Basic Electric Circuits
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Module 3
Network Theorem 1
Lecture – 15
Thevenin's Theorem with Dependent Sources

Namaskar. So, in this week we discussed few important properties of the network. We started with linearity property of the circuit where we discussed what is homogeneity and additivity and then we (proceed) proceeded towards the superposition theorem and then we discussed the other concepts like source transformation and the duality and then in the last class we started our Thevenin's theorem.

So, in the last class we basically discussed the Thevenin theorem aspect when the source is non-dependent source, so that can be like voltage or current both were independent sources. So, in this class we will continue our discussion on the Thevenin's theorem but we will discuss more about the dependent sources that may be the voltage or current.

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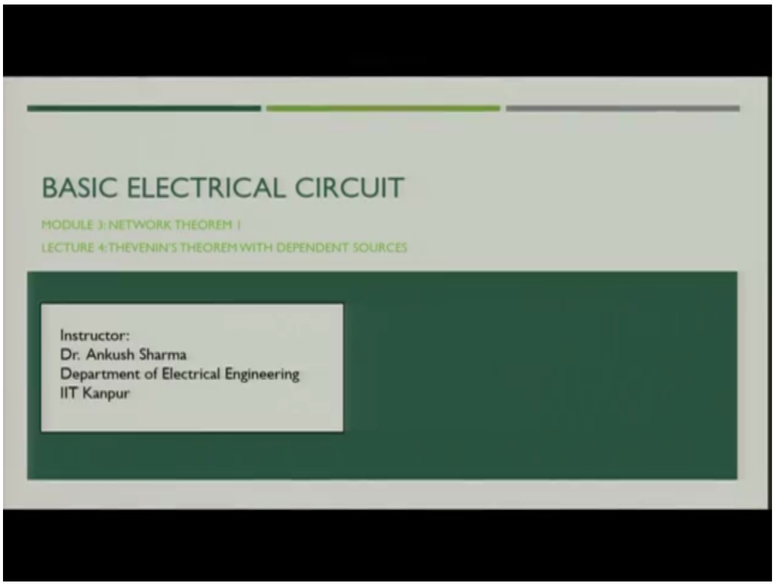

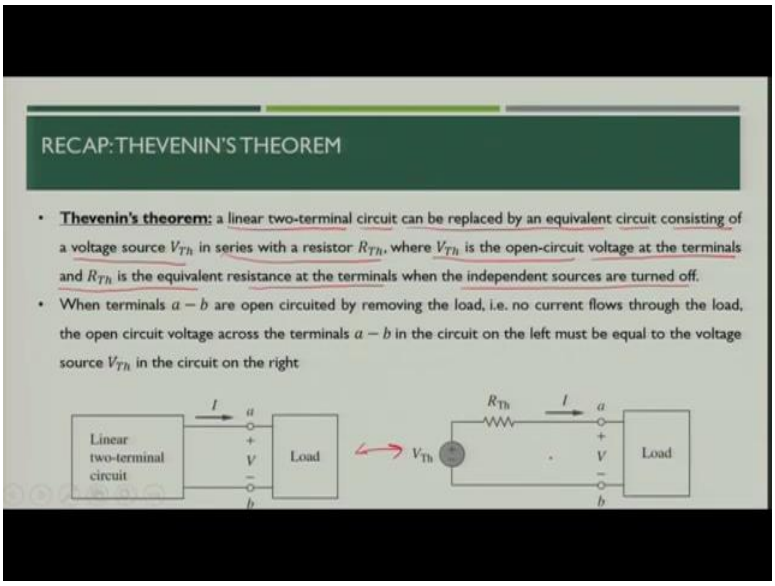

So, now let us start. First, let us recap what we discussed in the last class. We discussed Thevenin's theorem and we discussed that the linear two terminal circuit can be replaced by an equivalent circuit consisting of the voltage source V_{Th} in series with a register R_{Th} where V_{Th} is an open circuit voltage at the terminals and R_{Th} is the equivalent resistance at the terminals when the independent sources are turned off.

So, when we calculate R_{Th} we make the independence sources turned off, which means that independent voltage source would be short circuited and independent current source will be open circuited. So, if we have a fairly large circuit and we want to study the load connected across two terminals then the circuit which is left at the nodes a and b is equal to the voltage and resistances shown in the figure.

So, voltage would be can set as a Thevenin voltage and resistance would be consider as Thevenin resistance and we will see that the V-I characteristic across terminal a and b will be same in both of the cases. So, that means that both of the circuits are equivalent because their V-I characteristics are same.

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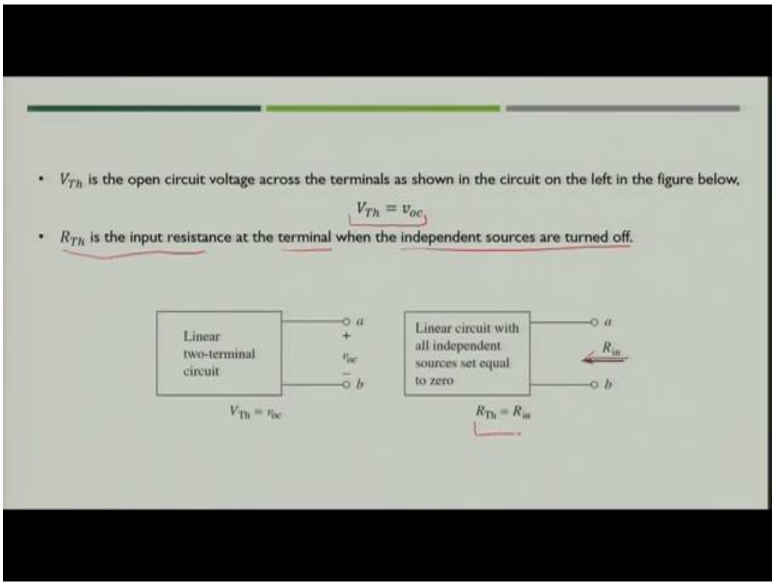

So, this is what we discussed in the last class and then we established that Thevenin voltage is nothing but open circuit voltage across the linear two terminal circuit and R_{Th} is nothing but the input resistance at the terminal when independent sources are turned off. So, if you look into the terminal a and b the equivalent resistance which you will see from here is nothing but the Thevenin resistance.

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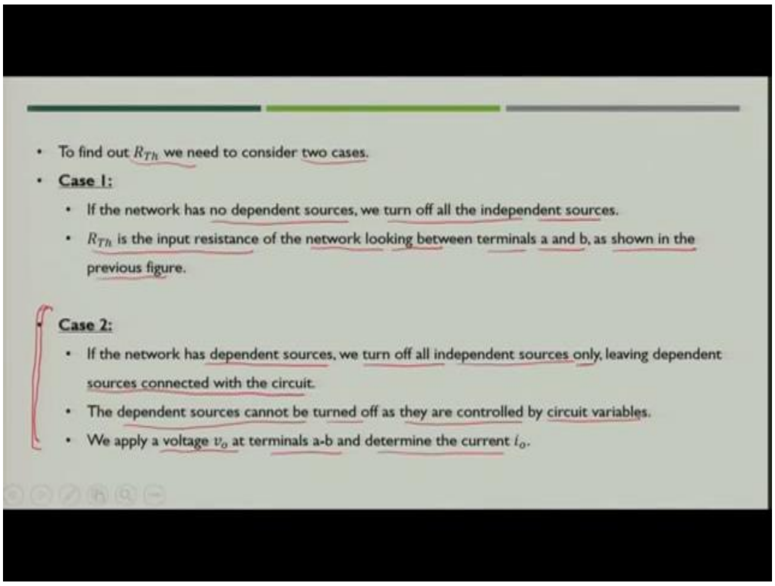

Now, we also discussed that for calculation of Thevenin resistance we need to consider two cases. What was the first case? In first case the network has no dependent sources and we turn off all the independent sources. As we just discussed that voltage source would be short circuited and current source will be open circuited and then R_{Th} is the input resistance of the network looking between the terminals a and b that was shown in the previous figure.

Now, in this session we will discuss more about the case 2. In this case the network has dependent sources, we cannot turn off the dependent source because it is dependent upon some circuit variable. So, we will keep dependent source connected with the circuit while we can turn off all the independent sources only and then since the dependent source cannot be turned off, because they are controlled by the circuit variables, we will keep them in the circuit and we will apply voltage that is v_0 at the terminal ab and determine the current i_0.

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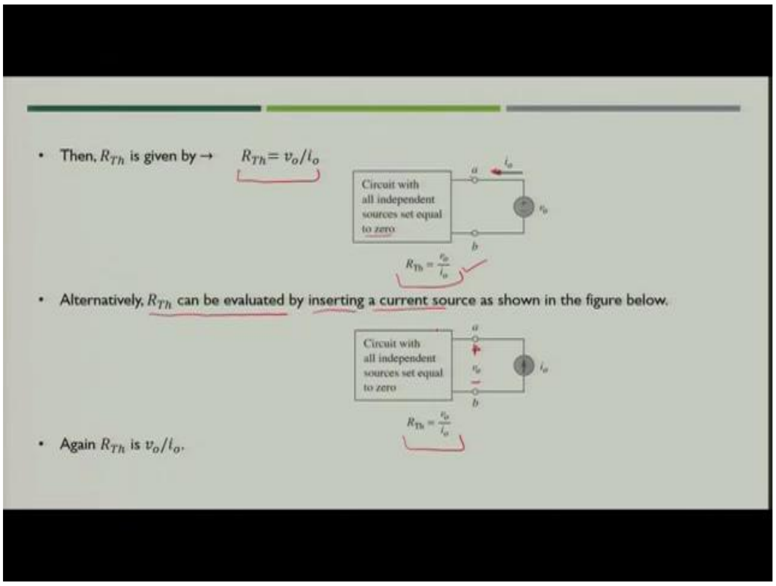

So, what does it mean? If you have circuit with all independent sources set equal to zero and the keeping all the dependent sources connected with the network, the terminal a and b would be supplied with voltage v_0 which will supply some current i_0. So, in that case if you solve the circuit with the help of either Kirchhoff Voltage Law or Kirchhoff Current Law you will be able to find out the value of Thevenin resistance which would be nothing but v_0/i_0.

So, when you have dependent source available in the circuit which you cannot remove you will first remove all the independent sources and then apply voltage v_0 and try to measure the current i_0 and accordingly you can find out the value of Thevenin resistance. Now, alternatively R_{Th} can also be evaluated by inserting a current source. So, instead of voltage source if you apply current source you will measure the voltage across these two terminals and when you measure you will get again the value of R_{Th} as v_0/i_0.

So, in both of the cases you can use either of them and you will get eventually the same result.

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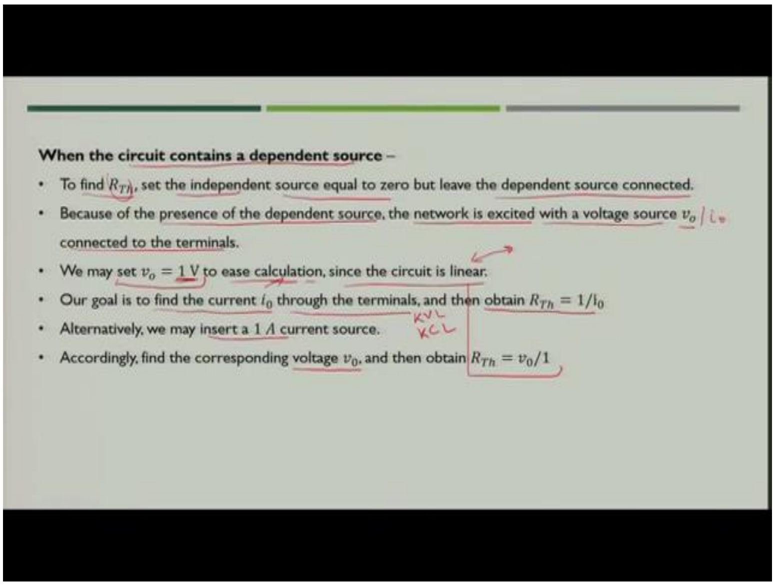

So, what the process we need to follow when the circuit contains a dependent source? We have to find R_{Th} that is Thevenin resistance we have to set the independent sources equal to zero and leave the dependent sources connected. Now, since the network has dependent sources we have to excite the network from outside with the help of either voltage v_0 or maybe the current source i_0 which would be connected to the terminals.

Generally, we set v_0 equal to 1 because it will be easier for our calculation and since the circuit is linear that means that we follow the homogeneity concept. So, whether it is 1 volt or 10 volt the response of the circuit will be scaled up accordingly and being a linear circuit, it is very easy for us to apply any voltage which we want across the terminal, but the easiest is 1 volt because it makes calculations easier.

Now, we have to find out the value i_0 because we have added voltage v_0 across the circuit. So, i_0 we can calculate which would be flowing through the terminals with the help of either KVL or KCL and then we obtain the value of Thevenin resistance that would be nothing but v_0/i_0. v_0 is in this case 1 so it will become $1/i_0$. We may insert 1 ampere current source so this will give you the R_{Th} value as $v_0/1$ because 1 is the value of current source.

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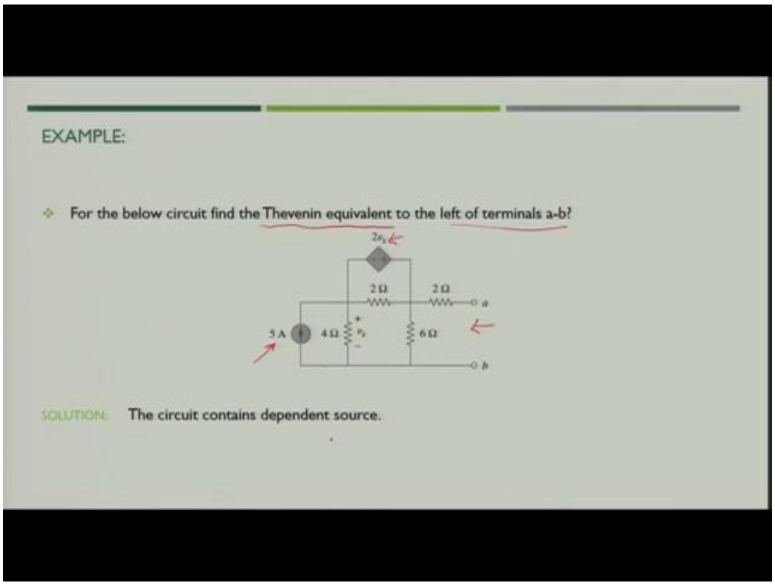

Now, let us understand this concept of how to solve the circuit when we have the dependent source with the help of one example. Now, let us see this particular circuit where you will see we have one independent current source of 5 ampere value additionally we have one voltage source which is dependent voltage source and the value of voltage is depending upon the voltage across 4 ohm resistance.

Now, what we have to do? We have to find the Thevenin equivalent to the left of the terminal ab. So, we have to find out the Thevenin equivalent which would be the equivalent of the complete circuit.

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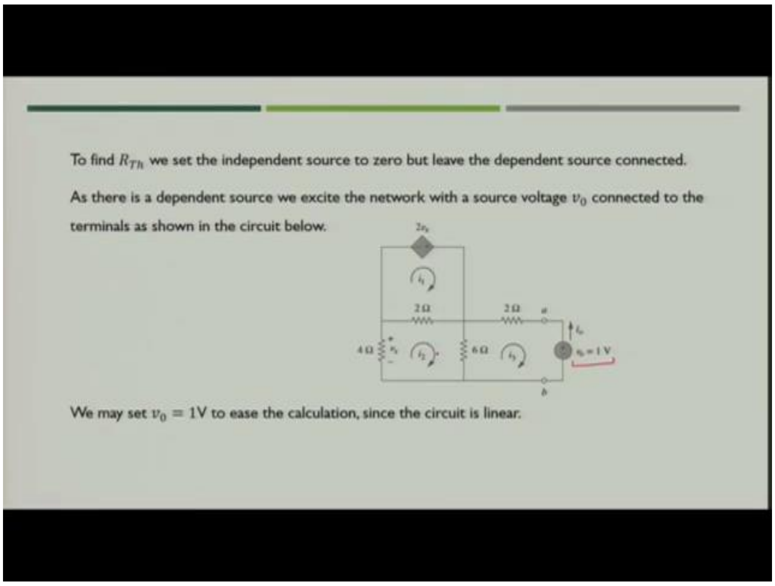

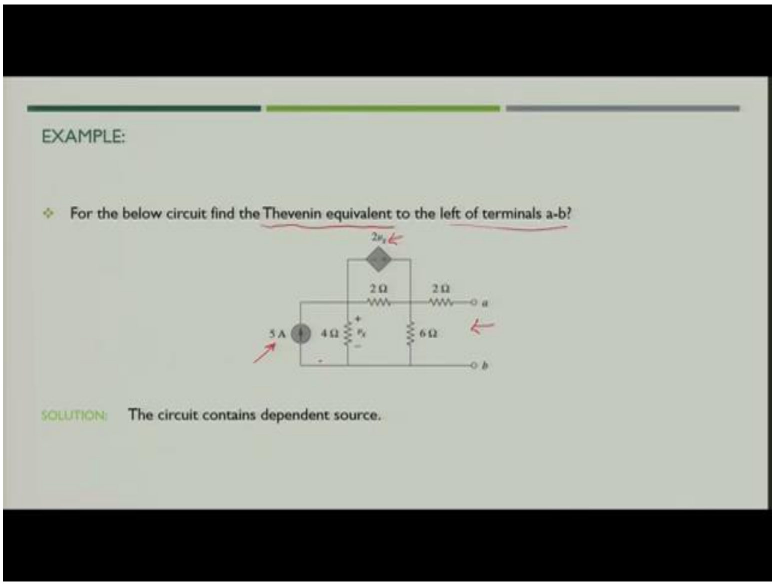

Now, let us solve it, what we have to do first? First, we have to solve for R_{Th} that is Thevenin resistance, in that case what we can do? We can remove all the independent sources, in this case this is a current source. You can make it open circuited. So, the resultant circuit would be like this and what we need to do is that? We need to add one voltage source v_0 that is equal to 1 volt which would be connected across the terminals a and b and then solve the circuit.

So, here we have made the current source as open circuit which was connected across 4 ohm resistance and additionally we have added one voltage source across terminal a, b that is $v_0 =$ 1 volt. So, the updated circuit is as shown in the figure then you will see that how you will solve? You can use Mesh Analysis which we discussed earlier. So, here we have three meshes connected so we will have three mesh currents like i_1, i_2 and i_3.

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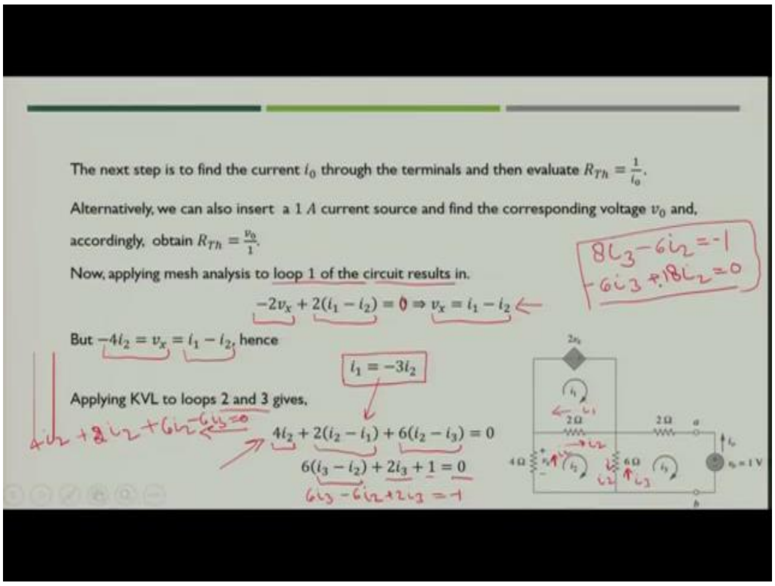

Now, what we have to do? We have to solve the circuit using mesh analysis. So, what would be the value of the mesh equation in case of loop 1? So this is loop 1 so when you solve since it is going from minus to plus across the dependent voltage source it will become

$$-2v_x + 2(i_1 - i_2) = 0 \Rightarrow v_x = i_1 - i_2$$

Now, if you see this particular mesh you will come to know the voltage drop across 4 ohm resistance is nothing but 4 ohm into current i_2. Now, since the direction of current is opposite of the polarity of the voltage so what will happen? You can simply say,

$$-4i_2 = v_x = i_1 - i_2$$

So, in that case you can simplify and you will get

$$i_1 = -3i_2$$

Next what we have to do? We have to apply KVL for loop 2 and 3. So, for loop 2 if you apply KVL what will happen?

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

For loop 3 what you will get?

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

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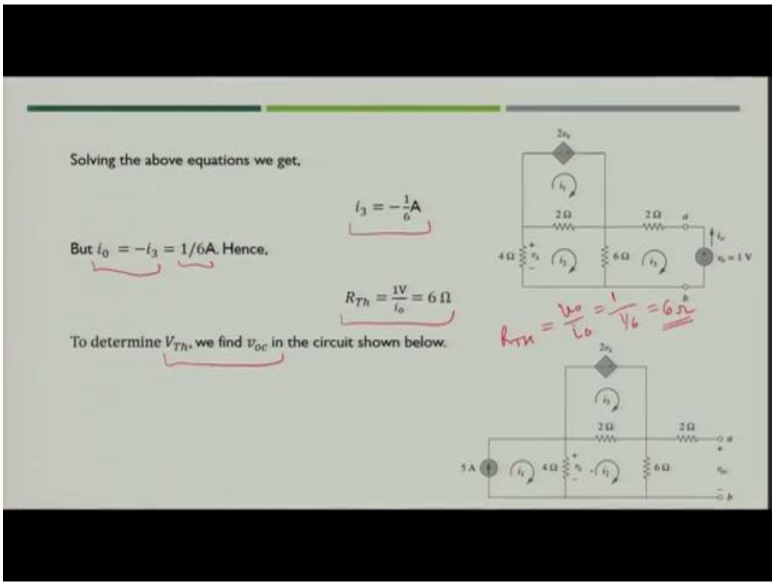

So, when you will solve you will get the value of

$$i_3 = -\frac{1}{6}\text{A}$$

Now, as we know that i_2 is equal to $-i_3$ if you see this circuit i_0 is in the opposite direction of i_3. So, we can say

$$i_0 = -i_3 = 1/6\text{A}$$

So, what you will get for R_{Th}?

$$R_{Th} = \frac{1\text{V}}{i_0} = 6\,\Omega$$

Now, we have to find out the v_{oc} because R_{Th} we have already found for finding out the value of v_{oc} what we have to do? We have to find out the voltage across terminal a and b. So, we will add the 5 ampere independent current source again in the circuit and then we will solve. So, again we have three meshes created in the circuit that is i_1, i_2 and i_3, we will write the mesh equation for all three meshes.

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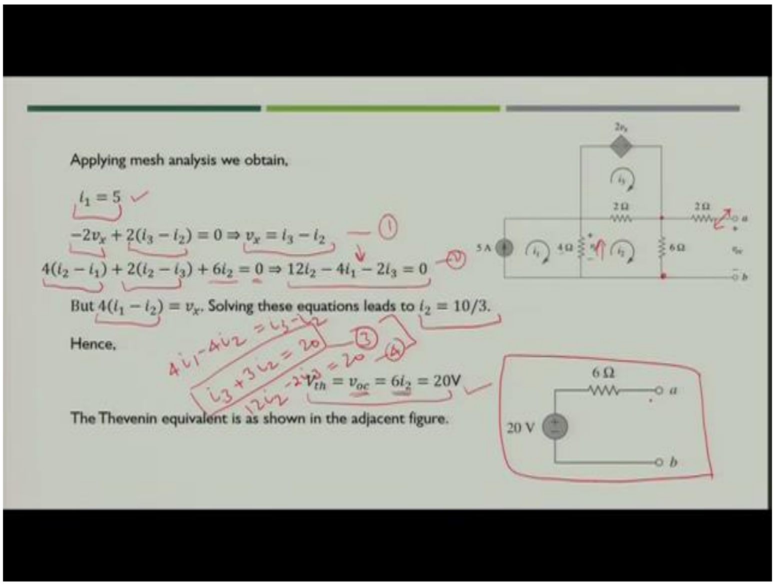

So, if you see this one you will simply get i_1 is nothing but 5 ampere because this is the independent current source connected to this. For next this mesh what you will get?

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_1 - 2i_3 = 0$$

Now, let us talk about the mesh number three what you will get?

$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2$$

Now, we know that value of $i_1 = 5$ and also, we know from the dependency because this is the constraint value that $4(i_1 - i_2) = v_x$.

So, when we solve these equations by putting the value of

$$4(i_1 - i_2) = v_x = i_3 - i_2$$

Now, $i_1 = 5$. So, this will become

$$i_3 + 3i_2 = 20$$

So, to get another equation which is only in terms of i_2 and i_3 you will use i_1 you will get

$$12i_2 - 2i_3 = 20$$

Now, you have these two equations where only unknowns are i_2 and i_3. So, you can simply solve and finally you get the value of $i_2 = 10/3$ ampere. Now,

$$V_{th} = v_{oc} = 6i_2 = 20\text{V}$$

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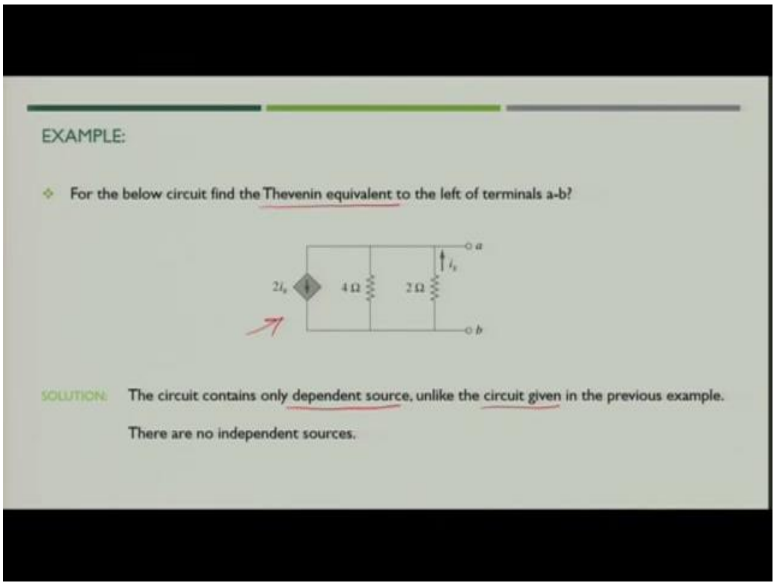

So now, let us see another example. In this circuit if you see the circuit we want to find out the Thevenin equivalent but in this particular circuit we do not have any independent voltage or current source, so the circuit would have only dependent source. So, here in this case we have the current source which is dependent.

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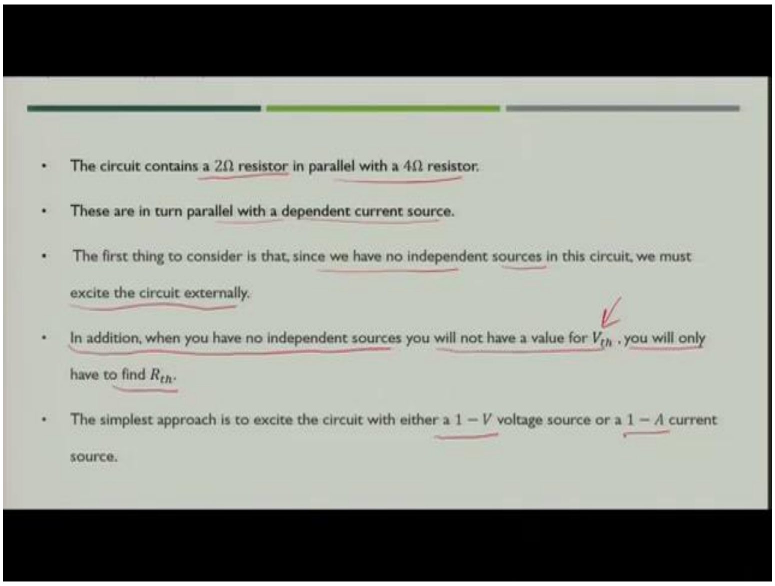

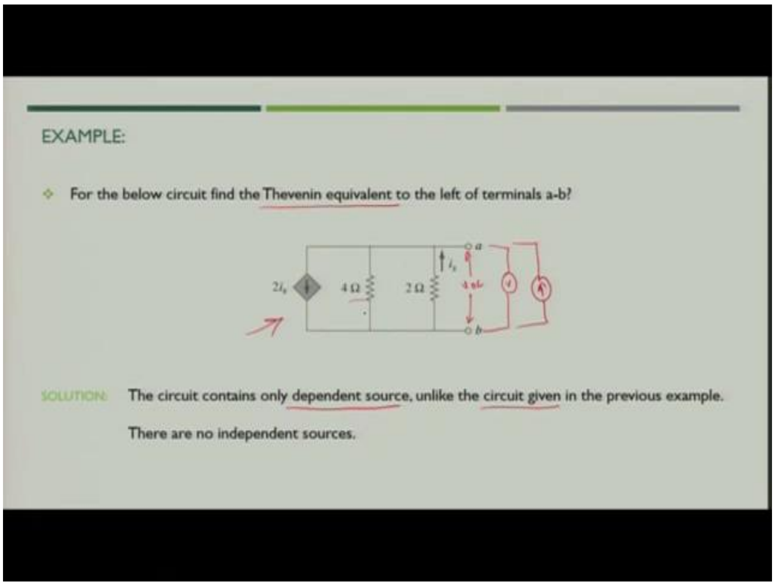

So, what we have to do? We just see that this circuit contains 2 ohm resistance in parallel with 4 ohm resistance so these two are in parallel and they in turn are in parallel with the dependent current source. Now, first thing which we have to consider is that since we do not have any independent source in the circuit we must excite the circuit externally, what does it mean?

Then we have to apply either the voltage or the current source across the a and b terminals so that we can excite this circuit from outside to find out the value of Thevenin equivalent. Now, another thing which we have to consider is that since we do not have any independent source we need not to find the value for Thevenin voltage, why? Because if you see this figure the value of this particular current source is depending upon the current flowing through 2 ohm resistance.

So, until unless we have any external source you cannot energize this particular circuit that means that if you are having the a and b terminal as open circuited and you want to find out the open circuit voltage across terminal a and b this will always be zero because there is no independent source to supply power to the circuit. So, in that way you can say that we do not have any we need not to calculate V_{Th} for this type of source, so what is left is that we need to find out the value of only Thevenin resistance.

So, next the simple approach is either you apply 1 volt voltage source or 1 ampere current source.

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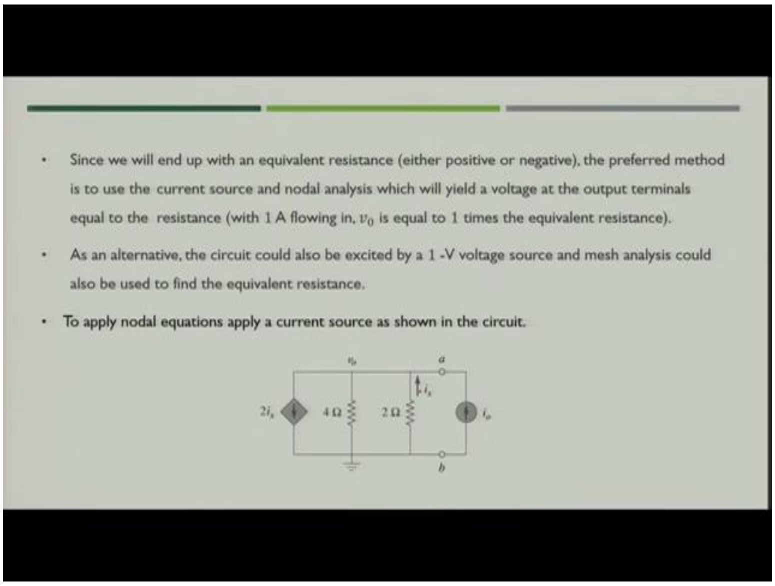

So, now let us apply one current source and try to solve it. Now, what we will do? We will see that this particular circuit has 1 node the voltage across this particular circuit a, b would be nothing but the voltage across both of the resistances also because all those elements are in parallel.

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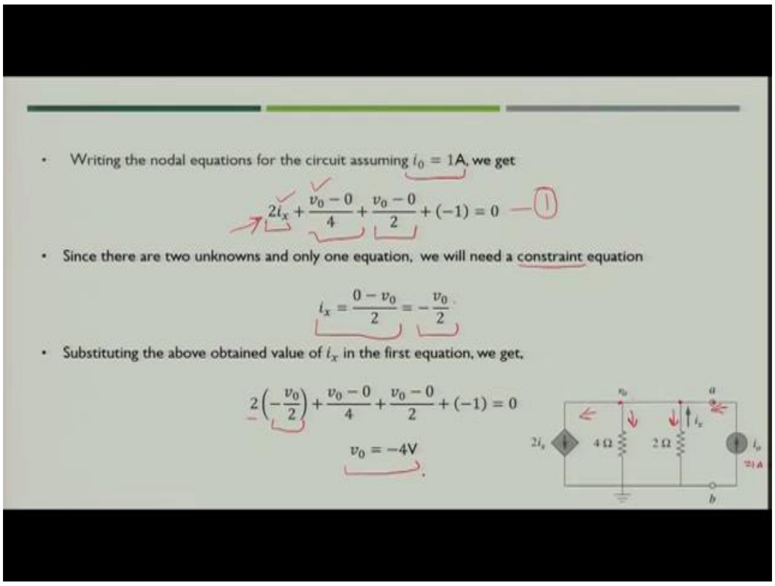

So, what we will do? We will solve it with the help of Nodal equation because it is easier to apply Kirchhoff Current Law in this particular circuit. So, let us try to apply Nodal equation while putting the value of i_0 as 1 ampere. Now, if you see this node whether this is a or this node or this node all are act same potentials so eventually this will be considered as a single node.

So, at this node if you apply the Kirchhoff Current Law first thing is that the at this particular node current going outside is $2i_x$, another current going outside is $\frac{v_0}{4}$, another current going outside of this node is $\frac{v_0}{2}$ and then the current going inside is the current i_0.

$$2i_x + \frac{v_0 - 0}{4} + \frac{v_0 - 0}{2} + (-1) = 0$$

Now, if you solve it what you will get? You will get the equation in terms of v_0 and i_x. Now, the problem would be that since we have only one equation and we have two variables means we need at least one more equation to solve this circuit. So, from where we will get the second equation? The second equation we will get from the constraint, that is, the constraint in this case is, current i_x which is defining the value of dependent current source that is $2i_x$.

Now, if you see the value of i_x, i_x is nothing but $-\frac{v_0}{2}$ because the direction of i_x is opposite to natural flow of current if voltage v_0 is having higher potential than the reference so in this way this will become $-\frac{v_0}{2}$. So, what we can do? We can simply put the value of v_0 in this equation so finally when you put these values here you will have only v_0 as an unknown in this equation.

So, you can simply solve it by putting the value of $i_x = -\frac{v_0}{2}$ and when you solve it you will get the value of $v_0 = -4$V.

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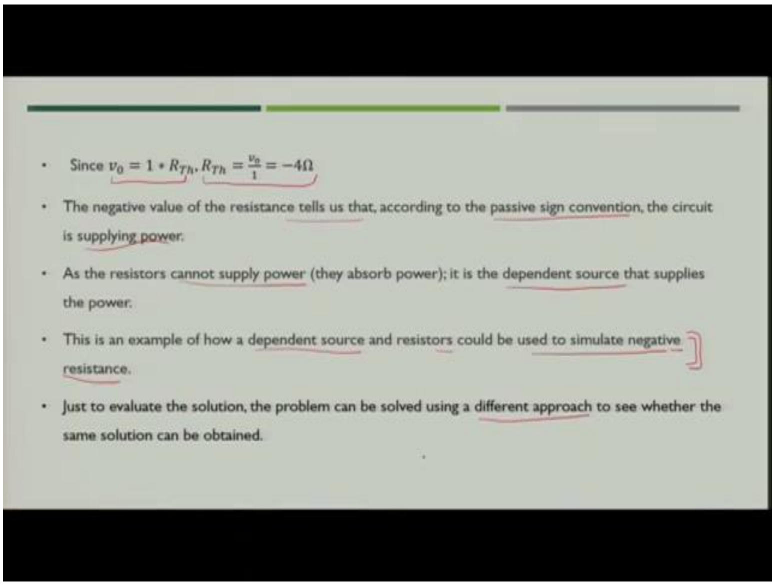

Now, since $v_0 = 1 * R_{Th}$ because $i_0 = 1$ampere simply the value of Thevenin resistance would be minus of 4 ohm. Now, the negative value of resistance will tell us that according to the passive sign convention the circuit is supplying power. Since the resistor cannot supply power who will supply is this power? The dependent source which is connected to the network.

Now, in this example we have seen how a dependent source and resistor could be used to simulate the negative resistance so these properties are very important sometimes you need to insert negative resistance in the circuit and you can realize with the help if this type of circuits. Now, if you solve this problem using different approach whether the same solution would be obtained or not let us see.

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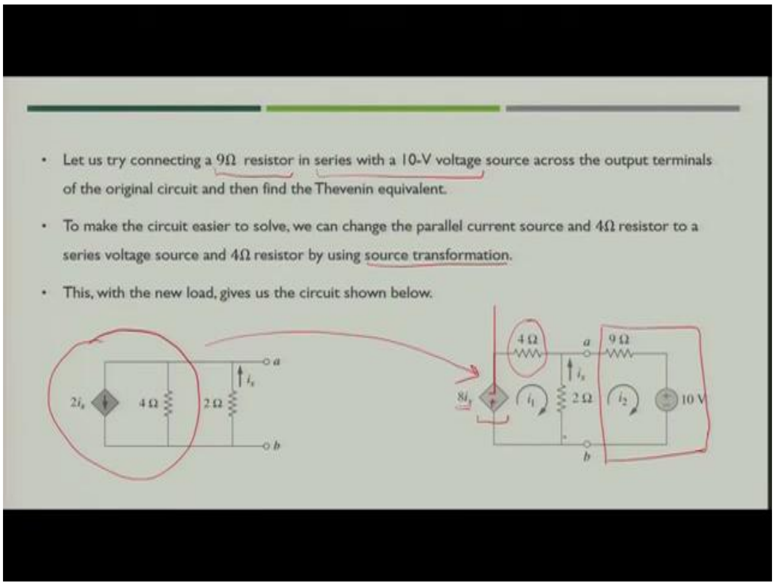

Now suppose we have load of 9 ohm resistance and 10 volt source connected across the terminal, so what happens? Suppose you are not following the Thevenin theorem and you want to analyze the circuit what you will do? You will see the circuit to find that one current source is in parallel with one 4 ohm resistance. Source transformation can be applied and you can solve it by converting the combination of current source in parallel with 4 ohm resistance with the voltage source in series with 4 ohm resistance, that is the source transformation technique which we studied earlier. So, what will happen in this case? The value of the voltage source that is definitely would be the dependent voltage source because this is the dependent current source.

So, the value of voltage source would become $8i_x$ and the direction of plus sign the polarity would be plus would be in the direction of flow of the current so that is why the below sign is plus up sign is minus and then in series with one 4 ohm resistance. So, this particular segment is now converted into voltage source in series with 4 ohm resistance then you add 2 ohm resistance that is the part of the circuit in parallel again and this is the load component which you have connected at terminal a, b.

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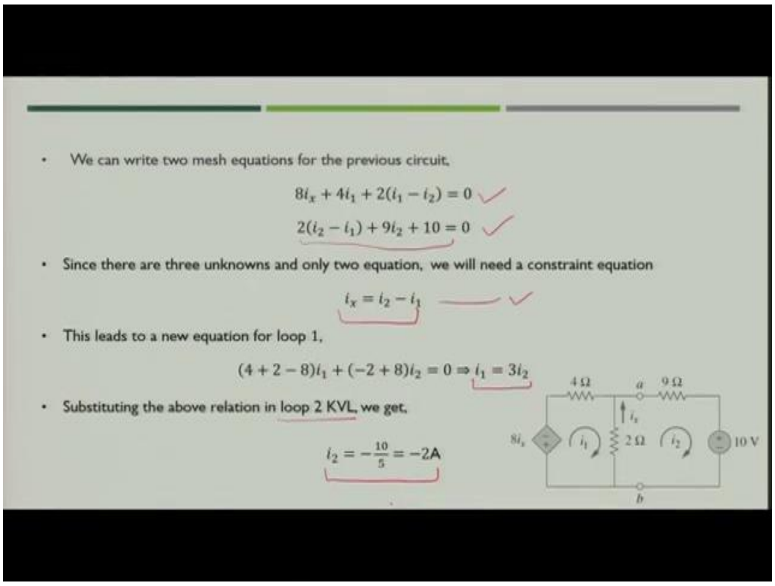

So, the updated circuit would be looking like this, here we have two meshes so what we have to do? We have to write the mesh equations for both of them so these would be the two mesh equations for these two meshes plus we will have another constraint for i_x that is nothing but $i_2 - i_1$.

Now, if you use these three equations and solve it you will get $i_1 = 3i_2$ and using KVL in loop 2 you will get $i_2 = -\frac{10}{5} = -2\text{A}$ because you solve it with the help of the circuit and you get the value of current i_2 because i_1 is nothing but $3i_2$ and you write the loop equation for this, this is what you have used in case of mesh two put the value of i_1 here, solve it you will get i_2 is nothing but -2 ampere.

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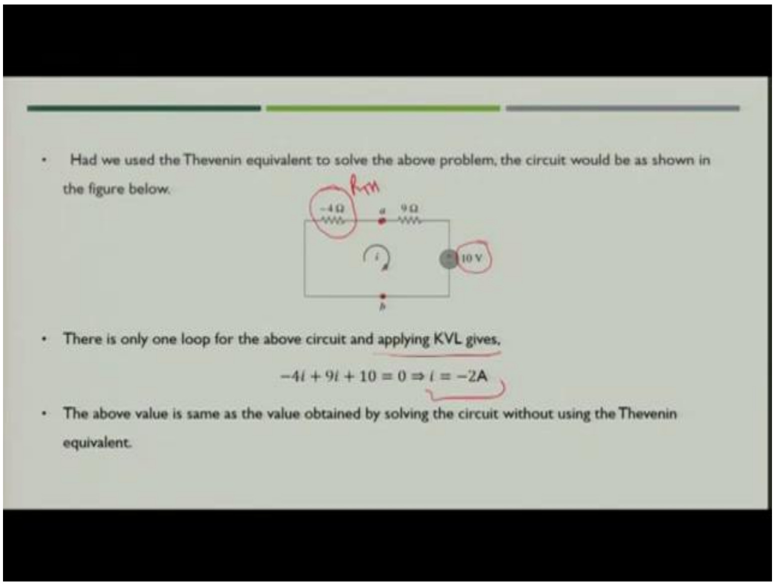

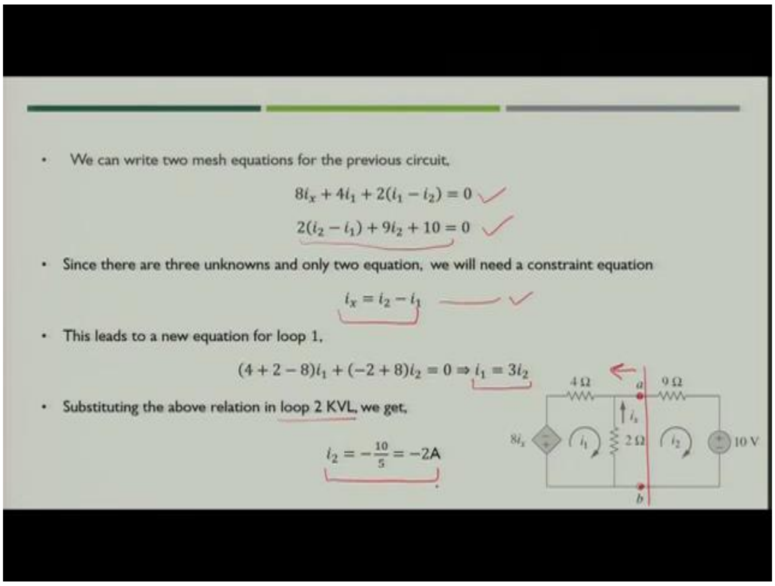

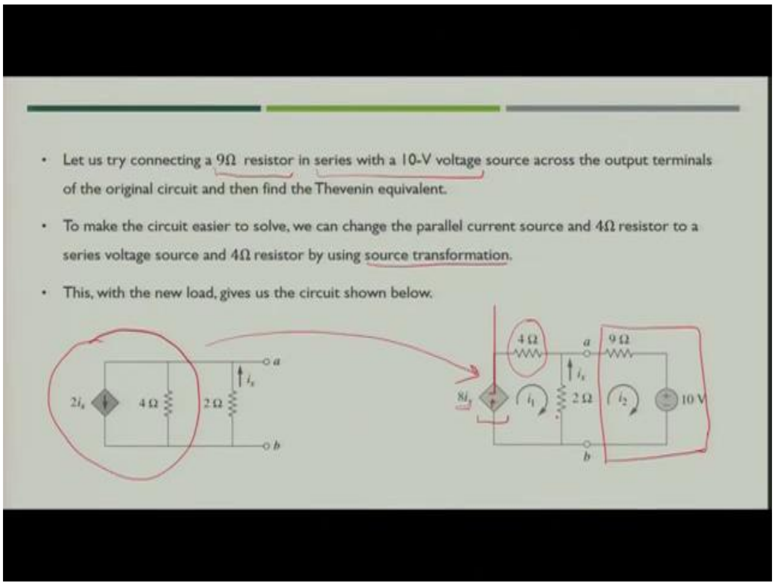

Now, this is what you got when you did not apply Thevenin theorem to solve this particular circuit. Now, you know that we have calculated the Thevenin resistance across these two terminals while there was no Thevenin voltage. So, if you use that concept of Thevenin theorem which we discussed we will have only one -4 ohm resistance as part of the Thevenin resistance with no Thevenin voltage.

So, the left side of this which was our original circuit can be replaced by only the -4 ohm resistance that is Thevenin resistance connected across terminal a and b. So, we have now very simple circuit when we consider the Thevenin theorem and we get the equivalent resistance of this particular loop as 5 ohm resistance and 10 volt voltage source is connected when you apply again the KVL in this particular circuit you will get again current $i = -2$A.

So, if you compare these two methods where you analyze the circuit with the help of source transformation and then you applied the mesh analysis method to find out the value of current i_2, here we used the Thevenin theorem and simplified the circuit very quickly as compare to the method where we were using the only the dependent voltage and current source when we connect them and how to create the Thevenin equivalent in that case and how to solve the circuit when we have dependent sources connected.

So, with this we close this week sessions where we studied various key concept of the circuit. We studied superposition as well as Thevenin's theorem. So, next week we will start with another theorem which is called as Norton's Theorem, thank you.