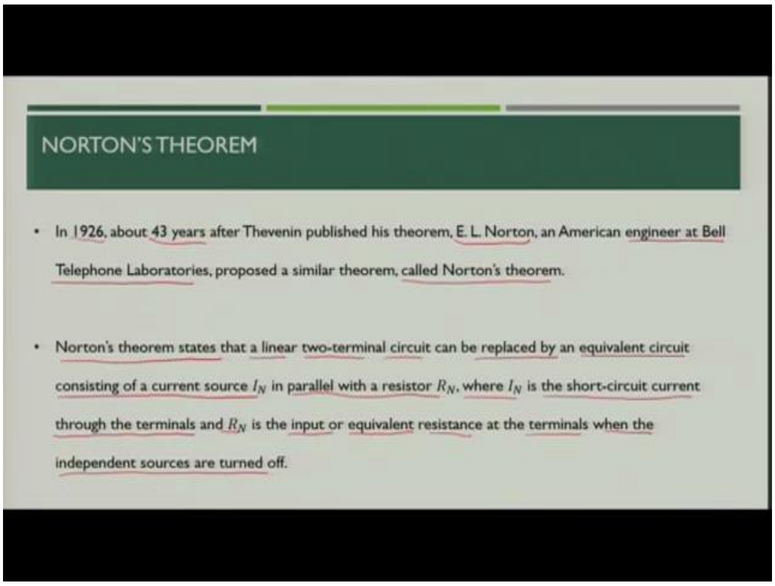

Let us see what does it mean. If you remember what we discussed in the last week about the Thevenin's theorem that Thevenin theorem was given by French telegraph Engineer in 1883. Then around 43 years after in 1926 another American Engineer called E. L. Norton gave the theorem called Norton's Theorem. So, Norton was an Engineer in the Bell Technical at Telephone Lab of USA.

So, what does Norton's Theorem means? So, Norton's Theorem says that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with resistor R_N where I_N is considered to be the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

So, again the turned off means the voltage source would be short-circuited and the current source would be open circuited.

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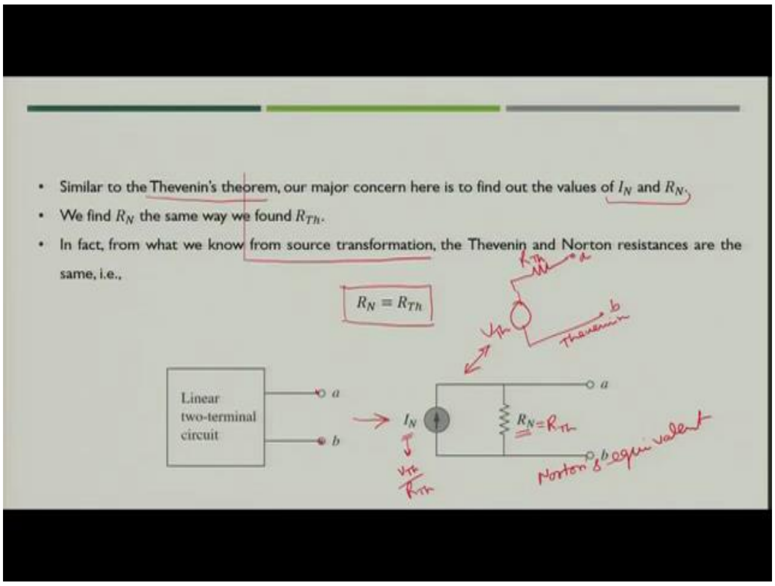

Now, if you correlate with the Thevenin's theorem you will see again as we saw in Thevenin theorem that our main concerned was to find out the value of V_{Th} and R_{Th}. Similarly, in Norton's Theorem also what we need to find out is I_N and R_N. So, now let us understand the Norton's Theorem from the linear two terminal circuit this is unknown circuit which is linear two terminal and we need to find out the Norton's equivalent of the circuit at terminal a, b.

So, if you recollect what we discussed in case of Thevenin's theorem that the linear two terminal circuit can be replaced with its equivalent or Thevenin equivalent circuit. So, what was that Thevenin equivalent? You saw that the Thevenin equivalent was nothing but V_{Th} in series with another resistance called R_{Th} and this was considered between two nodes.

Now, if you see again in case of Norton's Theorem what will happen? That you will change the circuit to a Norton's equivalent where the Norton's equivalent will have current source I_N in parallel with one resistance R_N so this is your Norton's equivalent of the circuit which is linear two-terminal.

Now, if you see this these two circuits that is Thevenin and Norton's equivalent they look very similar in the fashion that if you recollect the source transformation what we discussed in previous week, you can easily understand that if you carry out the source transformation from voltage source to current source what will happen? The voltage source divided by the R_{Th} that is the resistance in series would be nothing but the value of current source here.So, what you can say?

$$R_{Th} = R_N \text{ and } I_N = V_{Th}/R_{Th}$$

So, if you carry out the source transformation you will come to know that the Thevenin equivalent can be converted into Norton's equivalent. So, using this source transformation technique you can say that Thevenin resistance is nothing but Norton's resistance.

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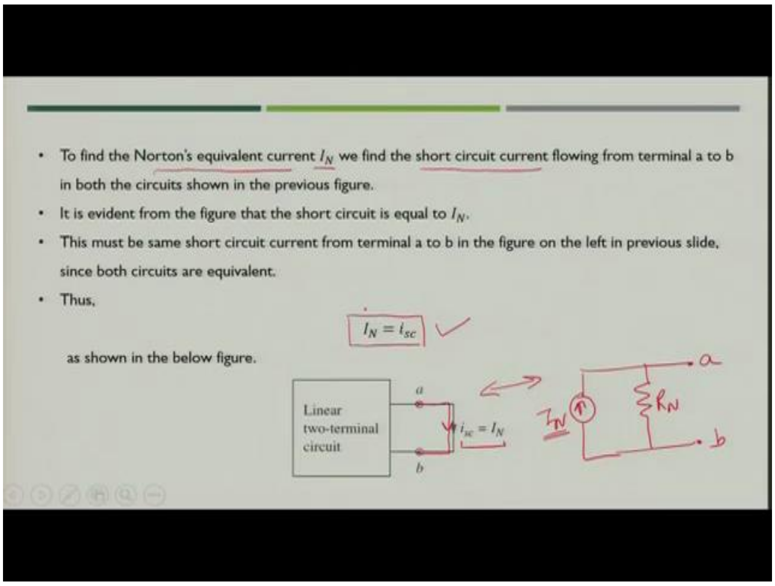

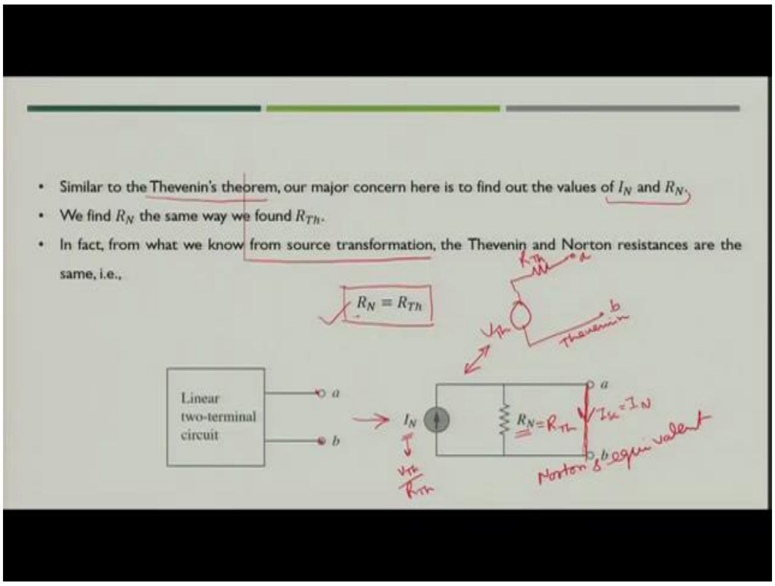

So, now the Norton's equivalent current how we will find out? We will find out while carrying out the short-circuit test on the network which is given to us. So, now if you see in this case if this is a two-terminal linear circuit we have to short-circuit these two-terminals and we have to measure the current that is short-circuit current. Now, when we find out the short-circuit current we will come to know that short-circuit current is nothing but the Norton's current, how?

Now, if you see the Norton's equivalent circuit now if you short-circuit a and b the current flowing through the short-circuit terminal that is between a, b would be nothing but I_N why? Because of the short-circuit the R_N will not be a significant component because it will again be short-circuited so if current I_N would be nothing but the short-circuit current. So, that is why when we short-circuit the terminal we get the Norton's current is nothing but short-circuit current.

So, in that way what we can say? We can say that the Norton's equivalent circuit if you draw Norton's equivalent circuit here the resistance R_N, current I_N that is between a, b. So, if you compare these two we can say that these two circuits are equivalent because their V-I characteristics at terminal a and b are same. So, in that way what we can say? We can say that the value of Norton's current that is I_N is nothing but the short-circuit current across which is flowing between a and b when it is short-circuited.

So, this you can calculate with the help of various circuit techniques like we discussed Kirchhoff Voltage Law or Kirchhoff Current Law and the R_{Th} that is Thevenin resistance which is nothing but the Norton's resistance also we can utilize the same technique which we utilize while we were discussing the Thevenm's theorem to find out the value of Norton's resistance.

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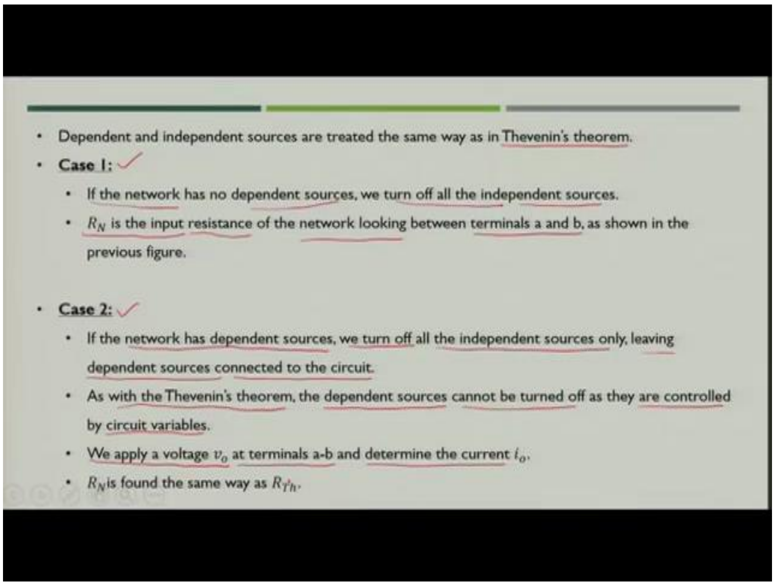

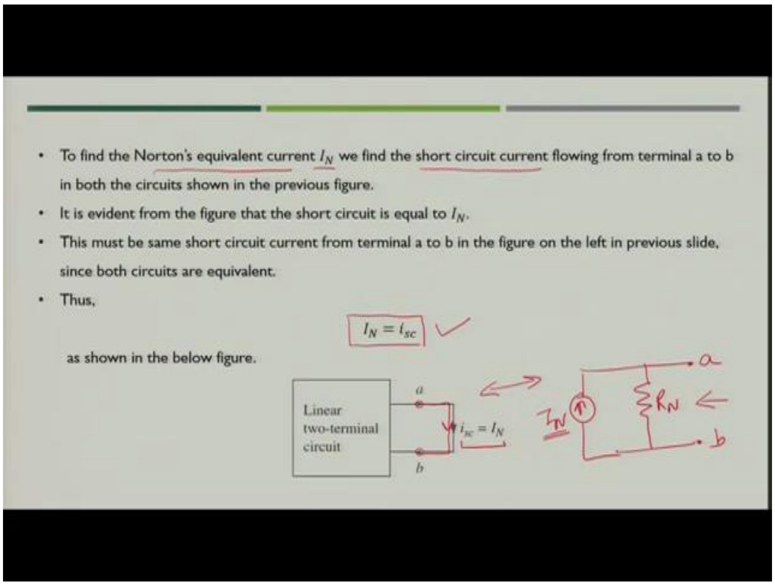

Now, as we discussed in case of Thevenin's theorem in this case also you will have two options rather we say two cases to find out the value of Norton's resistance. First case is that when you have the network which contains no any dependent source so in that case what we have to do? We have to simply turn off the independent sources. Now, what would be the value of R_N? R_N would be nothing but input resistance of the network looking between the terminals a and b so that is what we saw in the previous figure.

So, if you look from the side a, b the input resistance would be nothing but Norton's resistance. So, you will get R_N as an input resistance which would be between terminal a and b. Second case would be the case when network has dependent sources. So, what we have to do? We can only turn off the independent sources which are connected to the network while leaving dependent sources connected to the circuit.

Now, as we saw with the Thevenin's theorem in this theorem also the dependent sources cannot be turned off because they are controlled by the circuit variables. So, we will keep the dependent sources in the network and we will solve the circuits to find out the value of I_N that is Norton's current and R_N that is Norton's resistance.

So, what we have to do in that case? We have to again as we did in case of Thevenin's theorem here also we will apply voltage v naught at the terminals of a, b and determine the current i_0 and using these values we can easily find out the value of R_N which is nothing but equal to R_{Th}. So, Norton's resistance and Thevenin resistance would be same.

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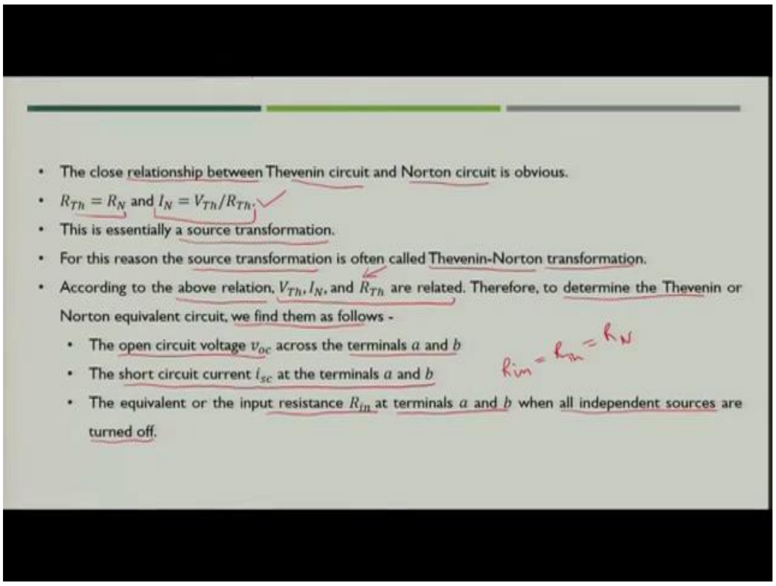

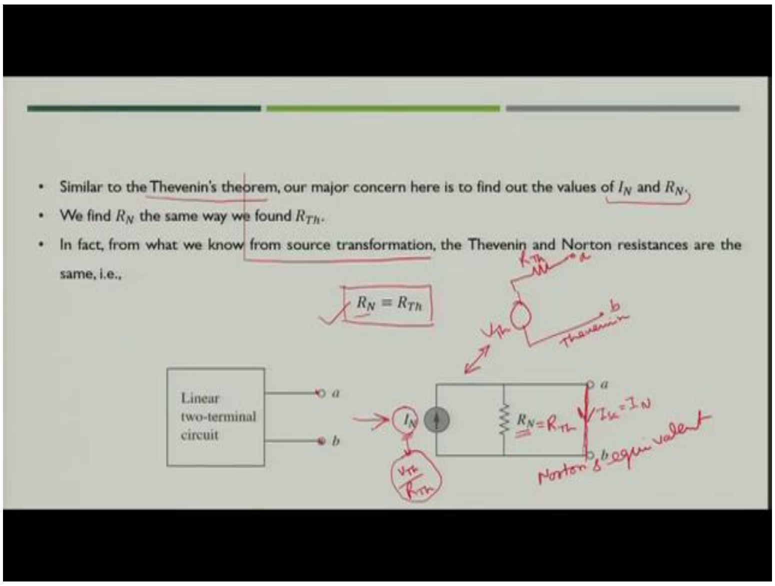

Now, let us understand the close relationship between Thevenin circuit and Norton's circuit. We have just established that R_{Th} that is Thevenin resistance is equal to Norton's resistance and as we discussed in case of source transformation this is what we discussed here we established that when we carry out the source transformation the Norton's current is nothing but V_{Th}/R_{Th}.

So, we can say that Thevenin and Norton's theorem are somehow related with each other with the help of source transformation. So, R_{Th} is equal to R_N and I_N that is Norton's current would be equal to V_{Th}/R_{Th}. So, you can simply say this is nothing but a source transformation that is why the source transformation is also called as Thevenin Norton's transformation.

Now, according to the above relationship what we can see that V_{Th}, I_N and R_{Th} are related. So, therefore to determine the Thevenin or Norton's equivalent circuit we need to only find them with the help of a circuit analysis technique that may be the Mesh analysis or a Nodal Analysis. So, we have V_{Th}, I_N and R_{Th} unknown variables which we have to find out.

So, what we generally do? We find out the open circuit voltage across the terminal a, b because in case of Thevenin voltage Thevenin what we have to find out? We have to find out the voltage that is open circuit voltage across terminal a, b and we need to find out short-circuit current at terminal a, b which is required when we are trying to find out the Norton's equivalent and then input resistance across seen through the terminal a and b when all independent sources are turned off.

So, these would be nothing but equal to R_{Th} or you can say that it is equal to R_{N} also because R_{Th} and R_{N} are essentially the same.

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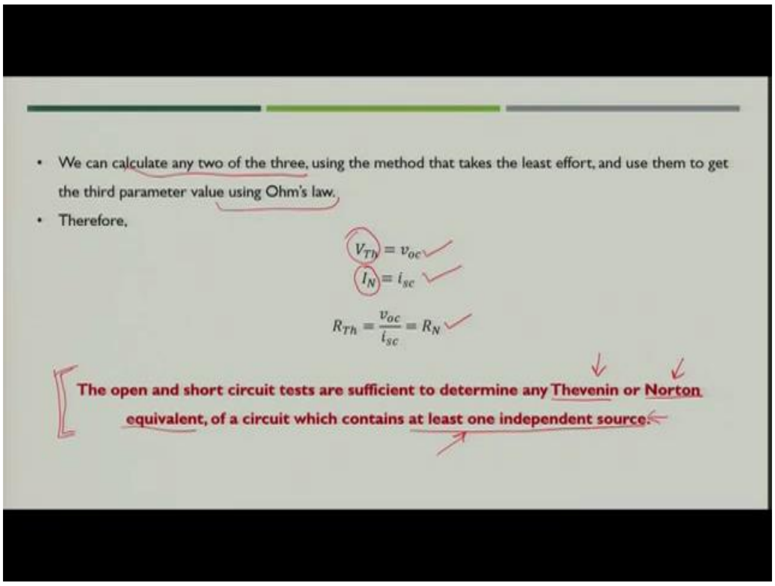

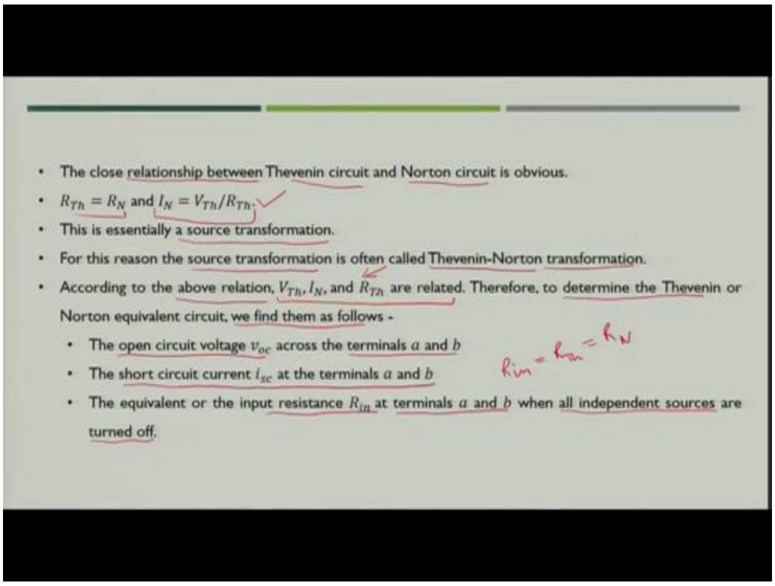

So, what we can do? These are the three unknown quantities like V_{Th}, I_N and R_{Th} so we need to calculate two of them and then we use the ohms law to find out the third one. So, what we get? We get V_{Th} is nothing but open circuit voltage across the terminal a, b Norton's current is nothing but short-circuit current flowing between a and b when both are short-circuited and then R_{Th} that is Thevenin resistance is nothing but open circuit voltage divided by short-circuit current and this would be equal to Norton's resistance.

So, now you can say in summary that the open and short-circuit test which we carry out at the terminal a, b are sufficient to determine any Thevenin or Norton equivalent of the circuit which contains at least one independent source. So, this is required because when we discussed the

Thevenin theorem and we discussed dependent sources we stabilized that if you do not have independent source then you cannot determine the value of V_{Th}.

So, similarly in this case also if you do not have dependent source, you do not have any independent source in the circuit you cannot stabilized the value of current I_N. So, this we have to keep in mind that there should be at least one independent source which helps you to determine the value of Thevenin and Norton's equivalent.

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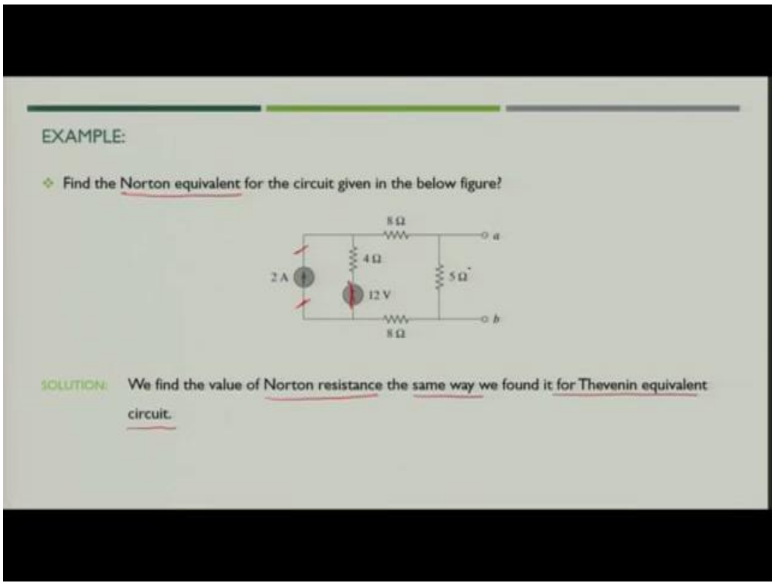

Now, let us understand the concept with the help of one example. So, the circuit which is given in the figure we need to find out the Norton's equivalent of the circuit. Now, we can also find out the value of Norton's resistance the same way as we found the value of Thevenin equivalent circuit that means what we have to do? We have to short-circuit the voltage source and open circuit the current source.

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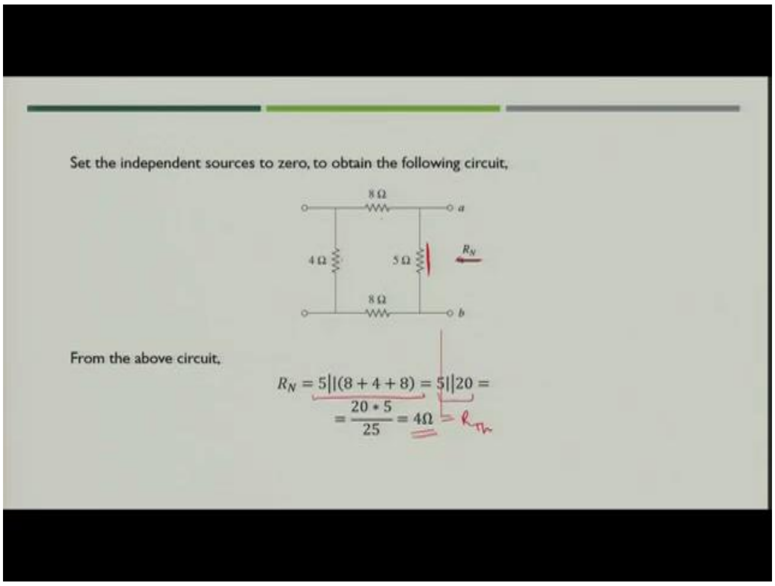

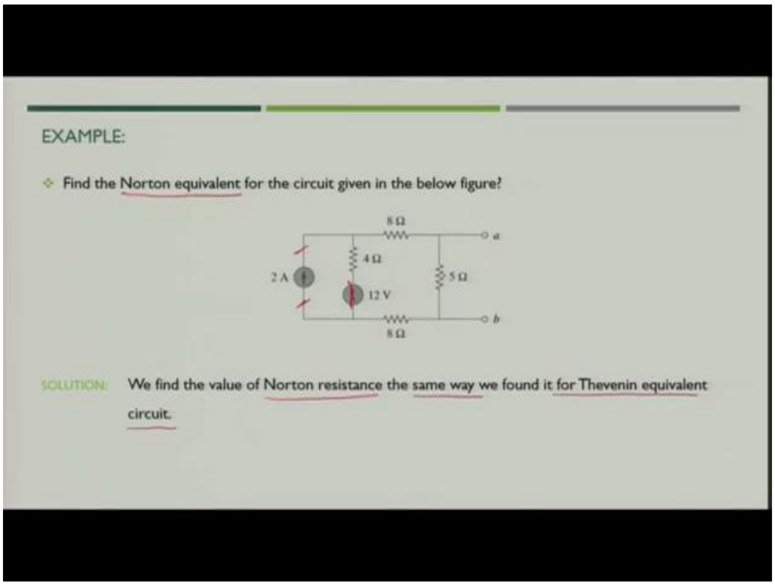

So, what would be the updated circuit? The updated circuit would look like as shown in the figure where voltage is voltage source is short-circuited and current source is open circuited and we are left with only the resistances. So, we have four resistances of 8 ohm, 5 ohm again 8 ohm and 4 ohm. So, these are in this combination when you see resistance that is input resistance across terminal a, b that would be nothing but the Norton's resistance or you can say Thevenin resistance, what would be the value? If you see the circuit you will come to know that

$$R_N = 5||(8+4+8) = 5||20 =$$

$$= \frac{20 * 5}{25} = 4\Omega$$

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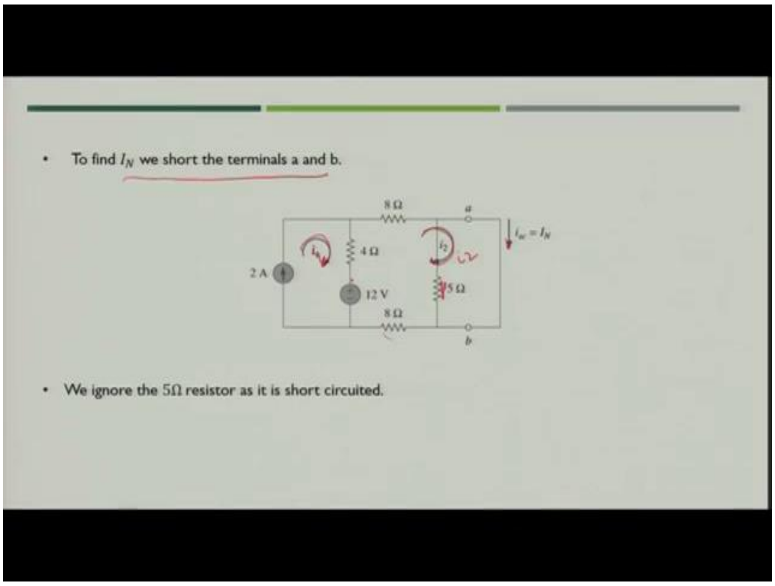

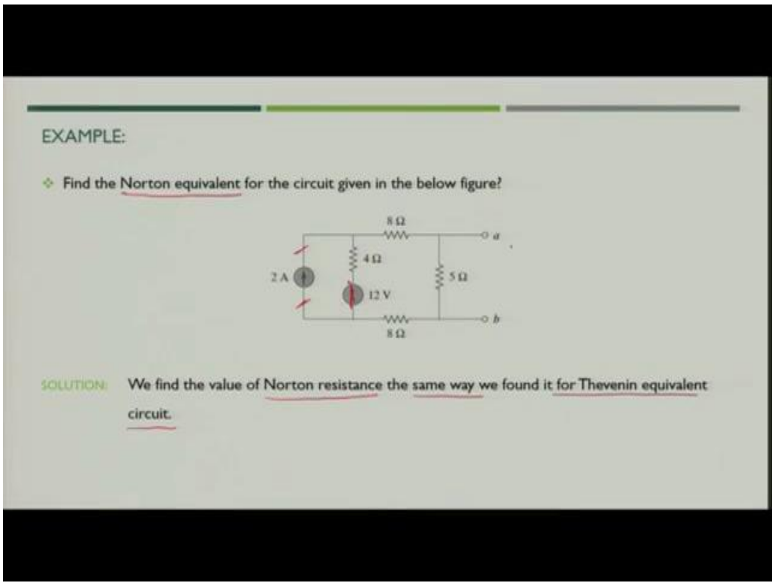

Now, the next task would be to find out what? The short-circuit current. So, what we have to do in that case? We have to first short-circuit the terminal a, b. So, we need to find out the short-circuit current flowing through terminal a, b and that would be nothing but the Norton's current. Now, if you see the original circuit we have just simply short-circuited the terminal a, b.

Now, when you will when you short-circuit the terminal a, b what will happen? If you see the circuit carefully you will see that 5 ohm is now in parallel with the short-circuit wire. So, that means that 5 ohms resistance is also short-circuited so you can neglect this 5 ohm resistance from the circuit. So, in that way we will be left with only two meshes which would be connected.

Let us say this is the mesh current as i_1 and this is mesh current as i_2 which is flowing through 4 ohm, 8 ohm and then this through short-circuit wire then again 8 ohm and 12 volt source.

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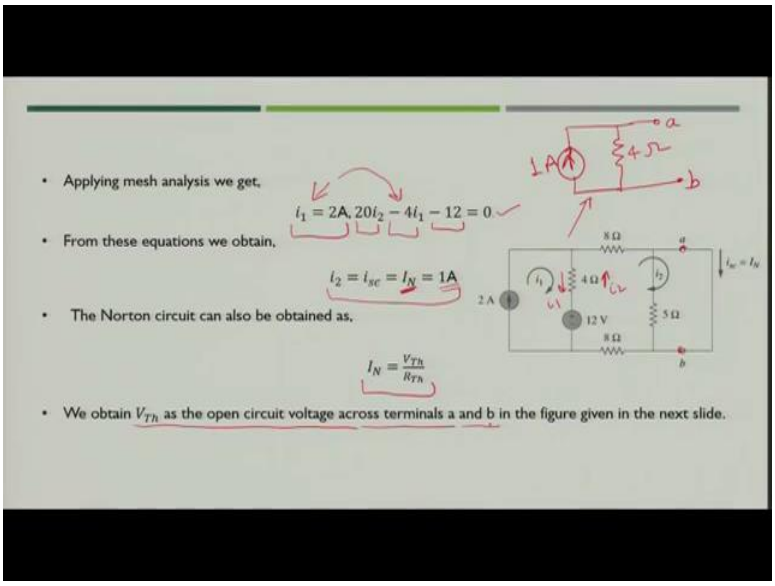

So, now you have two meshes in the circuit what we get from these two meshes? If you see mesh one the value of mesh current is i_1 is equal to 2 ampere. So, this you can easily see from the circuit because 2 ampere is the current source which is available in the mesh so the current i_1 was nothing but 2 ampere. Next is you need to find out the value of current i_2, so how you will get i_2? You have to just simply write the mesh equation for this mesh.

$$20i_2 - 4i_1 - 12 = 0$$

Now, you know the value of i_1, which you use get the current i_2 that is nothing but the short-circuit current because i_2 is nothing but the short-circuit here this is also a Norton's current and you get the value of Norton's current as 1 ampere. So, what Norton's equivalent you will get? You will get Norton's equivalent as shown in the figure. Now, you will have terminal a and b this you have just calculated equal to 4 ohm and this is 1 ampere.

So, basically the Norton's equivalent of this circuit when it is not short-circuited would be 1 ampere in parallel with 4 ohm resistance. So, now you got the Norton's equivalent next what you can do? Norton's current can be calculated with the help of Thevenin's voltage divided by Thevenin resistance.

So, what we have to do? We have to utilize the knowledge of Thevenin's theorem and we need to find out what would be the open circuit voltage across terminal a and b.

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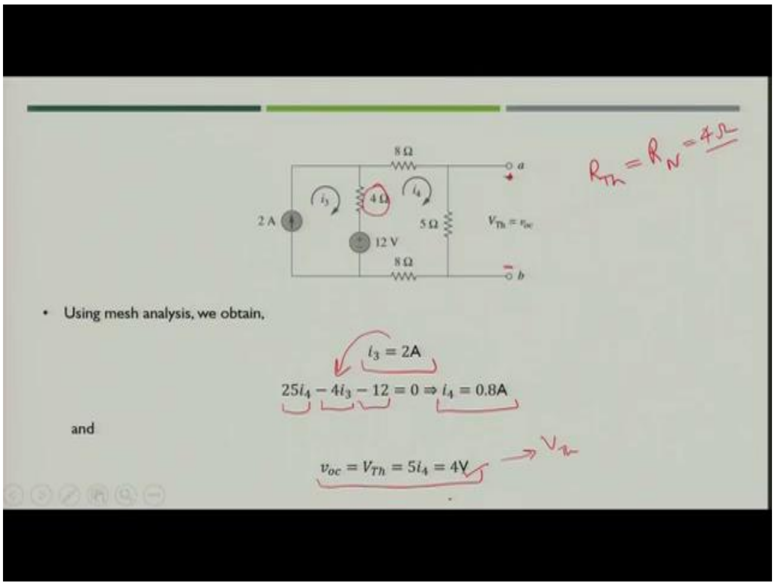

So, now if you see the figure again we are not going to find out the value of Thevenin resistance because we know that Thevenin resistance is equal to Norton's resistance and which we obtained as 4 ohm. Now, next task what we have to do is that we have to find out the value of Thevenin voltage and that is nothing but the open circuit voltage across terminal a, b.

If you see the circuit you will get two meshes in the circuit. Let us say the mesh current is i_3 and i_4 now, what would be the value of i_3? Value of i_3 would be 2 ampere which you can see from this mesh. Now, again to find out the value of i_4 you will start with -12 that is you are starting from minus to plus when the direction of current is seen from this side then you have now four resistances in this resistance your current is i_2 in this direction, i_1 is in this direction.

So, you will get i_3 in this direction and i_4 in this direction so you will get $-4i_3$ as a component and then you will sum up all the resistances multiplied by i_2, so what will you get? You get $25i_4$. So, for this mesh you have developed the equation like $25i_4 - 4i_3 - 12 = 0$ and since you know the value of i_3 you put the value of i_3 here and you will get i_4 is nothing but 0.8 ampere.

Now, what would be the value of open circuit voltage? The open circuit voltage value would be v_{oc} that is $5i_4$ which is nothing but your Thevenin voltage.

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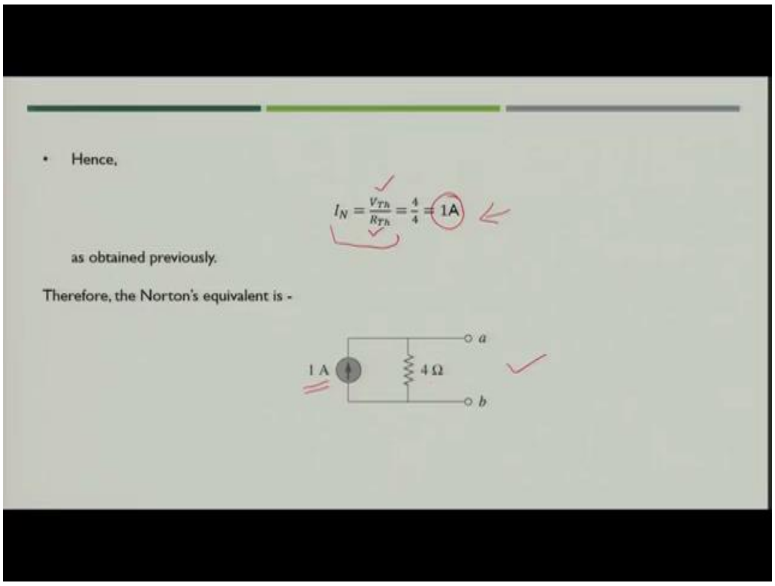

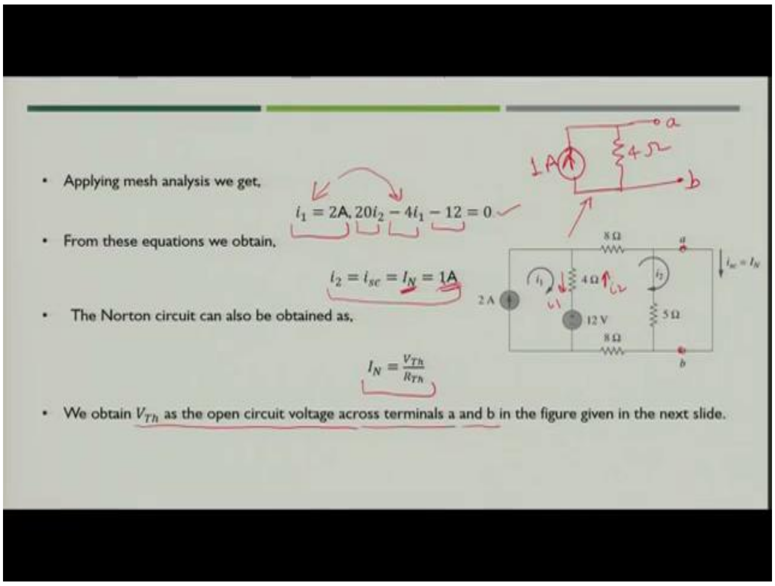

Now, we have got the Thevenin voltage, we have got the Thevenin resistance so using this formula $I_N = \frac{V_{Th}}{R_{Th}}$ you get again the value of Norton's current as 1 ampere and this is what we calculated when we directly applied the Norton's theorem. So, we can say that the Thevenin theorem and Norton's theorem are dual to each other you can simply use the source transformation and convert them into the from Norton's to Thevenin and Thevenin's to Norton equivalent circuits.

So, finally you got the Norton's equivalent where you have 1 ampere in parallel with 4 ohm resistance.

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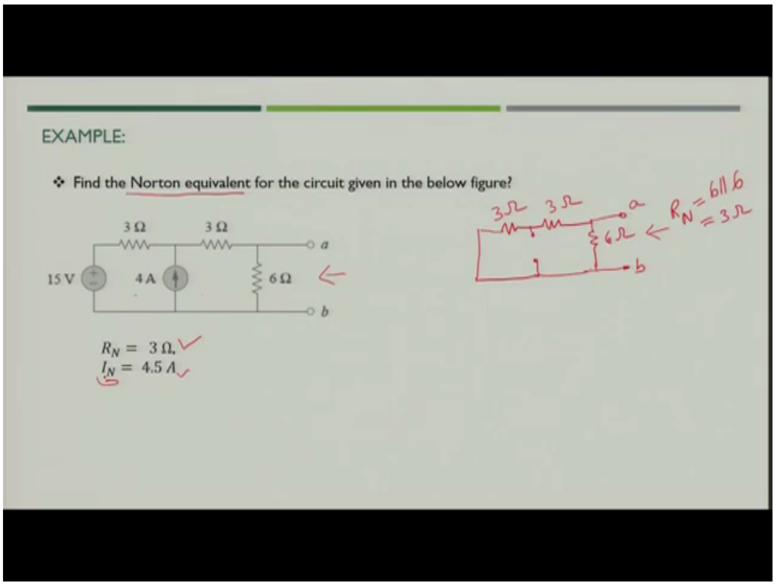

So, now let us see another example where you need to find out the Norton's equivalent for the circuit given in the figure. Now, here the important thing is that the source which is 4 ampere current is common to both of the meshes. So, what we can do? We can utilize our source transformation technique and then we can find out the values of Norton's current. So, first let us try to find out what would be the value of Norton's resistance that is input resistance when you will see from this through this a, b terminal.

So, what we have to do? We have to short-circuit the voltage so what we will get? We will get the circuit like this so this is 3 ohm this would be open circuited because current source would be open circuited then you have 6 ohm, this is 3 ohm and this is the terminal a and b. So, when you see this updated circuit you will come to know that 3 ohm and 3 ohm are in series and the series combination of these 3 ohms is in parallel with 6 ohm.

So, what you get? You get eventually the 6 ohm in parallel with another 6 ohm resistance. So, you can simply find out the value of R_N is nothing but 6 ohm in parallel with 6 ohm that is nothing but 3 ohm so you got now Norton's resistance. Now, next task is to find out the value of Norton's current.

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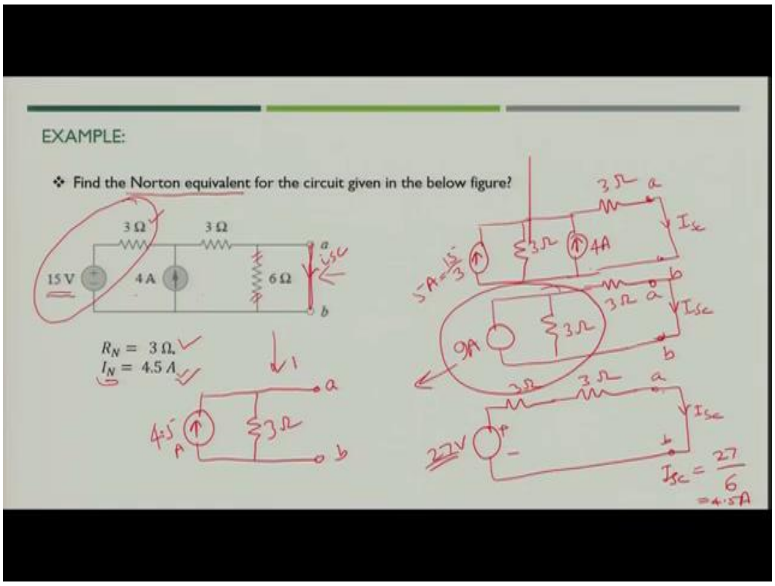

So, what you will do? Let us do the calculation of Norton's current with the help of the circuit we will just simply short-circuit the terminal a, b so the circuit would be let us apply here the short-circuit current so here you have short-circuited current i_{sc}. Now, when you have this terminal a, b short-circuited the 6 ohm resistance will become irrelevant. So, you have just simply 4 ampere current source 3 ohm resistance short-circuit and this 15 volt voltage source in series with 3 ohm resistance.

Now, if you see this particular segment of the circuit this segment of circuit is nothing but voltage source in series with 1 current voltage source in series with 1 resistance you can apply source transformation so what will happen? You will convert this into current source in parallel with resistance what would be the value of this current source? Current source value would be 15 by 3 that is nothing but 5 ampere and in parallel with one resistance that value of resistance would be 3 ohm.

Now, you have the given current source and you have another 3 ohm resistance and then you have terminal a, b and this is the short-circuit current. Now, if you see these two current sources are in parallel so updated current source you can add both of them and it will become 9 ampere then you will have another resistance 3 ohm in parallel 3 ohm resistance in series and then you have short-circuited the terminal a, b.

Now, you know you can again transform the circuit back into voltage and one resistance in series so what will happen? If you transform again into voltage source so this will become 9 into 3 that is 27 volt then in series with 3 ohm resistance again in series with 3 ohm resistance which is given in the figure and then you are short-circuiting the terminal a, b. Now, it has become very simple which you can see simply from the circuit itself that what would be the value of short-circuit current? Here the short-circuit current value would be that is the voltage source 27 divided by 6 so what you got is 4.5 ampere.

So, now you got Norton's current that is the short-circuit current as 4.5 ampere. So, finally what would be your Norton's equivalent? Norton's equivalent would be 4.5 ampere current source in parallel with the resistance that is 3 ohm across terminal a, b. So, you get the Norton's equivalent of the circuit. So, with this we close our today’s session. So, in the session we discussed about the Norton's theorem, a particularly in those cases where the independent voltage or current sources available.

So, we will continue our Norton's theorem discussion in the next class where we will discuss more about the conditions when the dependent sources are available in the circuit and we will come to know how we will solve those circuits, thank you.