Basic Electric Circuits
Module II
Mesh and Node Analysis
Lecture-10
Nodal Analysis
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Namaskar. So, yesterday we discussed about mesh analysis and we also discussed that how the mesh analysis would be done if current sources available. Current source may be the independent or dependent, so that particular aspect we discussed in yesterday's class. So, today we will discuss about the Nodal Analysis. So, let us see.

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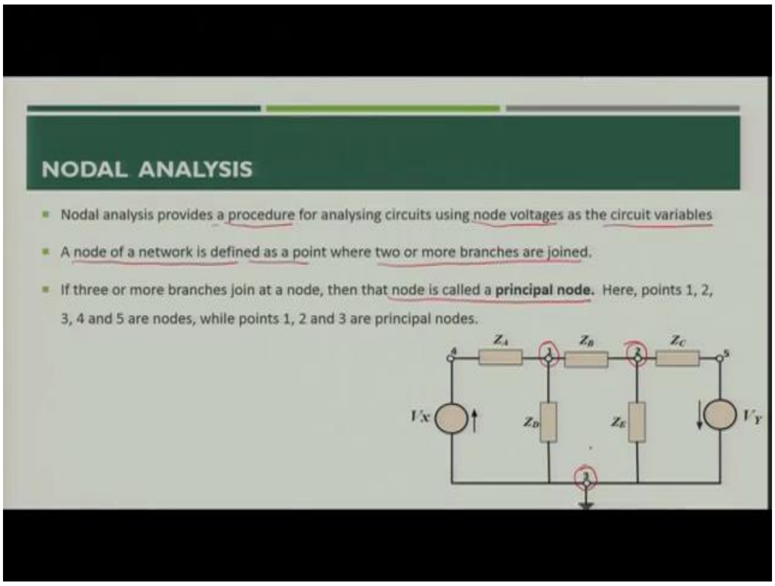

What do you mean by nodal analysis? Actually, nodal analysis provides a procedure for analyzing circuit using node voltage. So, in the mesh analysis we are more concerned about the mesh current and we were utilizing mesh current to solve the circuit. Here we will use node voltage as a circuit variable to solve the circuit. Now, node is the network node of a network is defined as a point where two or more branches are joined.

So, if you see this particular figure, in this figure you will see 1, 2, 3, 4, 5 are the total nodes. Now, if you see for this particular node called node 4 you are joining the impedance Z_A and the voltage source V_X. Similarly, in node 1 you are joining Z_D as an impedance and Z_A as an impedance and Z_B as an impedance. So, node 1 has three connections, similarly node 2 also have three connections that is Z_B, Z_E, Z_C. Node 5 will have only two connections that is Z_C and V_Y. Out of all those nodes one node is considered as a reference node that is node 3.

So, now this node is different from this node in the sense that this node contains at least three branches and this node contains only two branches. So, what we will call this particular node? This particular node would be called as principal node and rest of the other nodes would be called as a simple node. So, what are the principal node in this particular circuit? So, in this particular circuit principal node would be node 1, node 2 and node 3 while node 4 and node 5 are the simple nodes.

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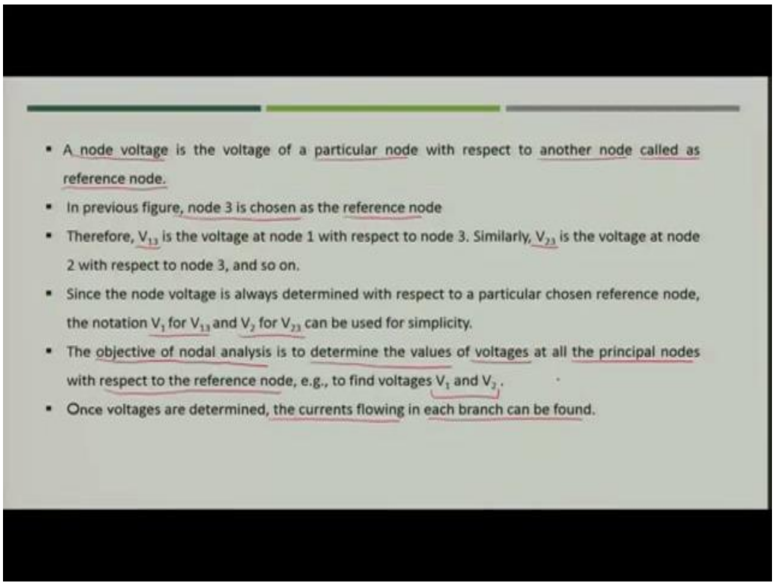

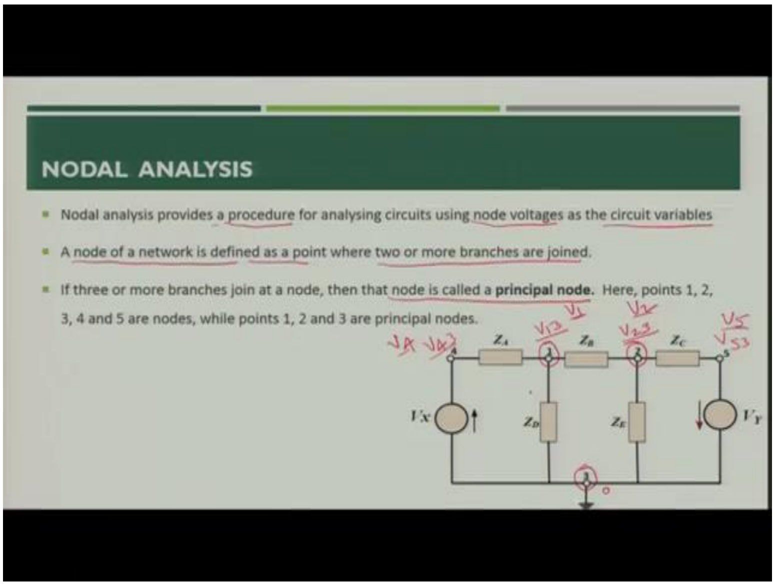

So, let us see what is the significance of the principal nodes. What is the node voltage? Node voltage is the voltage of a particular node with respect to another node which is called as a reference node. So, what we can see from this figure? If you say this as a V_{13} that means that V_1 voltage with reference to node number 3.

So, these would be the nodes, node voltages with respect to the reference node that is node 3 in our particular case. Now, what we have to do? We have to only take those nodes which are principal nodes into consideration and the voltages which we assign to those nodes would be the circuit variables for nodal analysis. So, if you see in this particular figure V_{43}, V_{13}, V_{23} and V_{53} are the four node voltages, node 3 will always be at zero potential because we considered it as a reference, so with respect to this node all would be considered.

So node 4, node 5 are the simple nodes while node 1 and node 2 are principal nodes. So, in circuit analysis using nodal analysis we are more concerned about the voltages at principal node. So, we have seen that the node 3 we have chosen as a reference node. Now, we have seen that we are mentioning V_{13} and V_{23} that is the voltages at principal node 1 and 2 with respect to node 3.

Now, since we know that node 3 is the reference node so what we can do? For simplicity we can write V_1 as in place of V_{13} and V_2 in place of V_{23} because we know that this is always be with reference to reference node. So, V_1 and V_2 would be the node variables basically we can say it as a circuit variable for this particular circuit.

Now, what is the overall objective of our nodal analysis? The nodal analysis objective is to determine the values of voltages at all the principal nodes that is with reference to reference with respect to reference node. So, in this case the objective is to find the voltages of V_1 and V_2 and then we can determine the currents flowing in each branch.

So, that means if you see this particular circuit if you are able to find the value of V_1 and V_2 you can simply find out the value of V_4, so here V_{43} can be written as V_4 and V_{53} can be written as V_5. So, when we know the value of V_1 and V_2 we can find the value of V_4 because that is simply $V_1 - V_x$ and for V_5 also if we know the value of V_2 we can find the value of $V_5 = V_2 + V_y$.

So, when we get these node voltages? You can easily find out the value of currents using Ohm's law.

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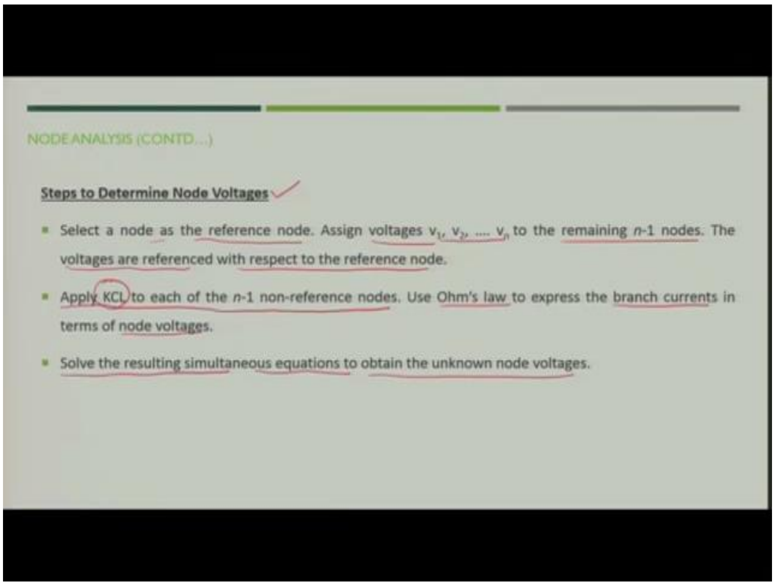

So, now you can summarize that what would be the steps to determine the node voltages. First you have to select a node as the reference node then assign voltages say V_1, V_2, V_n to the remaining n minus 1 nodes. The voltages are reference with respect to the reference node. Now, you have to take n minus 1 non-reference nodes that is the nodes which are other than the reference nodes and you have to apply Kirchhoff Current Law and use Ohm's law to express the branch currents in terms of node voltages.

So, here you can write equations for all the non-reference nodes but while analyzing the circuit you would be more concerned about the equations you write for principal nodes because the equations which you write for principal node would be solved first then you can find the value of other non-reference nodes voltage value. Now, you have to solve the resulting simultaneous equations to obtain the unknown node voltages.

So, these would be the three major steps which you will follow when you have to find the node voltages.

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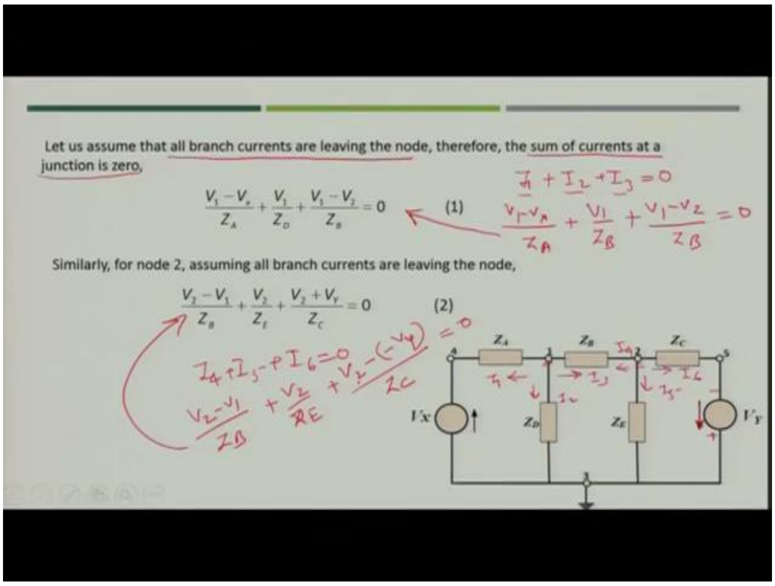

Now, how will we carry on this particular operation? Let us assume that this is the circuit which we saw previously and we will apply the nodal analysis and try to find out the values of node voltages V_1 in V_2 and then accordingly you can find the voltages and currents for say that is V_5 and V_4 and the currents flowing in the branches. Now, let us take the first node that is node 1.

Now, if you see node 1 you can see you can assume that all branch current which are leaving the node you will assume that all branch currents are leaving the node. So, as per Kirchhoff Current Law some of the currents at that junction would be 0, so what you can say? You can say these are the three currents you can write maybe I_1, I_2 and I_3, then, $I_1 + I_2 + I_3 = 0$.

Now, you have applied KCL now you have to put the value of I_1, I_2 and I_3. What is the value of I_1? I_1 value would be the voltage difference between V_1 and V_4 divided by Z_A, so what is the voltage V_4? V_4 is nothing but V_x because the V_x is connected between V the node 4 and node 3. So, what you can write for I_1? I_1 you can write $(\mathrm{V_1} - \mathrm{V_x})/\mathrm{Z_A}$.

For I_2 what you can write? I_2 is simply the voltage $\frac{V_1}{Z_D}$ and current I 3, I 3 would be $\frac{V_1 - V_2}{Z_B}$, and $\frac{V_1 - V_x}{Z_A} + \frac{V_1}{Z_D} + \frac{V_1 - V_2}{Z_B} = 0$ so this is what you will get in case of node 1. Similarly, for node 2 $\frac{V_2 - V_1}{Z_B} + \frac{V_2}{Z_E} + \frac{V_2 + V_Y}{Z_C} = 0$

So, what eventually you will get? You will eventually get the same equation which is mentioned in this slide, so you will get these values.

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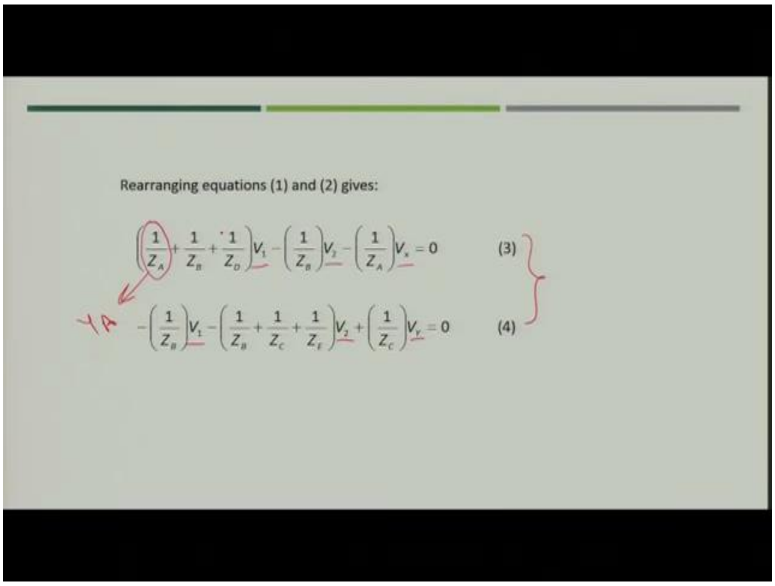

Now, what you can do? You can rearrange all the variables like V_1 you can put together V_2 you can take together and then V_X and V_Y in both of the equations. Now, if you know the values of Z_A, Z_B, Z_D, Z_C, Z_E so you can simply put the value it will be very simple equation, sometimes it is difficult to write in this form what you can alternatively do? You can convert impedance into admittance that is Y_A, Y_A is nothing but 1 by Z_A. Similarly, Y_B is 1 by Z_B and Y_D is 1 by Z_D.

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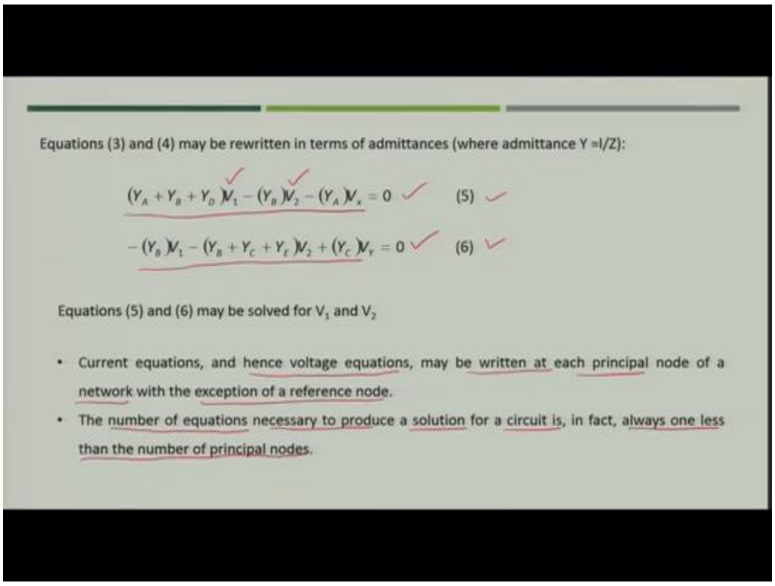

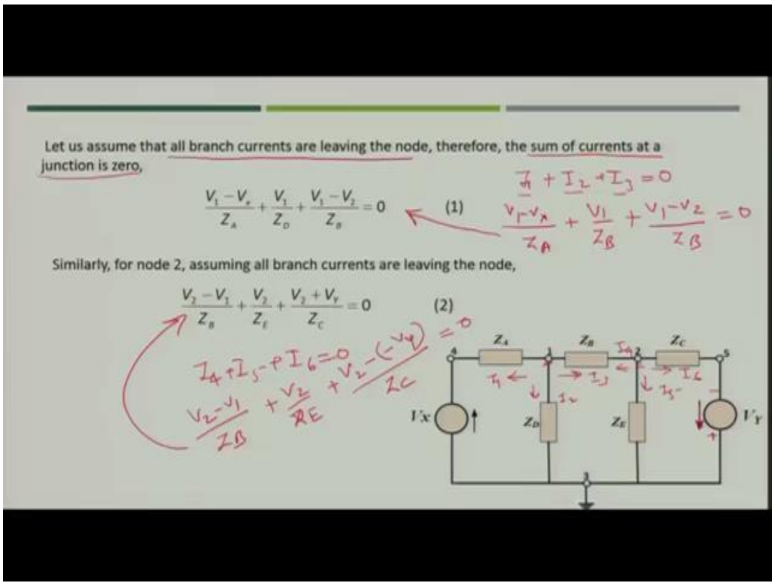

So, what you can write now? You can simply write the equation in the form of admittances and the now you have simpler equations you can solve it for unknown variables which are V_1 and V_2. So, you have two unknown variables and two equations which can be easily solved. So, the current equations enhance the voltage equations may be written at each principal node of a network with exception of reference node.

So, you have written for principal node 1 and 2 and you need not to write any equation for reference node, so you got two equations. Now, the number of equations necessary to produce a solution for a circuit is in fact always 1 less than the number of principal nodes. So, in this figure the principal nodes for 3; 1, 2 and 3, so number of equations required would be 3 minus 1.

So, you need two equations to solve the circuit and you got these two equations. So, with this equation creation you can solve the circuits which are having node voltages, which are having node voltages as a variable.

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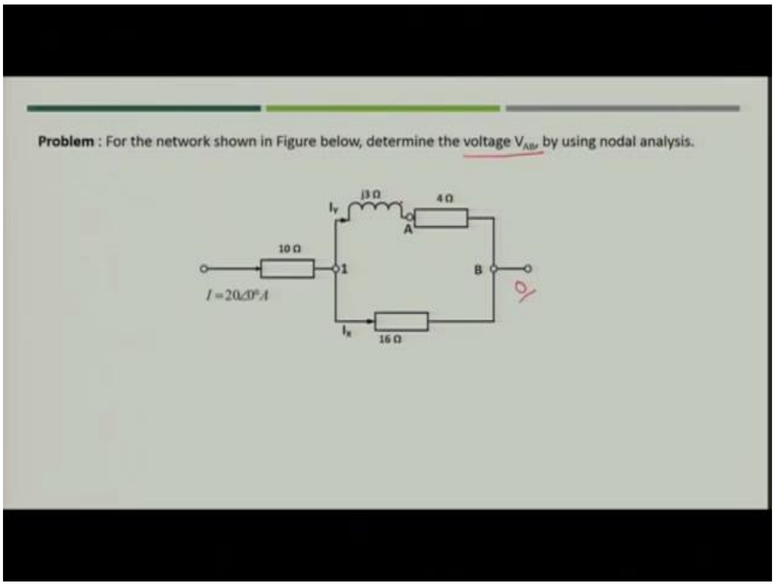

So, let us take an example so that you can understand the concept clearly. For the network shown in the figure below we need to determine the voltage V_{AB}, so voltage V_{AB} means V_A - V_B. So, if you take B as a reference node then it is potential can be at 0 degree, so at 0 volt and voltage V_{AB} is nothing but simply voltage at node A.

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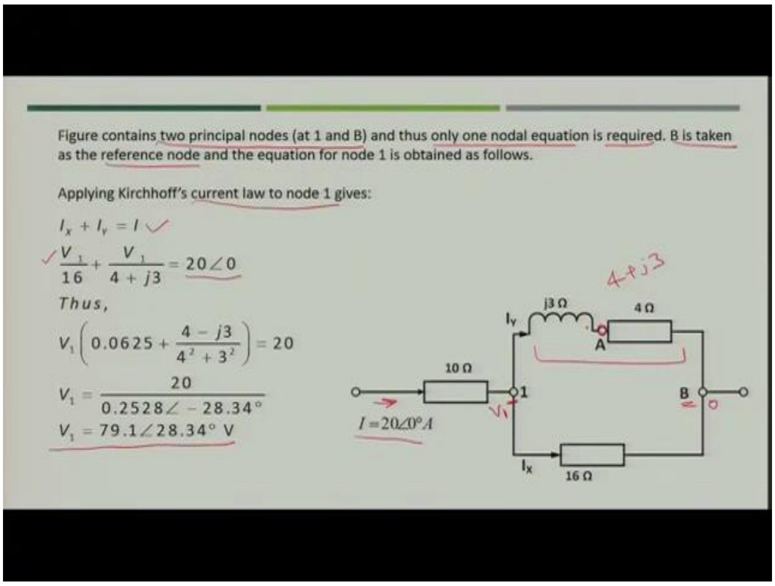

So, how you will solve? You have to again find out the principal nodes, so what are the principal nodes? The principal nodes here are 1 and B because these are the only two nodes where number of branches connected at three, so you have got principal nodes. Now, out of that only one nodal equation is required because you have taken B as a reference node. So, you need to write only one equation to solve the voltage at node 1 that is V_1.

So, if you apply Kirchhoff's Current Law at node 1, what will happen? This current is I_Y, this is I_X and this current is I. So, if you write I is going in I_X and I_Y are coming out, so I would be equal to $I_X + I_Y$. Now, what you have to do? You have to write the values of currents in terms of node voltages. So, for I_X what is the value? That is basically the node you have already taken as a reference node, so the I_X would be V_1 divided by the 16 ohm resistance.

Similarly, for I_y, what you can write? You can write V_1 divided by the impedance of the complete branch. So, what is the impedance now? This is resistance, so this is 4, this is inductance so you can just simply write j3. So, what would be the current I_Y? It will be V_1 upon 4 + j3 and this will be equal to the current value which is given in the example.

So, now what you have to do? You have to just simply solve it because here you have only 1 unknown and you have got one equation to solve it, you simply take V 1 out and solve it you will get the value of V 1 as 79.1 with angle 28.34 degree. So, you got the value of V 1 but your objective is to find the value of V A.

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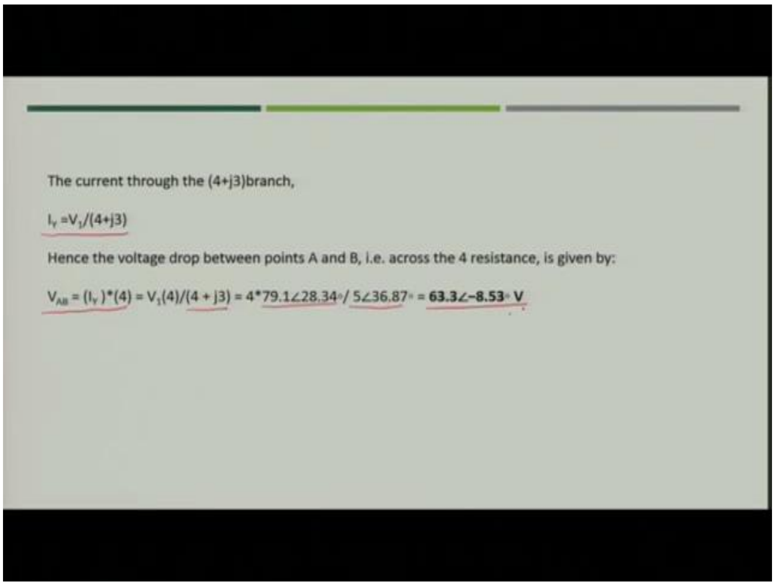

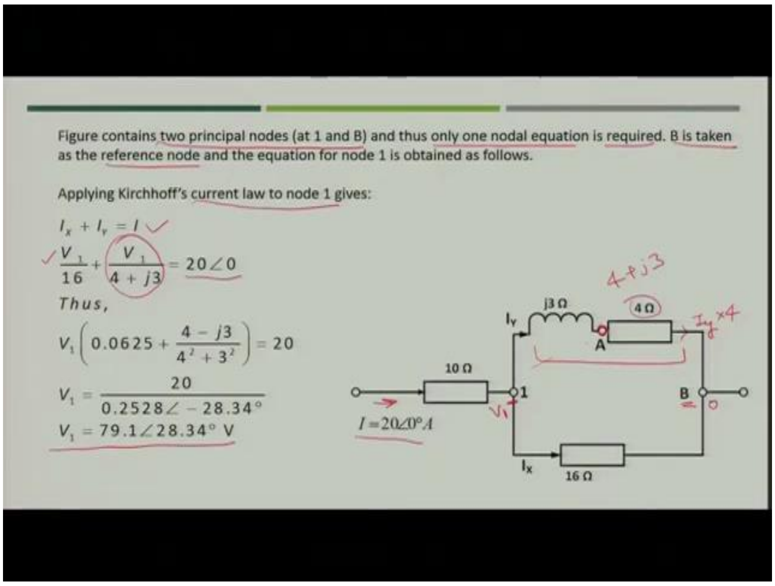

So, what you will do? You have to first find out the value of current $I_Y = V_1/(4 + j3)$. Now, we have to see what is the drop in this particular segment that is between A and B. So, what you will do? You will use Ohm's law and whatever the current is flowing that is I_Y and multiplied by the resistance 4 would be the voltage at node A.

So, what will be V_{AB}? $I_Y * 4$ that is the resistance, so you will simply put the value of I_Y that is $V_1/(4 + j3)$, V_1 you have just found that is $79.1\angle 28.34^\circ / 5\angle 36.87^\circ$ = **$63.3\angle -8.53^\circ$ V**.

So, this you can easily solve and find out the value.

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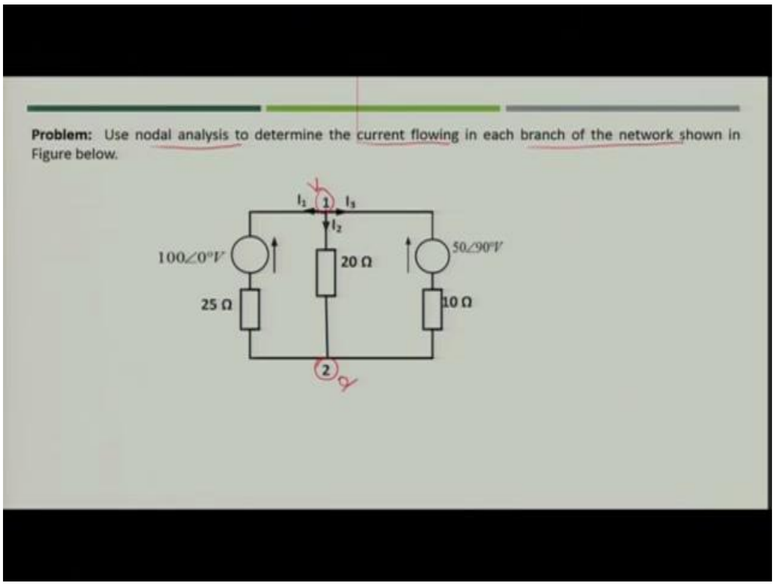

Next let us take another example, if you are asked to use nodal analysis to determine the current flowing in each branch of the network which is shown in this figure. So, what you can see? You can see from here there are two principal nodes that is node 1 and node 2, node 2 you can take it as a reference node.

So, you need to first find out the value of V_1 when you find the value of V_1 you can simply find the values of current that is I_1, I_2 and I_3. Here I_1, I_2 and I_3 all three have taken out of the node 1, so you can simply write I_1+I_2+$I_3 = 0$.

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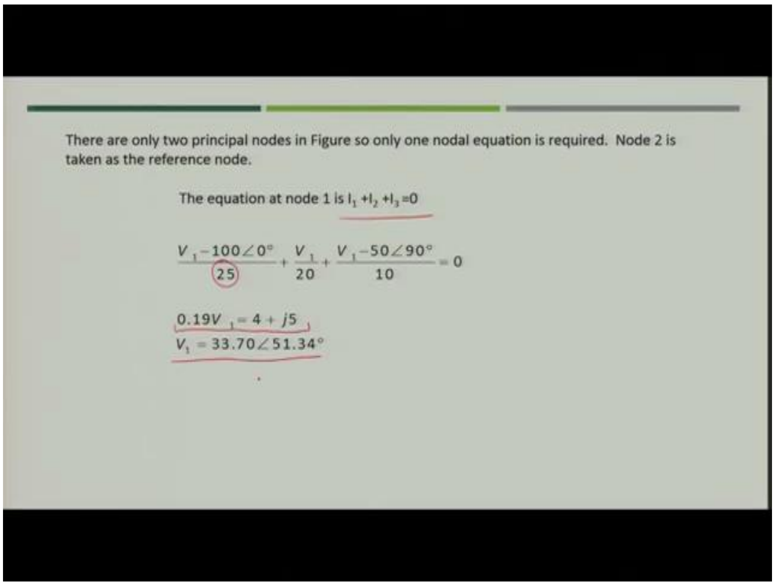
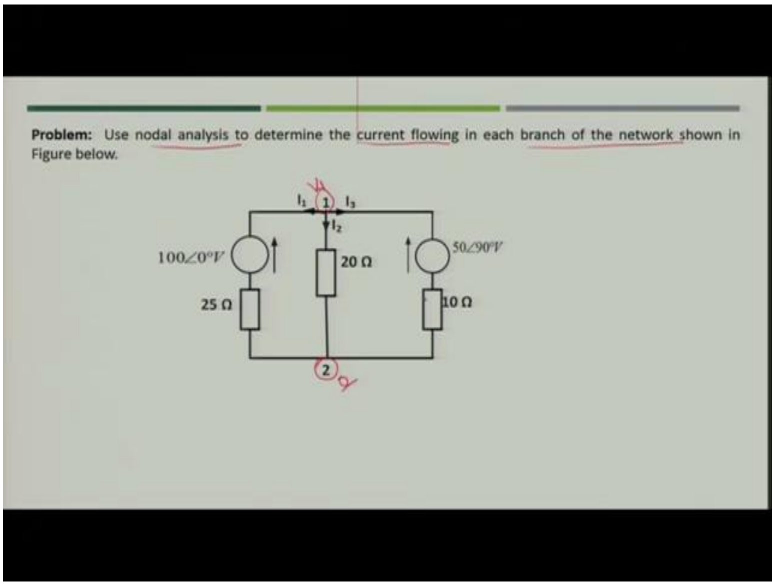


Now, you have to put the value of I_1, I_2 and I_3, what is I_1?



$$I_1 = \frac{V_1 - 100\angle 0°}{25}$$

$$I_2 = \frac{V_1}{20}$$

$$I_3 = \frac{V_1 - 50\angle 90°}{10}$$

Now, if you solve what you will get? You will get the value of V 1 as 33.70∠51.34°.

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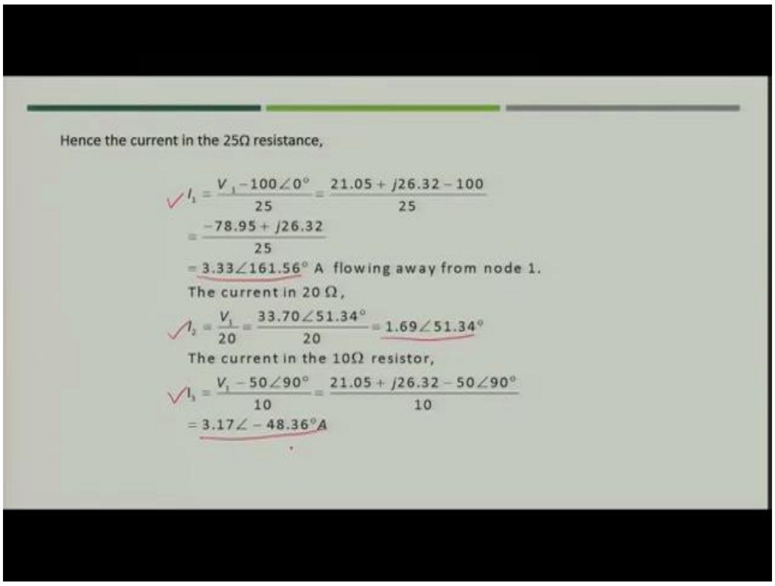

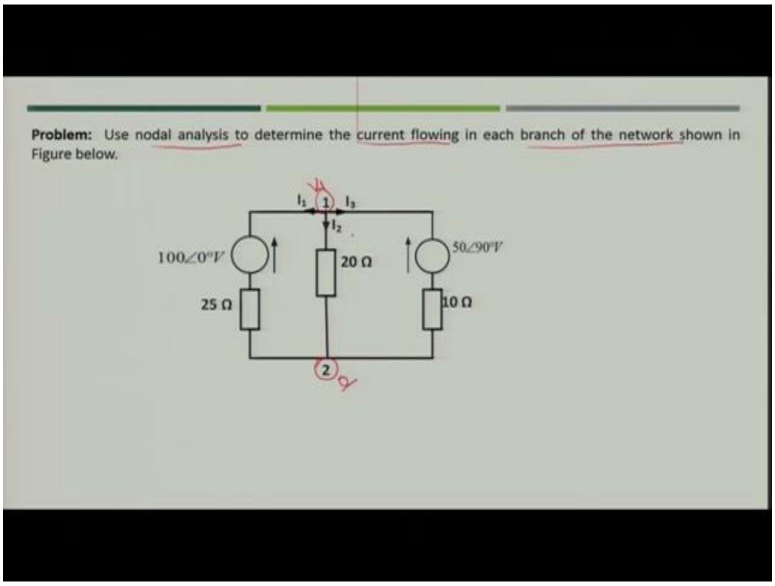

So, you have got the value of V_1 simply you have to put the values of V_1 in I_1, I_2 and I_3 and when you will put the value you will get the values of currents, right. So, in this way you can easily find out the value of currents of those are the branch currents in this case and you can solve the circuit.

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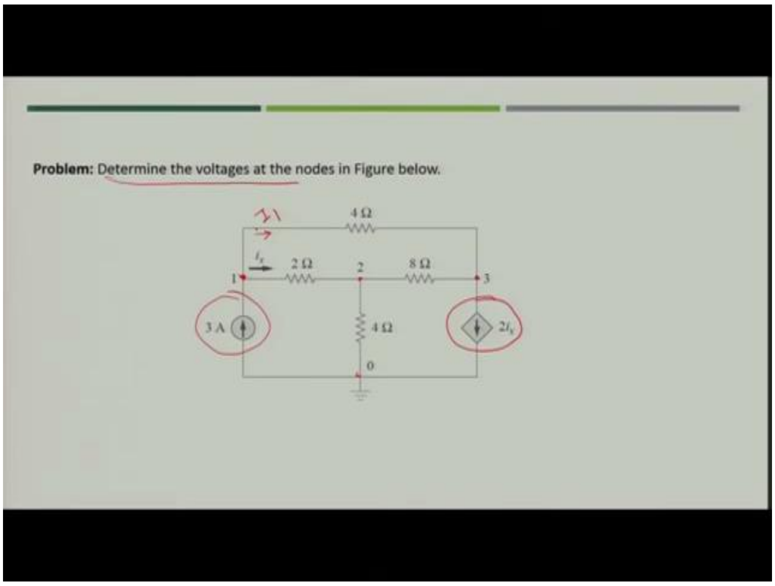

Now, let us take one another example in this case the node voltage needs to be determine but here the source is one is independent current source and second is the dependent current source. So, in this case when you have dependent and independent current source, how you will solve the circuit? You will follow the same procedure what we did in the previous example, you have to first find out the principal nodes.

So, what are the principal nodes in this case? In this case this would be principal node because here 1, 2 and 3 branches are joining, 2 is also principal node because all three resistances are connected here, node 3 is also principal node because two resistances and 1 dependent current source is connected and 0 is anyway principal node but this is reference node.

So, we have considered this as a reference node and now we will try to solve the circuit.

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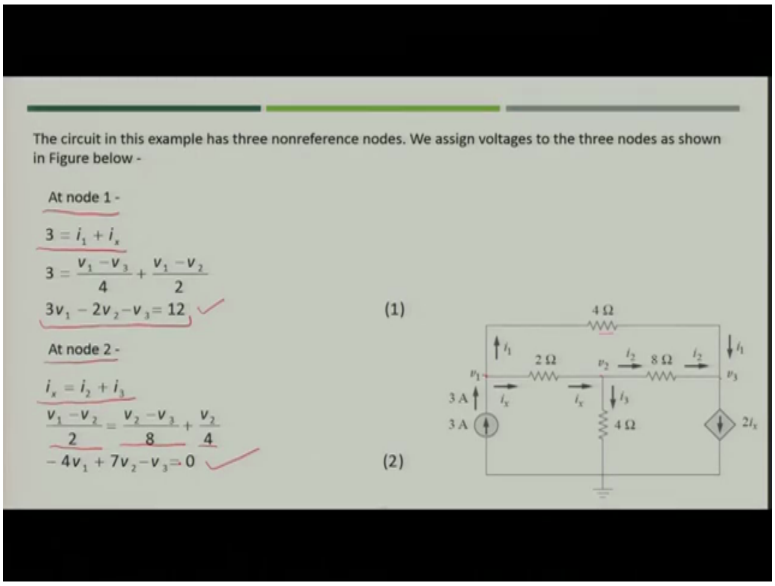

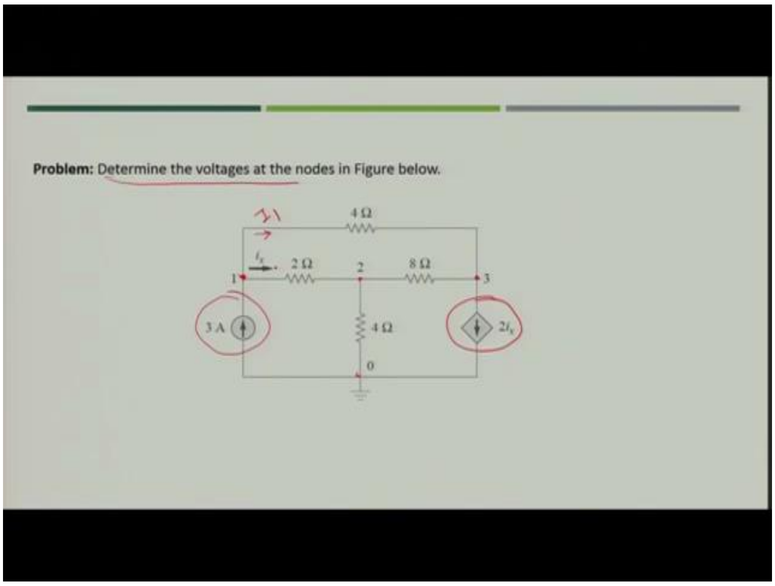

What you will get from node 1? In case of node 1 the value of current say this is i_x, this is I and this may you may take another value as I_1, so what will happen? That you can simply write it the 3 ampere current is going in and I_1, and i_x, are going out, so you will write as $I_1 + i_x$. Now, you have written this value in terms of current converts them in terms of voltage, so what you will do? You have assigned the value of node voltages as V_1, V_2, V_3 that means you need three equations to solve because you have now three unknowns V_1, V_2, V_3.

So, when you write current I_1 in terms of the voltage V_1, what you can write? $I_1 = (V_1 - V_3)/4$ and for $i_x = (V_1 - V_2)/2$. Another equation which you get is $3V_1 - 2V_2 - V_3 = 12$, so this is the equation you gt while applying KCL at node 1.

Similarly, for node 2, you can write i_x is going in and I_2 and I_3 are going out, so i_x would be equal to $I_2 + I_3$. i_x is also equal to $(V_1 - V_2)/2$, $I_2 = (V_2 - V_3)/8$ and $I_3 = V_2/4$. So, when you solve you get another equation in terms of V_1, V_2, V_3.

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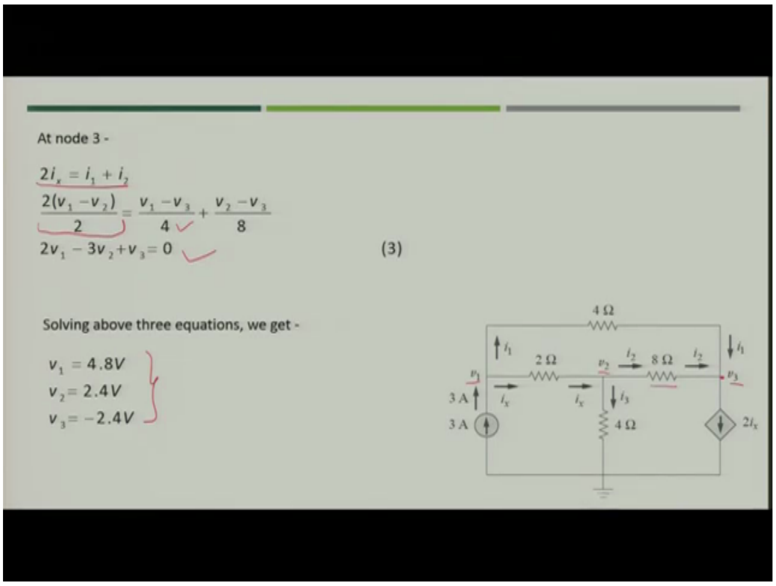

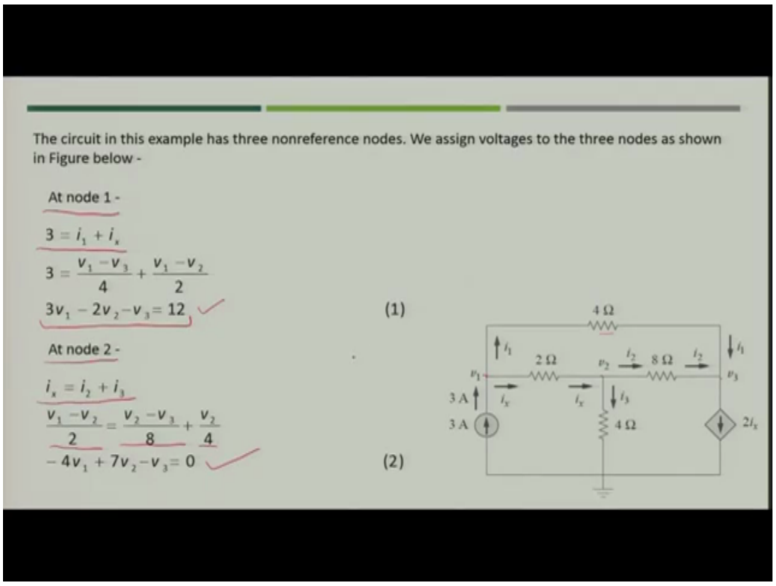

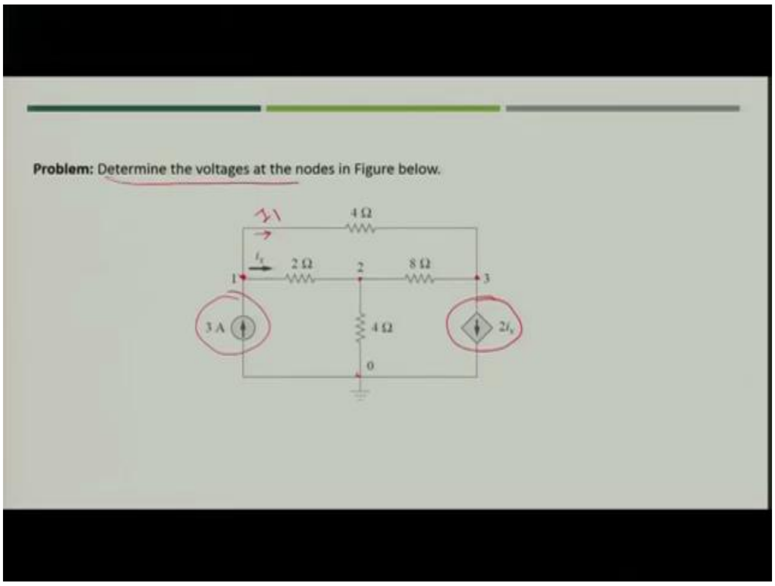

Similarly, for node 3 if you write in this case node 3 would be $I_2 + I_1$ plus since it is in downward direction so you can write this is going out, these two are going in the node, so you can write $2i_x = I_2 + I_1$, right. But $i_x = (V_1 - V_2)/2$. So, you will just place the value of i_x in this dependent current source, so you will get the value of $2i_x = 2(V_1 - V_2)/2$ and then simply $I_1 = (V_1 - V_3)/4$ and $I_2 = (V_2 - V_3)/8$, so you got another equation from node 3.

Now, you have three unknowns V_1, V_2 and V_3 and you have got three equations that is $3V_1 - 2V_2 - V_3 = 12$ and $-4V_1 + 7V_2 - V_3 = 0$ and $2V_1 - 3V_2 + V_3 = 0$. So, now you have three equations, three unknowns if you solve all those three equations you will get the simply the value of V_1, V_2 and V_3. So, the node voltages which are asked to determine have been calculated in this way.

So, with this way you can solve any type of (equation) any type of circuit network with the help of nodal analysis. So, in this particular week we discussed about node voltage, nodal analysis and mesh analysis. So, whenever you see that you need to find out the circuit parameters in terms of currents you can use mesh analysis and when you are asked find out the node voltages you can use nodal analysis.

So, if you compare both of the analysis you will come to know that mesh analysis is depending upon Kirchhoff Voltage Law and nodal analysis is depending upon Kirchhoff Current Law but sometimes in both of the cases you might need both of the Kirchhoff's Laws that is voltage law as well as current law to solve the circuit. So, with this we will close today's session, in next week we will discuss about the various network theorems, thank you.