Basic Electric Circuits
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Module 3: Network Theorem 1
Lecture 03: Duality

Namaskar, so in today's lecture we will discuss about another concept called duality. Duality is applicable for various engineering application but in this particular course we will particularly discuss about duality in relationship with the electrical circuits, so let us understand what is duality.

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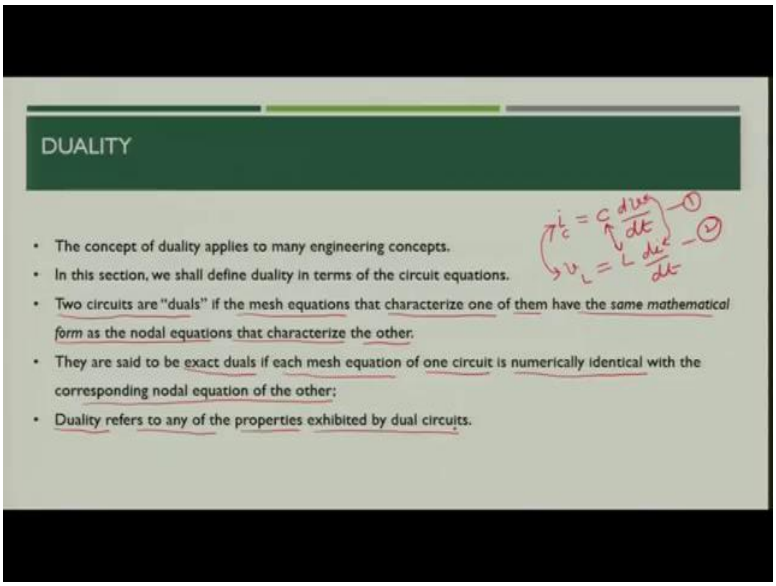

So, when two circuits are dual that means the mesh equation that characterize one of them have the same mathematical form as the nodal equation characterize the other one, what does that mean? It means that if you write the mesh equation for one circuit and write the nodal equation for another circuit, the mathematical form of both of the equations would be same. If you recollect, what is the current across the capacitance?

$$i = \mathrm{C}\frac{dv}{dt}$$

Similarly, what is the voltage across an inductor?

$$v = \mathrm{L}\frac{di}{dt}$$

So if you see these two equations both looks similar in mathematical terms because both have one term before the derivative that is d by dt of v and d by dt of current i, so from mathematical point of view this two are having the same mathematical form. That is why these are dual so what happens in that case, you can say that current through the capacitor would be dual of voltage across inductor, capacitance would be dual of inductance and similarly voltage across capacitor would be dual of current flowing through the inductance, so in this way you can say that the various elements and the various electrical variables of the circuit are dual.

Now these are exact dual, each mesh equation of one circuit is numerically identical with the corresponding nodal equation of the other, in that case we can say that both of the circuits are exactly dual, so what does duality means, duality refers to any of the properties exhibited by the dual circuit.

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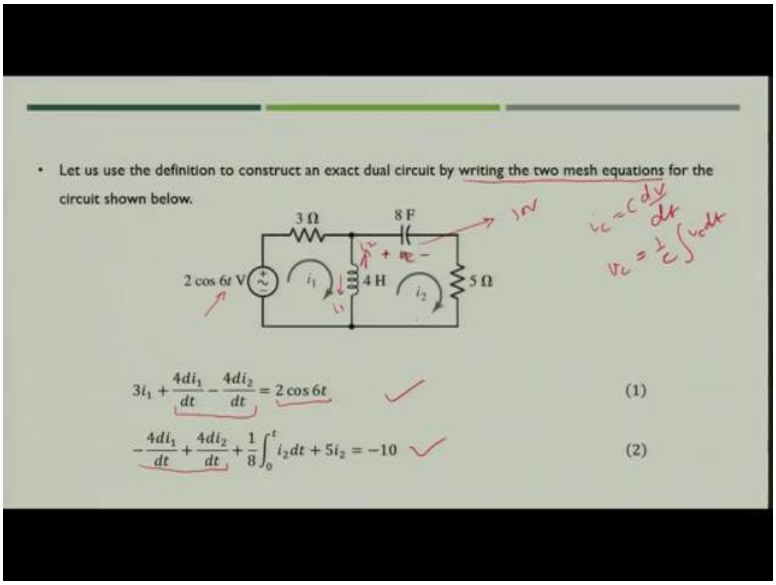

Now let us understand this concept with respect to one case study, so let us see one circuit which is shown in the figure, what we can do we can write first two mesh equations, so recollect of what we discussed in case of mesh analysis although we did it for mainly the DC circuit but same is applicable for the circuit where the source is not DC, in this case it is sinusoidal but the principle will be the same as in case of DC as well as AC, so what we will do in this case if you take two circuits and we define mesh current like i_1 and i_2 in his case we can write the mesh equation, how we can write?

$$3i_1 + \frac{4di_1}{dt} - \frac{4di_2}{dt} = 2\cos 6t$$

Now in this particular mesh what we have assumed that there is initial voltage across this capacitor which is given as v_C and the value of $v_C = 10$ volt, so how you will write the mesh equation, you will write the mesh equation for this second mesh as

$$-\frac{4di_1}{dt} + \frac{4di_2}{dt} + \frac{1}{8}\int_0^t i_2 dt + 5i_2 = -10$$

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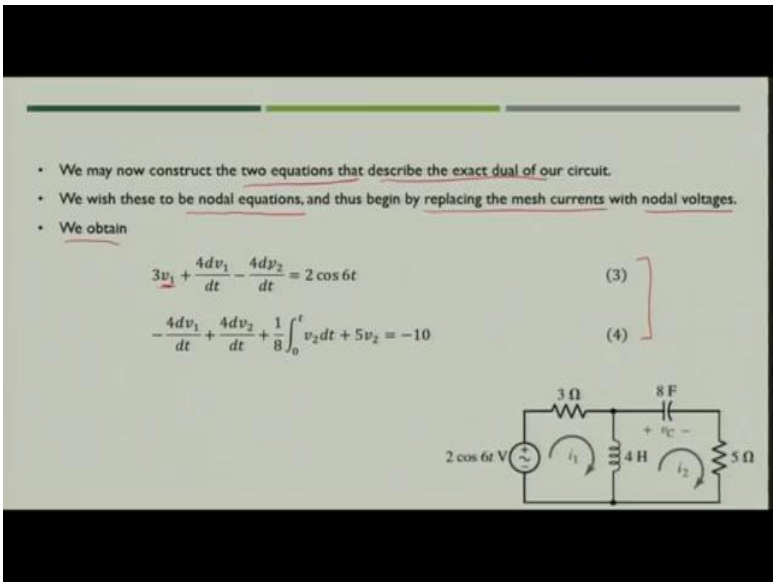

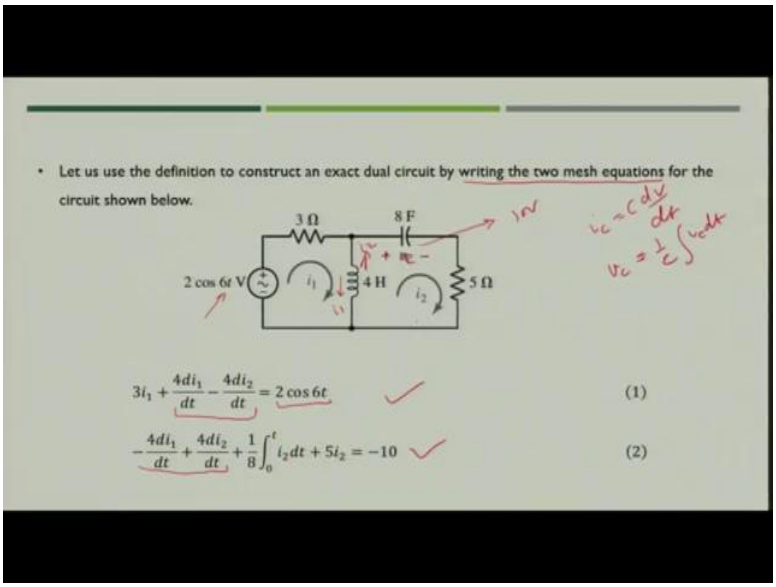

Now what we have to do, we have to construct two equations that describe the exact dual of our circuit. So, since the pervious equation were mesh equations so dual of mesh equations are nothing but node equation, so what we have to do now we have to write the nodal equations, so what we can do, we can just replace the mesh current with the nodal voltage and we can

write it, so in this case i_1 is the mesh current, so we can simply replace i_1 with v_1 and i_2 with v_2 and we can write the exact dual of these two equations.

$$3v_1 + \frac{4dv_1}{dt} - \frac{4dv_2}{dt} = 2\cos 6t$$

$$-\frac{4dv_1}{dt} + \frac{4dv_2}{dt} + \frac{1}{8}\int_0^t v_2 dt + 5v_2 = -10$$

So now we have got the two equations for nodes, now if you see these two nodal equations you can easily verify that there are essentially two nodes.

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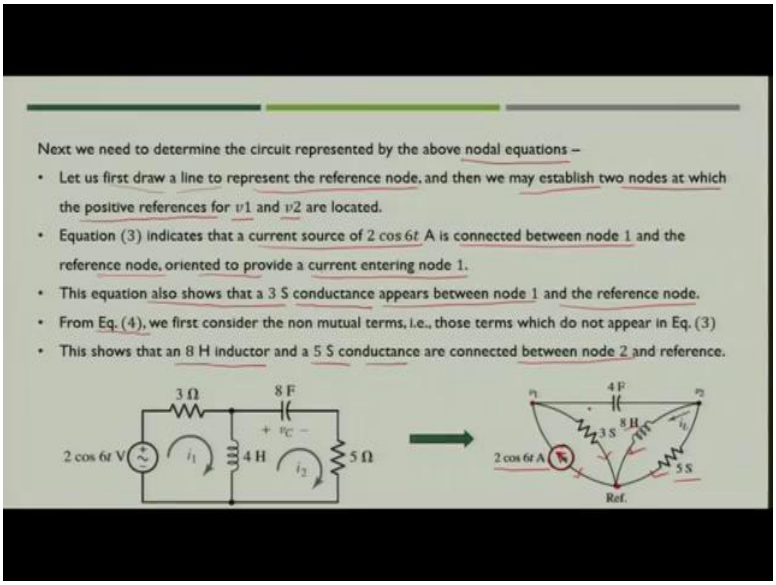

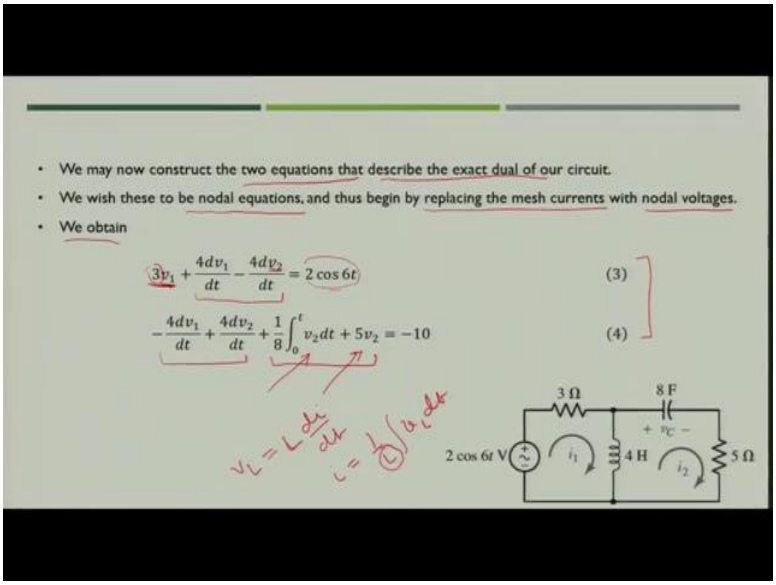

So, what do we do? We will first analyze these two equations and try to find out what does it mean, so when we see this two equations we come to know that there are two nodes, so what we will do first we will draw a line to represent the reference node because the reference node is anyway required and then we establish two nodes at which the positive reference for v_1 and v_2 are located.

So what we are doing, we are creating one reference node that is anyway required and then since we are using i_1, i_2 the dual of them would be v_1 and v_2. Now the current source of $2\cos 6t$ ampere is connected between node 1 and reference node because this is nodal equation and this representing the current value which is anyway representing for only for node 1 it

means that this current source is connected with the reference node and the node1, so what we can write we can create one current source with equal value because voltage is $2\cos 6t$, current will also be having the same magnitude that is $2\cos 6t$ but in this case this is now current source and this will be connected between node 1 and the reference node.

Now how will define the orientation, if you see the voltage this voltage is giving the positive current that is the current in the same direction which we have assumed i_1, if this is the case then corresponding dual current source would be nothing but the giving current to the node, so it means that the current source having direction going inside the node v_1. In this way we will say the current would be entering node 1. Now if you see the equation it has another component $3v_1$, what is 3, 3 is nothing but conductance because this represents the current value in node equation so this will become current and 3 would be conductance, so 3 Siemens conductance would appear between node 1 and the reference node because in this case it is connected only in mesh 1, so 3 S conductance would essentially be connected with reference and node 1 only.

Now let us come to the next equation. Let us concentrate on those terms which are not common to both of the nodes, so this two are nothing but the value of currents and these are nothing but 8 henry inductor and 5 Siemens conductance which is connected to only node 2, why it is now the inductor because if you see the voltage across inductor it is L di by dt, so i would be nothing but 1 upon L integral v_L dt, since this is the voltage term so if you compare both of them the value of inductance would be 8 and v_L is nothing but voltage at node 2 so this will now become the inductance and this will be again the conductance as we saw in the equation 3, so this two are only connected to node 2 so this would be coming in parallel, 5 Siemens conductance and 8 henry inductance. So we got 1, 2, 3 and 4 elements of the circuit.

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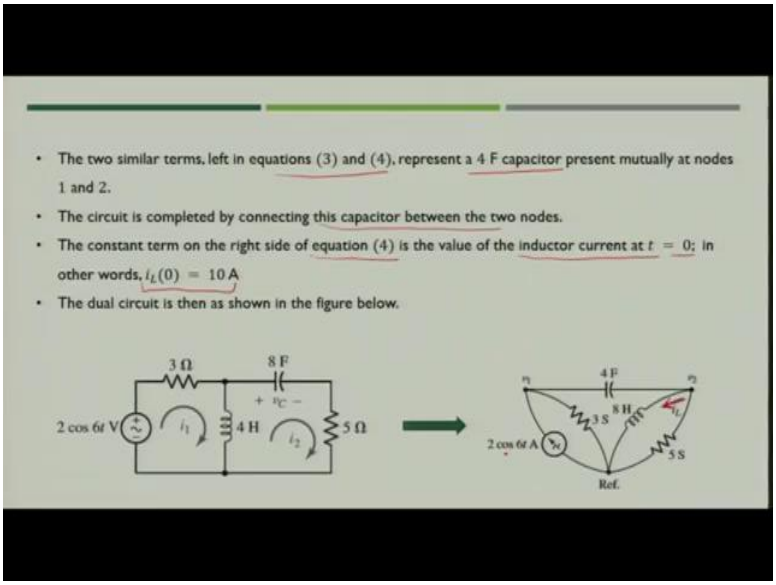

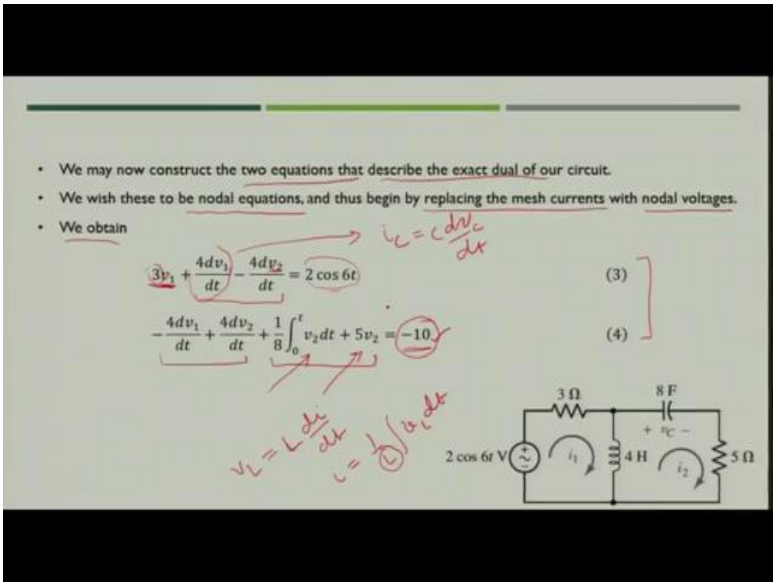

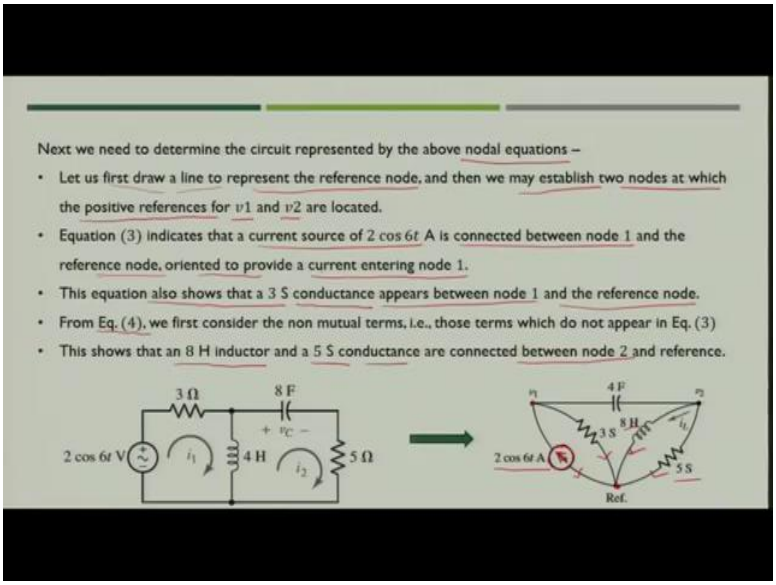

So, now coming to the element which is common in both equations and what it will represent, it will represent 4 farad capacitor, how? If you see this, this is nothing but the value of the current which is connecting two nodes, so if this is the value of current it signifies what, if you correlate with capacitor current i_c is nothing but C dv by dt, so if you compare this two this will be something called i1 this will be i2 and i1 minus i2 means the values which are connected between two nodes, so finally what we got is the capacitor which is connected between voltage v1 and v2.

So we connect the capacitor also now, having value of 4 farad, finally what is left, we left with the equation 4 value that is inductor current at t is equal to 0, because here if you see the value of minus 10 is coming and since this is the equation for node and this is representing the current value so what is left is that we will say that the dual of that is the capacitor was the voltage across the capacitor was minus 10 volt, so in case of dual this would be current through the inductor, so we will assume that this is again the initial case because for capacitor also we assume that voltage across the capacitor is at initial time t is equal to 0, so here also you can say that the value is inductor current at t is equal to 0 and value is 10 ampere, so we will represent the initial current i_L at time t is equal to 0, now how you will, find out the orientation, again if you see this circuit voltage v_C is opposite of the mesh current direction i2 so that means now in this case the current would not go inside the non-reference node v2 but it will come out of the non-reference node v2, so that is why the direction of current $-i_L$ is in this direction.

Now this is the circuit we came out while analyzing the mesh and nodal equations, so this sometimes is very difficult when you have more nodes and more meshes, then what you can do.

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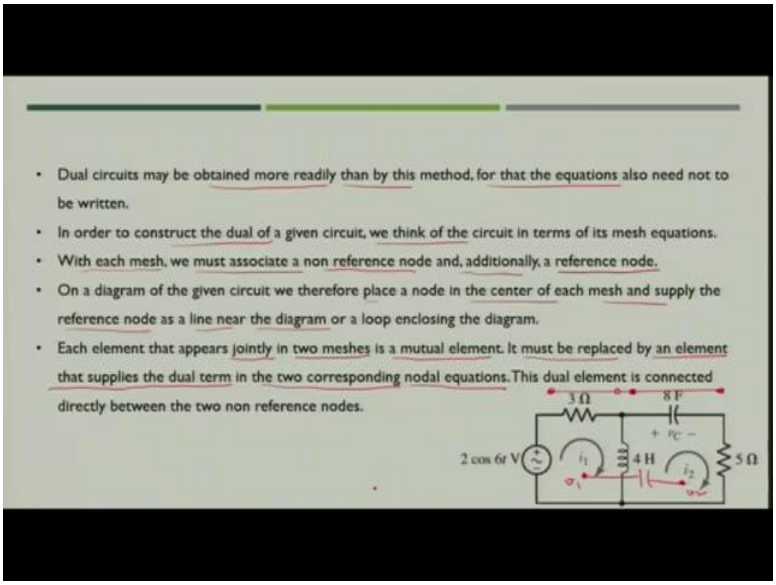

You can use one analogy which we will discuss that you can obtain this particular circuits more readily by writing the equation and you need not to write the equation why because you will construct the dual circuit by inspection, how will do that, now with each mesh we must associate one non reference node and additionally a reference node, so what will happen in this case there are two meshes which are there in this circuit, so for each mesh let us assume there is one node in both of the meshes and there is one difference node also, so let say the voltage is v1 and v2.

So, by inspection also you can say that there would be two non-reference nodes because there are two meshes being constructed plus you need to additionally have one reference node, now on the diagram we place node at the center of each mesh and supply the reference node as a line near the diagram or the loop enclosing the diagram, so you can say like simple node or you can create a line for nodes, because it is easier for you to connect so you can represent reference in any form like long line having nodes at each note having same potential, now each element that appears jointly in two meshes each a mutual element, so it must be replaced by an element that supplies the dual term in the two corresponding nodal equation.

So, what will happen here this is the element which is connected between two meshes, so the dual of this would be what? The dual of this is the capacitance so what you can write here you can say that dual of this is capacitor so since it is connected between two mesh in the nodal case it will be connected between two nodes, so you can simply add these two nodes through one capacitor.

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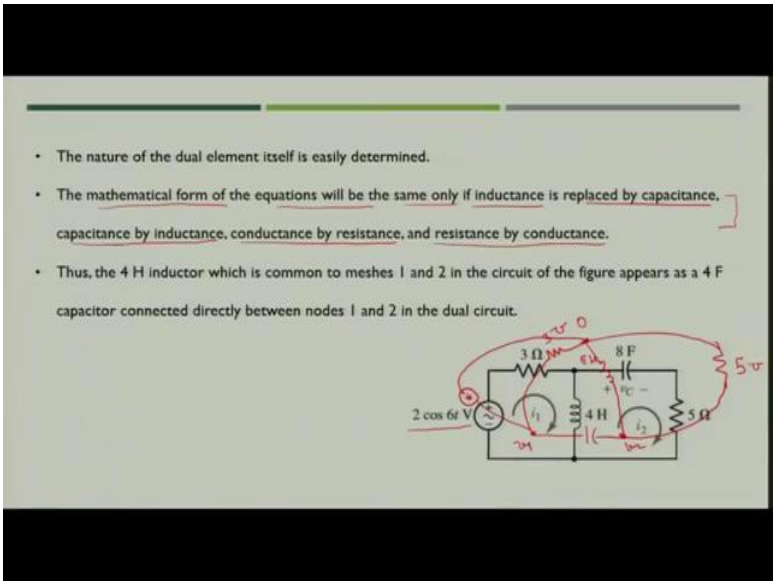

Now what next? The mathematical form of the equation will be same only if the inductance is replaced by the capacitance, capacitance by inductance, conductance by resistance, resistance by conductance, it means that this are the duals of each other, so what you will do this you have already identified that this will be the capacitance so you will make a capacitor, now the 3 ohm resistance 8 farad capacitor and 5 ohm resistance are only in their respective meshes, it means that in dual term they will represent only respective they will be connected only with respect to their nodes.

So, what will happen you can just connect resistance to the reference node and this value would be this will not be resistance this would be conductance having value of 5 moh or 5 Siemens, similarly for this capacitor what you can say, you can say that the dual of this capacitance is inductance, value would be 8 henry and it will be connected between the reference node and the non-reference node v2, similarly for this resistance what you can say, you can say the dual of this is a conductance having a vale 3 moh.

Now finally what is left, the left value is the voltage source, so in this case it will be nothing but the current source now what would be the direction because direction of voltage is in such a way that it is giving positive current so the current will also be in such a way that it is going inside the node, so the direction would be like this.

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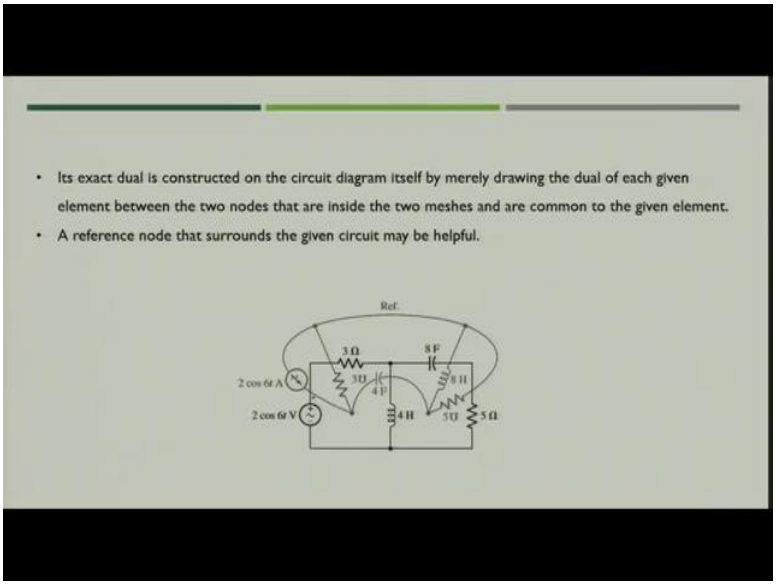

So in this way you can simply represent the values of the equivalent, dual network, so what we have discuss you can simply see from here that this are the values which we have seen and now this are connected to the reference node.

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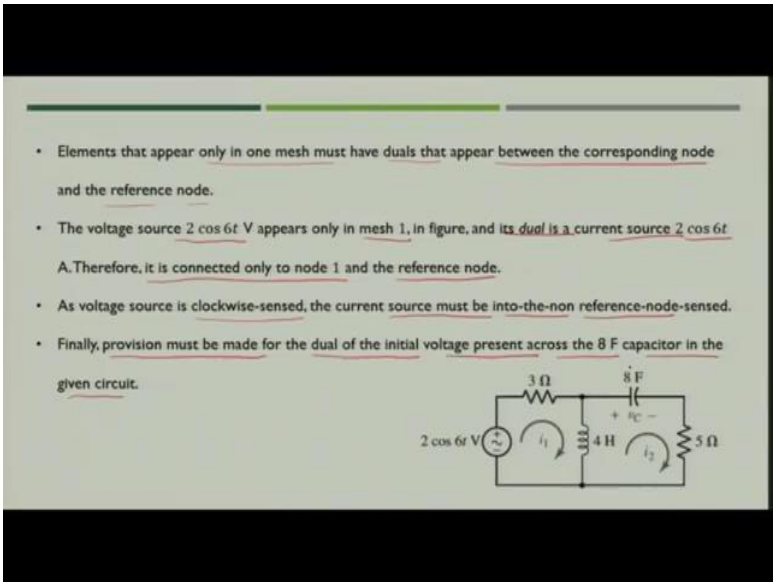

So now let us summarize what we have done the elements that appear only in one mesh must have dual that appear between the corresponding node and reference node only.

Now voltage 2 cos 6t we appear only in mesh 1 so it's dual would be current source and the value would be 2 cos 6t therefore it is connected only to node 1 and the reference node. Now the clockwise voltage source which we have seen that means the voltage is giving the current which is flowing in a clockwise, so the dual would be current should be going inside the non-

reference node and provision must be made for the dual of the initial voltage present across 8 farad capacitor, so in this case what will happen the 8 farad capacitor voltage would be represented as 10 ampere current through the inductor which is the dual of the capacitor,

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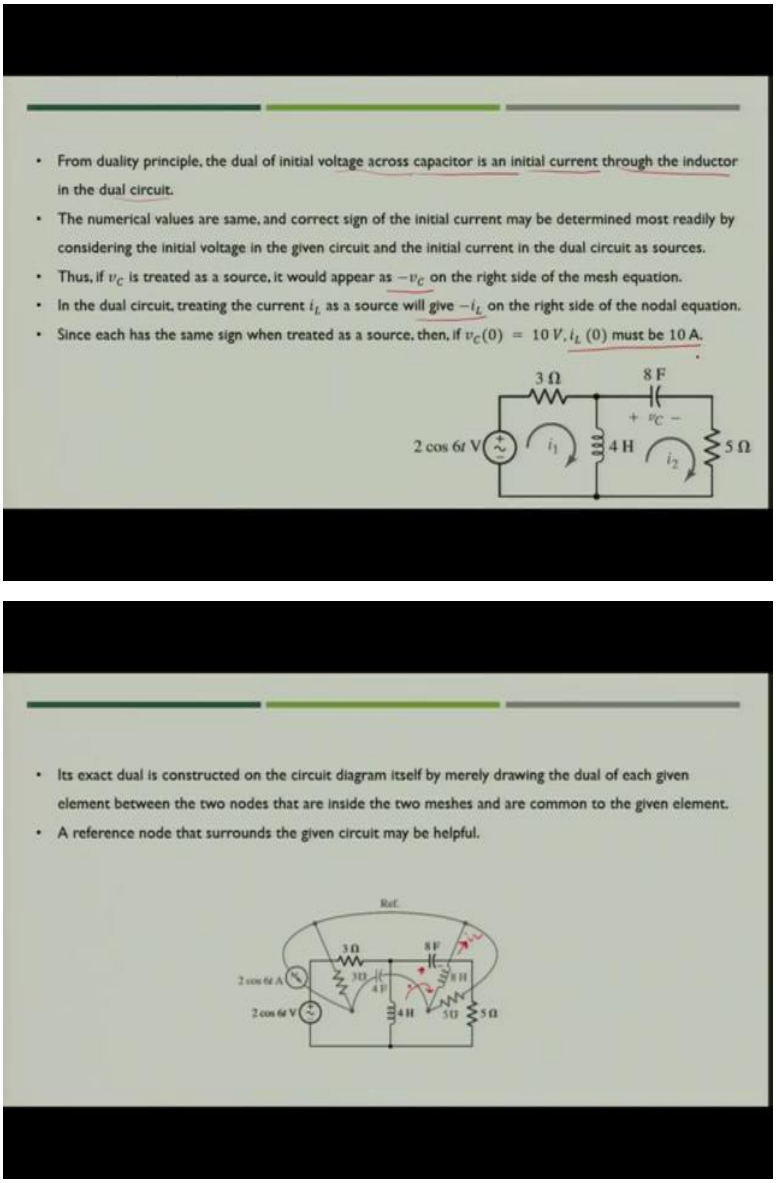

So initial voltage across the capacitor is an initial current through inductor in the dual circuit. Now what would be the value, here the value is $-v_C$ because it is opposite to the direction of the current, so the current would be $-i_L$ or alternatively you can say current of 10 ampere but it is coming it is coming out of non-reference node, so in this way the direction of the current would be this, because this would be against what we have taken the assumption in the case of mesh analysis because voltage across capacitor is opposing the flow of mesh current so that is why the dual value of the dual current value would be the direction of that dual current through

that inductor would be coming out of the node, so this way by analogy also you can simply represent the dual of the particular circuit.

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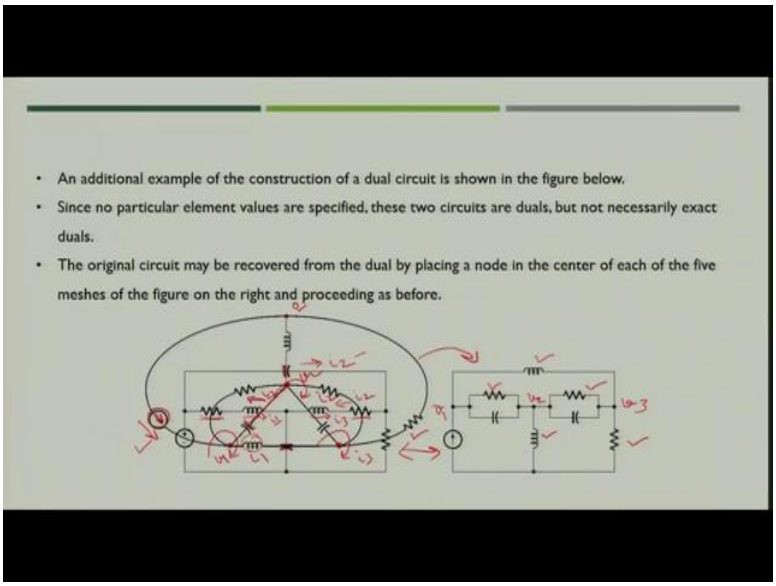

Now let us take one example so that you can further understand what we discus till now, let us say that we have a circuit given in the figure where we have resistance, inductance are connected in series and we have one capacitance, one resistance and one capacitance here between this two nodes, so what are various mesh currents? So you say that there are 3 meshes that is i1 may be i2 and i3, so rather than writing the mesh equation and correspondingly finding out the nodal equation what we will simply do we will simply use the analogy, we will say what is the dual of which particular circuit element and then we will draw the dual circuit.

So in this case for this inductor the dual is capacitor so we have first what we will do, we will first identify the nodes, so there would be 3 nodes because there are 3 meshes and plus 1 reference node between this current mesh current i1 is flowing in the inductance in this direction while current i2 is flowing from the upper mesh in this direction, so it means that these are the current flowing through this inductor would be i1 minus i2 so that means that in nodal term when you replace this inductor with capacitor the capacitor should be connected between node v1 and v2.

So, in this way we have connected node the capacitor between node v1 and v2, similarly for this resistance also the current is i1 minus i2 that means that this resistance the dual of this resistance that is conductance would also be connected between node 1 and node 2, so we have connected this conductance in parallel with capacitance in this particular node. Now if you see this two inductors and resistors both are connected between node 2 and node3, why? Because

we are assuming current i3 in this direction while current i2 would be in this direction, so this will show that the inductor dual that is capacitor again would be connected between node 2 and node3.

Similarly, for resistance dual that is conductance will also be connected between node 2 and node 3, now next what we have left with is the capacitance value that is between this side to this side of the circuit, which current is flowing here? If you see this is having only current i2, no any current is flowing it means that the dual of capacitance that is inductance would be connected between node 2 and reference node only, so that is why this is connected between node 2 and reference node.

So we have covered all the elements of this segment, finally we are left with this resistance where only node the mesh current i3 is flowing, so it means that this would be connected the dual of this would be connected between node 3 and reference only, so the conductance of this dual would be connected between node 3 and reference node, now we have left with the voltage source, so voltage source is giving the positive current to this mesh so that means that the dual of this would be a current source which would be coming inside the node.

So that is why the dual of this is represented as current source with current in this direction because if it is giving current in a clockwise direction to the mesh that means that the corresponding dual current source will have current flowing inside the node so using the circuit and corresponding the dual so each and individual element we can draw the dual circuit, now rearrange the what circuit would look like as shown in this figure, where you will see that capacitance and conductance these two are in parallel source, so you have this is v1, this is v2 this is v3 that means node1, node2 and node3 the capacitance and conductance are connected in parallel between v1 and v2, so this is represented in this way.

Now between v2 and v3 again you have one capacitance and one conductance so you have represented in this way, now the element which is inductance is connected between node 2 and reference, so you have got this conductance and then you have another inductor which is connected between v1 and v3, so this is the inductance which you have got which is between node 1 and node 3 and finally between node 3 and reference you have one conductance and the current source which is connected between reference and node 1, so this circuit you can rearrange and represent it nicely so that you do not have ambiguity in representing the dual circuit, so basically what happen that this circuit has become the dual of the circuit so the

important thing you need to figure out from this circuit is that most of the circuit elements which are in series are now converted into parallel.

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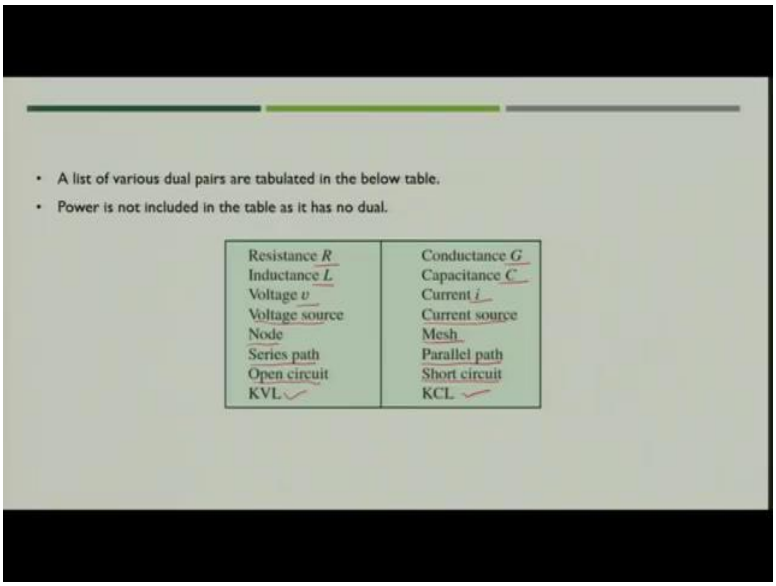

So what we can say what are the various dual parameters resistance are would be converted into conductance, inductance L would be converted into capacitance, voltage V would be converted into current I, voltage source would be current source in source of its dual circuit, node would be the mesh in the dual circuit, series path would be parallel path which we have just discussed, open circuit would become short circuit, KVL would be KCL in case of its dual so this is what we started initially with we started with mesh equations those are nothing but KVL and then we created dual with the help of KCL that this are nothing but the node equations and the accordingly we found the dual of the circuit. So now with this discussion on the dual circuit let us close the session of today's lecture, so in this lecture we discuss about what is the dual circuit, what is duality, in the next lecture we will start with various network theorems like Thevenin's theorem, Norton's theorem and so on, thank you.