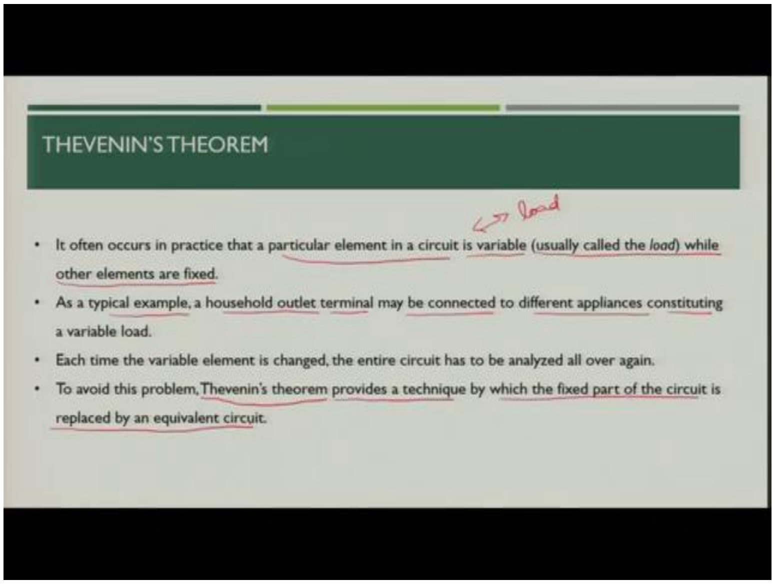
Basic Electric Circuits
Professor Dr. Ankush Sharma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Module 3 Network theorem 1
Lecture 14: Thevenin's Theorem

Namaskar, So in this week, we discuss about linearity of the circuit and then we discuss about the source transformation and in last class we discuss about duality. With the reference of these three important properties of the circuit let us start the network theorem which is called as Thevenin's theorem. Thevenin's theorem is very important for the circuit point of view.

(Refer Slide Time: 00:47)



THEVENIN'S THEOREM

load

- It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed.
- As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load.
- Each time the variable element is changed, the entire circuit has to be analyzed all over again.
- To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

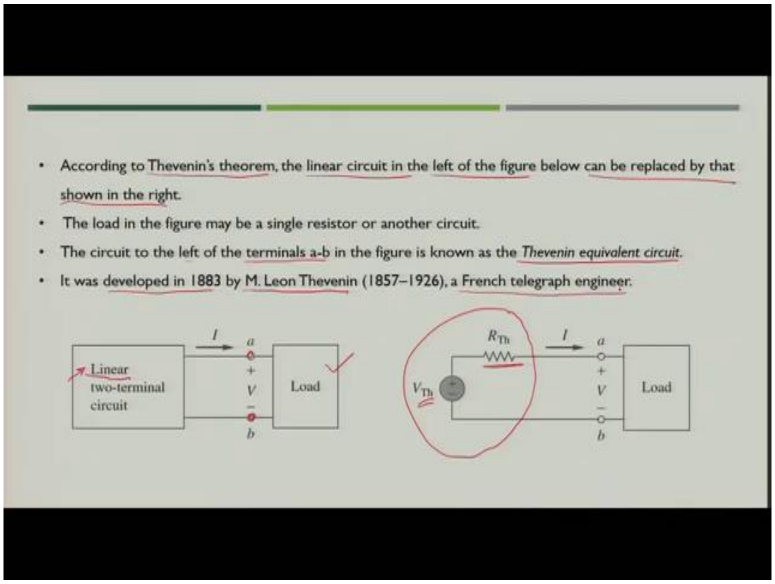
What happens when you plug in your appliance in the house. You will analyze the voltage and current parameters of that appliance, while neglecting the circuit which is outside the particular plug block plug point. So, what happens that you will analyze these circuit parameters like voltage and current while doing the analysis for various appliances.

So various appliances will have their own current level. Well voltage level will almost remain same in all the appliances. So, in that scenario if you are doing the study for appliances, how you will analyze the circuit? If you see the conventional way of circuit analysis what you have to do? You have to take the complete power system network.

That is very large in size and you will end up having hundreds of mesh networks or hundreds of nodes in the network. And if you start doing the mesh analysis or nodal analysis it will take a lot of time and it will be almost impossible to do it in a reasonable time frame. So, to address that problem the alternate ways that you can represent the equivalent of the sum of the power system network which is external to that particular plug point. And then analyze only that particular load or that particular variable element within which you are basically interested.

So, to solve that problem we use the theorem called Thevenin's theorem. Because it provides a technique by which the fixed part of the circuit is replaced by its equivalent circuit.

(Refer Slide Time: 03:15)



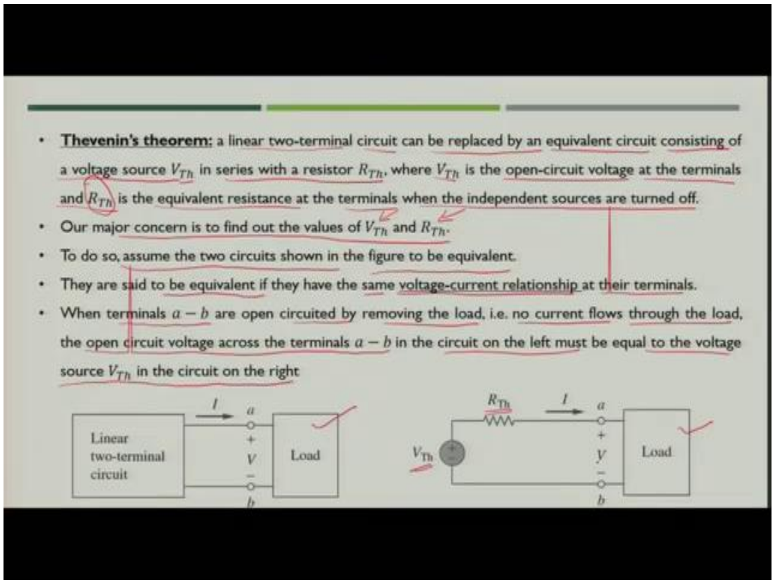
- According to Thevenin's theorem, the linear circuit in the left of the figure below can be replaced by that shown in the right.
- The load in the figure may be a single resistor or another circuit.
- The circuit to the left of the terminals a-b in the figure is known as the *Thevenin equivalent circuit*.
- It was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Now according to Thevenin's theorem, the linear circuit in the left of the figure can be replaced by that shown in the right. So, if you can imagine that these are the two points of your plug in the house. External to that you will see as a linear circuit. So, you have to keep in mind that the Thevenin's theorem works well when the circuit is linear.

You will assume that this circuit which is outside your plug point is the linear circuit and you will replace that particular segment with one equivalent circuit where you will have one voltage source and one resistance. So, in that way the load which you are analyzing for various circuit elements like the variables like voltage and current, you will keep that load connected with the equivalent of the circuit which is external to that particular load terminal.

So that circuit to the left of this the node a b is called as Thevenin equivalent circuit. This concept was developed by M. Leon Thevenin in 1883 who was a French telegraph engineer.

(Refer Slide Time: 04:50)



- **Thevenin's theorem:** a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the equivalent resistance at the terminals when the independent sources are turned off.
- Our major concern is to find out the values of V_{Th} and R_{Th}.
- To do so, assume the two circuits shown in the figure to be equivalent.
- They are said to be equivalent if they have the same voltage-current relationship at their terminals.
- When terminals $a-b$ are open circuited by removing the load, i.e. no current flows through the load, the open circuit voltage across the terminals $a-b$ in the circuit on the left must be equal to the voltage source V_{Th} in the circuit on the right

What does Thevenin’s theorem says: A linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} In series with resistor R_{Th} where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the equivalent resistance at the terminals when the independent sources are turned off.

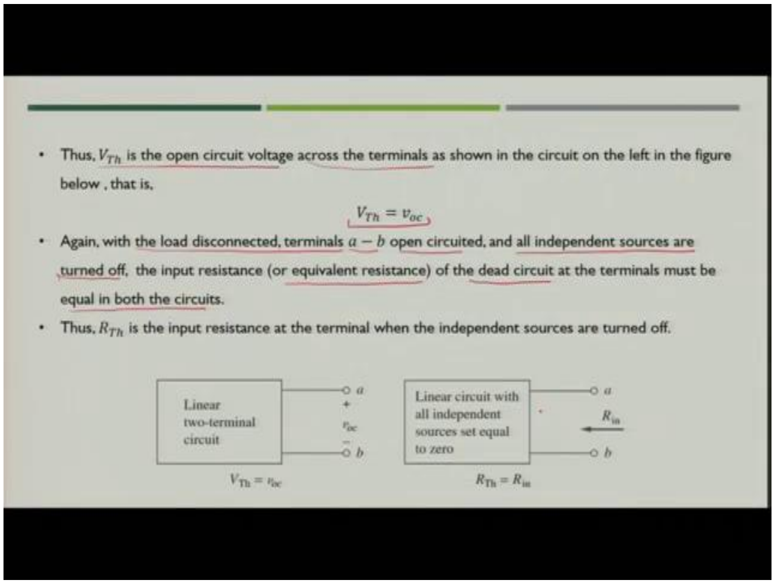
When we calculate the value of R_{Th} we will create the equivalent circuit by switching off the independent sources. Switching off the independent source means, if you have independent voltage source you will short circuit and if you have an independent current source you will open circuit. The problem with the Thevenin’s theorem is that you have to identify the ways how you will calculate the value of V_{Th} and R_{Th}. So, if you recollect what we discussed in case of equaling circuit, you can easily figure out that how you will calculate V_{Th} and R_{Th}. Now if you assume that there are two circuits which are shown below are equivalent that means there is an unknown linear network where we do not know what is inside and outside that linear two terminals circuit we have connected a load across terminals a-b.

Voltage across terminal a-b is V and current flowing into the load from the external circuit is current I. So, if we replace the external circuit with Thevenin equivalent that is Thevenin voltage V_{Th} in series with resistance R_{Th}. and if you have V-I characteristic at this terminal same while taking the load out then this particular circuit would be considered as the equivalent of that external circuit.

So, you have to keep in mind that these circuits are set to be equivalent only when you have voltage current relationship same for both of the circuit at the terminal. Now when the terminals a-b are open circuited by removing the load, no current will flow through the load. Now this open circuit voltage across the terminal a-b in the circuit on the left must be equal to voltage source V_{Th} in the circuit on the right.

So that means that if you remove load from here there would be some voltage across Terminal a-b and if you remove load from here there will be same voltage which would be coming across a b that is V.

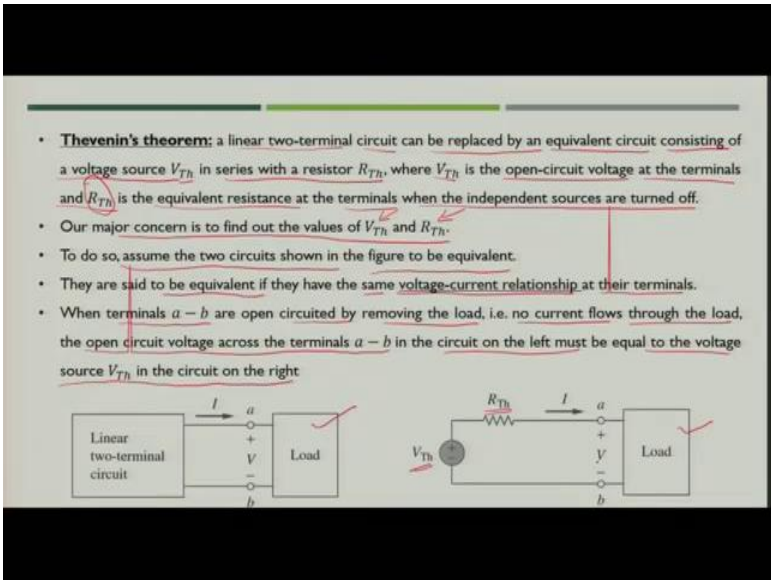
(Refer Slide Time: 08:04)



- Thus, V_{Th} is the open circuit voltage across the terminals as shown in the circuit on the left in the figure below , that is,

$$V_{Th} = v_{oc}$$

- Again, with the load disconnected, terminals $a - b$ open circuited, and all independent sources are turned off, the input resistance (or equivalent resistance) of the dead circuit at the terminals must be equal in both the circuits.
- Thus, R_{Th} is the input resistance at the terminal when the independent sources are turned off.



- **Thevenin's theorem:** a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the equivalent resistance at the terminals when the independent sources are turned off.
- Our major concern is to find out the values of V_{Th} and R_{Th}.
- To do so, assume the two circuits shown in the figure to be equivalent.
- They are said to be equivalent if they have the same voltage-current relationship at their terminals.
- When terminals $a - b$ are open circuited by removing the load, i.e. no current flows through the load, the open circuit voltage across the terminals $a - b$ in the circuit on the left must be equal to the voltage source V_{Th} in the circuit on the right

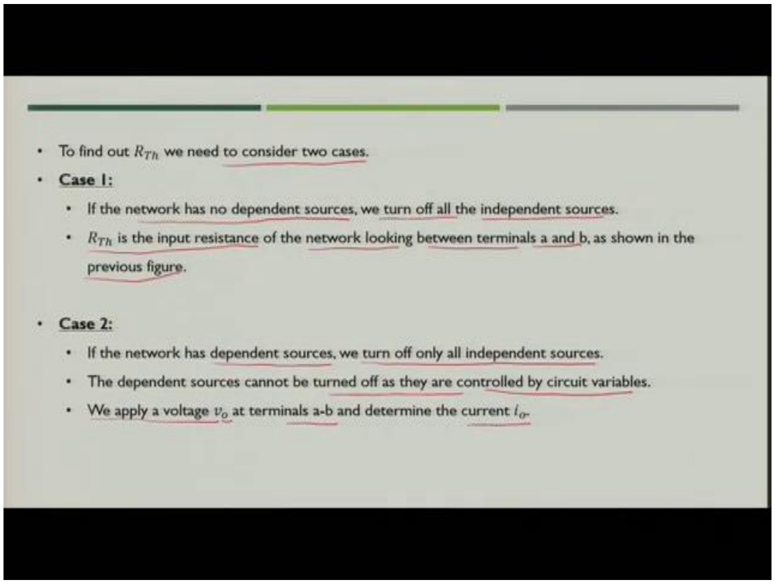
So, using this particular equivalency you can identify the voltage of V_{Th}. Because V_{Th} is open circuit voltage across the terminals. When you remove the load, you will see some voltage let us say it is V_{Th} similarly for the Thevenin equivalent that is here if you remove the load you will again get the value v_{oc} and these two v_{oc} are same because this v_{oc} would be nothing but V_{Th}. Because when it is open circuited there would be no current flowing in this particular segment right.

If you remove the load, no current will be flowing. In that case this particular voltage would be nothing but V_{Th}. So, what we can say that, the linear two terminal circuit, open circuit voltage across terminal a-b would be equal to V_{Th}.. Now again with the load disconnected because the terminal a-b then you have open circuit at the terminal a-b all independent sources you need to turn off. Turning off the independent sources means if you have the voltage source inside the terminal you will replace it with the short circuit and if you have current source you will replace it with the open circuit.

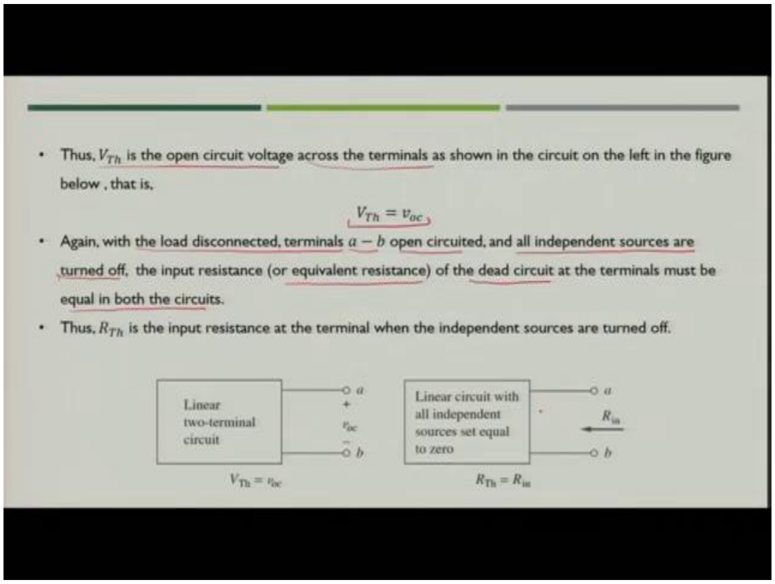
In that case what will happen is that this linear two terminal circuit would be nothing but a dead circuit because there is no source available. Now the equivalent resistance which you will measure between these two terminal a and b would be equal in both of the cases. The case where you have linear two terminals circuit you will take all the sources off. You will measure the R_{Th} here.

Similarly, if you measure R_{Th} here it will be this value because in that case this would be short circuited. So, in that case whatever the resistance you will see in both of the cases the input resistance which you will see across the linear circuit with all independent sources are equal to zero would be equal to R_{Th}. So, with these two conditions you can find the value of the important parameters which are required for Thevenin's theorem which are V_{Th} and R_{Th}.

(Refer Slide Time: 10:59)



- To find out R_{Th} we need to consider two cases.
- **Case 1:**
  - If the network has no dependent sources, we turn off all the independent sources.
  - R_{Th} is the input resistance of the network looking between terminals a and b, as shown in the previous figure.
- **Case 2:**
  - If the network has dependent sources, we turn off only all independent sources.
  - The dependent sources cannot be turned off as they are controlled by circuit variables.
  - We apply a voltage v_o at terminals a-b and determine the current i_o.



- Thus, V_{Th} is the open circuit voltage across the terminals as shown in the circuit on the left in the figure below, that is,

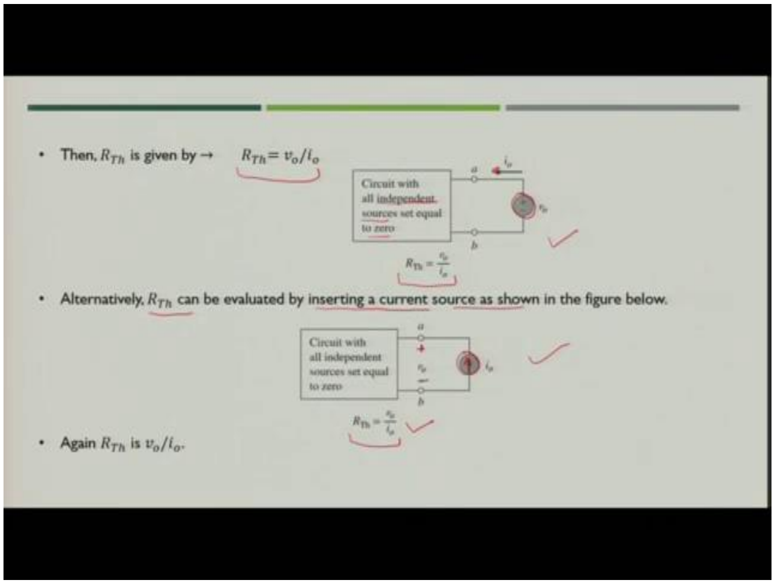
$$V_{Th} = v_{oc}$$

- Again, with the load disconnected, terminals $a - b$ open circuited, and all independent sources are turned off, the input resistance (or equivalent resistance) of the dead circuit at the terminals must be equal in both the circuits.
- Thus, R_{Th} is the input resistance at the terminal when the independent sources are turned off.

Now to find out the value of R_{Th} there are two different cases. What are the two different cases? First case is that if the network has no dependent sources we can simply turn off all the independent sources like we can make the voltage as short circuit and current source as open circuit and then R_{Th} is the input resistance of the network looking between terminal a and b. So that means that whatever the input of the resistance you will see from terminal a-b while keeping all independent sources equal to zero. That will give you simply the Thevenin resistance.

Now in case 2 when the network has dependent sources also. We cannot switch off the dependent sources because they are anyway controlled by some circuit variable. So, what will happen in that case? You can simply turn off only the independent sources while keeping dependent sources on. And then you will analyze the circuit. How? You have to simply apply one voltage v_0 at the terminal a-b and determine the current i_0.

(Refer Slide Time: 12:21)



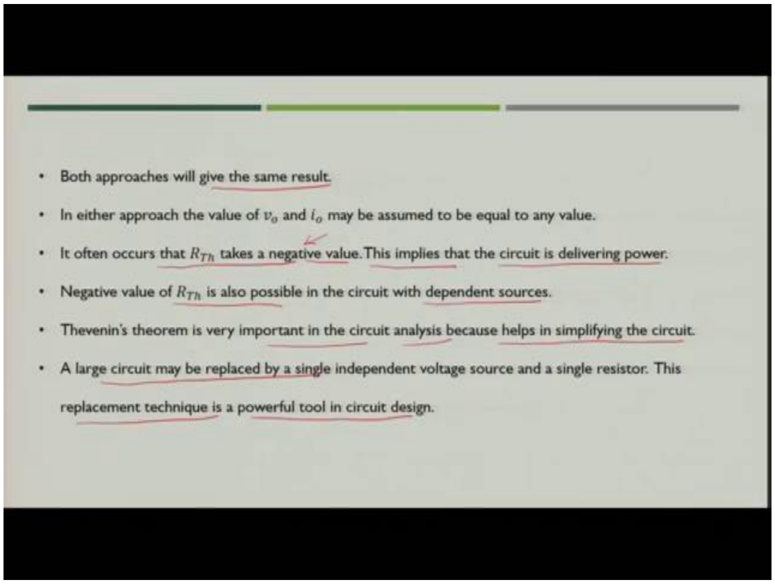
- Then, R_{Th} is given by → $R_{Th} = v_o/i_o$
- Alternatively, R_{Th} can be evaluated by inserting a current source as shown in the figure below.
- Again R_{Th} is v_o/i_o.

So, what will happen in that case? Here the circuit with all independent sources equal to zero are present. And the dependent sources are also available. In that case you have to apply one voltage v_0 across the terminal of a-b and measure the value of current i_0.

What will happen in that case? Whatever the current which is flowing inside the terminal a-b divided the voltage which you are applying then v naught divided by I naught would be giving you the value of Thevenin equivalent resistance that is R_{Th}. Alternatively, the R_{Th} can also be measured by inserting a current source. So instead of voltage source across these two terminals you will apply one current source.

Now what do you have to do? When you apply the current source instead of the current which you have measure here you have to measure the voltage across terminals a-b. What will happen in that case? Again the R_{Th} would be given as the voltage across terminals a-b divided by the current supplied by current source. So, in that case the R_{Th} would be nothing but v_0/i_0.

(Refer Slide Time: 14:15)



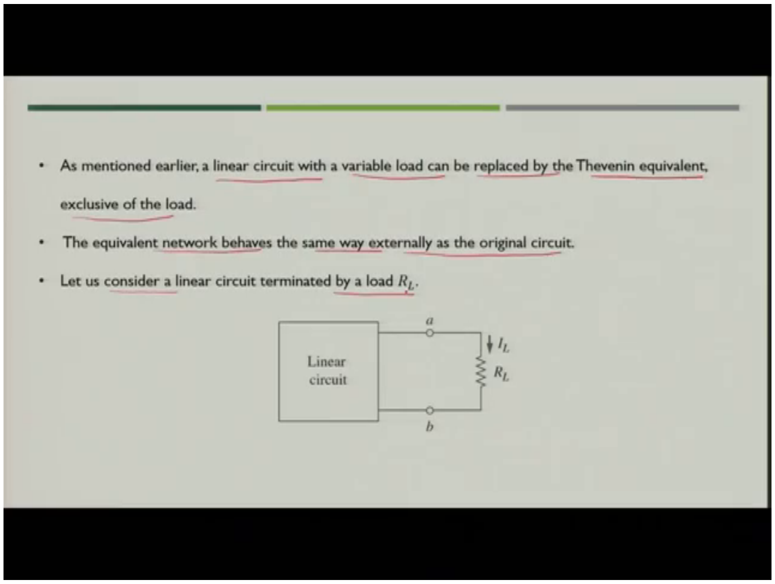
- Both approaches will give the same result.
- In either approach the value of v_o and i_o may be assumed to be equal to any value.
- It often occurs that R_{Th} takes a negative value. This implies that the circuit is delivering power.
- Negative value of R_{Th} is also possible in the circuit with dependent sources.
- Thevenin's theorem is very important in the circuit analysis because helps in simplifying the circuit.
- A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

Now in both of these cases you can use any one of the methods. In both of the cases you will get the same value of R_{Th}. So, you will get eventually the same result. Now it often occurs that the R_{Th} which you calculate will become negative. So, it implies that the circuit is delivering power.

You need not worry whether your solution is correct or not if your R_{Th} is negative. So sometimes R_{Th} would be negative. And it is also possible when the circuit is having dependent sources. Now this theorem is very important in the circuit analysis why? because it helps in simplifying the circuit.

So, whatever the larger network outside the area of your study is present you will simply equal that network which is outside the area of your study by Thevenin equivalent. You will replace with the V_{Th} and R_{Th} and you will use that particular equivalent circuit to analyze the circuit which is of your interest. So that is why this Thevenin equivalent replacement technique is very powerful tool in our circuit design.

(Refer Slide Time: 15:35)

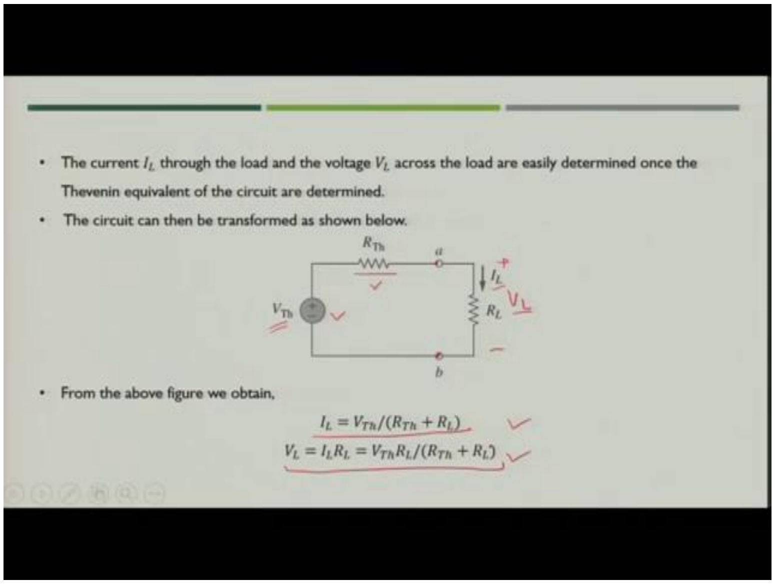


- As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.
- The equivalent network behaves the same way externally as the original circuit.
- Let us consider a linear circuit terminated by a load R_L.

Now as we have discussed that linear circuit with variable load can be replaced by Thevenin equivalent excluding the load. Now this equivalent network will behave as same way as the external circuit is behaving, because you are creating the Thevenin equivalent of the external circuit by keeping vi characteristic equivalent.

So how you will calculate the different load voltage and load current values. Let us take one simple circuit and we will try to find out the value of current flowing through the load Resistance R_L.

(Refer Slide Time: 16:26)



- The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit are determined.
- The circuit can then be transformed as shown below.

- From the above figure we obtain,

$$I_L = V_{Th}/(R_{Th} + R_L)$$

$$V_L = I_L R_L = V_{Th} R_L/(R_{Th} + R_L)$$

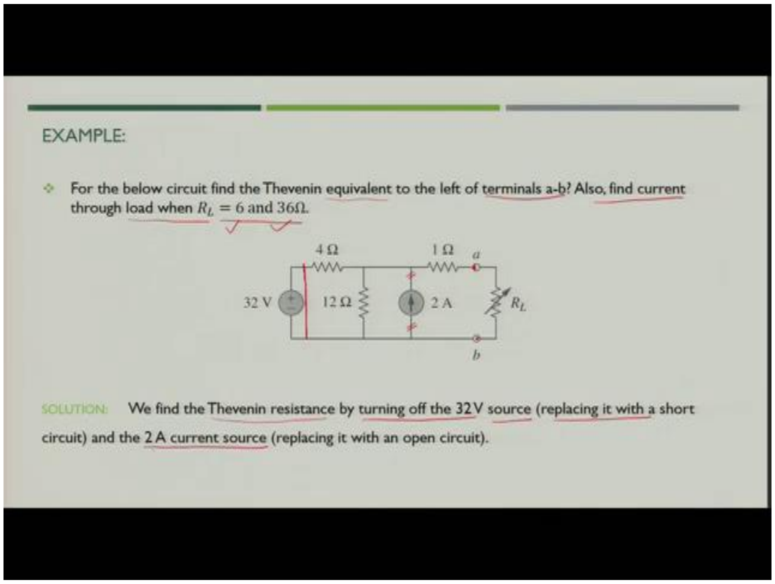
Now what you will do, the exact external to your load R_L would be replaced by its Thevenin equivalent that is V_{Th} and R_{Th}. So, what will happen in that case is R_L would be your load and there will be one resistance R_{Th} would be the Thevenin equivalent resistance and it would both would be connected through voltage source which is nothing but Thevenin Voltage.

So, this would be the equivalent circuit of the original circuit where the linear circuit is not known to us. We are just simply replacing it with the Thevenin equivalent. Now what we will, how we will calculate the value of current I_L?. Simply the value of current I_L would be $I_L = V_{Th}/(R_{Th} + R_L)$. So that is simply the Kirchhoff's voltage law. You can apply and get the value of current I_L. Now the voltage across load $V_L = I_L R_L = V_{Th} R_L/(R_{Th} + R_L)$.

When you get the values of R_{Th} and V_{Th} you can easily find out the variables across the load. That is voltage and current. Now if you have multiple cases or multiple loads through which you want to find out the current and across those load voltages, you need not to change anything in this circuit that is R_{Th} and V_{Th} you have to just keep on changing the loads across terminal a and b.

So, based on the resistance of the load your load current will keep on changing. You need not to rearrange the circuit or you need not to reprocess the complete information again and again. You have to just simply change the load and find out the updated value. In this way it will be very handy to process the multiple cases when these are connected to two terminals.

(Refer Slide Time: 19:13)



EXAMPLE:

- For the below circuit find the Thevenin equivalent to the left of terminals a-b? Also, find current through load when $R_L = 6$ and 36Ω.

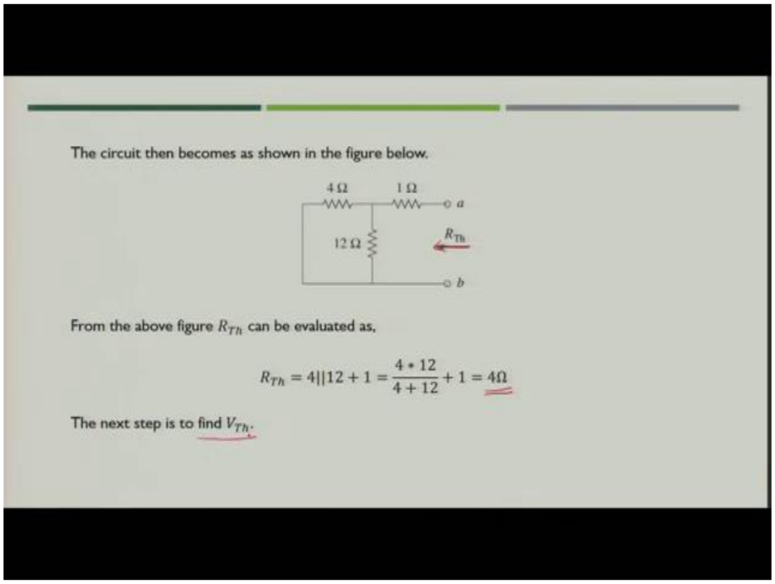
SOLUTION: We find the Thevenin resistance by turning off the 32V source (replacing it with a short circuit) and the 2 A current source (replacing it with an open circuit).

Now let us understand this particular aspect with the help of one example. Now you see there is a circuit given in this figure. We need to find out Thevenin equivalent to the left of terminal a-b. So, these are the terminals a-b and we have to find out the current through the load when R_L is equal to either 6 ohm or 36 ohm.

This is the variable load it will keep on changing. You need to find out the value of current through the load under various loading conditions. For this particular case we are considering two different loading conditions where R_L is 6 ohm and 36 ohm. We find the Thevenin resistance by turning off the 32 volt source.

Now when you replace the voltage source it will become short circuited and we have another source which is current source that is 2A current source. So, we will simply make it open circuit. What will be the updated circuit in that case? We have the 4 ohm resistance in parallel with 12 ohm resistance and then finally the parallel combination in series with 1 ohm resistance.

(Refer Slide Time: 20:55)



The circuit then becomes as shown in the figure below.

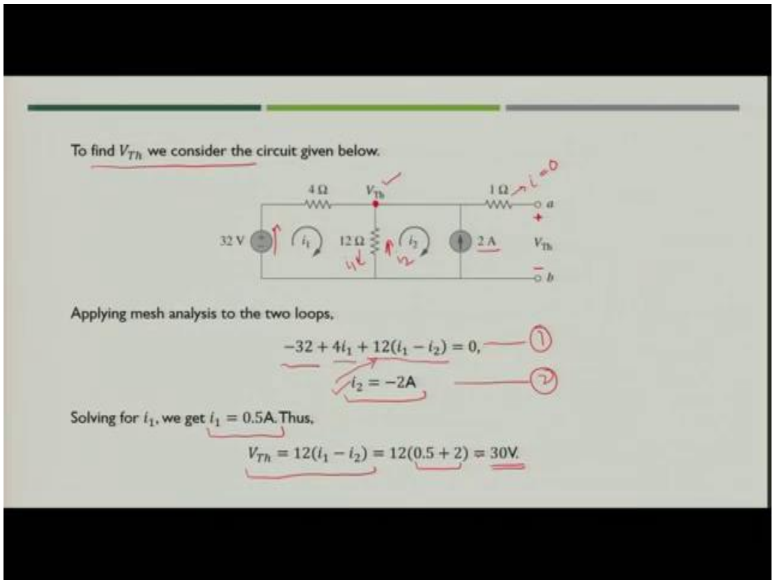
From the above figure R_{Th} can be evaluated as,

$$R_{Th} = 4||12 + 1 = \frac{4 * 12}{4 + 12} + 1 = \underline{\underline{4\Omega}}$$

The next step is to find $\underline{V_{Th}}$.

The resistance which you will see through these particular terminals that is a and b you will get the value of R_{Th} as 4 ohm because the parallel combination of these two resistances would give you 3 ohm plus 1 ohm in series. So finally, the R_{Th} value is 4 ohm. Now next step is finding out the value of Thevenin Voltage.

(Refer Slide Time: 21:32)



To find V_{Th} we consider the circuit given below.

Applying mesh analysis to the two loops,

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \text{———} ①$$

$$i_2 = -2A \qquad \text{———} ②$$

Solving for i_1, we get $i_1 = 0.5A$. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = \underline{\underline{30V}}.$$

To find the value of Thevenin voltage we have to consider the circuit and whatever we discussed in previous classes we have to just recollect our study specially the mesh analysis which we discussed. Because here it is you can see that you can create two meshes. You can apply mesh current i_1 and i_2 across these two meshes and you need to find out the voltage V_{Th} across terminal a-b.

Now important thing which you need to see from this particular circuit is that there would be no current flowing through 1ohm resistance because the terminal a-b is open circuit. So, there would be basically no current so current i would be zero. Now when current i is zero the V_{Th} would be applied straightaway across this. That means the same would be applied across this because all are in parallel.

So V_{Th} would be nothing but the voltage which is coming at this particular node. So V_{Th} would be the value of this node. Now let us apply the mesh analysis for these two loops.

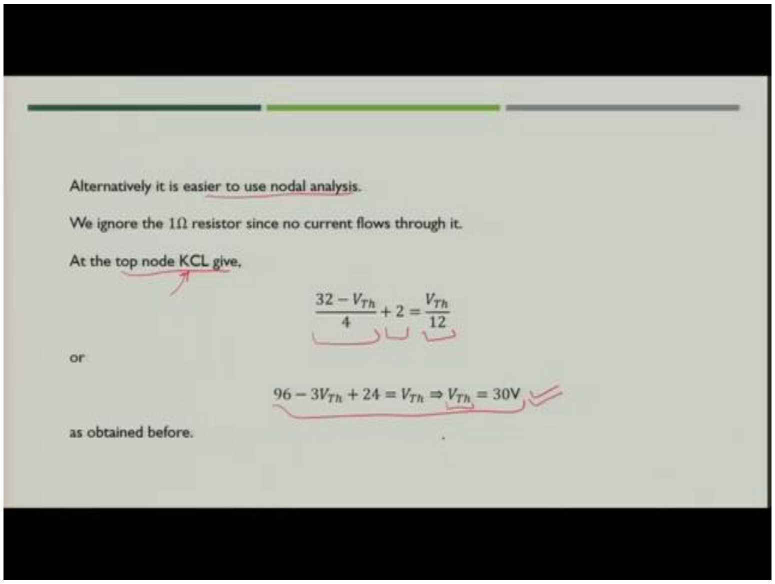
$$-32 + 4i_1 + 12(i_1 - i_2) = 0$$

$$i_2 = -2\text{A}$$

So now you have got two equations. i_2 you have a straightaway found, next you have to just place the value of i_2 in this equation and you will get current i_1 as 0.5ampere. Now as you have got value of current i_1 and i_2, V_{Th} would be

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30\text{V}$$

(Refer Slide Time: 24:55)



Alternatively it is easier to use nodal analysis.

We ignore the 1Ω resistor since no current flows through it.

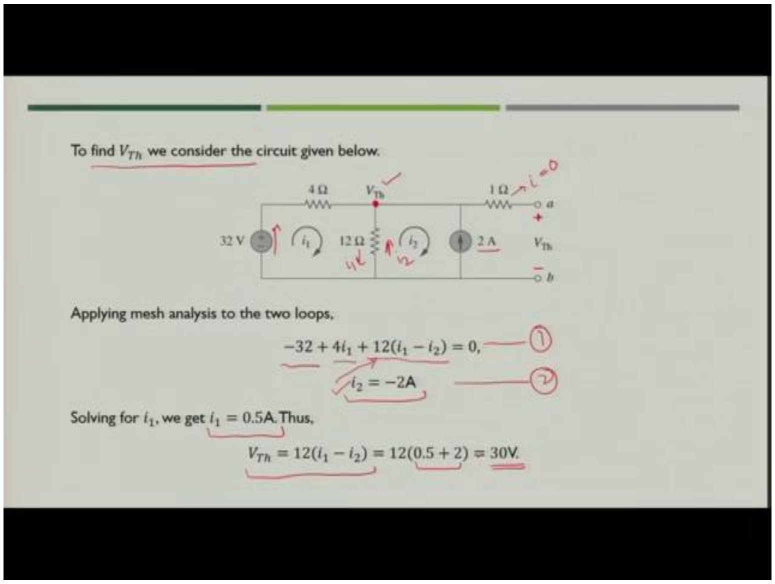
At the top node KCL give,

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \Rightarrow V_{Th} = 30\text{V}$$

as obtained before.



To find V_{Th} we consider the circuit given below.

Applying mesh analysis to the two loops,

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \text{ — ①}$$

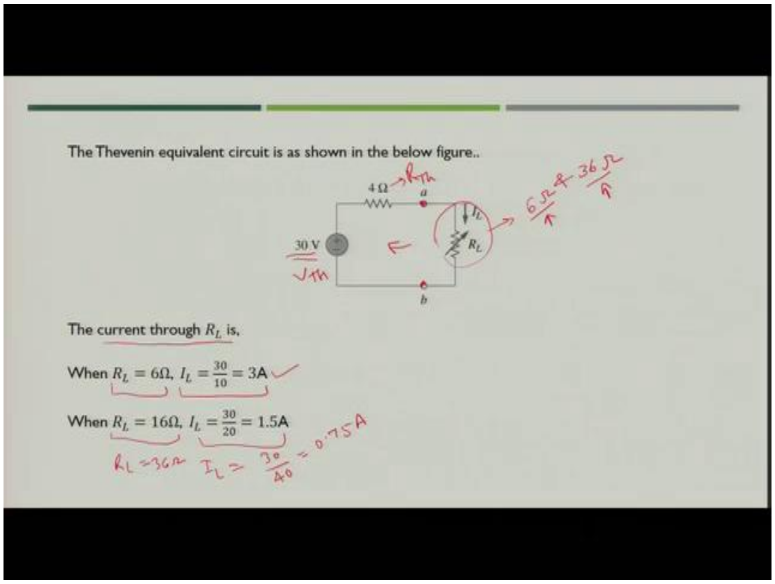
$$i_2 = -2\text{A} \text{ — ②}$$

Solving for i_1, we get $i_1 = 0.5$A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30\text{V}.$$

Now instead of Mesh analysis you can also apply Nodal analysis as well. So, if you apply the nodal analysis at this node because this is the node where you are seeing the voltage V_{Th}. If you apply nodal analysis that means that you have to use Kirchhoff's current law here. So, what will happen?

(Refer Slide Time: 25:26)



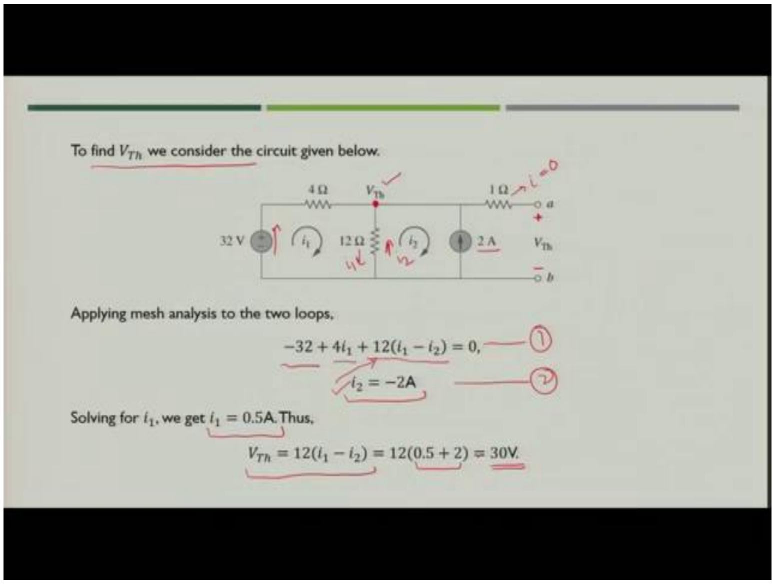
The Thevenin equivalent circuit is as shown in the below figure..

The current through R_L is,

When $R_L = 6\Omega$, $I_L = \frac{30}{10} = 3\text{A}$ ✓

When $R_L = 16\Omega$, $I_L = \frac{30}{20} = 1.5\text{A}$

$R_L = 36\Omega$ $I_L = \frac{30}{40} = 0.75\text{A}$



To find V_{Th} we consider the circuit given below.

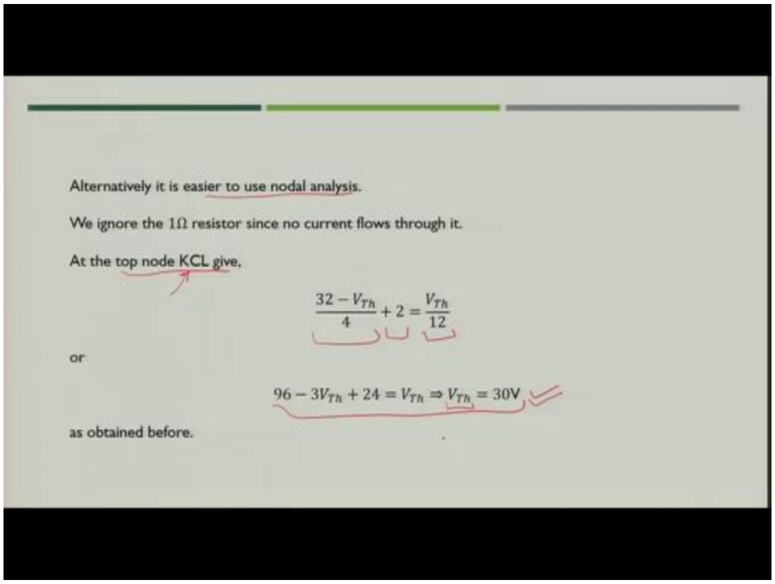
Applying mesh analysis to the two loops,

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \text{——} ①$$

$$i_2 = -2\text{A} \text{——} ②$$

Solving for i_1, we get $i_1 = 0.5\text{A}$. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30\text{V}.$$



Alternatively it is easier to use nodal analysis.

We ignore the 1Ω resistor since no current flows through it.

At the top node KCL give,

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \Rightarrow V_{Th} = 30\text{V}$$

as obtained before.

You will simply see what are the various currents flowing into the node and it equate it equal to zero because total sum of current at that particular node would be zero. What would be the current coming inside from left? That would be $\frac{32-V_{Th}}{4}$. So, you have got first term of the equation.

Next is 2A current which is flowing in this direction. So, you can simply put the value of 2 ampere. Third is the current which is flowing in this direction. So, the current flowing in this direction would be some of these two currents. So finally, this would be equal to V_{Th} by 12 because the voltage at this is V_{Th} and the resistance connected between the reference node and node in the question is 12 ohm.

You will simply get this equation when you apply the Nodal analysis. Now if you rearrange the terms and simplify it you will again get the value of Thevenin voltage that is 30V. So, in that way either you apply Mesh analysis or the Nodal analysis in both way you will get the Thevenin voltage same.

Now your equivalent circuit of the particular figure which we discussed is 30V that V_{Th} and 4 ohm that is R_{Th} and across the terminal a-b you will apply variable resistance where you have to find out the value of current, load current for 6 ohm and 36 ohm. So, what you will do? You will just put the value of R_L. When R_L is equal to 6 ohm you will simply get I_L is nothing but you will simply apply Kirchhoff's voltage law and you will get the value of current I_L as 3A.

Similarly, if R_L is 16 ohm you will get current I_L as 1.5A. When you will replace R_L with 36 ohm you will get I_L is nothing but 30 divided by the value 40 so you will get 0.75 ampere. So, in this way the benefit which you will get is that you will keep the left of this particular segment, the left of the terminal a b same and you will keep on bearing the load resistance and you can find out the value of load current when various load resistances are given.

In this way you can calculate the value of the load current for various Loading conditions. We stop here with the initial discussion on the Thevenin Theorem when we considered only the independent voltage or current sources. In next session we will discuss the case where we have dependent voltage or dependent current source is also available in the circuit. In that case how we will analyze the circuit? Thank You.