Basic Electric Circuit
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Module 3 Network Theorem 1
Lecture 11 Linearity Property and Source Transformation

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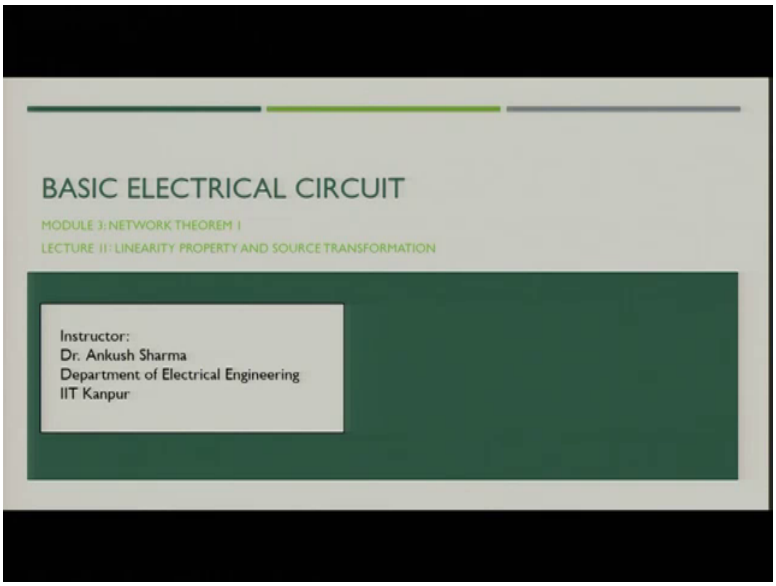

Namashkar, so welcome to the 3rd week of our course on basic electrical circuits. So today we will start the discussion on network theorem, and in this particular lecture we will discuss about the linearity properties and the source transformation.

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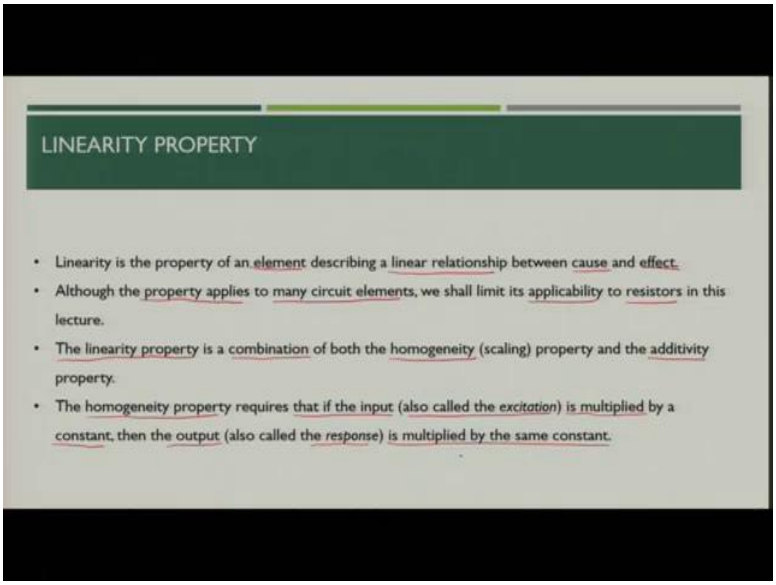

So let us start the topic on linearity property. What is linearity? Linearity is the property of an element describing a linear relationship between cause and effect. So, cause can be an input

like if you provide voltage or current input which causes the element to either absorb or dissipate the energy. Voltage or current sources is causing some phenomena inside the circuit and as a result that is the effect you will get some output voltage or current a particular element. The cause and effect will be having the relationship, which is linear in this case. Now although the property applies to manage circuit elements but in this particular lecture we will limit our applicable to resistors only, because it is easier for us to understand the property in terms of resistors and then we can consider for other linear elements like inductors and capacitors.

Now linearity property is a combination of both homogeneity and additivity. Now what is homogeneity? Homogeneity means the property which requires that if the input, or you can say that, excitation of the network is multiplied by a constant then the output is also multiplied by the same constant. That means that your excitation and the response will have homogeneity, means if you scale up a particular voltage input by say constant K. So, output effect that may be the voltage across a particular element or current flowing through that element will also be multiplied by the same constant K.

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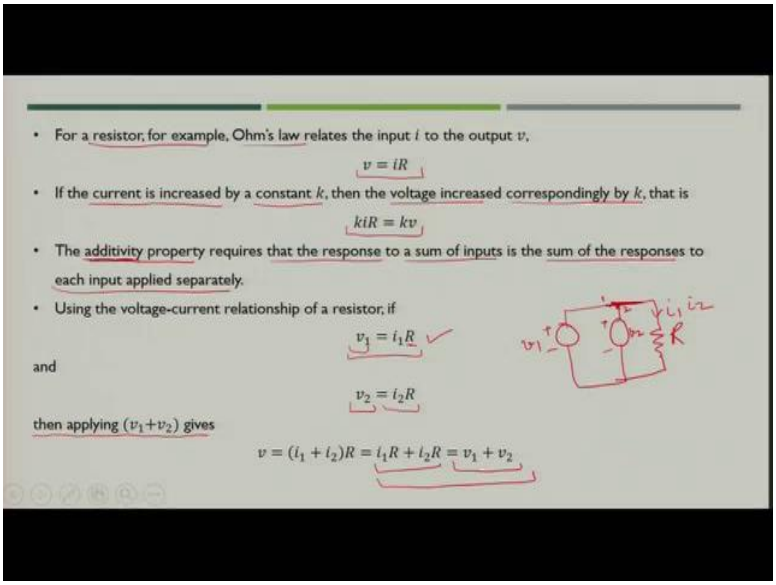

So now let us see how we will define it. Let us take the example for a resistor following Ohm's law. As for Ohm's law what you can write for voltage, voltage would be current I multiplied by resistance R. Now if it is following the homogeneity property it means that if current is increased by constant K, then voltage also will increase by constant K. In that way what you can write, you can write K * I R is nothing but K * V.

Additivity property will say that the response to multiple inputs is the sum of responses to each input applied separately. Now suppose in the circuit you have 2 voltage sources that maybe

you can say V_1 and V_2 which are applied across a resistor R. V_1 and V_2 are applied separately to a resistor R, let us take 1 dc source and we have resistor R, we have another dc source and suppose it is connected thorough some switch. So, you have switch connected either for voltage that is V_1 or voltage V_2. So now when it is connected to V_1 then you will have some current i_1 flowing through it and the V_1 will be across it and you will get i_1R after getting this value if you switch over from point 1 to point 2 it means that now it is connected to voltage V_2.

So, it will become say this current is now i_2, so V_2 would be i_2R. Now when you apply both V_1 and V_2 at the same time suppose if you are current should be sum of individual currents provided by individual voltages and it will follow this equation like $V_1 + V_2$ would be equal to $i_1R + i_2R$. So, this is basically called as additivity property means you can add the responses of the individual voltage or current sources and get the final response that may be in the form of current or voltage.

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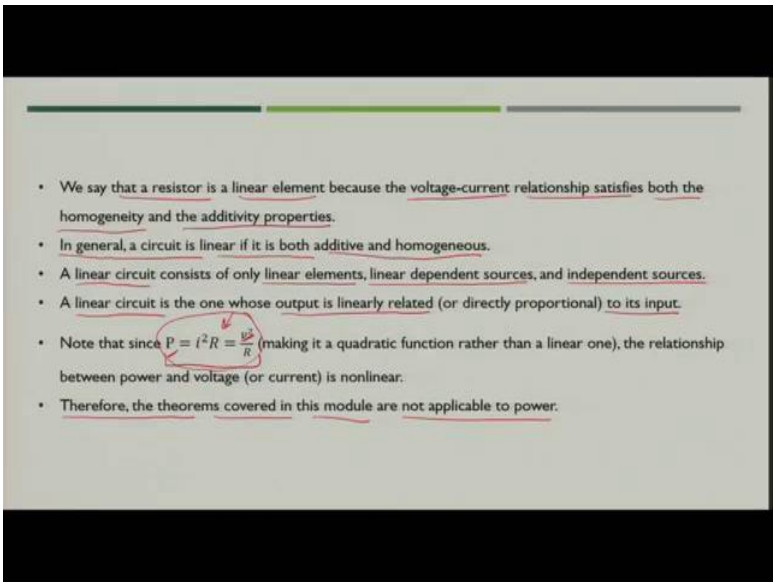

Now we say that a resistor is a linear element, why, because the voltage and current relationship of the resistor satisfy both the homogeneity and additivity properties. So, what we can say in general this circuit is linear if it is both additive and homogeneous. Now what is a linear circuit? Linear circuit is the circuit which contains only linear elements linear depended sources and independent sources.

So, in the linear circuit the output is linearly related to it input. Now these, although this circuit may be linear but its input output relationship may become nonlinear. So, can we apply the

homogeneity and additivity in that case, so if suppose your input is voltage and output is power across a resistor, so what relationship they will follow? They will follow relationship like

$$p = i^2R = \frac{v^2}{R}$$

So now if you see this particular equation the relationship between voltage and power is not linear. So, in that case you cannot apply the homogeneity and additivity property for getting the power across the resistor because these are not linearly related. So that is why whatever the theorems we will cover in this particular module will not be applicable to power because the relationship between input and output is not linear.

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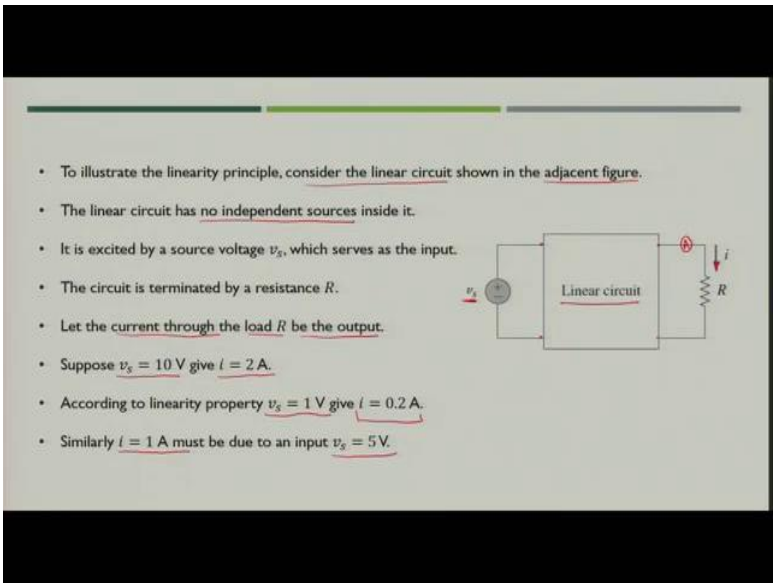

Now let us take one example to better understand the linearity property, let us consider one linear circuit shown in the figure below. Here we have linear circuit which is inside this particular box we do not know what is the configuration of that circuit we know that some input is being applied that v_s and we are finding out what is the value of current flowing through the resistor R?

Now, the assumption is that there is no independent source inside the linear circuit. So, the source is only v_s and the output which we want to measure is current flowing through resistor R. Now if current through the load R is the output and in one case when you apply $v_s = 10$ V give $i = 2$ A.

Suppose you are doing an experiment way you do not know what is inside but you are applying one voltage source across its input terminals and connecting a load R across its output terminal

and you are measuring current i. So, you can say that if you added one ammeter here and you are measuring the value of current through this ammeter.

Now if you have applied 10 volt to the input of linear circuit and you got current is 2 ampere. So, suppose if you are asked to find out the value of current i when voltage v_s is 1 volt and you know that this is a linear circuit without doing experiment you can straight away say that current flowing through that resistor would be 0.2 ampere when voltage source of 1 volt is applied across the input terminals.

Similarly, is you want to get current as 1 ampere through the resistor what would be the value of the voltage input, the voltage input value would be 5 volts because this is a linear circuit and it will follow the homogeneity property which says that if you scale up or scale down through some parameter K the input output relationship should not change. So, if input is increased or decreased by constant K then output will also be increase or decrease by the same constant K.

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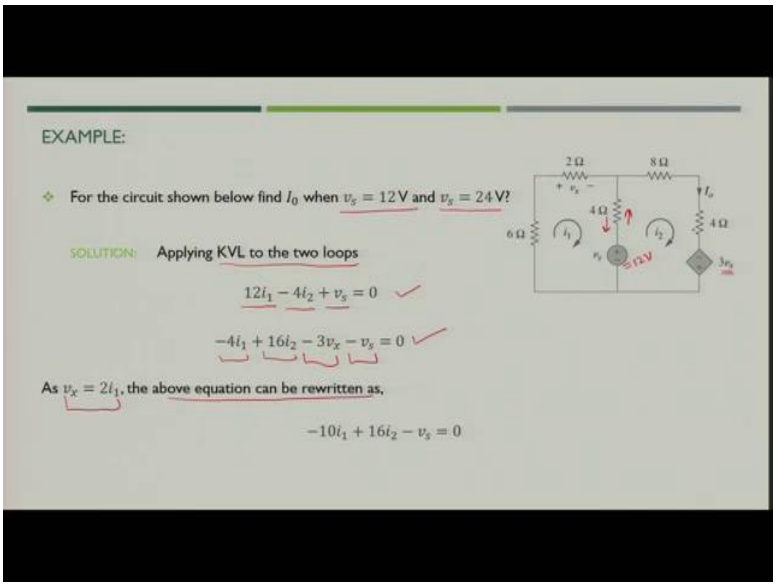

Now let us take one example to understand the properties which we discussed like linearity and the additivity and homogeneity. Suppose the voltage source $v_s = 12$ V in this circuit and we have to find out the value of current I_0 flowing through 4 ohm resistance when voltage is 12 volt and when voltage is 24 volt. So this would be the case of homogeneity what we have to do? As we discussed in last week we did mesh analysis to solve this kind of circuits.

So here we will apply match analysis again and try to find out the current value I_0 in first case that is when $v_s = 12$ volt. So, if you apply KVL to the 2 loops that is suppose if you take i_1 and i_2 mesh currents. So, for this particular mesh you can write the

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0$$

Now if you see what is the value of v_x?

$$v_x = 2i_1$$

So, the equation can be written as

$$-10i_1 + 16i_2 - v_s = 0$$

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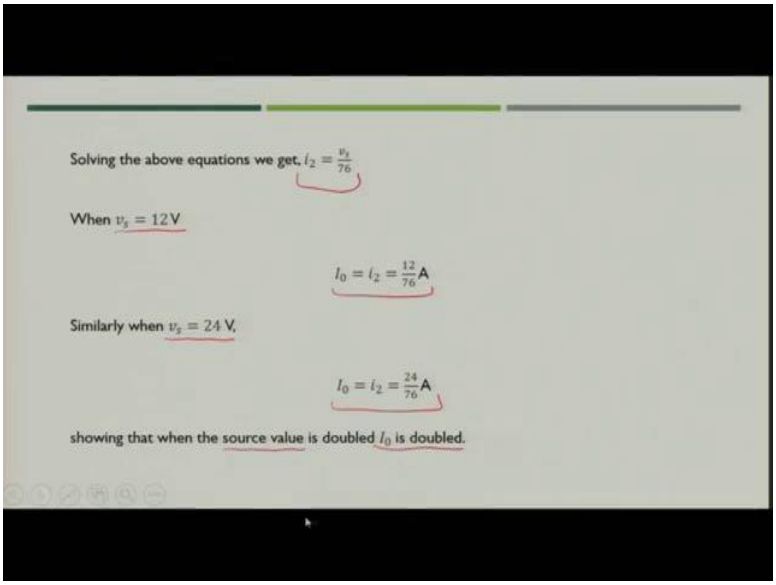

And you can use the both of the equation and solve it you will get

$$i_2 = \frac{v_s}{76}$$

The case when you take $v_s = 12$ V you will

$$I_0 = i_2 = \frac{12}{76}\,\text{A}$$

Now you need not to calculate again the complete circuit equation for Vs is equal to 24.

Because you know that this particular circuit is a linear circuit. So, what you will do you will simply use the homogeneity property and you can say that the I_0 in that case when $v_s = 24$ V would be $\frac{24}{76}$ A. So, it means that when sources value is double I_0 will also be double because of homogeneity property.

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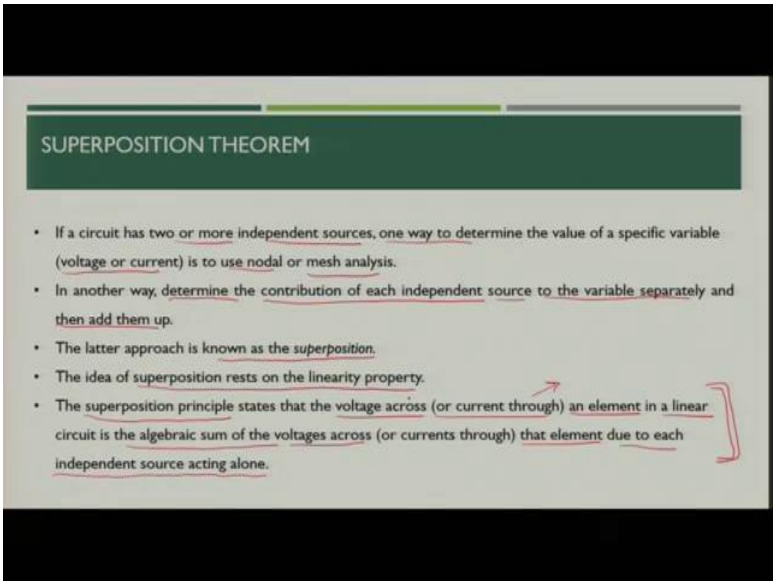

Now let us talk about one important theorem called superposition theorem. Now if a circuit has 2 or more independent sources the one way to determine the value of a specific variables that is may be voltage or current is to use nodal or meh analysis, which we discussed in the previous week. Now in another to determine the contribution of each independent source to the variable separately and then you add them up to get the final variable value.

So, in this way when you determine the contribution of each independence source separately and then adding up is called as a super position theorem. So, what you do in superposition theorem? In super position theorem the voltage across an element is the algebraic sum of voltage across that element due to each independence source acting alone.

So this will give a complete property of super position theorem where if you have multiple voltage sources or multiple current sources in the network you have to take a 1 source at a time and find the voltage across a current through an element of your interest means the given element property which you want to find out and then when you get the variables value independently for each and individual voltage or current source you will add them up get the final value of the variable.

That may be the voltage or current why? Because this superposition is following the linearity property. So, the homogeneity and additivity both would be applicable when you have to use superposition theorem to solve circuit.

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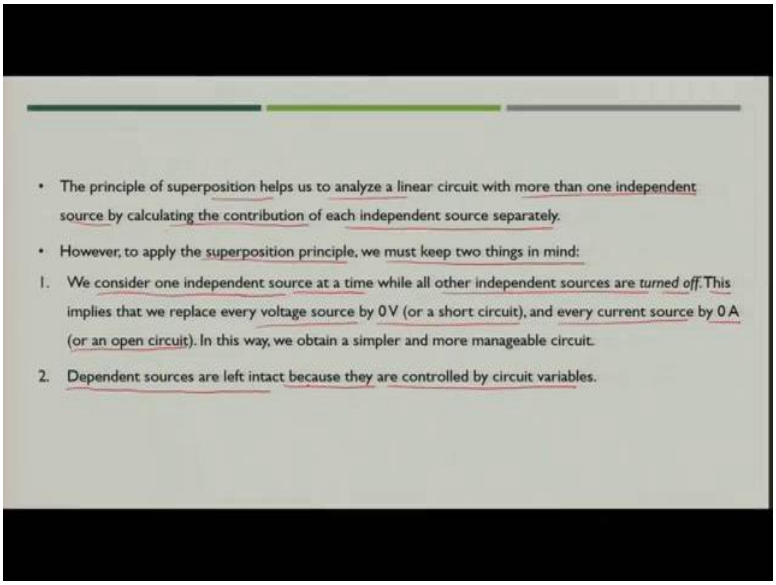

Now this principle of superposition theorem helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. Now before applying this superposition theorem we have to keep 2 things in mind that when you try to solve the circuit you will consider only one independent source at a time while the other independent sources are turned off.

So, turned off means what? In the case of voltage source, it will be short circuited and in case of current source it will be open circuited. So, if you have multiple voltage and current source in the network you will take one source at a time and other sources would be either short circuited if they are a voltage sources and the current source is would be open circuited.

Now second important thing is that you cannot apply that particular assumption in case of dependent voltage or current sources and you have to keep them with the circuit and solve them as you have solved for others circuits like nodal analysis or match analysis. So dependent source may be voltage or current source should be left intact in the circuit.

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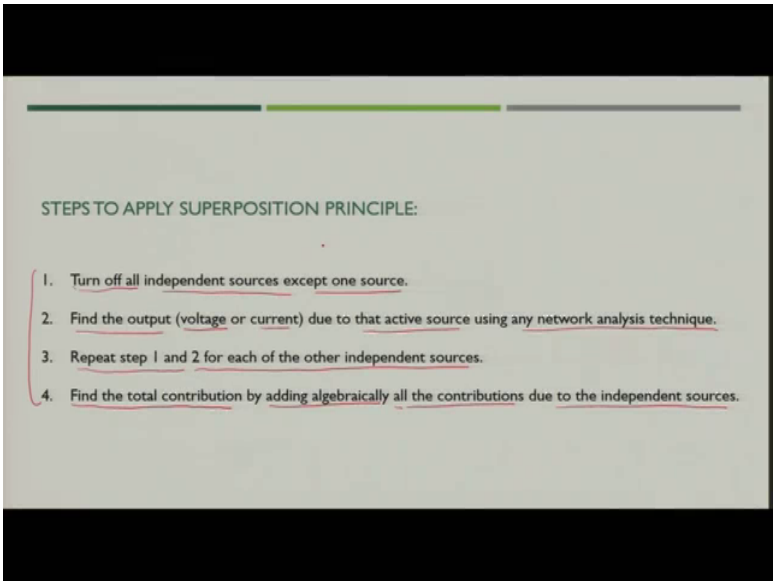

So, in summary what you can say for applying the superposition theorem you will first turn off all the independent sources except one, find the output that may be the voltage or current due to that active source using any network analysis technique. Now you will repeat its step 1 and 2 for other independent sources. So, these 2 steps would be repeated for other independent sources taken one at a time. Then find the total contribution by adding them algebraically so that you get the contribution of all the independent sources. So, these 4 steps are utilized to apply the superposition theorem and finding out the circuit variables.

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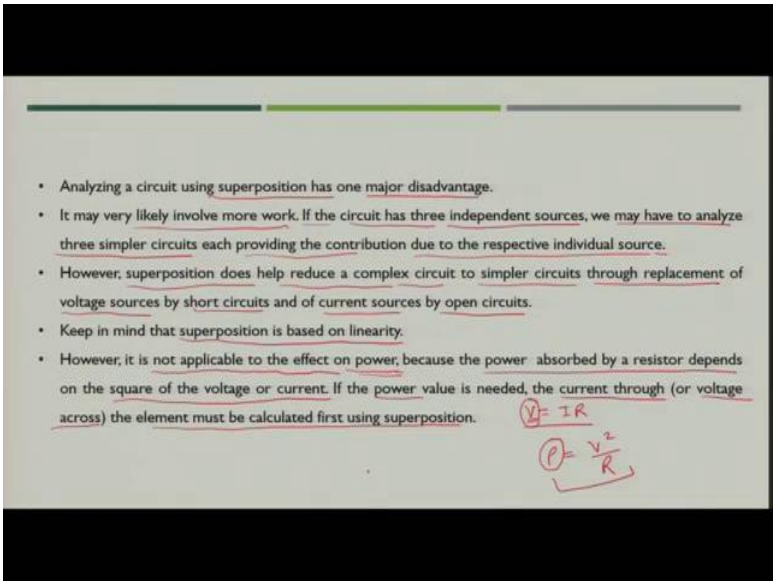

Now one major disadvantage you will see when you apply superposition theorem that is that you need to have more work when you are using superposition. The reason is simple because when you apply superposition theorem you have to take one source at a time so it means that the same circuit have to be analyzed multiplied by number of independent sources in the network.

Suppose if there are 3 independent sources that means that you have to analyze 3 circuits and after that you have to combine to get the final output. So, it means that the same circuit you have to analyze 3 times by making them simpler, simpler means the current voltage sources would be either short circuited or current source would be open circuited and voltage source would be open circuited except the one which you are going to analyze in that circuit.

So, this will be giving you more work to analyze the circuit but the important thing is that it helps us to reduce very complex circuit to very simple circuit because you are replacing the voltage source by short circuit and current source by open circuit. So, using super position theorem all though you are analyzing multiple circuits by taking 1 voltage source or 1 current source on at a time but eventually you may get the solution very early because the circuits are now simpler as compared to having the multiple sources connected and you are analyzing that circuit.

And super position theorem is based on linearity, so it is easier to analyze and then at the responses individually to get the final outcome. Now that we have discussed earlier that the relationship between voltage and power and current and power is nonlinear. So, you cannot apply super position theorem to find out the value of power. Why? Because power absorbed

by resistance depends on square of the voltage and current which is nonlinear relationship between voltage and current.

Now if you need to find out the value of power what you can do? You can either find the value of current or voltage with the help of superposition theorem and then finally calculate the value of power after getting the value of current and voltage. So, what you can do you can use simple relationship is Ir and using super position theorem you can find out the value of voltage across resistor using superposition theorem.

When you get final voltage, you can simply calculate the value of power with the help of this equation that

$$p = i^2R = \frac{v^2}{R}$$

So you need not to go for finding out the value of power p directly using superposition theorem but you can use the linear relationship of VI and use super position theorem to solve the circuit.

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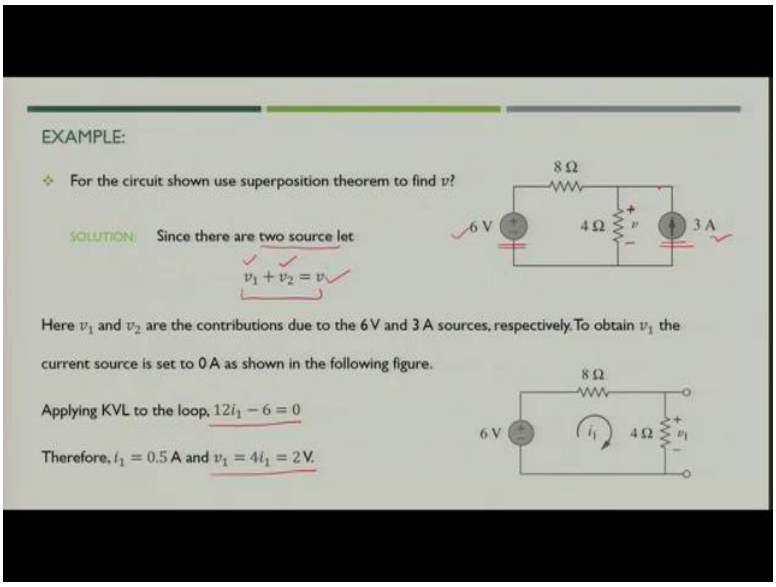

Now let us take one example so that you can understand the super position theorem more clearly. Now if there are 2 sources, if you see in this particular circuit you have 2 sources one is voltage source other is current source. Now you need to find out the voltage across 4 Ohm resistor. So, you can say that you are dividing the circuit in 2 parts you are taking 1 source at a time so first you will take voltage source and second you will take as current source and analyze the circuit.

Suppose when you take voltage source first and analyze this circuit you have got voltage across 4 ohm resistor as v_1 when you apply circuit apply current source in the circuit you get voltage across resistor as v_2 since it is a linear circuit the final voltage across resistor would be $v_1 + v_2 = v$. So, let us take 1 source at a time. Suppose if you take voltage source first then what you will do you will current source as an open circuit.

The updated circuit which you will see here would be like this where you will have 6 volt and then need to find out the value of voltage across 4 Ohm resistance. So, this will be a simpler circuit so you have to just apply Kirchhoff's voltage law and find out the value of voltage v_1 which comes outs to be 2 volt in this case.

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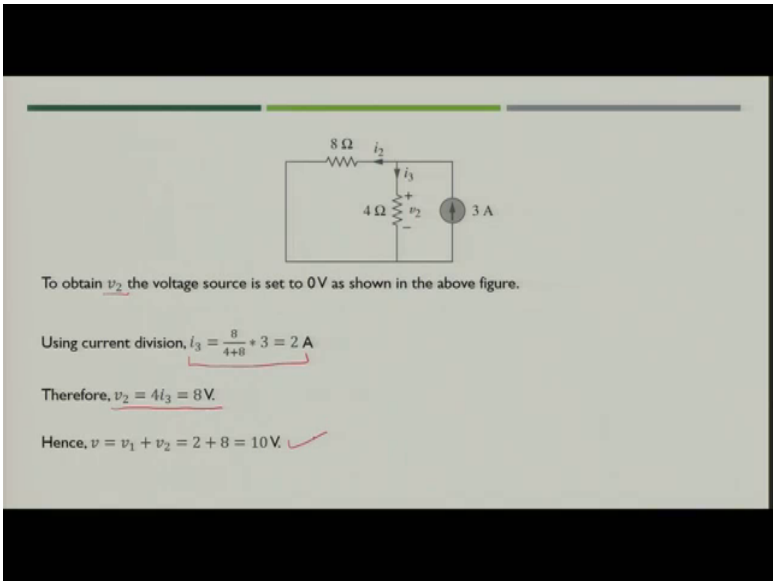

Now to obtain v_2 what you will do? You have to simply short circuit the voltage source and apply the current source. So, in that case what will happen you have to use current division at this particular note you will get the value of current flowing through 4 Ohm resistor that is nothing but 2 amperes. So, in that case volt is v_2 you will get as 8 volt and finally using super position theorem what you can say the total voltage in this particular circuit across the 4 Ohm resistance would be 10 volt.

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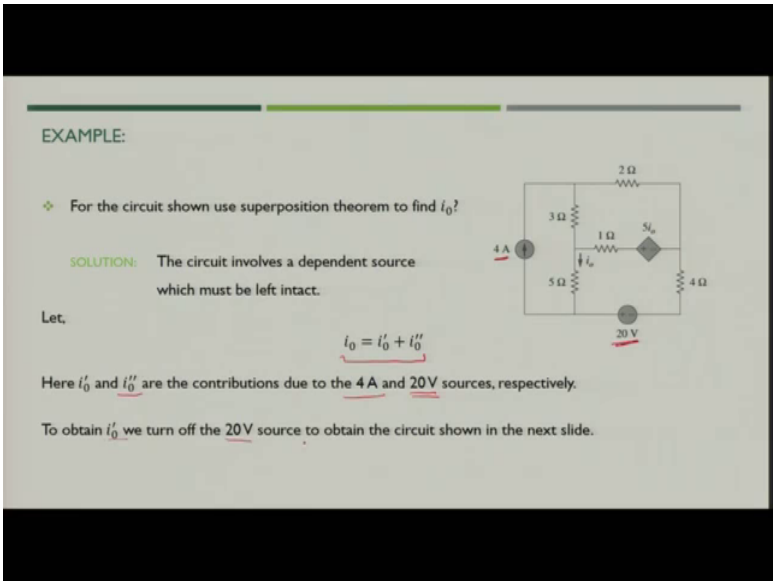

Now let us take slightly complex problem where you have 1 dependent voltage source in the circuit. So now you have to apply the super position theorem, here you will see that you have 1 current source 1 voltage source and your objective is to find out the value of current i_0 which is flowing through 5 Ohm resistance. So, you divide current $i_0 = i_0' + i_0''$, i_0' is contribution due to 4 ampere current source and i_0'' is the contribution due to 20 volt voltage source.

Now to obtain first i_0' what we have to do? We have to short circuit the 20 volt voltage source.

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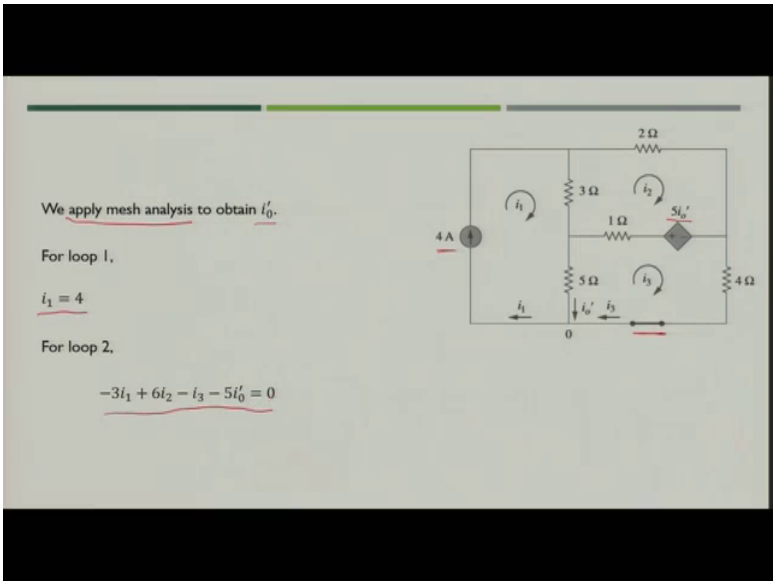

So, we will get this circuit where we have short circuited the voltage source. Now we cannot change or we cannot short circuit or open circuit the dependence sources. So, in this case this is depended voltage source so we keep intact with the circuit, the value of the voltage source

will be $5i_0'$ because right now i_0' is flowing in this particular circuit which we are analyzing while short circuiting the voltage source.

So now what we have to do? We have to apply mesh analysis to obtain the value of i_0'. So, we see there are 3 loops now for each loop if we apply the Kirchhoff's law then for i_1 the current would be 4 ampere which is straight away you can see from the figure. For loop 2 if you apply Kirchhoff's voltage law you will get

$$-3i_1 + 6i_2 - i_3 - 5i_0' = 0$$

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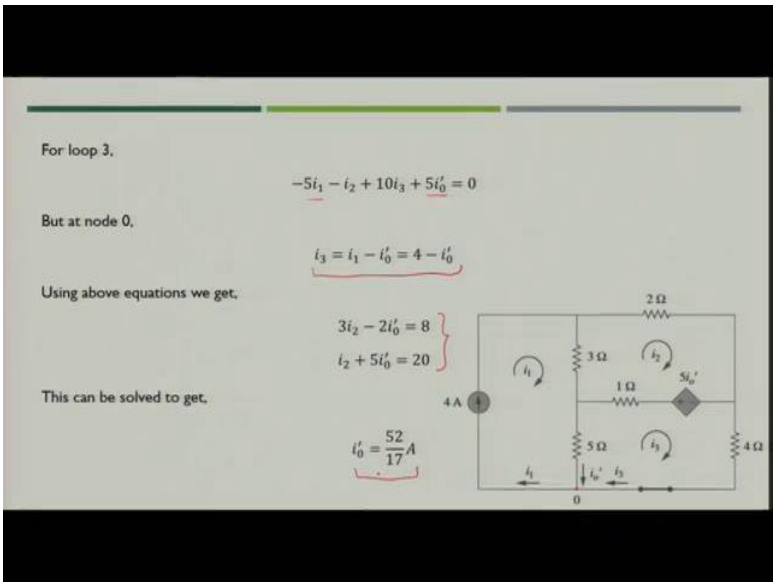

For loop 3, if you see you can write the mash equation using KVL

$$-5i_1 - i_2 + 10i_3 + 5i_0' = 0$$

Now since it is dependent source loop equation alone is not able to solve this circuit we have to take one nodal and apply KCL also to find out the current relationships.

So, if you put KCL at node 0

$$i_3 = i_1 - i_0' = 4 - i_0'$$

Using the above you will get 2 equation which are in terms of i_2 and i_0'. So, if you simplify you will get value

$$i_0' = \frac{52}{17} A$$

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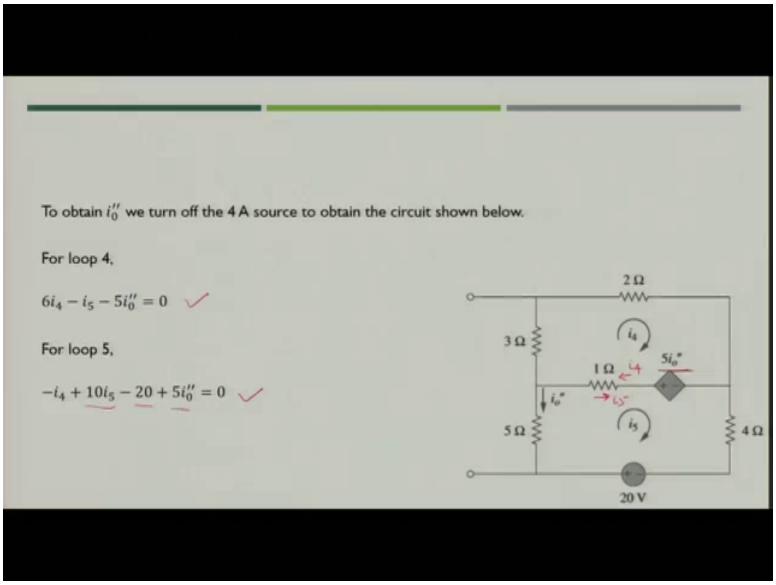

Now to obtain i_0'' that is the contribution of voltage source what you will do in this case your current through 5 Ohm resistance is i_0''. So, your dependent source value will become now $5i_0''$. Now if you write again the mesh equation for mesh 4

$$6i_4 - i_5 - 5i_0'' = 0$$

Similarly, for loop 5 you will write the mesh equation as

$$-i_4 + 10i_5 - 20 + 5i_0'' = 0$$

So why $-i_4$ because current flowing through 1 Ohm is for i_5 it is in this direction and i_4 it is in opposite direction. So -i_4 would be because of this one open resistance $10i_5$ because there are 3 resistance of total value of 10 Ohm, -20 for this voltage source and $+5i_0''$ for this dependent voltage source.

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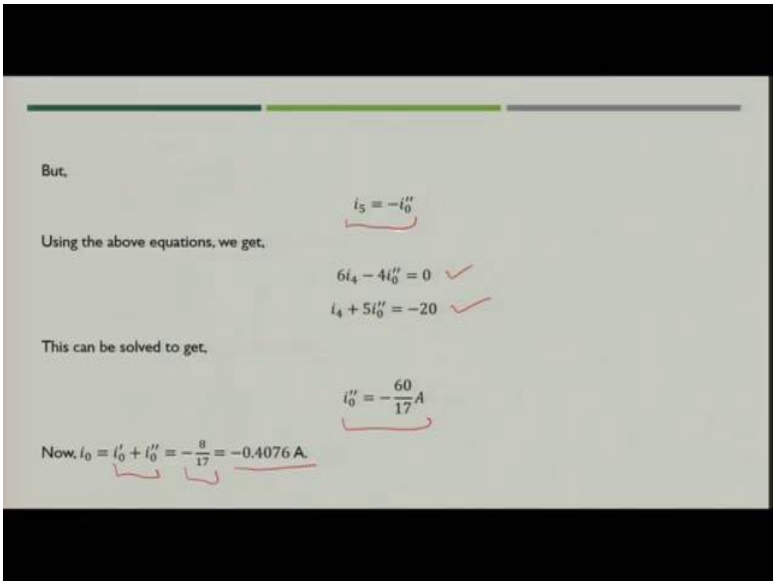

Now $i_5 = -i_0''$ which you can see from here because i_5 is flowing in opposite of $-i_0''$. So instead of i_5 we can write $-i_0''$ in these equations. So, when we put the value of i_5 you will get these two equations and when you will solve you will get

$$i_0'' = -\frac{60}{17}A$$

So finally, when you will take this particular circuit that means the final circuit you will come to know that the $i_0 = i_0' + i_0''$.

You have found the value of i_0' and i_0'' you just add the value, you will get final value of i_0 that is -0.4076 A. So, in this way you can use this superposition theorem to calculate the circuit variable. So, in this case the circuit variable was current flowing through 5 Ohm resistance, so using super position theorem first you removed the voltage source and then you remove the current source and finally rent flowing through 5 Ohm resistance because of both of these sources independently were summed up because it will follow linearity property.

So with this week close our today's session next session we will talk about few other theorems thank you.