INDIAN INSTITUTE OF TECHNOLOGY KANPUR

NPTEL

NPTEL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

Course Title
Electromagnetic Waves in Guided and Wireless

Lecture - 09
Time Domain Analysis of Transmission Line - 1

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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media, this is module 9, and in this module we switch the domains of the transmission line discussion.

So far we have been working with what is called as the phasor domain or the frequency domain, wherein we dealt with, we didn't really talk much about the time aspect, we assume that all the voltages and currents were you know just free phasors, meaning that they were, they all had the same sinusoidal functional dependence or time dependence, but in this module onwards we will look at time domain applications and this set of behaviors, when we are looking at the time domain behavior is very important in many classes of transmission line problems.

While the frequency domain approach is quite popular and works very well in case of RF frequencies and RF circuit design, microwave system design and antennas for example, when you come to the time domain analysis, this is very important in any modern processors or some kind of a communicating elements where you are communicating you know pulses, let's say on printed circuit board between IC's or it could be from one you know PCB to another PCB, so where the lines essentially become transmission lines, so in all this applications it is necessary for us to look at the time domain behavior of the transmission line, okay, for the responses in time domain are going to be very different in the frequency domain response, but it will, but this time domain behavior is essential in designing this interconnection systems.

And also looking at situations where the interconnection instead of becoming just a wire becomes a transmission line and starts to distort all the pulses that are transmitting, okay, so this broad area I would call it as signal integrity, meaning that you're looking at how you know what is the integrity of the signal for the pulses which are propagating on the transmission line between any two communicating chips, okay, it could be on chip interconnection as well, so many VLSI systems will have connection from one transistor to another transistor, but as the frequency increases on the on-chip even though small wires essentially become you know parasitic transmission lines I would say, and then they will start to distort and you know the voltages and currents and that has to be understood in the time domain, so this is the complimentary behavior or rather complimentary understanding of the transmission lines as in from the frequency domain analysis, of course you could technically use another domain called as Laplace domain to understand the time domain behavior and that is in fact done at you know in some special cases, but we will not do that here because we are you know this is an introductory course and we don't want to go into too much of a depth invoking this Laplace transform.

So we will look at some simple time domain analysis, that would be sufficient for us to get the flavor of this signal integrity and then we will see what the signal integrity means later on itself, okay.

So we start off with a transmission line which I'm drawing, now previously I would give the transmission line in terms of its characteristics impedance Z naught and perhaps the phase velocity, I would not have given the phase velocity normally because that's really not that of an

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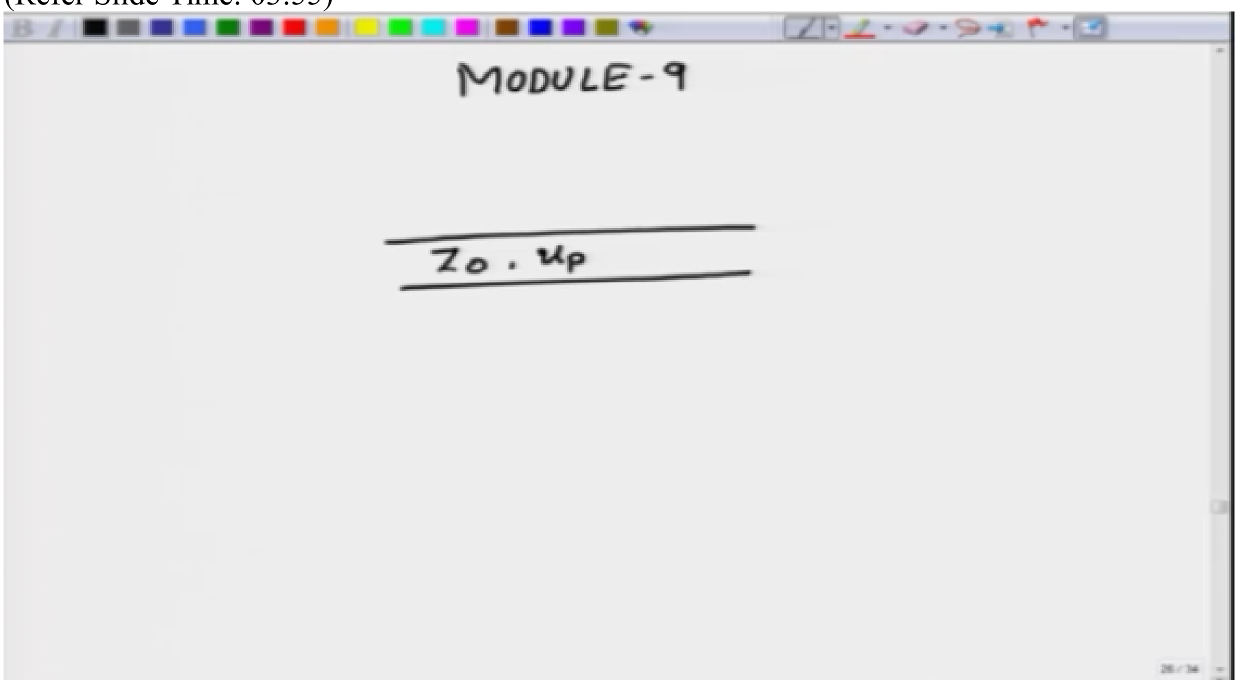

important criteria but sometimes if the transmission line has a different you know directory constant then you may give the phase velocity between only sub to fix the wavelength lambda, okay, here we are not interested in this lambda wavelength, but we still need to know what is the phase velocity for a slightly different reason which I will tell you now, okay.

So I will give Z naught, I will give UP and I will give you the length of the transmission line, right, separately much kind of the same parameters as you would do in describing this in the frequency domain, you will have this terminating loads as well, but this time the terminating loads will not be complex, meaning that we are not exciting our transmission line with a sinusoidal signals and therefore you know an inductor subjected to a sinusoidal signal force will actually look like omega L that is J omega L and a capacitor would look like 1/J omega C and therefore the overall impedance was turning out to be a complex number, right, but now we don't have this complex number thing because we want to keep our excitations not as a sinusoidal excitation, but a slightly different one, you know in general we would like to have a pulse as well,

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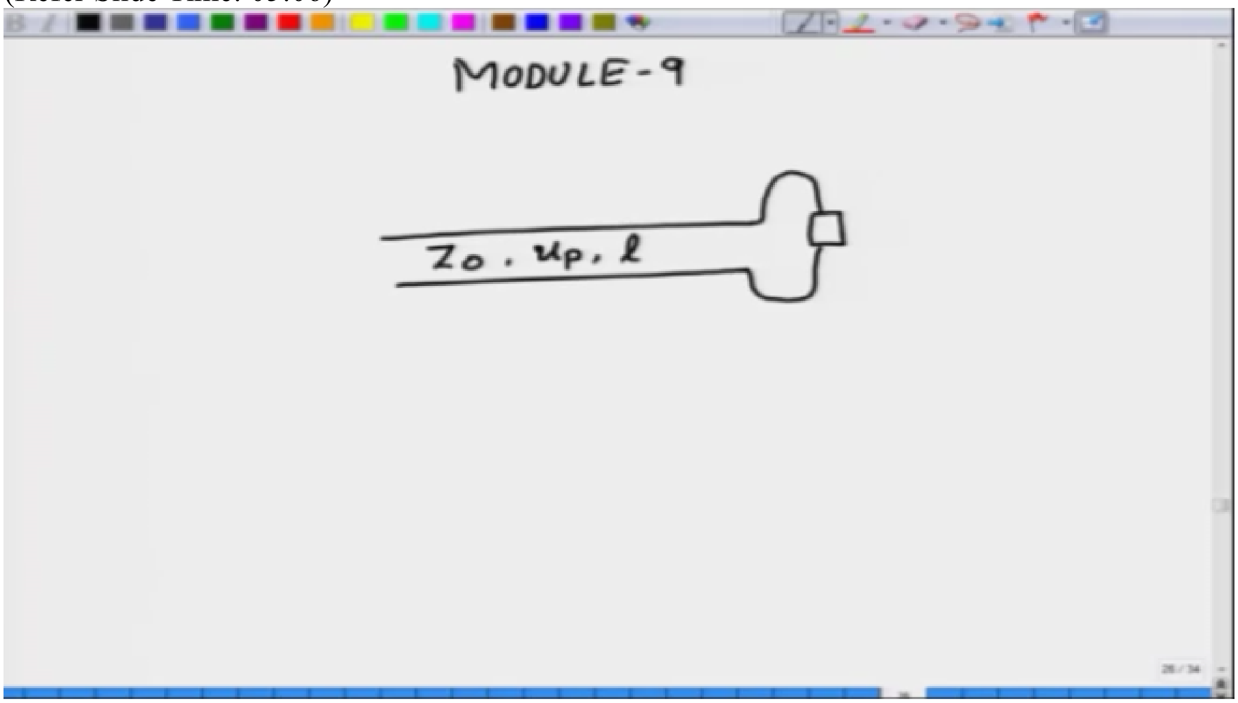

okay, so the load will be the resistor or it would be a combination of a resistor and a capacitor or it could be a resistor in parallel with a capacitor or any kind of a network which will be a lumped parameter network, okay.

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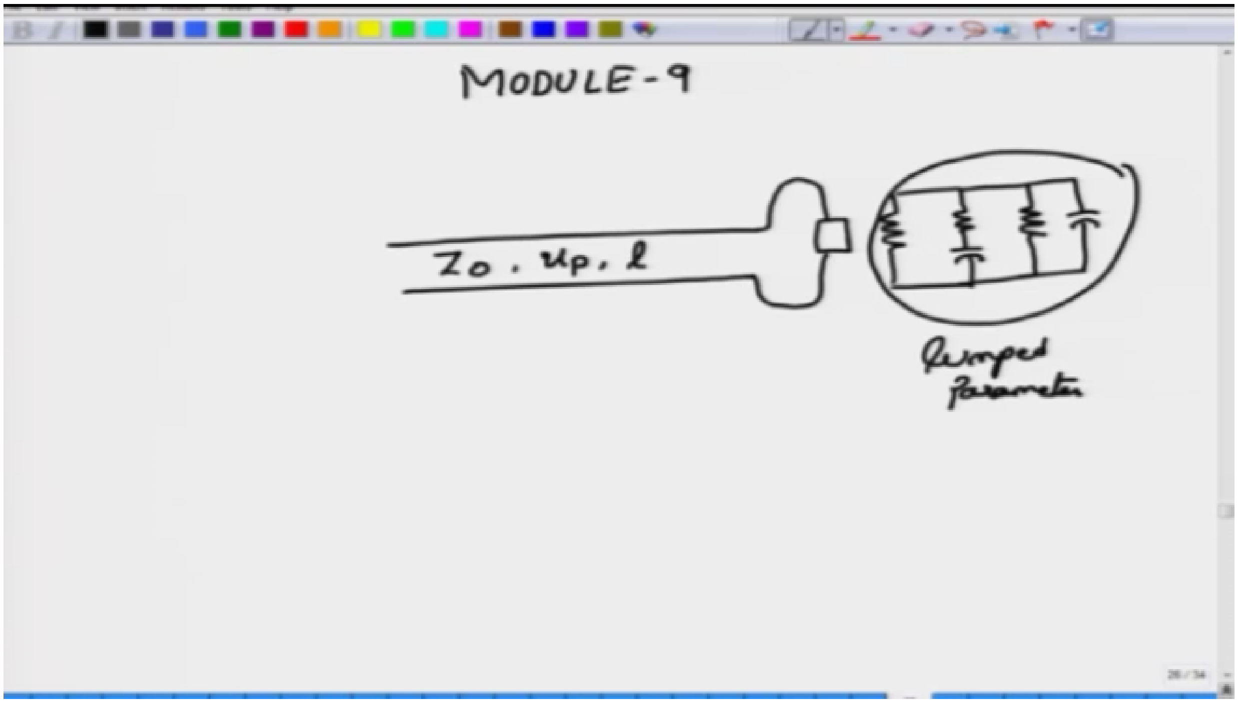

For simplicity we will start with only resistively terminated transmission lines, because the methods that we developed here can be applied, I mean cannot be actually applied directly to the cases where there are reactances, we need to be little bit different there, so instead of introducing this complex case of having reactive elements or capacitors or inductors into the load we will start with the simplest case of resistively terminated transmission line, okay.

So that is our basic problem setup, I still haven't of course talk to you about how I am going to you know what quantities I'm interested in, but I'm just interested in this load which I will still call as ZL, but please understand that this ZL is actually equal to RL for at least a few more modules, this module and probably the next two modules as well, okay, or probably until the next module.

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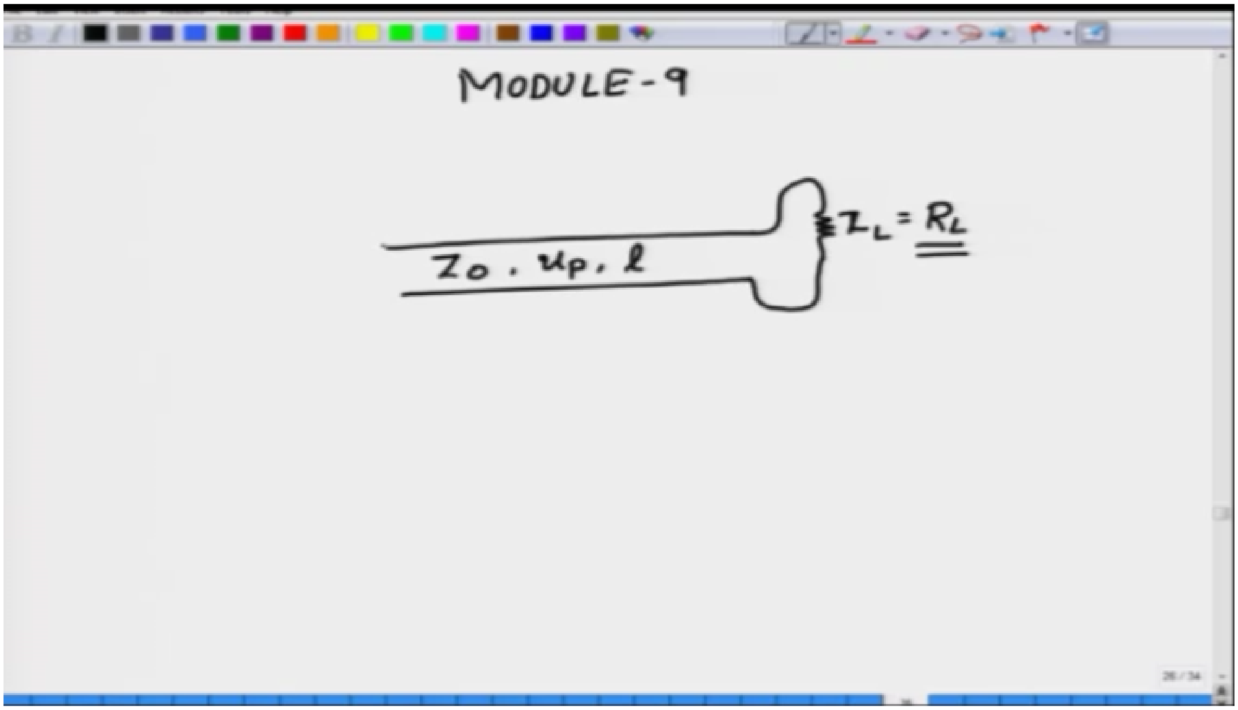

The excitation that we are going to you know give to the transmission line will again have some load, okay, which could be just a resistive load in this case, I will call this as ZS, please note that I'm not calling this as ZG, you could of course call it as ZG as well doesn't really matter, I'm calling this as ZS to indicate that these are source quantities, because this terminology is quite common, your signal integrity applications, okay, or in the time domain behavior applications,

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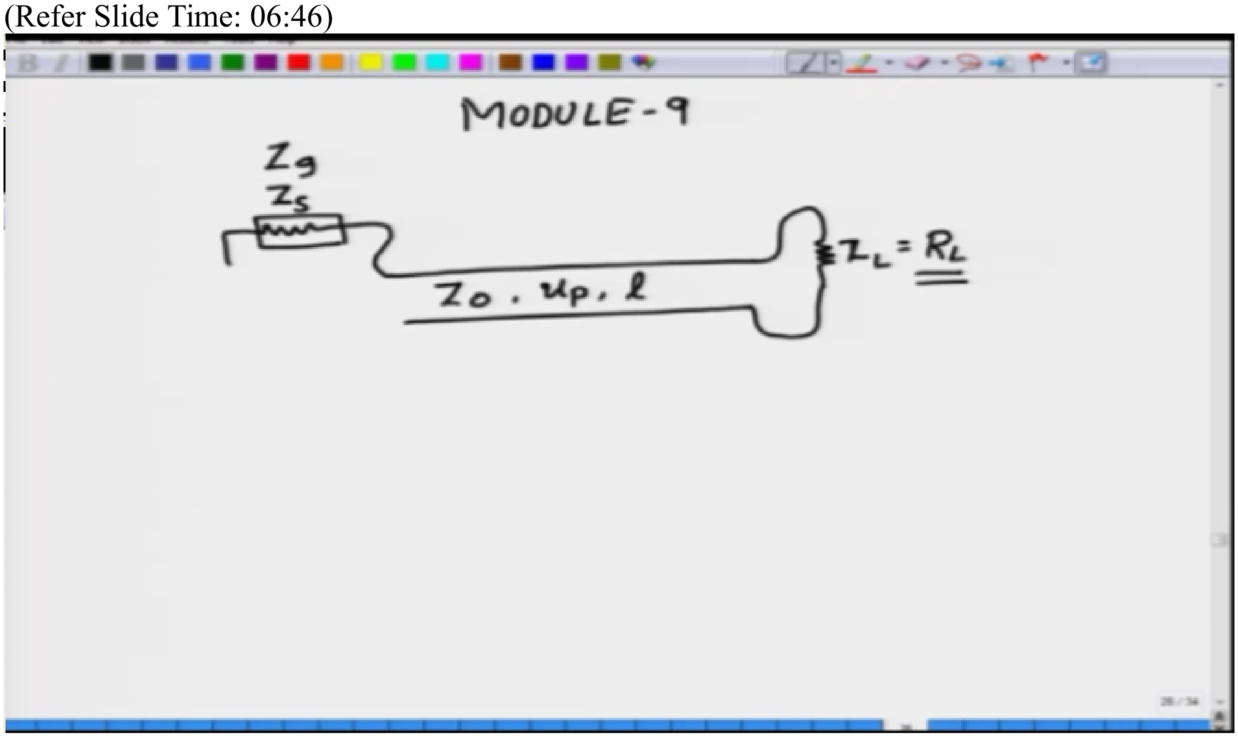

you normally don't write it at ZG because ZG is usually reserved in that RF circuit literature, so I have a source which could be a pulse source as well which we will call as VS, and then connected that VS up to Z naught, okay.

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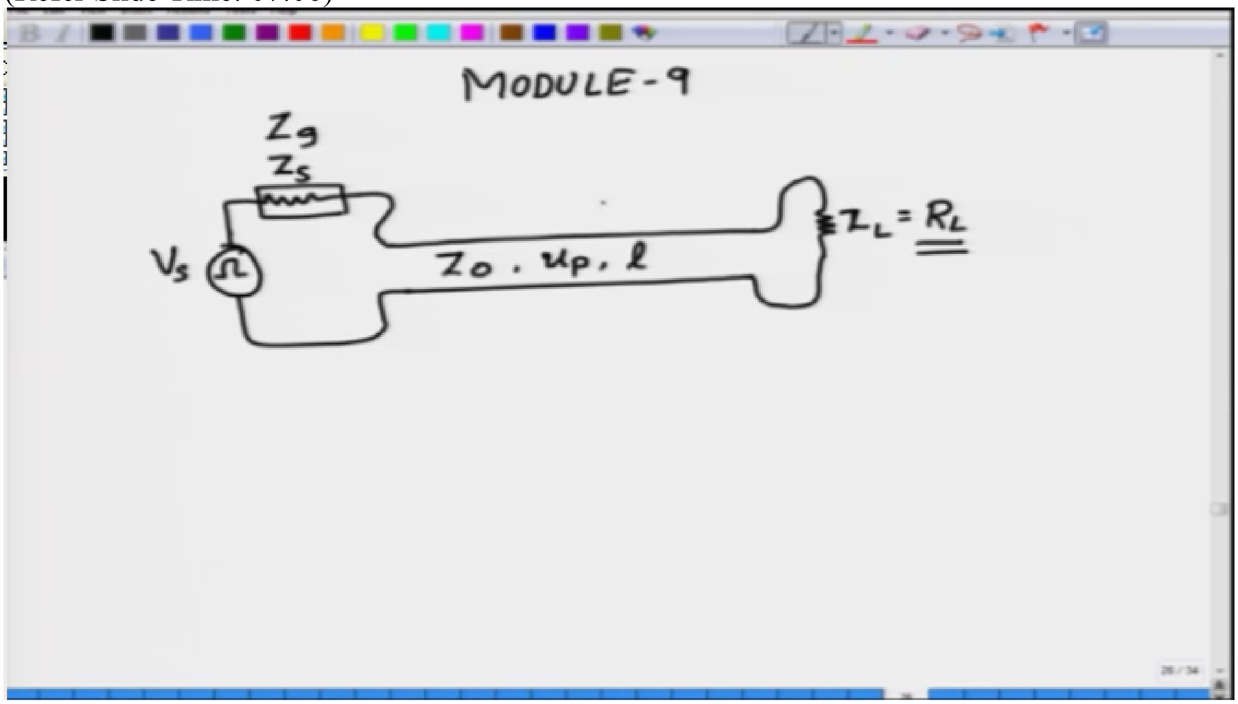

Now why is this UP and L important? UP denotes the phase velocity, but I really don't have a sinusoidal signal for the phase velocity, so specifying this UP here is really not a good idea, okay so I'm not going to specify this UP instead I will tell you that if I have a rising edge here launched on the transmission line then that rising edge will be available at the output of the transmission line just before it goes to the load it would be available at a length, or sorry at a delay off TD which is given by L/U, okay,

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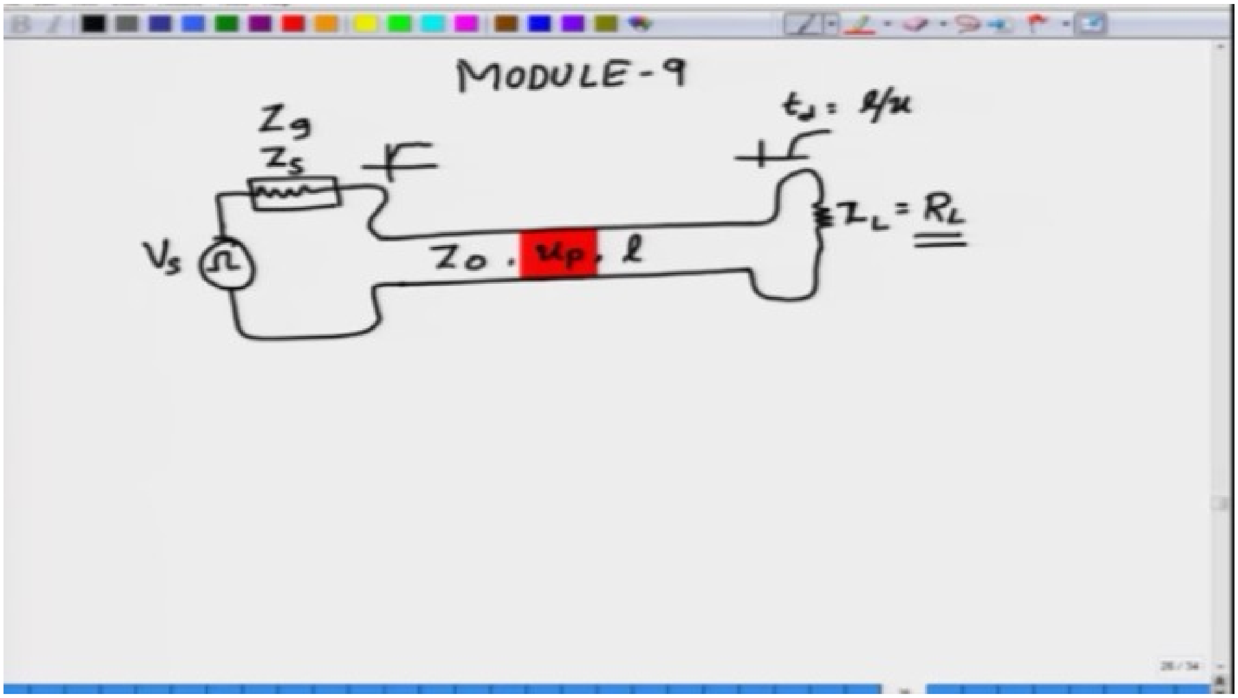

I won't call it as phase velocity we'll just simply call it as the velocity U that would tell you what is the delay through the transmission, so if I have a rising edge of the pulse then that rising edge will not be available immediately at the load side or at the load end, but it would be available at a delay, after a delay of TD seconds, okay, and that delay is the one way propagation delay which is given by the length of the transmission line divided by the velocity U, okay.

So you don't need to really know what is the velocity U here, you can of course still use the same 1/square root LC for the lossless transmission line and then write that as the velocity or alternatively if someone is already given you TD by doing this kind of a measurement then you can simply use that value of TD, so normally we give you Z naught and TD, we rarely give you the length and the velocity but if it is given then it is easy for you to find out what is TD from this relationship, and it is important to note what is TD, okay, all these things are coming simply by assuming the you know the time domain behavior of the voltages are actually given in the form of V(T-Z/U)
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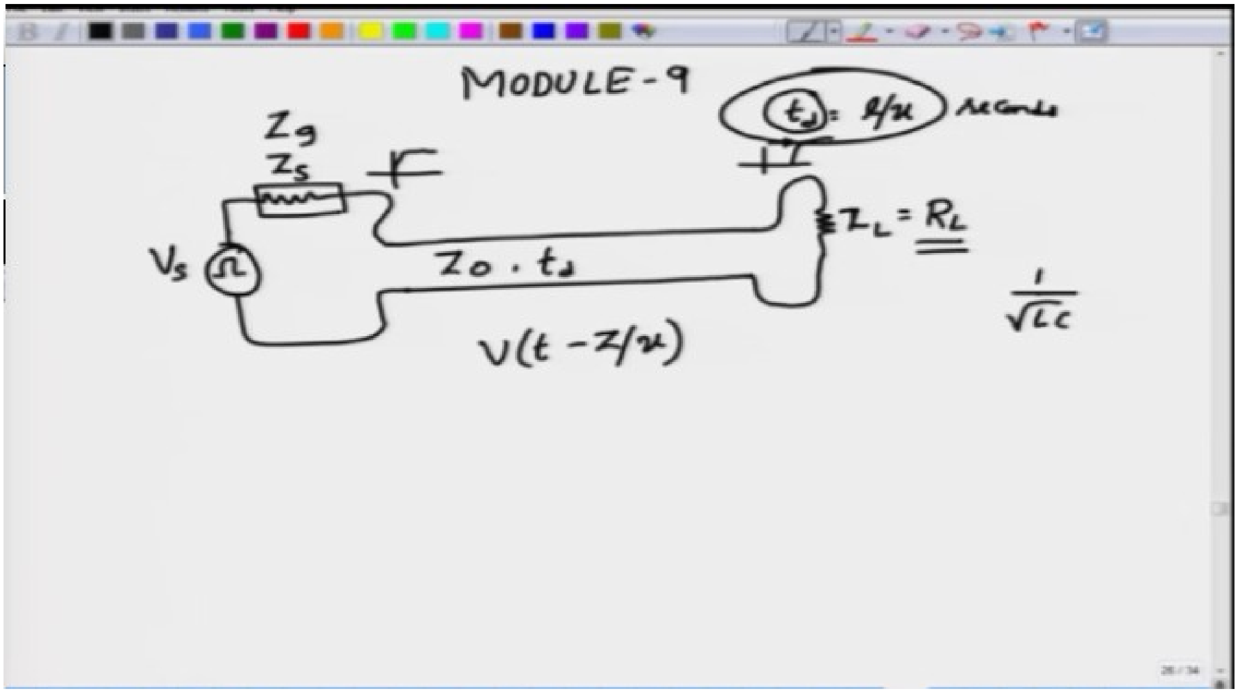

where V is basically the voltage that is propagating, okay, so if at all this load were to be moved all the way to infinity, then there will only be a positively traveling voltage which would be a function given by T-Z/U and at any plane Z = L this would simply indicate that, the wave form is delayed by a certain time TD given by L/U, okay.

Now we are not going to consider infinitely long transmission lines, we will consider finite length transmission lines, okay, and what we are interested in this finite length transmission lines is, on the transmission line at the load or at the source or at any point along the transmission line, okay, at the load you are definitely interested because that voltage will you know is the one that would be delivered to the load, but you are also interested in the voltages that would normally comeback if at all there is a mismatch then this load would come back, I mean the way it would comeback voltages and currents both will come back, and by monitoring this voltage at the you know, input end of the transmission line you can tell a lot about the load itself as well as the line itself, okay, so monitoring the voltages that actually come back at the receiver side, sorry at the source side and using that information to infer characteristics of the load is called as function of a device called as timed domain reflectometer, okay.

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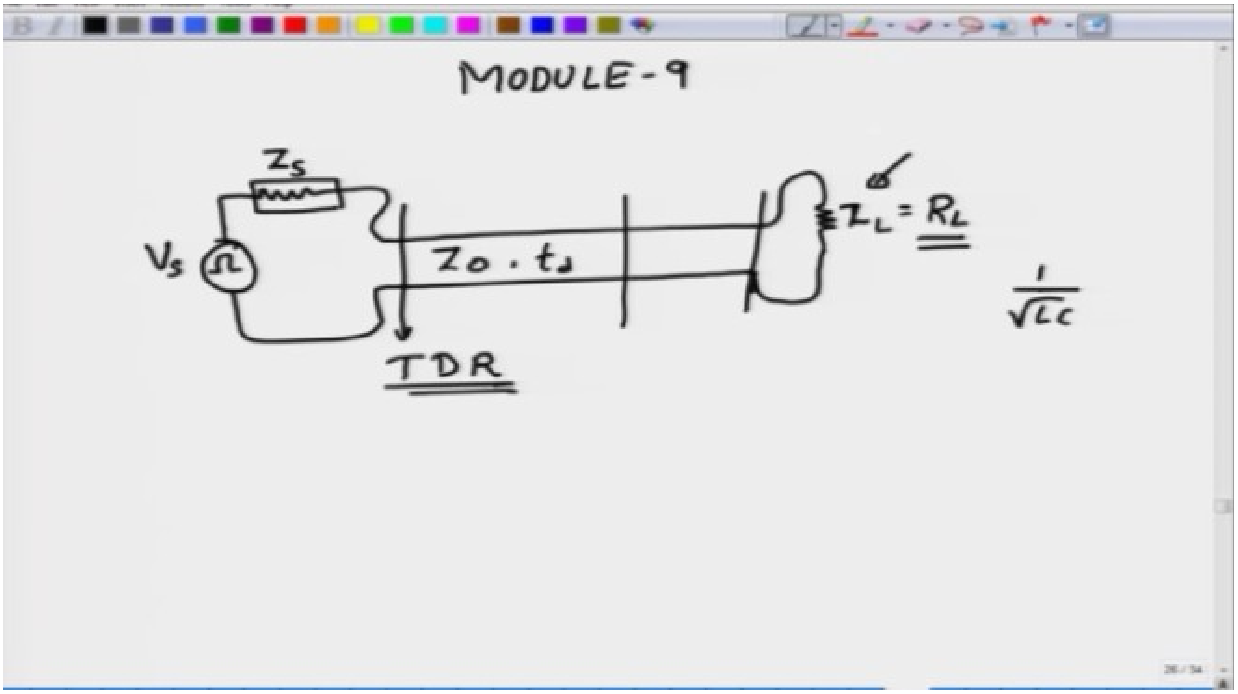

A time domain reflectometer is just like a radar, its sends out the pulse and then monitors any return pulse, okay, the changes in the shape of the return pulse as well as you know which includes change in the amplitude as well as change in the, or the distortion that would be induced would tell you lot about what is the nature of the load ZL, okay, so we will look at some TDR basics later on, but before we can do so or we need to know how the voltages are going to be changed by this kind of a connection, okay, so that's all that's what we are interested in.

So the problem as of now that we are interested is that I have a resistively terminated transmission line of appropriate length L or equivalently at time, I mean time delay of TD connected to the source VS and ZS, okay.

For simplicity let us assume that the source is off a step voltage, okay, meaning that there is essentially a switch here, okay, there is essentially a switch here and this switch would be closed at some time origin which we will call us T = 0, connecting both the voltage and ZS on to the transmission line,
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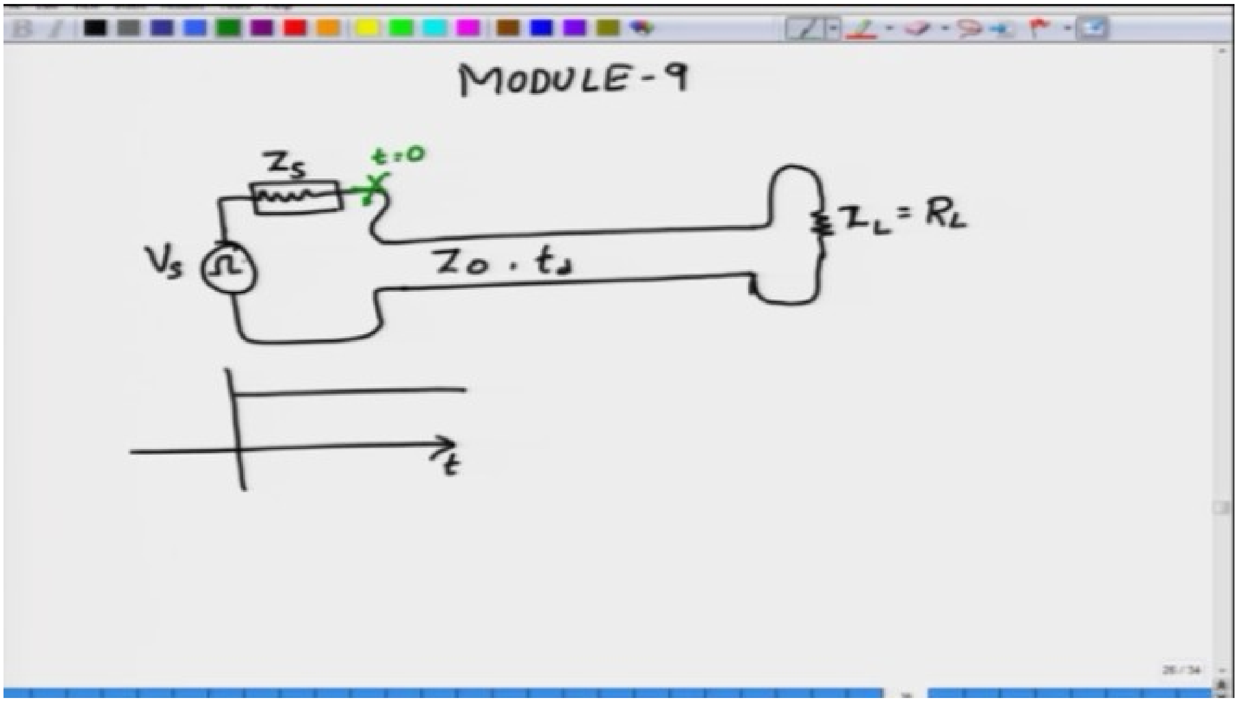

okay, so that's what we are fully have.

Let's take the amplitude of the source voltage as say some capital VS okay, and if you are interested you can give this as VS into U(T) indicating that this is a switch that were close at T = 0, and connects both the voltage source VS or with its internal resistance ZS on to the transmission line which is resistively terminated with ZL, okay, this is what we have, we will also assume that there is no initial voltage on the transmission line,

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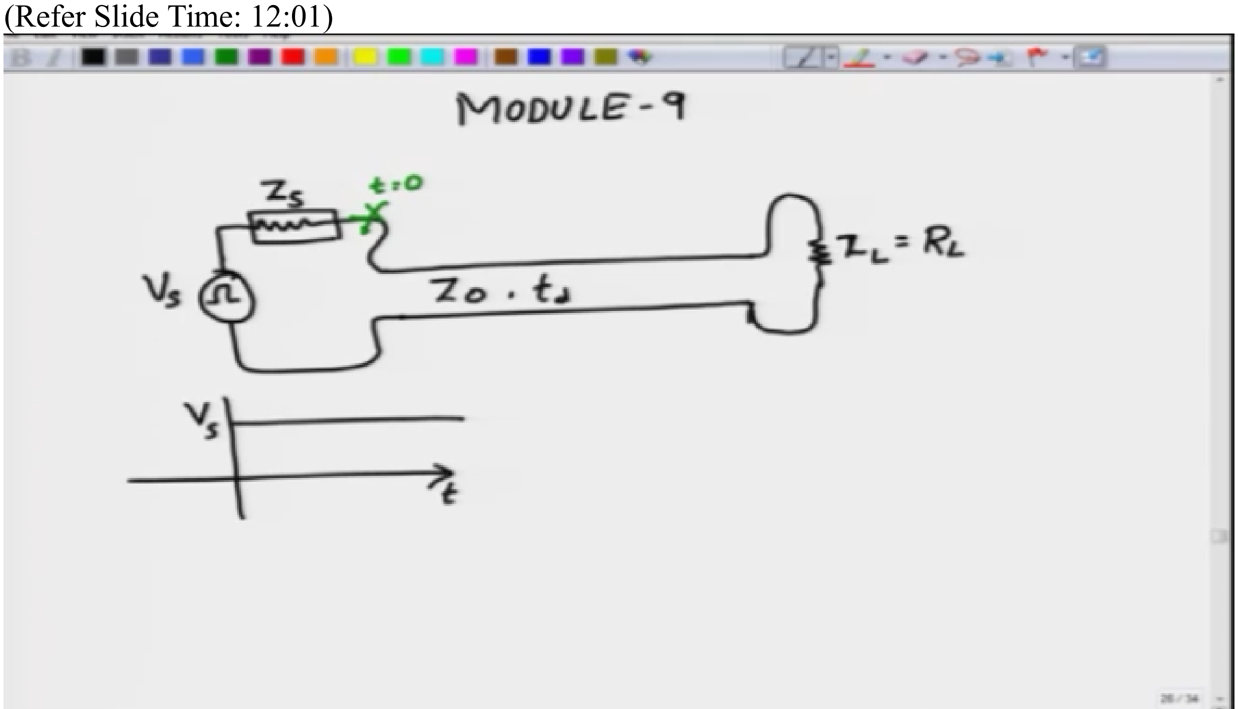

we say that the transmission line is uncharged initial, okay.

What happened? Well, just at T = 0 when you close, remember there is no line voltage, there is no voltage anything or there is no current on the transmission line, at T = 0 this will just closes and then this VS is connected, ZS is connected to the rest of the transmission line.

Now think about this for a minute, will this source voltage, right, this step voltage see the load immediately or not? It will not see the load immediately because the voltage has to travel all the way to the load right, and then if there is a mismatch, only then this load voltage is actually seeing, in fact the travel voltage is seeing this load only after a delay of TD, and then when it actually starts to come back because assuming normally that there will be some kind of a reflection then any changes that should happen at the generator or the source side will happen after at 2 times propagation delay which is 2 TD, so the immediate reaction or the immediate equivalent circuit that the source VS and its internal resistance ZS is will simply be something like this,
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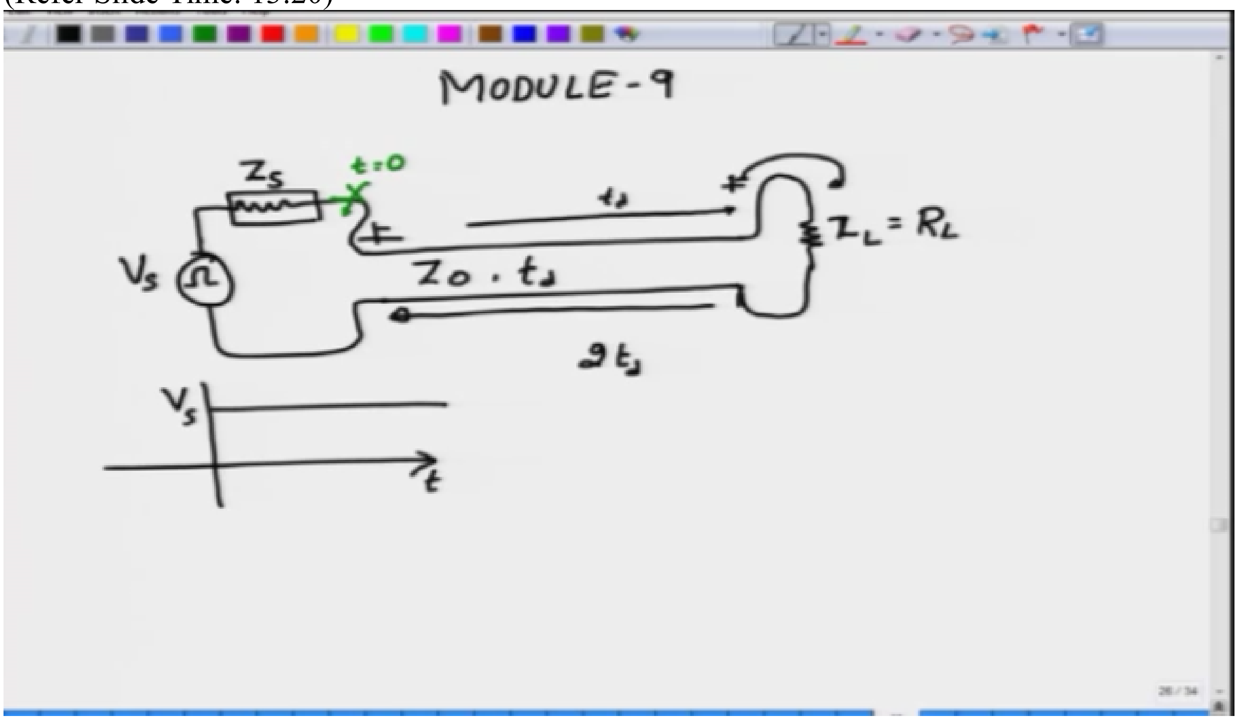

so this is your DC voltage, so I'll call this as plus minus, with an amplitude of VS that we have connected, so I have this VS and I have this resistor, internal resistor ZS connected immediately to the transmission line whose characteristics impedance Z naught will be the one that the source will see, so this is the equivalent circuit that the source VS, ZS is going to see at T = 0, okay.

Now what would be the voltage that would be launched on to the line right away at T = 0 which we will call us V1, okay.
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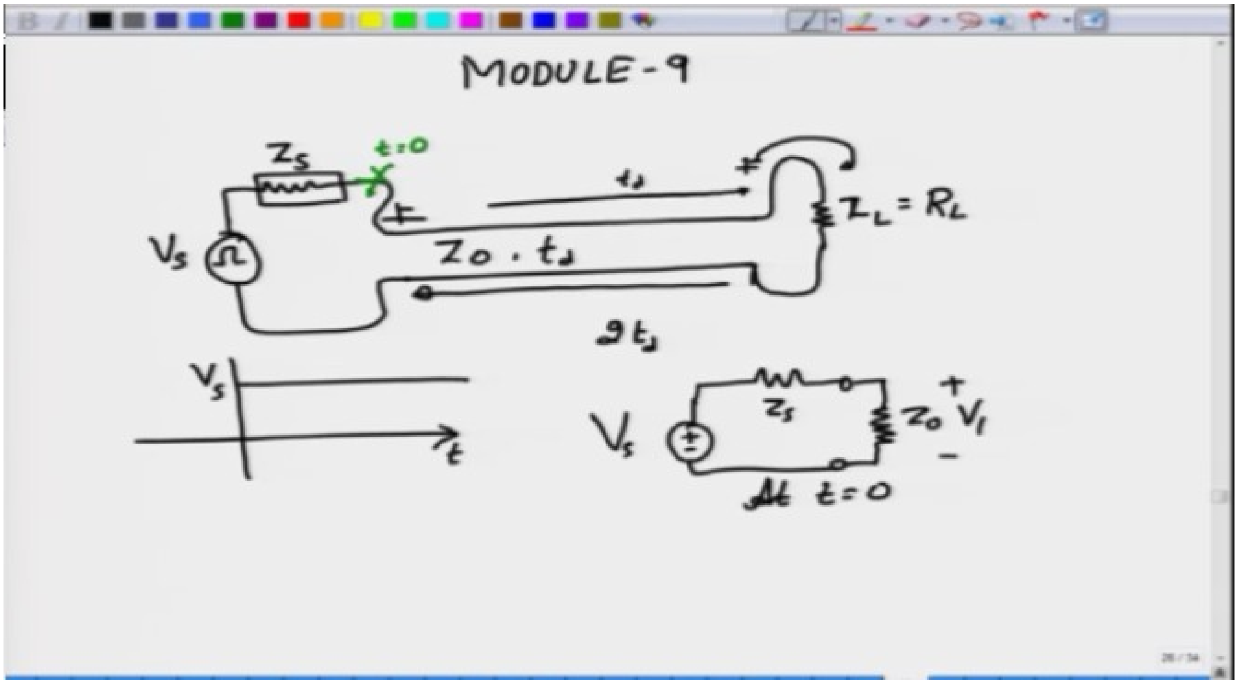

V1 would correspond to the amplitude of the voltage that is actually launched on the transmission line at time T = 0 by the action of this switch being closed, and this V1 is now simply given by the voltage deviation formula, right, which is given by VS times Z naught by Z naught + ZS, in fact this is the voltage that would be launched on to the transmission line at T = 0,

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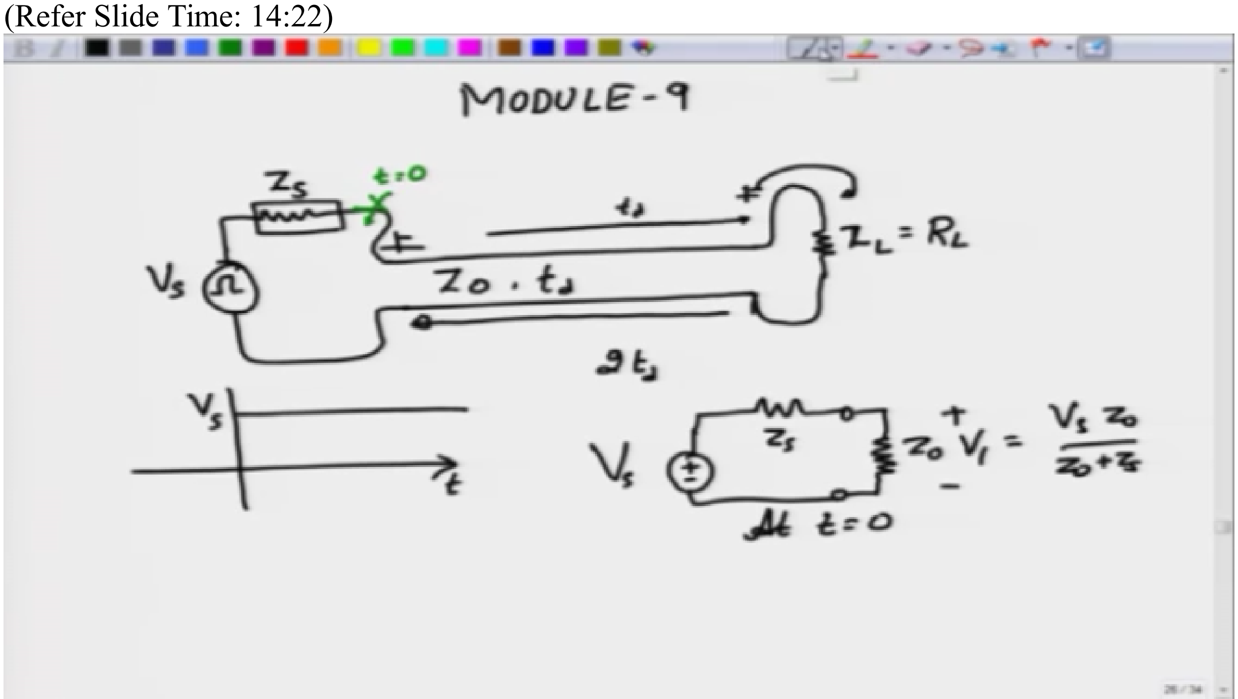

okay, while the source may be at VS, usually the voltage that is launched on the line will be less than that VS and it will be given by V1 to begin with, and this V1 is the step voltage that would start to propagate, so this part of the voltage would reach the load after a distance or after a length of TD, so at which point it begin load, or it tries to go into the load, so this delay of TD is the delay, the edge or the voltage here the point which I've marked in the red would actually start reaching and then start flowing into the load.

But at the point TD that is at time T = TD, right, the equivalent circuit at the load would look something like this, you have V1 here, I've just indicated with the DC source, because that's what we are using and then you have a characteristic impedance Z naught, and then you have a load impedance ZL, okay, it's not exactly like this you know I would, so I will call this as Z naught, but it's not exactly like this, but there is a voltage and there is a characteristics impedance Z naught, and then it sees the voltage ZL, okay.

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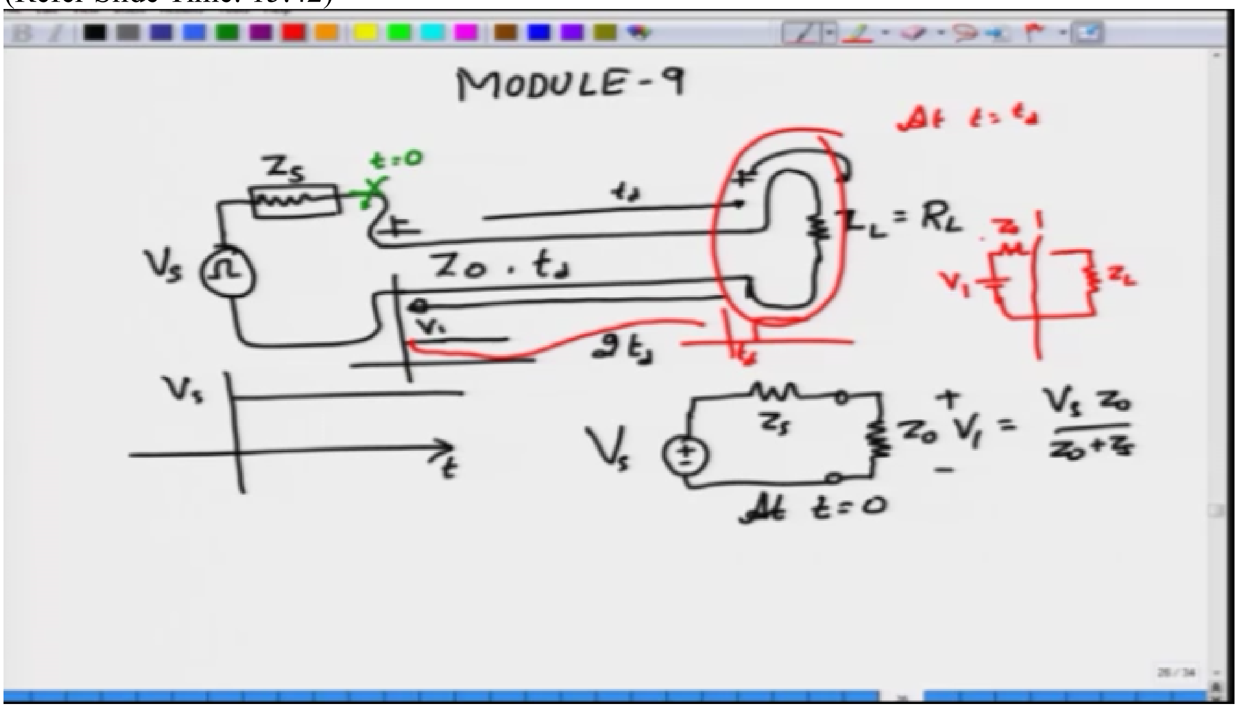

When Z naught and ZL are not matched which normally happens then only the incident wave and the transmitted waves, voltage waves cannot exist, there has to be a reflected voltage as well, so in fact at T = TD there will be a reflected voltage, and what would be the amplitude of that reflected voltage which we will call us V2, okay, V2 and we will give it a direction V2 – similarly we could give a direction V1 + indicating that this is the positive going voltage or the forward voltage, and this is the backward traveling voltage, so V2– will be given by the load reflection coefficient gamma L times whatever the voltage that was launched or that was available at time T = TD at the load end and that is given by V1+.

So V2- is gamma L V1+, so what is gamma L? Gamma L is basically ZL – Z naught divided by ZL + Z naught, this number can be positive or negative and this would be a real number now,
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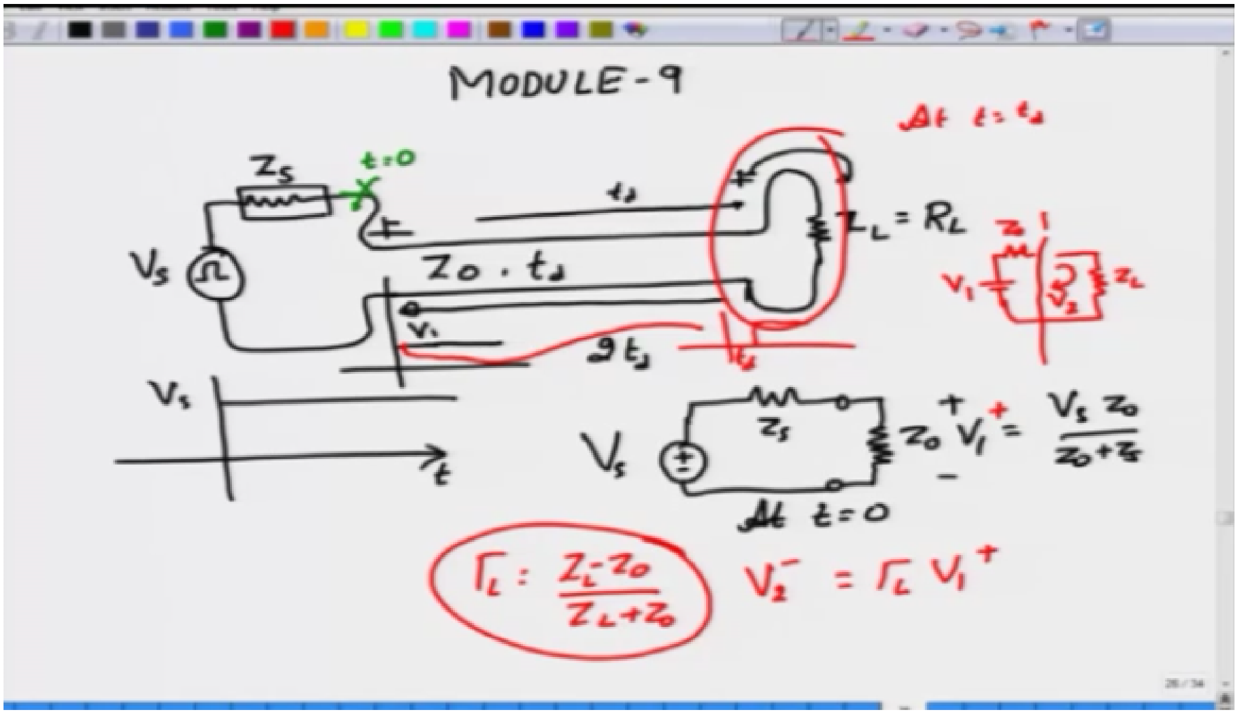

because we have assume resistive termination as well as the lossless transmission line which means its impedance is also characteristics impedance also real, okay, so this amount of propagate back, okay, and reaches this source side or the source end at T = 2TD.

At T = 2TD you will now have you know you can simply think of the source as a load now and that would generate another reflected voltage which we will call us V3+, and V3+ will be equal to gamma S or gamma G doesn't matter which one you use, gamma S times V2-, okay, so this is what the reflected voltage V3+ would be,

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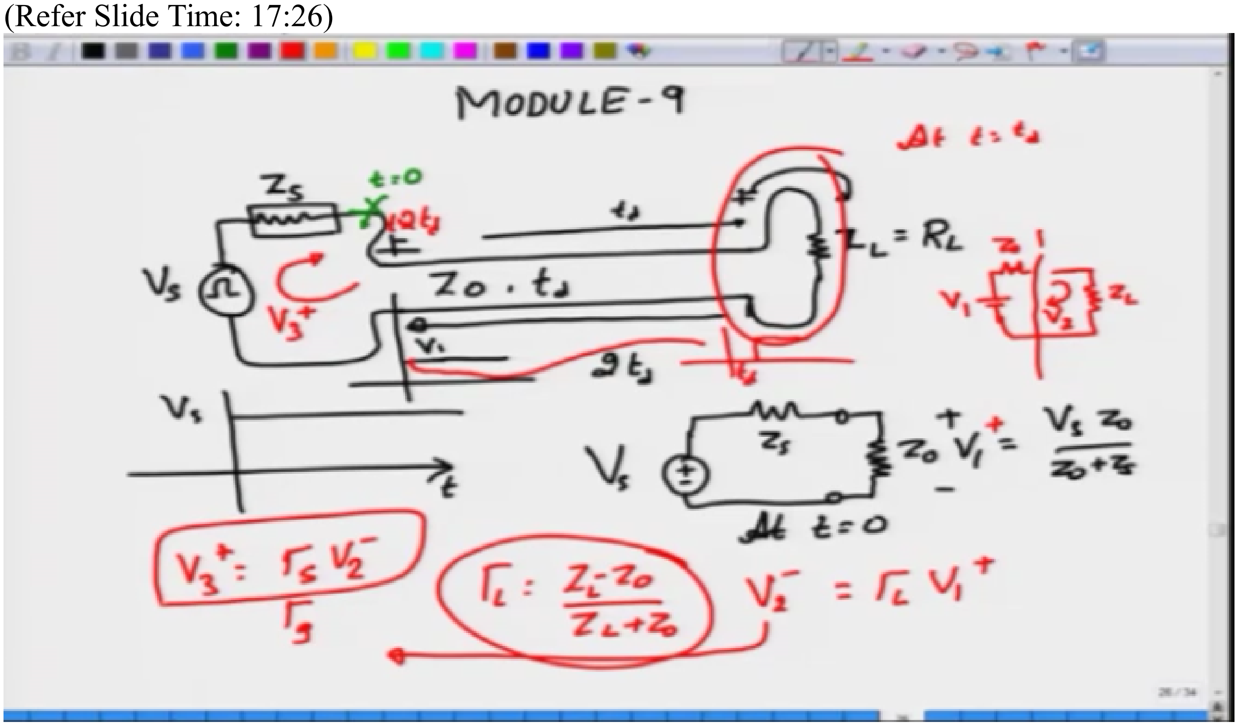

because gamma S is usually less than 1, gamma L is less than, the amplitude V3+ is usually smaller than the incident voltage, in some special cases it would be larger but we will see what that special cases later on, so V3+ and then it travels generate V4-, then V5+ and so on and so forth it continues in this manner until your tired off tracking these voltages, okay.

Now this may seems slightly complicated way of you know solving and finding out the voltages and moreover we have just been seeing the amplitudes of the voltages, we don't know the shape of the voltage that we are going to see finally at the load, and finally at the source if you are using a TDR meter and you can actually solve for the shape of the voltages along with the amplitudes in a correct manner by utilizing the tool called as space time diagram for the transmission line or sometimes simply called us lattice diagram, okay.

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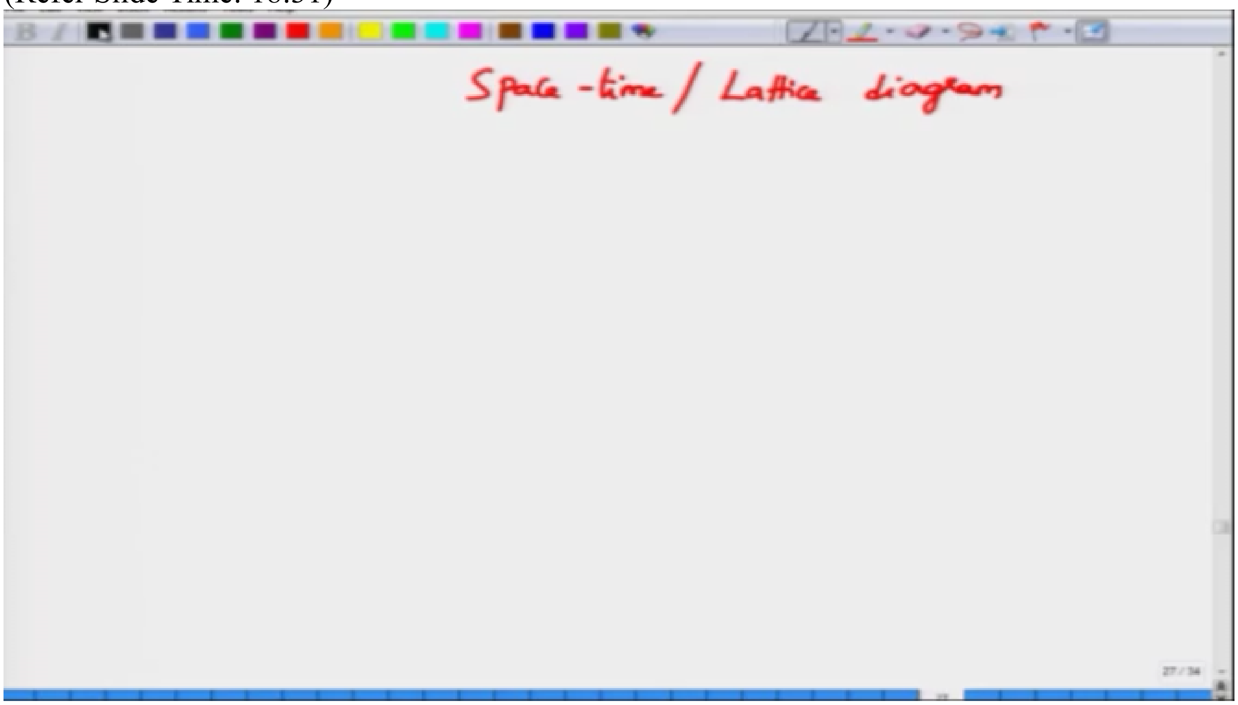

I am going to look at this lattice diagram and show you how it can be applied for a simple transmission line, we will have to see more about lattice diagrams in the next module as well.

To begin with lattice diagram is simply a two dimensional representation, okay, the two lines that I have drawn the vertical lines correspond to the load, sorry source as well as the load ends of the problem, so Z = 0 is the source end and Z = L is the load end so you can see that the coordinate systems have been slightly changed so I have Z = 0 as the source, Z = L as the load, so this is the Z axis which you know which will tell you where the load is located and this is the time axis, usually you give the time in terms of time delays, because those are the intervals where something significant happens on this problem, so you can write this as 1, 2, 3, 4, 5, 6,
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so when I write this 1, 2, 3, 4, 5, 6 I'm actually implying that, I'm looking at time instance TD, 2TD and so on up to 6TD, okay, so 2TD will correspond to voltage going from the source to the load and then coming back, and so by 6TD you have actually done 3 round trips

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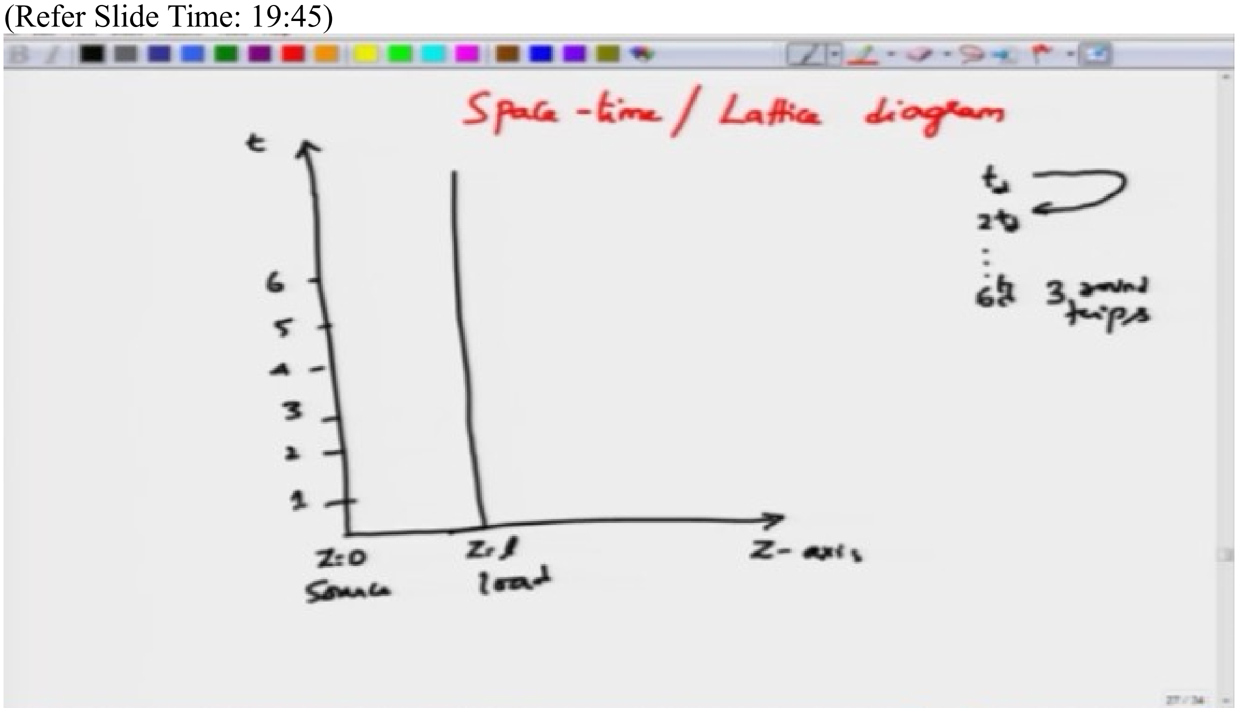

and usually this amount of calculation is sufficient for you to get an idea of what would be the voltage, okay.

Of course on this side also you can write 1, 2, 3, 4, 5 and 6, and now you are begin to, and now you are ready to begin the lattice diagram.

Now well, quite not ready because you also want to know what is gamma L? Gamma L will be given by ZL – Z naught/ZL + Z naught, gamma S will be given by ZS – Z naught/ZS + Z naught, okay, and you need to know what is the equivalent circuit that you are going to get at T = 0, so that would be VS with ZS connected to a transmission line of characteristics impedance Z naught, therefore V1+ is given by Z naught/Z naught + ZS times VS as the voltage division formula would tell you, okay,

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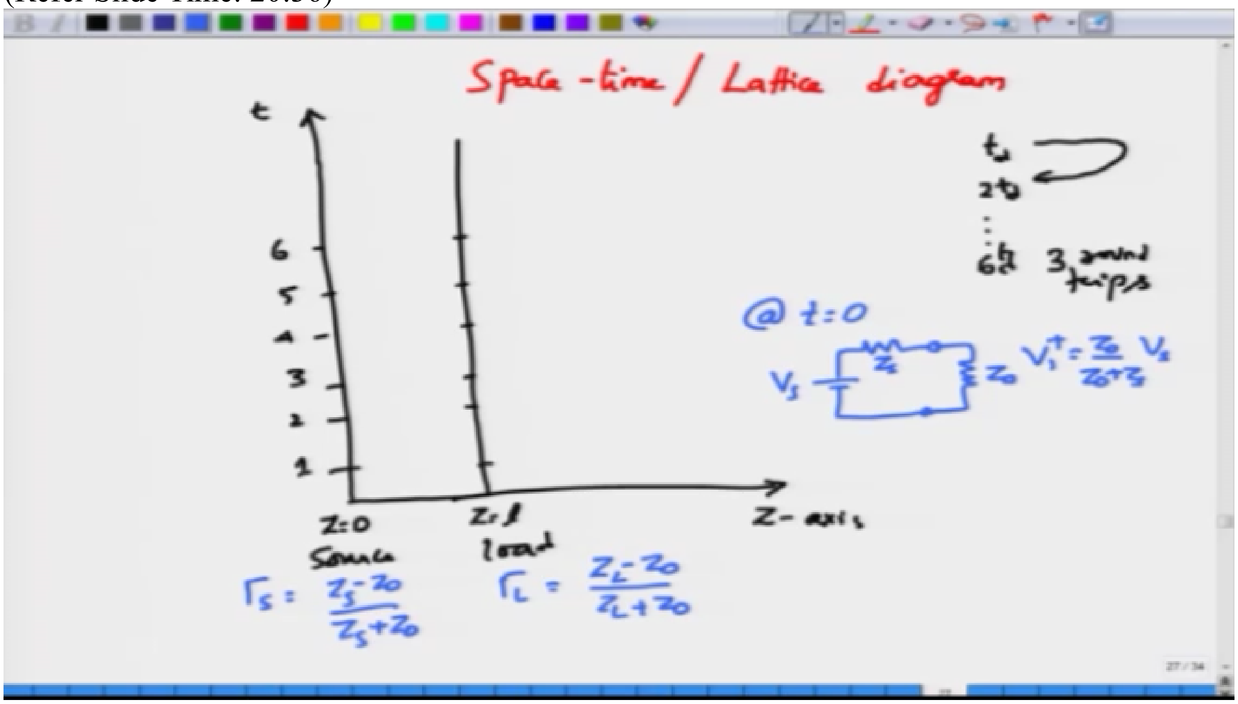

so now you are ready to begin.

So at T = 0 you start off the voltage V1+ that would be launched and of course at the source it simply continues right, so this would be VS, it continues all the way to infinity, but this starting point of the voltage VS, instead of VS being completely available to the load, what you actually get is V1+, right, so the first voltage V1+ also start with an amplitude of V1+ and continues forever to infinity, right, but this won't be available to the load at, after a time TD, so after a time TD we will now see this voltage appearing at the load, okay,

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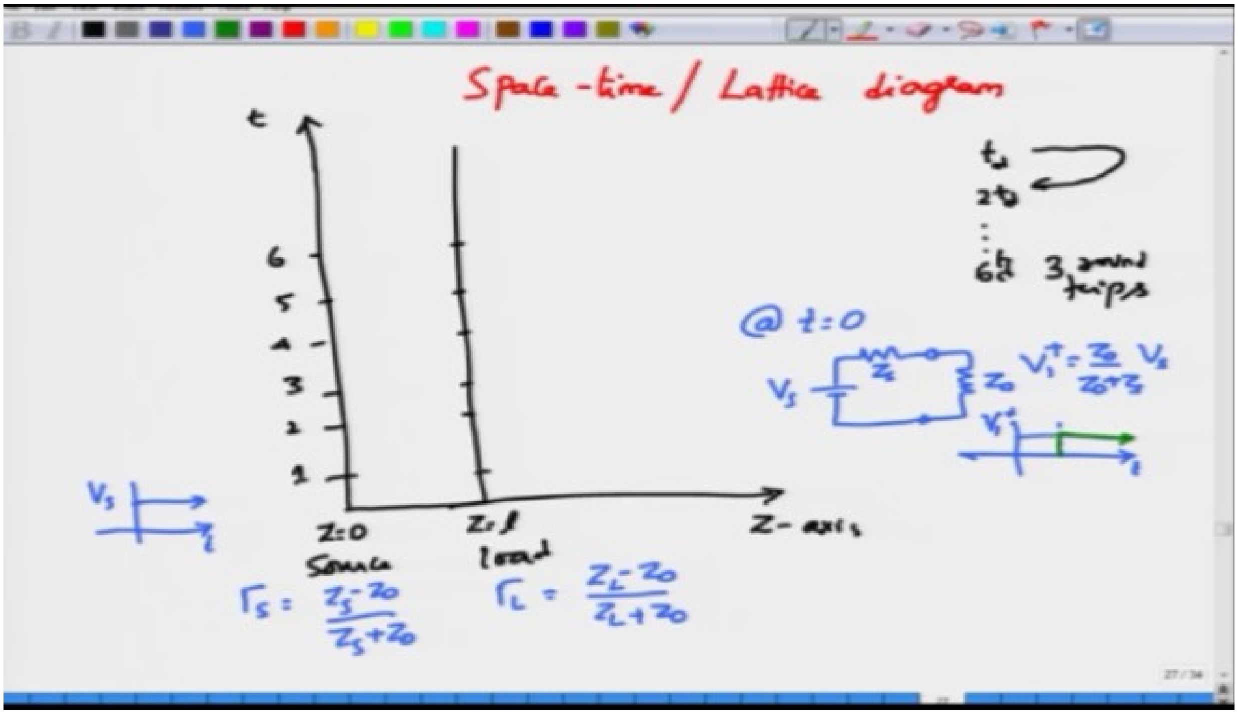

and this portion is actually keeps of, what if the load is now instead of keeping at Z = L you keep it at Z = L/2, then the time delay would reduce from TD to TD/2, right, so you can actually you know capture this entire thing by drawing the straight line, okay, and marking the voltage value V1+, starting from Z = 0 source to the Z = L time, and when you do that so this two points are marked because the voltage V1+ will reach the load at T = TD.

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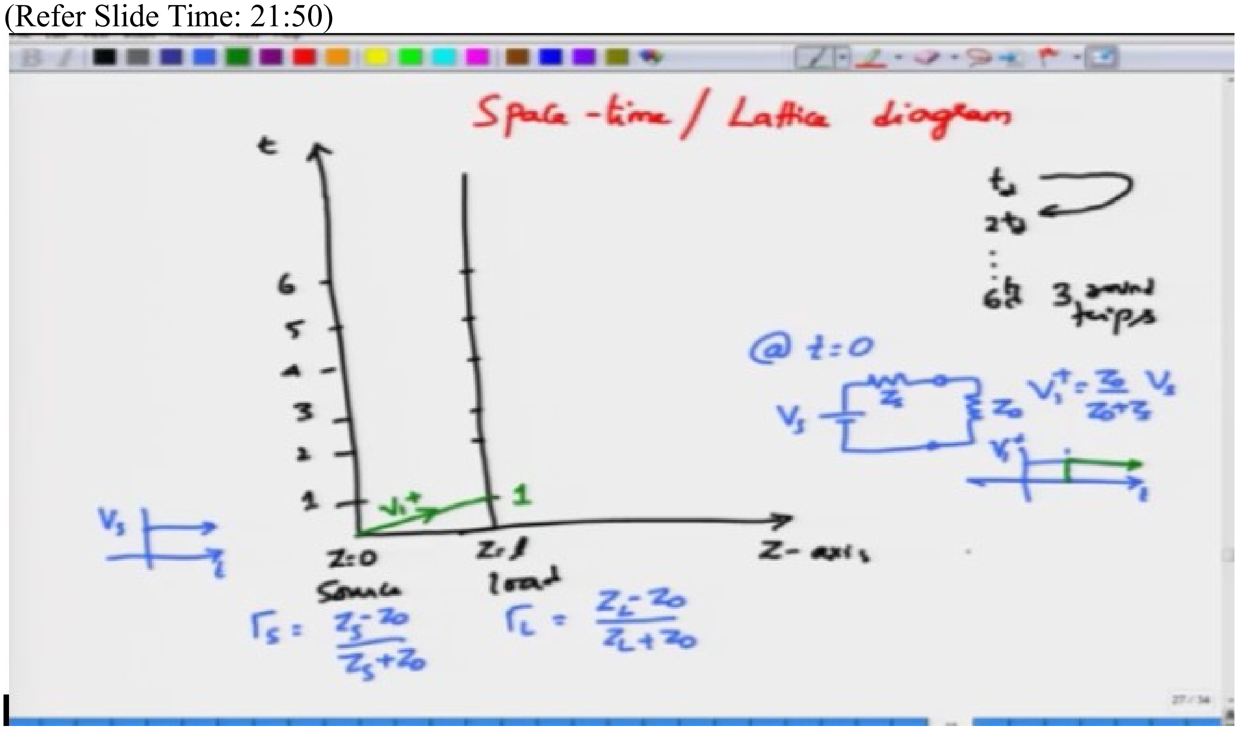

So now it reaches here, but as we have seen a voltage now coming through a transmission line of characteristics impedance Z naught terminated in ZS load, sorry ZL load will actually see a reflected voltage of the amplitude V2– which is given by gamma L V1+, you know V1+ from this diagram, you know gamma L return directly below that one, so you now start a V2- which would move in the direction from load to source, so V2- = V1+ times gamma L, you can actually write this separately factored in this manner,

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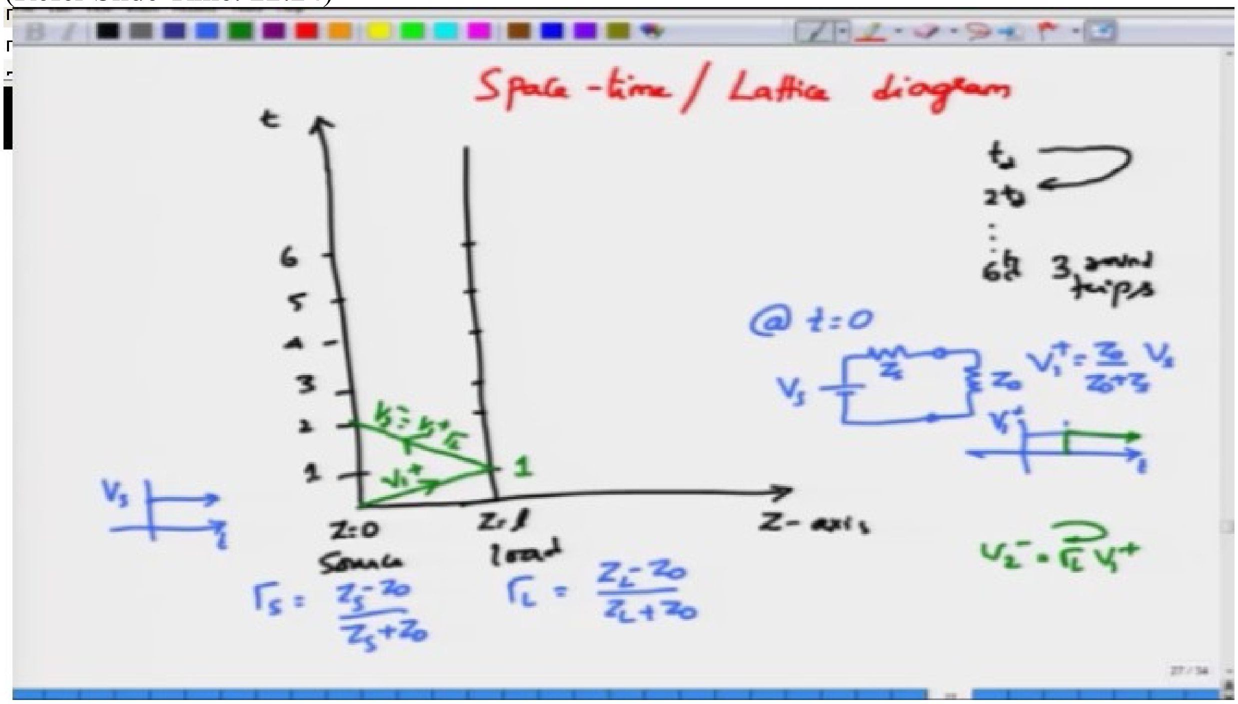

so that you know your problem will be slightly simpler to fall, okay, all information are being properly given, so you now have V2-, so what would be the total voltage that you would see at the end of T = TD on the load you would actually see V1+ + V2- which is actually V1+ into 1 + gamma L, and this 1+ gamma L is called as the transmission coefficient,

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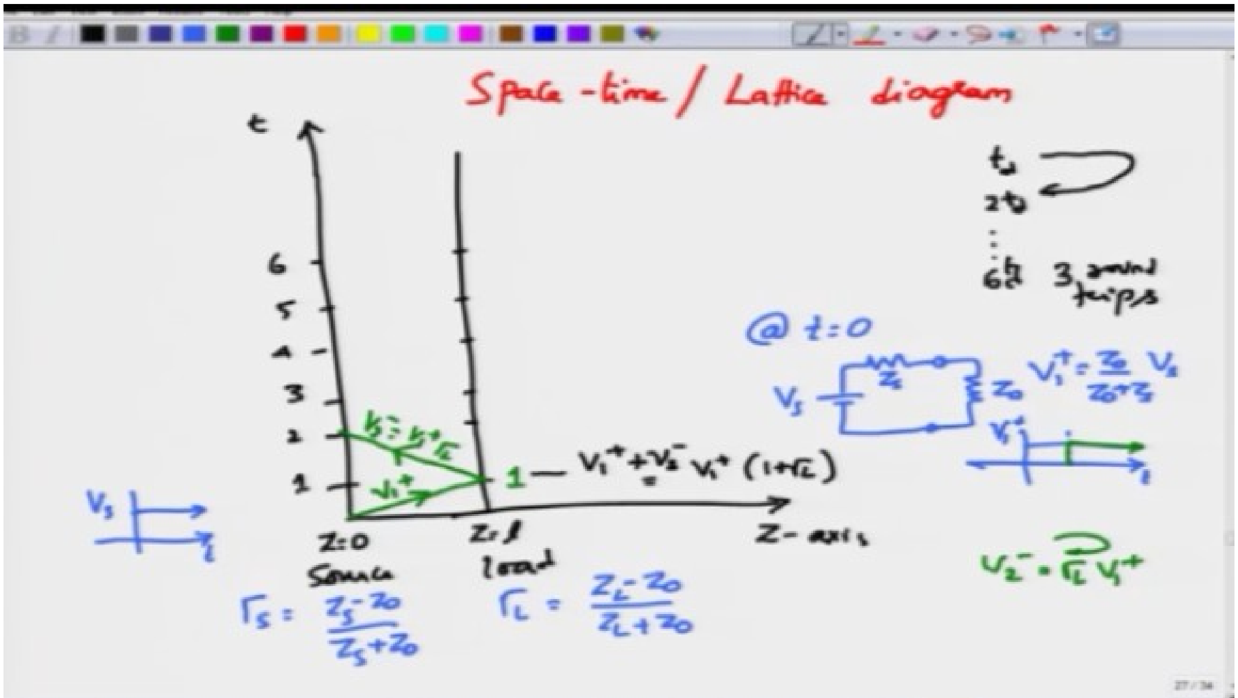
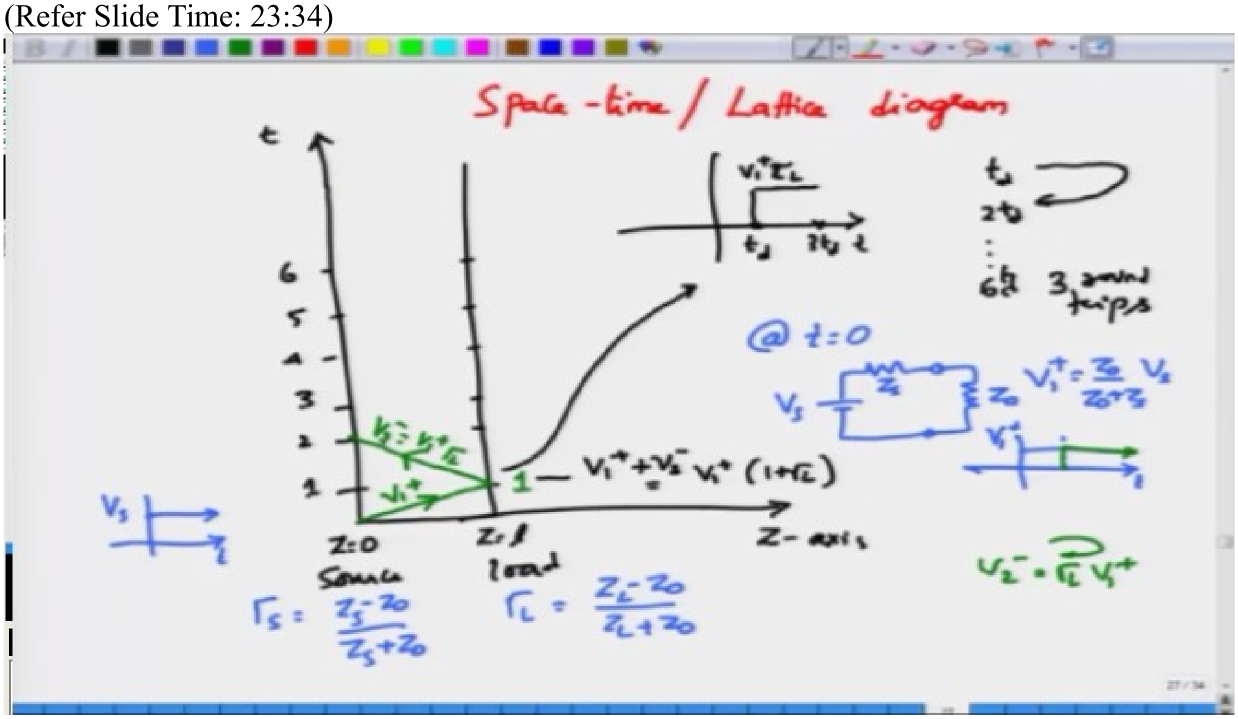


okay, so at the end of T = TD if you are actually looking at you know you put a oscilloscope at the load and you are looking at the voltage changes you will see that the voltage suddenly changes at T = TD to a value that is given by V1 + times tau where tau L, where tau L = 1 + gamma L,.

If you don't like tau L you can simply write down what is the, calculate the voltage and just put the voltage value down there, and until that time 3TD nothing really happens on the load, why because there is no voltage coming back, right, the partial voltage will come back at 3TD, (Refer Slide Time: 23:34)



because V2- which has now reached the source will be reflected off and forms V3+ right, which will then be launched at T = 2TD and then goes all the way towards the load at this point where it would be reflected off to give you V4-, therefore at 3TD the voltage will jump from V1+ tau L to V3+ +V4 - + whatever the earlier voltage that you had that is V1+ tau L, so this would be the total voltage, so it could be jumping down or it could be jumping up depending on the signs of tau, I mean depending on the signs of gamma L and gamma S, so I will just assume that for this problem where I'm not given you the numbers, I will simply assume that it has actually jumped by an amount, okay.
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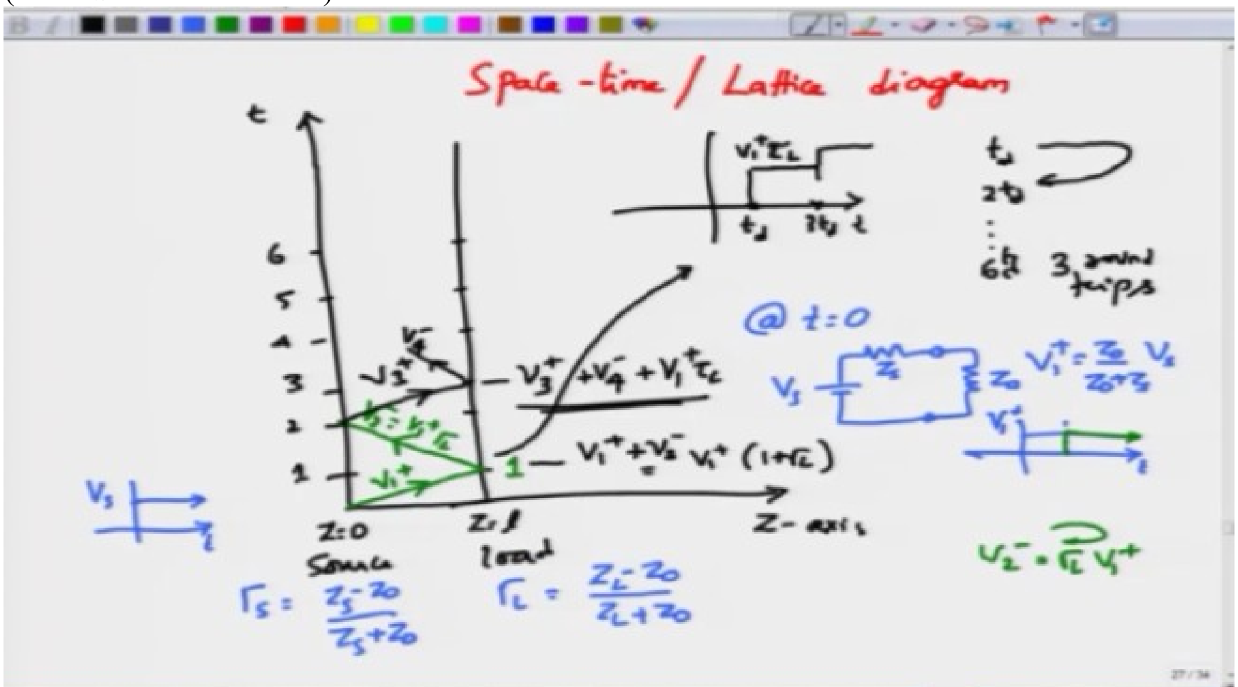

And this continues up to 5TD at which point you'll again see a jump, perhaps this is you know the jump will be smaller and eventually after certain jumps you will reach a final value or the asymptotic value which in fact would simply be given by, so if you call this as V infinity, that V infinity is simply given by ZS, ZL okay, connected to VS, that is the value of at that you are going to see, V infinity will simply be ZL/ZL+ZS times VS, this is also making sense because the line is lossless, so eventually the total voltage that you are going to see on the load has to be equal to the chase where there was no line, right, so line is lost, it's not storing any energy, it's not dissipating any energy I should say, right, but it does takes it time when the conditions are not right, it does takes it time to reach that asymptotic value, and that is where the transmission line is going to be, I mean the transmission effect is actually being seen.

If there was no transmission line effect then the voltage would immediately jump as soon as the switch is connected the voltage of the load would jump to the final value of V infinity, but now it will take many, many times T, so maybe at least 4 or 6TD times for it to actually change or be very closed to the final value V infinity, okay, so this is the voltage that you would see on the load and at the source what you would see is, initially you would start seeing this VS-, but at 2TD because there will be a voltage V2- that comes in from reflected from the load you will see a total voltage change of V2- + V3+,

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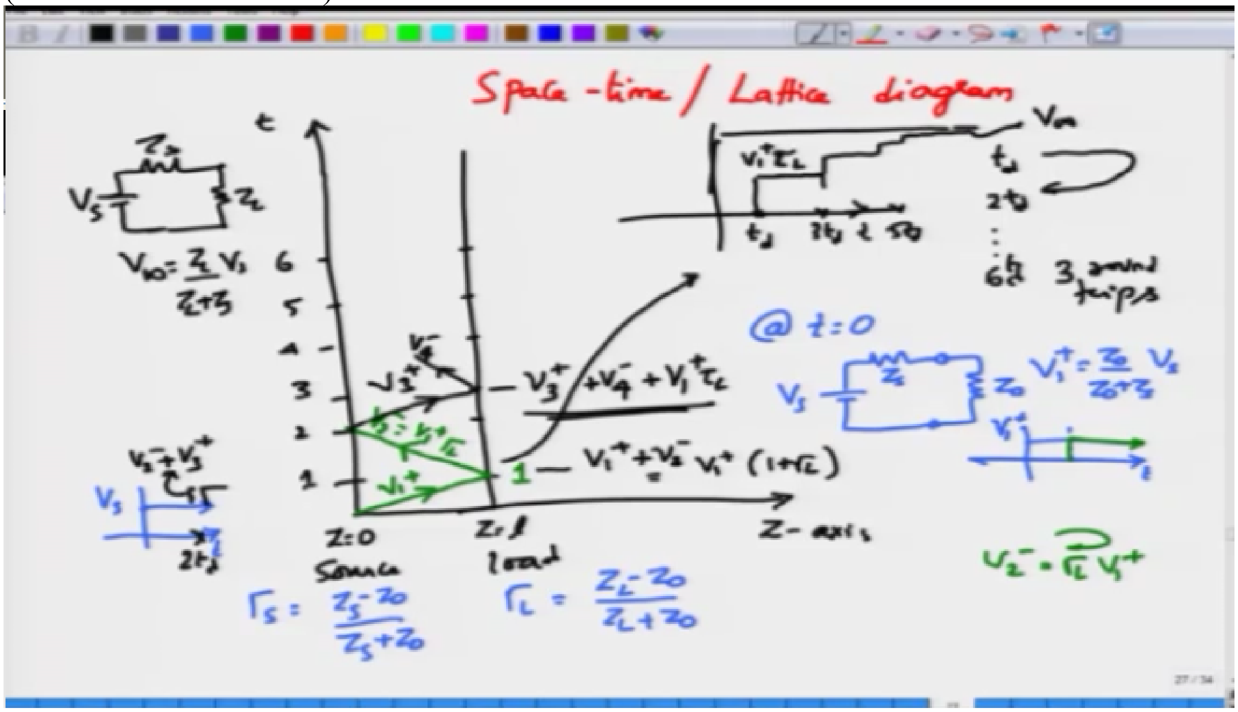

so this step change that you are going to see will be V2- + V3+, why V3+? Because that would be the partial voltage that is reflected at the source, so V2- then V3+ which is reflected, that would be the step voltage.

And again you know I'm not really looking at the numbers correctly, we will do that number calculations in the next module, but this would be what you would actually try to see, right, so you would again see that voltages are actually changing at the source as well as they are changing at the load, okay, not just that sometimes you're interested in knowing what is the voltage at the midpoint of the load let us say,

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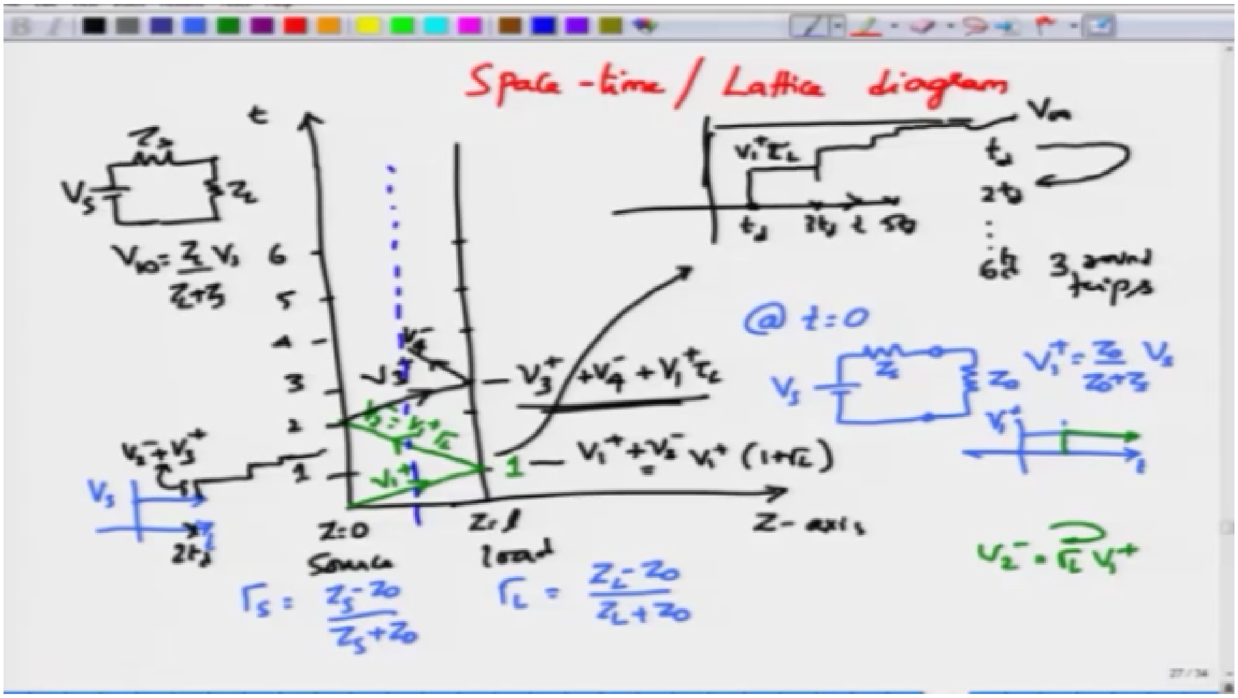

so at the midpoint meaning at Z = L/2, you are going to wait until a length, I mean until a time delay of TD/2 right for the voltage to change, and at that point it changes to V1+, then you will wait until you know up to that 1TD later, so that would be safe, 1.5TD, so at 1.5TD you will see that it would change a value from, so this is 1/2 TD, this is 3TD/2, where you will see a jump of V2+ this is V1+, and then similarly V3+ and so on, okay,

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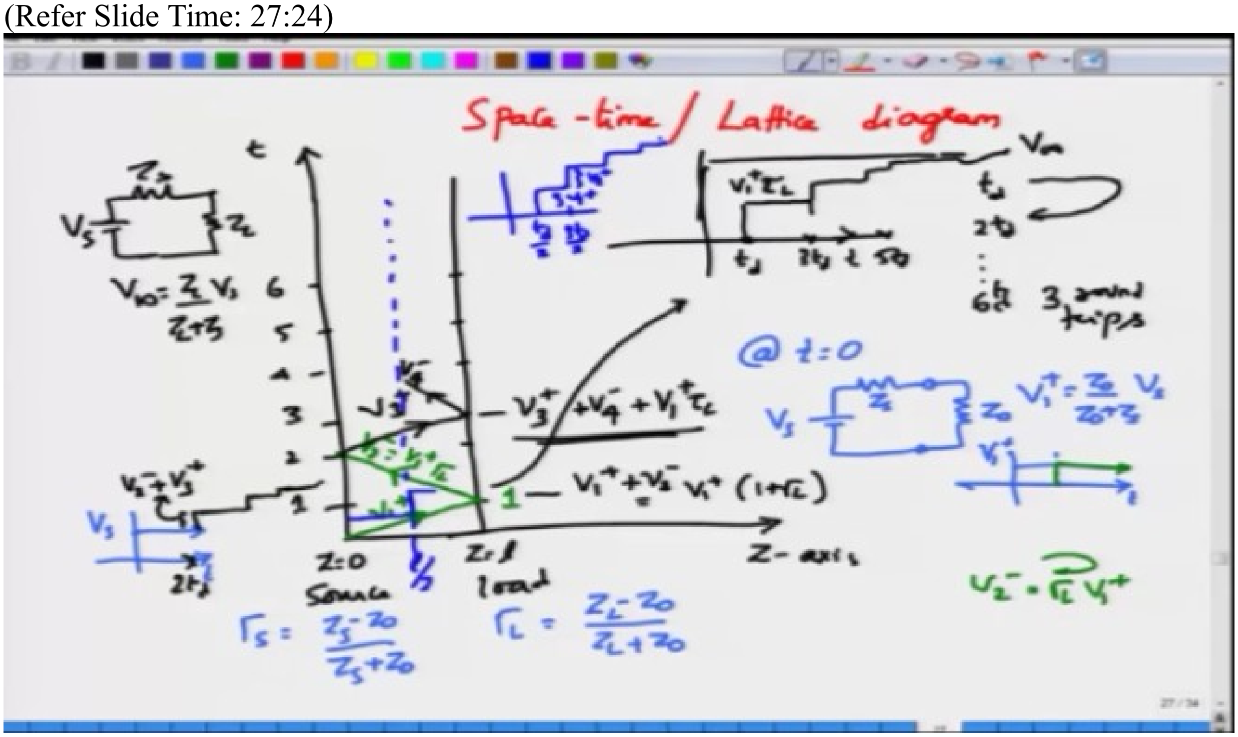

again I'm showing all of them to be increasing, but in the next module we will actually put down the numbers and then considers some very special cases which are also very important to see what is the effect of this different types of loads or rather different values of loads, and how they would affect the signal integrity.

To give you a basic idea before we go, the idea here is that if I have you know a logic gate, another logic gate connected by a transmission line then and if I say that I need a certain voltage at the load IC for it to be considered to be a digital 0 or digital 1, so let's say digital 1, so at digital 1 requires a certain threshold voltage V, in the case where there was no transmission line, any switch change or any sudden change at T = 0, because of the source IC changing would be immediately available to the load, there is no ambiguity whether the voltage that the load IC is seeing is actually less than threshold or greater than threshold that would have been immediately available, however now although the final voltage maybe greater than the threshold voltage because it takes sufficient time, so let's say it took about 5TD time of the voltages because of the non-desired but present transmission line, you would see that the voltage reaches its threshold voltage at 5TD, right, so if the receiver would wait or the receiver IC would wait until 5 time delay then it would know that the actual voltage coming from the source IC is greater than the threshold and it has to be considered to be a logic one, but that would either mean slowing down the devices so that you give sufficient times to account for the transmission line effects or not to slow down and not to slow down the devices but run it at regular speed and then incur lot of errors, right, so that is where the signal integrity is going to hit you, it's going to give you lot of errors which otherwise would not have happened if the transmission line itself were not to be present. Thank you very much.

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