**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

**NPTEL**

**NPTEL PROGRAMME ON**
**TECHNOLOGY ENHANCED LEARNING**

**Course Title**
**Electromagnetic Waves in Guided and Wireless**

**Lecture - 07**
**Smith Chart**

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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media, this is module 7 where the primary focus will be on Smith Chart and its use in solving transmission line problems, okay.

Now let me warn you that Smith Chart is not the magic technology, one has to understand certain basic relationships on a transmission line to effectively use Smith Chart and moreover Smith Chart can be used in most modern transmission and applications to only give you a qualitative answer, okay, if you want exact numbers which in many practical you know products, hardware equipments are required then you have to go back to equations write appropriate programs and then get you know correct numbers, so if that is the case why do we even want to use Smith Chart? Well, the answer is that Smith Chart gives you an excellent intuition behind what is happening on this transmission line and microwave circuits, okay.

So if for example you want to you know just quickly change the parameter and then note its effect on the Smith Chart, I mean note its effect on the circuit then a very quick way of doing that would be to actually use a Smith Chart, in fact you can program Smith Charts that is you can write programs that will generate Smith Chart and then you know do all the manipulations for you, but you have to interpret those results, and interpreting those results actually gives you lot of physical intuition behind what is happening in the transmission line problems, and that is the main reason why we want to go for Smith Charts.

Now even before we go to Smith Chart I would like to you know just answer couple of questions related to earlier transmission line thing, one when we talked about lossless transmission line we represented, I mean we have the characteristics equation of the transmission line right which was real, Z naught was given by square root of L/C and it was a real quantity, but then impedances are kind of resistors or resistive impedances, so then why do we want to call this as resistive you know, I mean why do we want to use a resistive impedance or a resistor like value to actually talk about lossless transmission and because we know that resistance dissipate energy.

The answer here is that if you take an infinite length transmission line, okay, whose characteristics impedance is Z naught and it is lossless and then you launch a certain voltage and current at the generator end, okay, meaning you're actually launching some amount of power in to the transmission line.

Do you think that you will ever get that power back? No, the power is actually lost because this waves will continue all the way to infinity and there will be no return signals carrying the power that you have launched, so in a sense you have kind of have a you know black body so to speak wherein your launching the power on the infinity long transmission line and nothing is actually coming back, so to represent this fact that you know an infinitely long transmission lines actually you know take out, I mean take the power without giving any power back to the system we have this kind of a resistive interpretation, I mean so that is not exact answer as such, but this is one way you can think about why a lossless transmission line should be represented by real impedances, okay, so that is the first question that I wanted to you know answer for you, okay.

Now I mean not that you ask for it but you know this question comes up so often that I thought I will give you an answer before we move on to Smith Chart, okay, as we have said Smith Chart is essentially a relationship between gamma and Z, correct, of course this is normalized Z, the Z represents the load impedance, line impedance or the input impedance depending on what position you are actually measuring or you are actually denoting this one, right, and in terms of the you know chart itself which I hope you have downloaded, you will see that there will be two kinds of you know, I mean there will be two kinds of chart actually if you download you will find that in one chart the circles shrink onto the right and the other chart circles shrink on to the left, okay, so this is one of the popular charts that you will see, so this chart, I'm drawing only a few circles so obviously you would have a full chart with you and I'm just drawing a few circles just to make my point clear, okay, so this is the chart that you have you normally would see this one and you can clearly see that the circle shrink in size as we go from left to right, okay.
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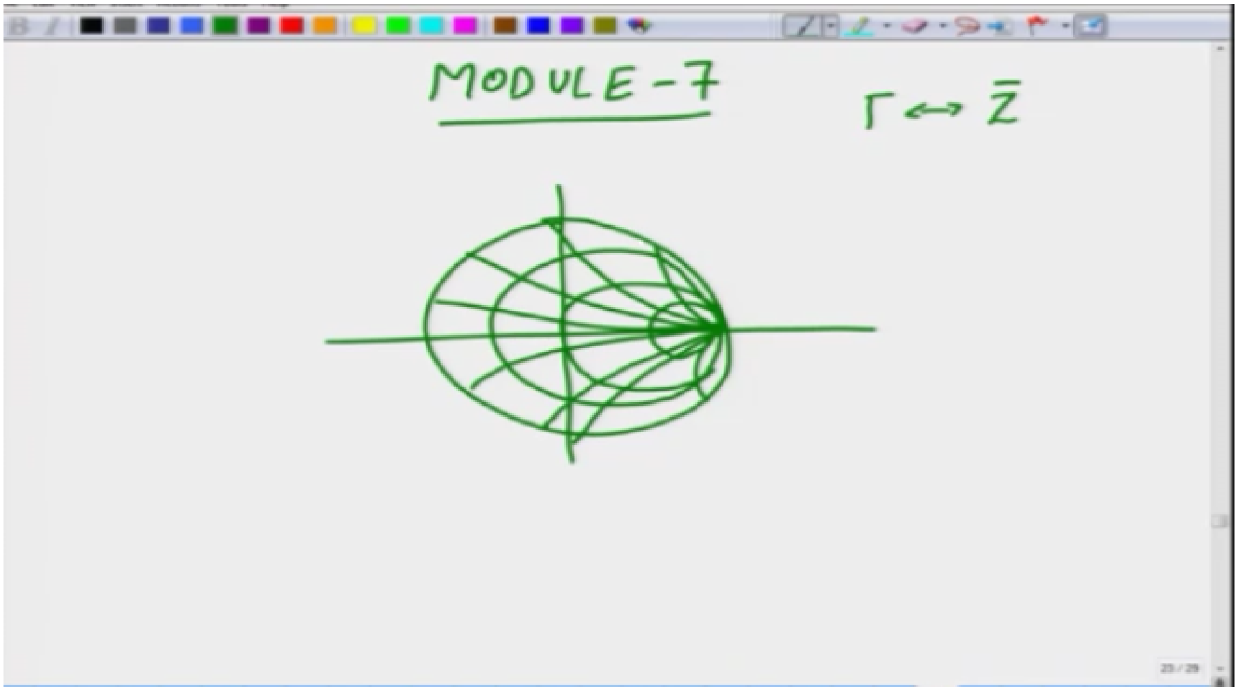


An entirely different chart can be you know downloaded or you can find out in commercial applications, and commercial stores, you will actually see that the circles actually shrink on to the its little hard for me to write, but you get the idea I think, so the circles are kind of rotated by 90 degrees, right, and the arcs are also in the same manner so you have this arcs which are constant X circles, they also look in this manner,

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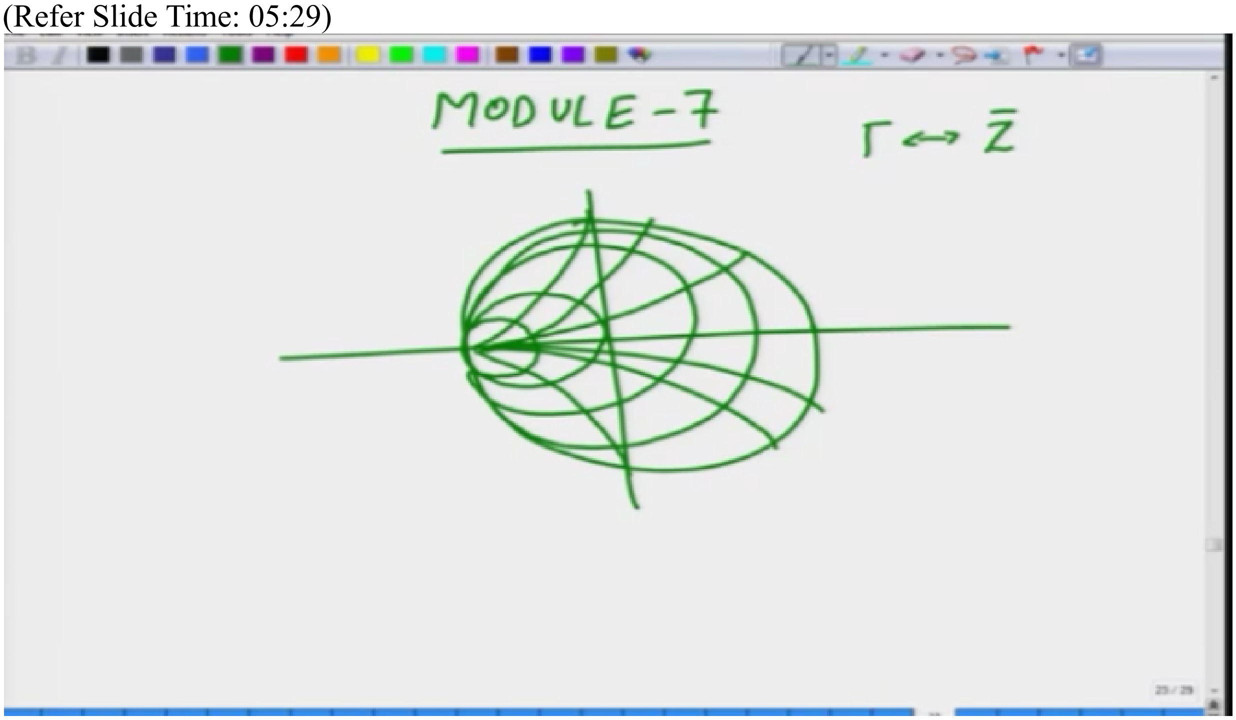


so it's like a mirror image of the other Smith Chart, this Smith Chart is actually called as admittance chart, okay, because when you want to deal with many applications where you know it would be sometimes very easy to work with admittances you want to be able to use these chart, okay, but you will also get a commercial chart in which you have both these you know, both of these circles super imposed usually with different colours if you print out a colour circle for you, it looks very messy but you can switch between impedance and admittance, okay, so you don't have to worry about changing the admittance to impedance,

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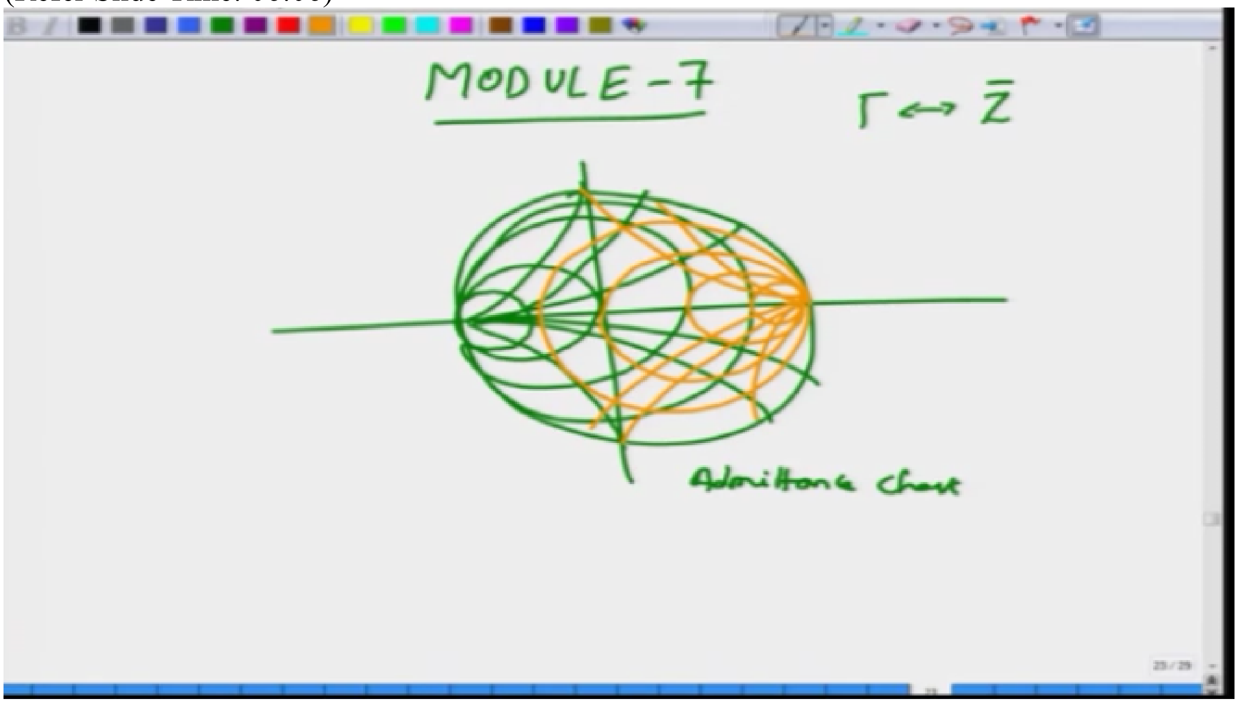


impedance to admittance as you would do with a normal Smith Chart, so this Smith Chart that you have is called as ZY chart, Z obviously standing for impedance and Y is standing for admittance, I would not prefer using the ZY chart because of the so many circles that are present it becomes quite hard to find out where the lines are moving around, okay, but most professionals used this ZY chart, because that will save them time of conversion from impedance to admittance, and admittance back to impedances, okay, so this you have to keep in mind, but for this course we will work with only the Smith Chart which anyway I have drawn it several times earlier, I'm going to just draw a few basic circles and that indicate but the actual values I would you know expect you to note those actual values from the Smith Chart that you have, keep a compass ready with you and then you can actually start working on this Smith chart, they are actually quite fun if you really think about it.

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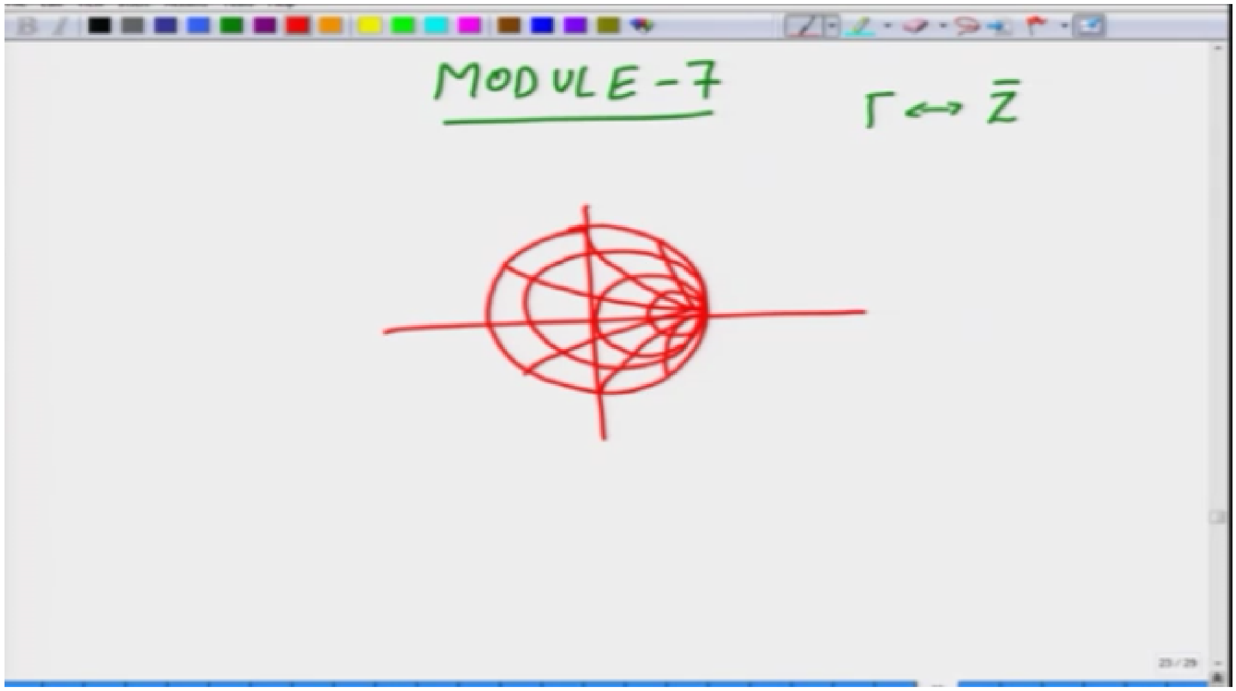


Now before we start talking about how to solve the problems, let me also give you one you know interesting bit of information which you can, yourself find out later on, suppose I arbitrary locate my gamma here, I can always do this because this is simply a gamma R and gamma I plane, so this gamma A which we will have given, we'll give a value of say 3 + J4 is perfectly valid value of gamma, right, as far as the mathematics is concerned there is nothing which stops from having gamma A = 3 + J4,

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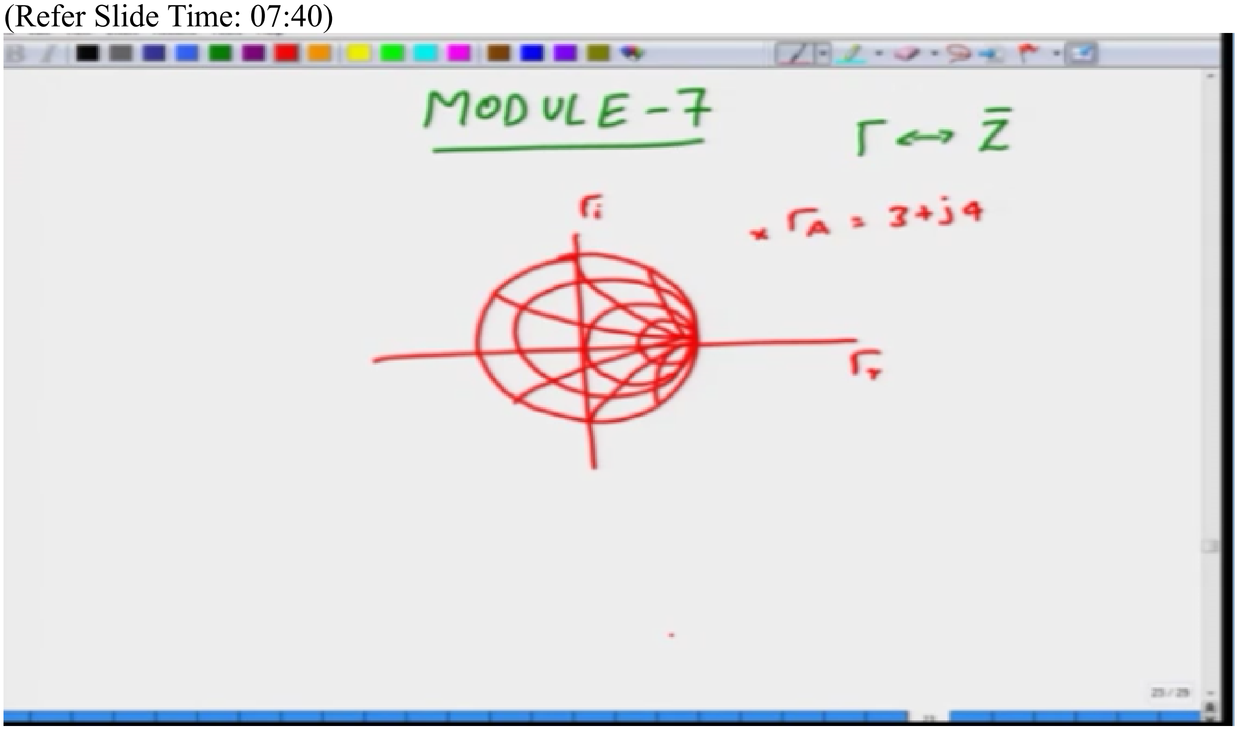


and because there is a one to one relationship between this one and the normalized impedance you can also calculate normalized impedance from gamma, right, you know how gamma is related to normalized impedance, this is the equation, so you invert this equation and then find out what is ZA bar?

Now if you take a little bit of a time and actually do the calculations you will see that, the real part of this ZA will actually be negative, okay, so in sense what you are going to get in the region outside is a negative resistance you know that you are going to get, a negative resistances don't exist physically,
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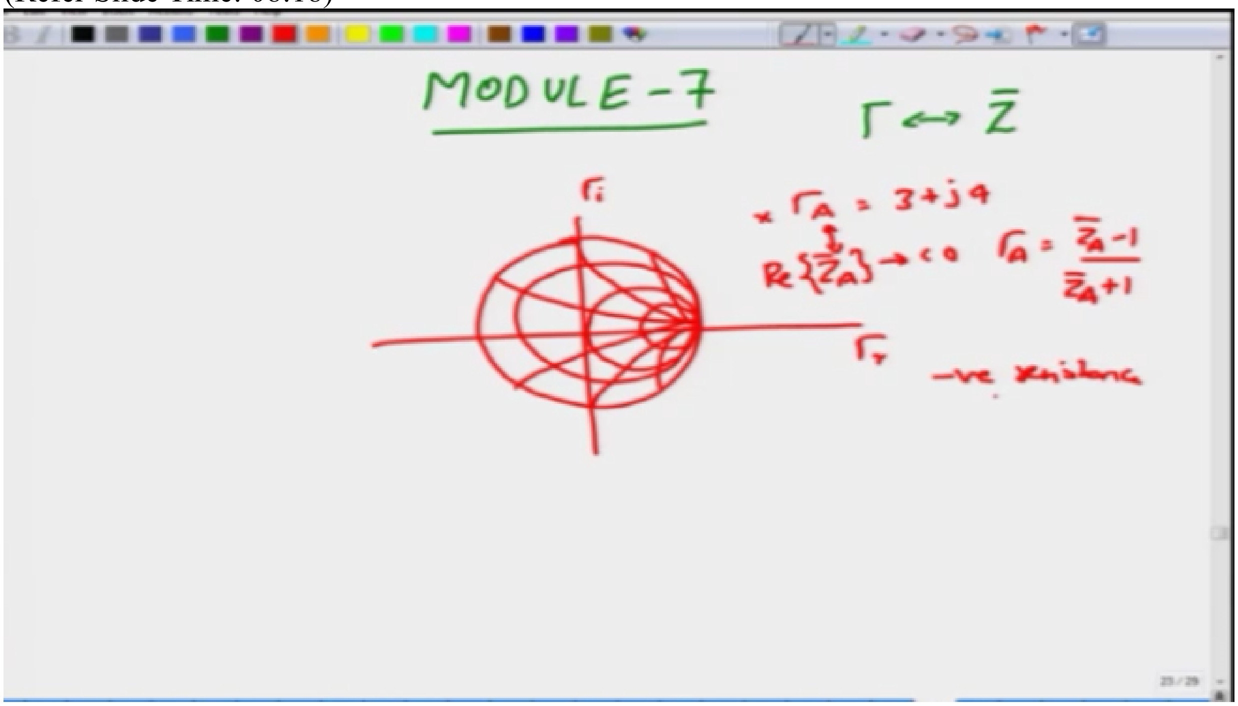


but they are excellent models for amplifiers, okay, that is the reason why I said that this region outside that chart that we have like vertical chart or the of the maximum magnitude gamma L equal to 1 chart corresponds to active circuits or when you work with active circuits, we will be working around region external to the Smith chart as well, but because in our course we don't go to active circuits we will not deal with this one, any good microwave course that is being currently offered on NPTEL also will have more discussions on this, okay.

Now this is our chart which is very simplified fine, no problem, so what are the possible transmission and problems that I can solve with this chart, let's begin with a very simple problem, given the load impedance ZL is 50 + J100 ohms at some frequency where the transmission line characteristics impedance is 50 ohm, can you find out what would be the admittance YL? Well you can do so mathematically, right, you can first you know you have ZL you can take 1/ZL
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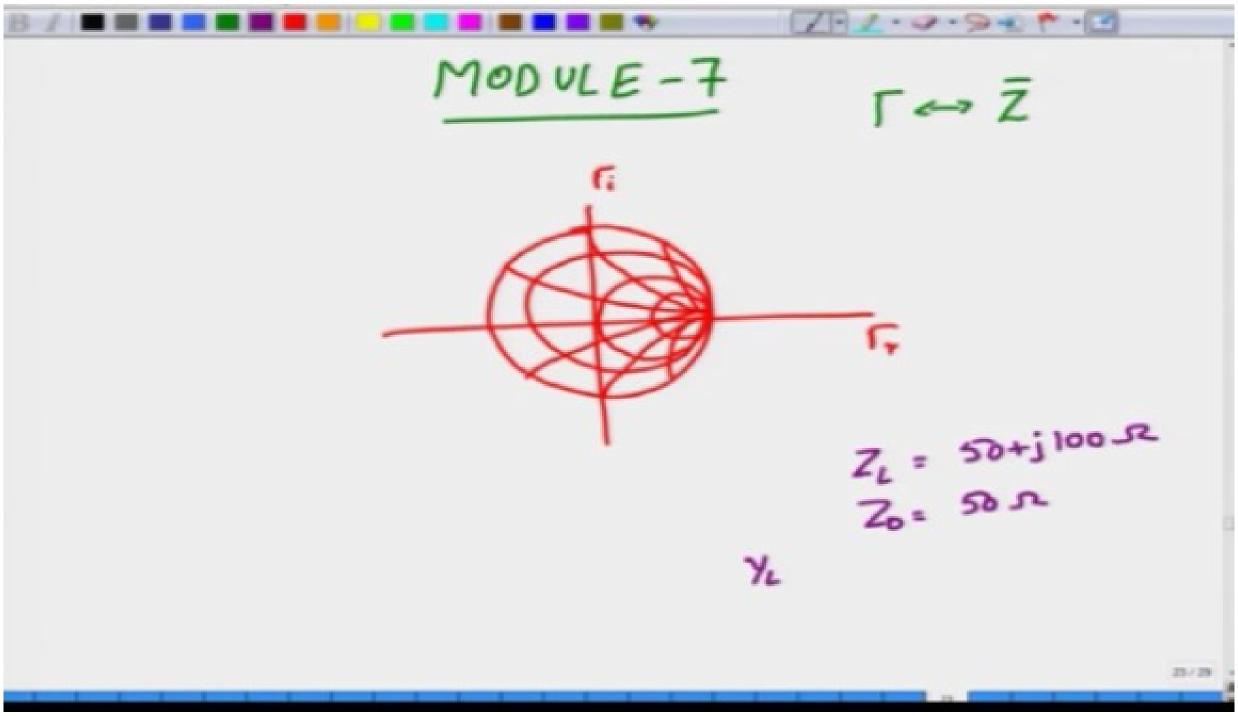


and then you know complex conjugate multiplication and other things, and you can do it and a computer I mean, on a calculator you may even get this answer in a second or so, or less than a second or so, right, but imagine that you did not have calculator or you want to anyway convert impedances to admittance.

The idea here is that imagine that there is a transmission line, okay, whose length is lambda/4 and it is terminated by some load resistance ZL, calculate what would be the equivalent input impedance here if you wish you can also calculate the normalized value here, okay, to your surprise what you would actually get here will simply be the load, normalized load admittance, okay,

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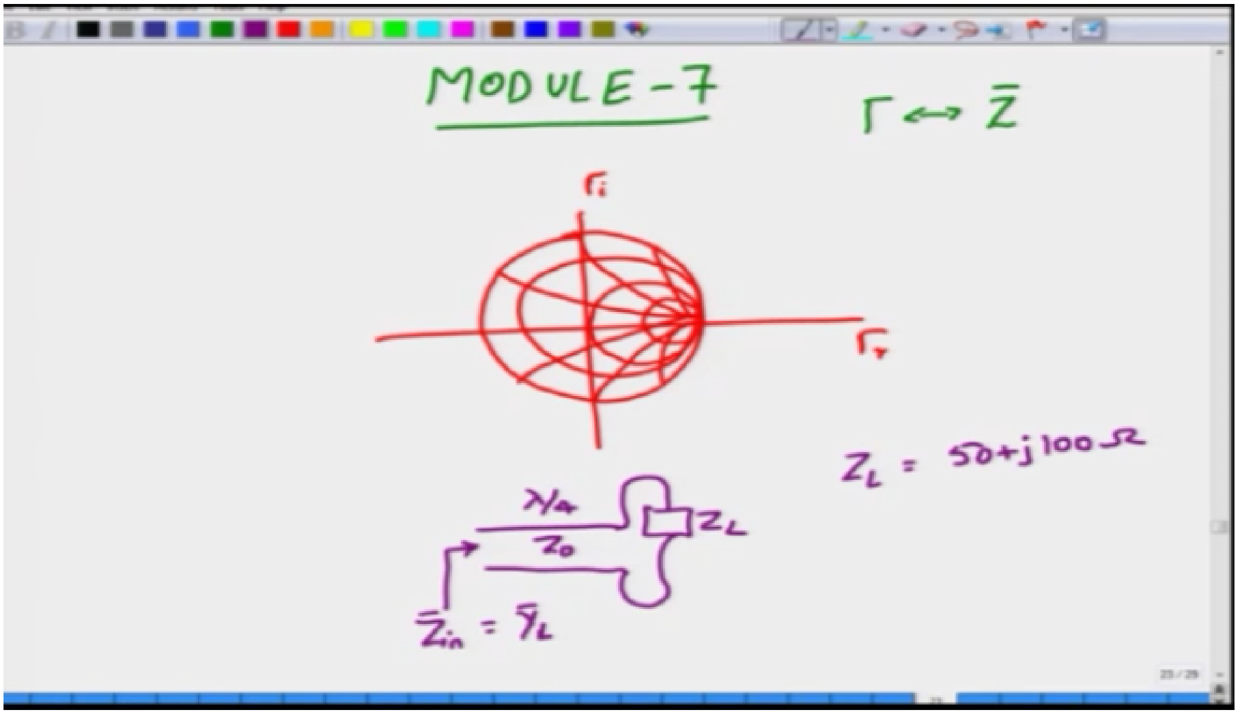


so if you want to un-normalized it you simply multiply admittance which is normalized with Y0 which is your admittance of the transmission line, okay.

Impedances normalized can be un-normalized or restored its value by multiplying it by Z naught, so this is an interesting thing and I will leave this as an exercise for you to figure it out for yourself, so all that you have to do is to first enter the Smith chart with the normalized load impedance, draw a constant SWR circle and move a distance of lambda/4.

What will you do, I mean where will you land up when you move a distance of lambda/4? Will actually land up exactly on the opposite sides of the circle, on the same circle we'll land up on the opposite side and that would correspond to the normalized load admittance, so let's imagine that you know because ZL is given here that would be 50 + J100, the normalized ZL will be given by 1+J2 this will be opined by intersection of R = 1 and X = 2 circle, okay, so this is R = 1 circle that I have, and X = 2 circle would lie somewhere over here, so let's say this is the point A which corresponds to the normalized load impedance.

Now here is a tip, please always refer all points in a sequential manner, okay, so that praising the steps becomes easier for you, I use A, B, C, D, you can use 1, 2, 3, 4, you can use any other symbol that you want, but you always go sequentially, okay.

Now that you've located A as ZL bar, okay, all you have to do is to draw the constant SWR circle, so let's say this is the SWR circle that we have drawn, so someone might ask you what is the magnitude of gamma L or what is the full gamma L of corresponding to this ZL that we have written, ZL magnitude that, or the normalized ZL that we have written is 1 + J2, what would be the value of gamma L? Well, all you have to do is to measure this arc, okay, arc I mean sorry the length of this radius from origin to A,

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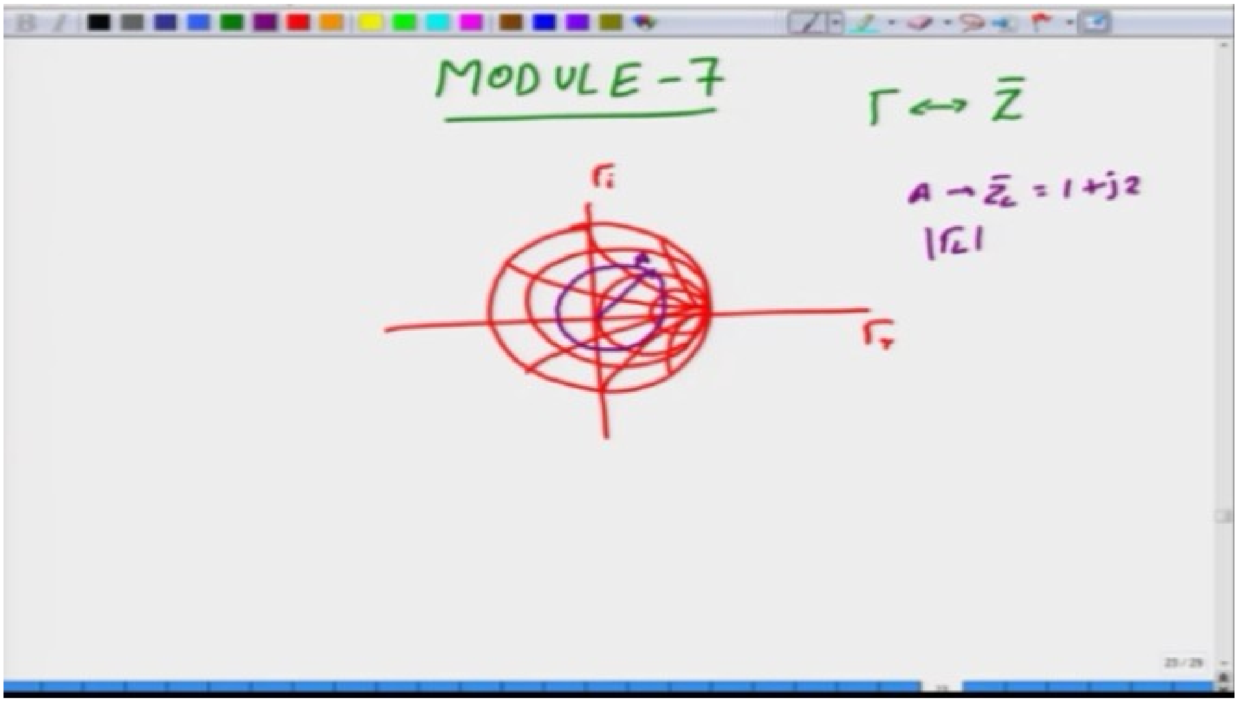


in fact commercial Smith Charts actually have a scale given down, okay, which will you know indicate both magnitude as well as angle, so there will be magnitude gamma L as well as the angle theta gamma in degrees, okay, so you simply take your compass measure this length here from 0 to A, and then cut that on the two axis, read the value here you will get gamma L and you will also get theta gamma, okay, so this is as simple as that,

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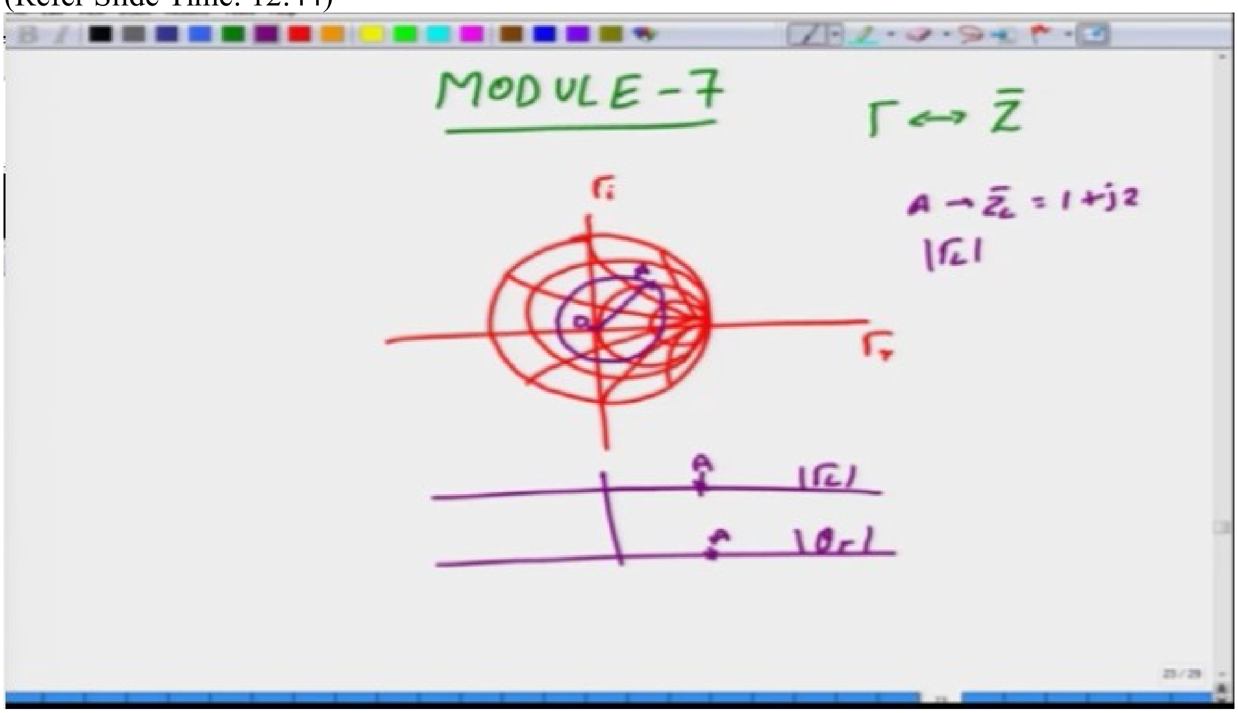


so all you have to do is to have a compass scale and then I mean a compass and a scale and then you are done, right.

So you got ZL bar information, you got, I mean you started off is ZL bar information, you found out what is the load reflection coefficient, okay, you found out specifically what is magnitude of gamma L as well as the angle theta gamma both, okay.

Now how do you find you know admittance? You simply move along this line, you simply move a distance of lambda/4 which exactly equals landing on the opposite side of this chart,
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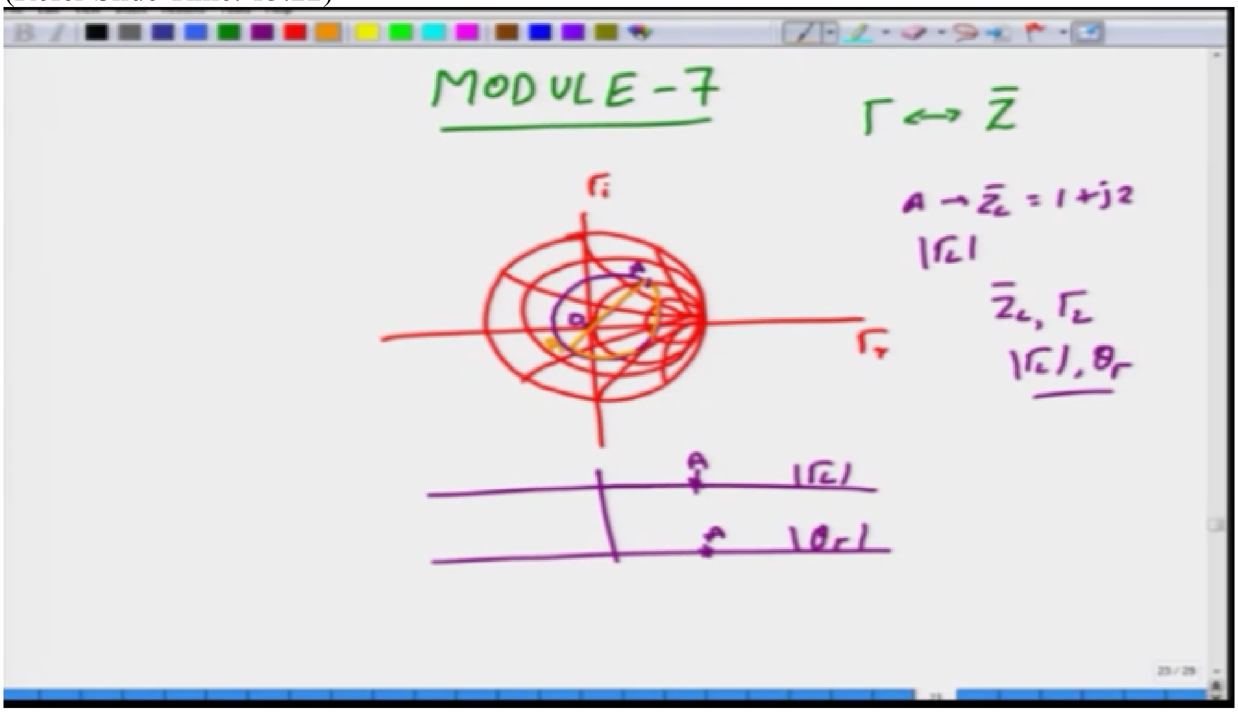


so I have landed on the opposite side of this chart which I will denote as B and you simply read the value of B, okay, so when you read the value of B you will find out what would be the admittance, I will leave that as an exercise for you to figure it out, I will supply the answers during the course and this B point is simply your normalized admittance, as I have told you if you want to obtained the actual admittance you simply multiply this YL bar with Y naught to un-normalized everything.

So seems very interesting that we are able to start with load impedances and then go all the way to admittance and reflection coefficient. What if someone asked you what is your SWR on the line? Well, do you want to do a calculation? You don't have to do a calculation interestingly why because SWR is actually the ratio of maximum voltage to minimum voltage, correct, and what is the maximum resistance that you have, on the line when you measure the line what would be the maximum resistance that you will measure? Maximum resistance that you will measure will be the voltage maxima, right, divided by the minima of the current and maximum voltage is basically 1 + magnitude gamma L times V0 + no surprises here, minimum current will be when 1 – magnitude gamma L V0+ divided by Z naught, okay, clearly V0+, V0+ will cancel
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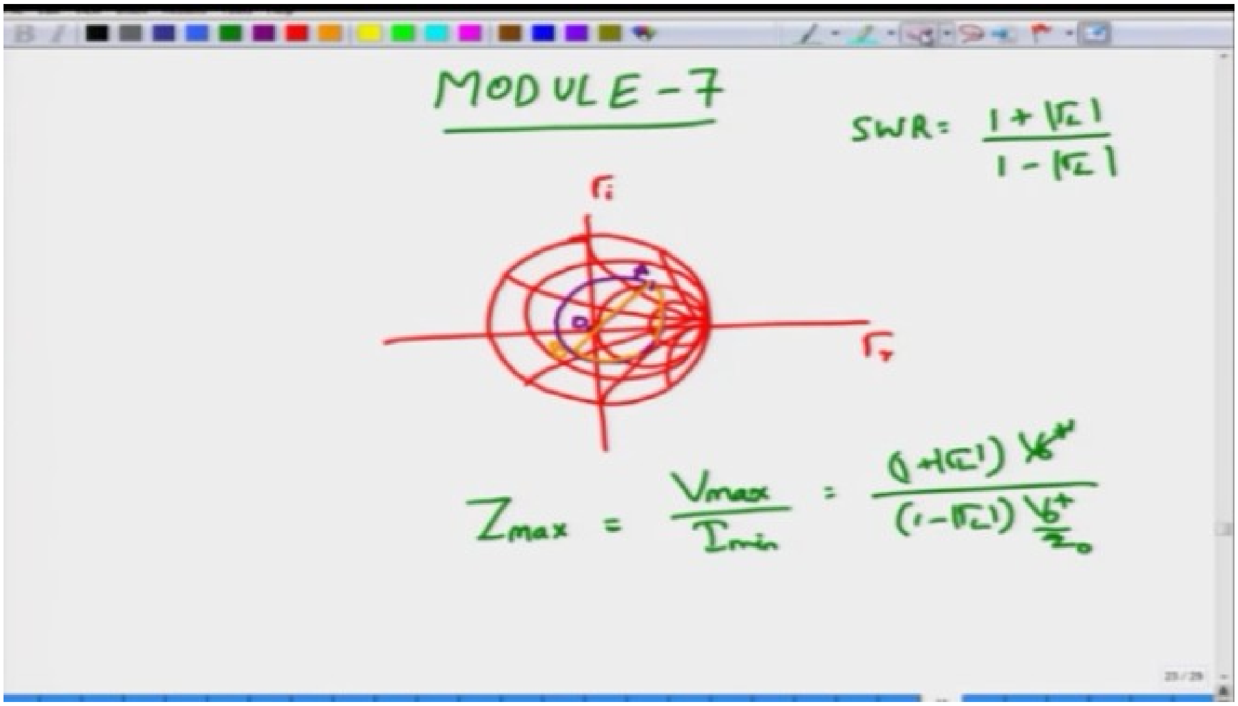


I can move this Z naught from denominator to the numerator and what I get is this quantity.

But what is this 1+ magnitude gamma L/1-magnitude gamma L? That is nothing but SWR, right, so Z max is actually equal to SWR times Z naught, interesting. Now what is the normalized Z max?

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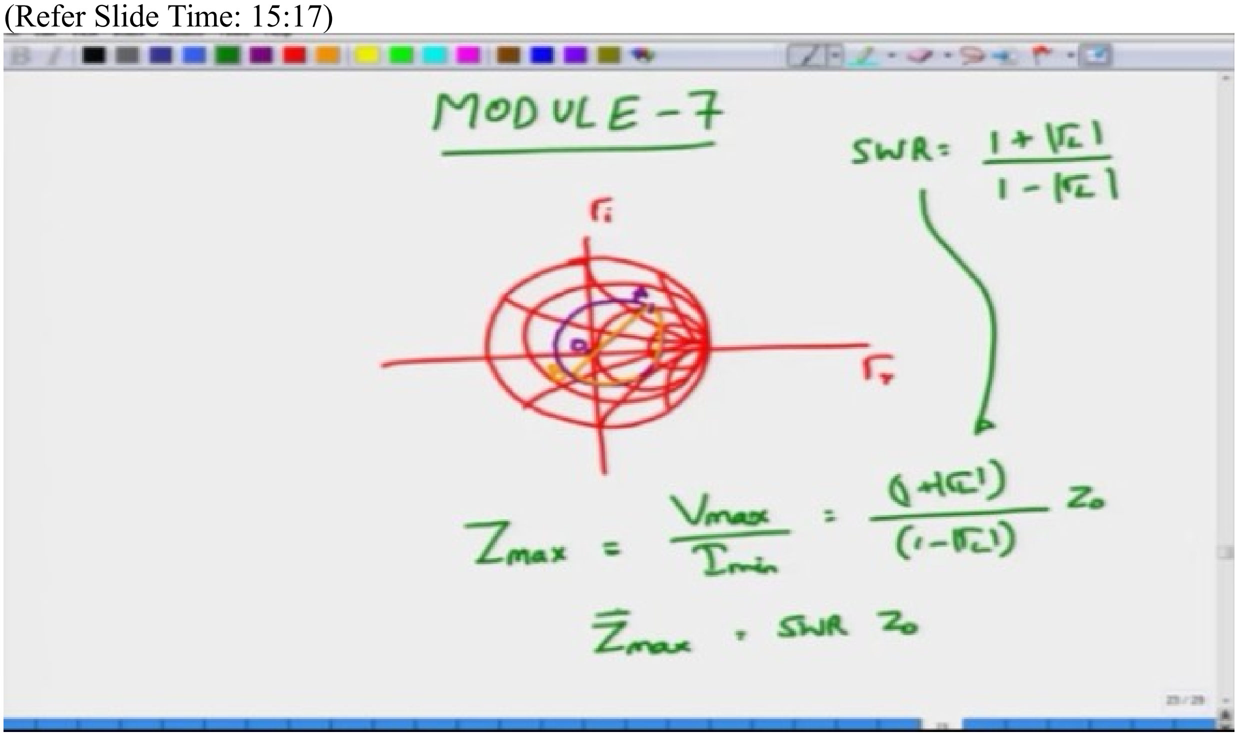


Normalized Z max is exactly equal to SWR, okay, you can in fact show that normalized Z min that is impedance will actually be equal to 1/SWR, now you don't have to do any other calculation, what you have to do is to simply move from point A and then C where you cut the real axis, right,

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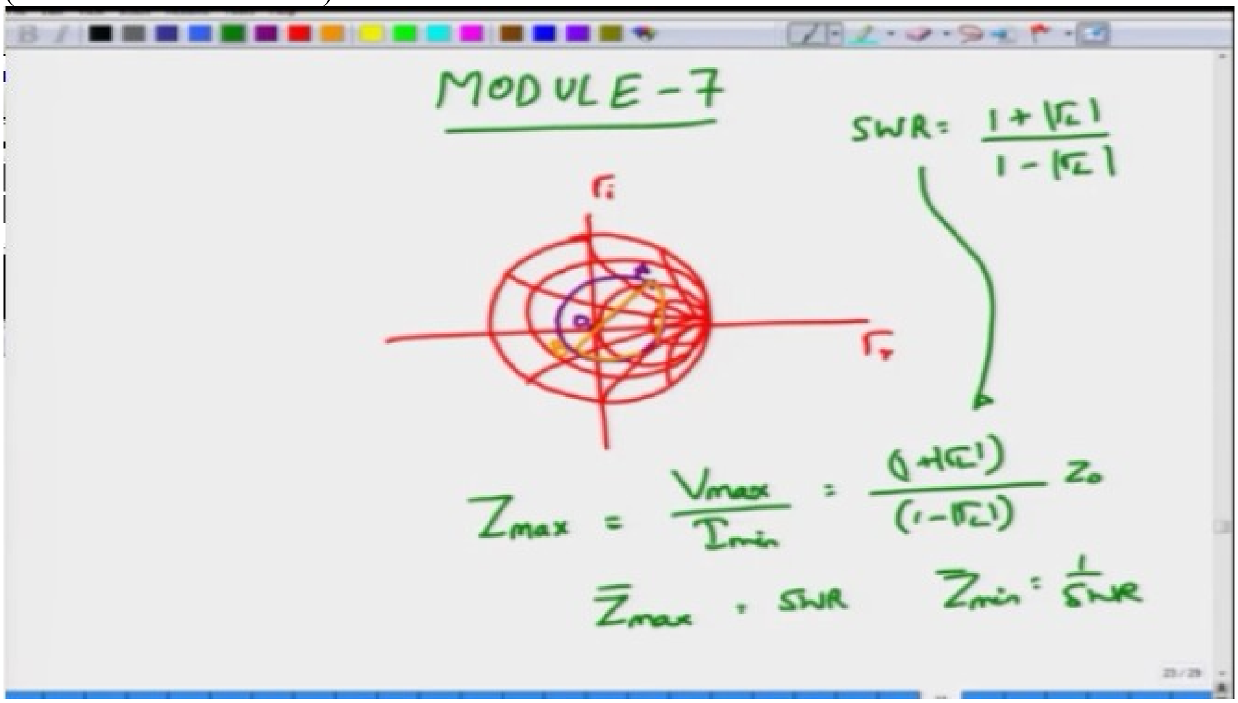


so when you cut the real axis here that would be the point where you will get the maximum value of small r, which is basically the normalized maximum resistance which is also equal to SWR, so in one point you actually killed 2 stones, right, you killed 2 stones with a single bird normally you kill 2 birds with a single stone, now you did the other way around and what you did or what you obtained was both information on Z max as well as SWR, okay.

Again without using another calculator, if instead of moving on to this one and you know terminate you know meeting the real axis here if you now move 180 degrees away and then meet the real axis on this side, we will actually obtained 1/SWR which is actually equal to Z min, you see you are actually killing 4 stones by just moving along this particular circles, right, so this is interesting, so you are able to use Smith chart to do this calculations, but the real power of Smith chart comes when you can solve much more complicated problems, okay, so we will solve slightly different problems here in the remaining time of our module, and this will tell you most of the things that you are actually interested, I mean how to work most of that things, okay.

So again let me draw the skeletal Smith chart here, full Smith chart you have it with you, now let me draw the chart, okay, yes, now we are ready, what problem should we consider? Well, I want to consider slightly difficult problem, I have a transmission line okay, okay, I have terminated the transmission line with some load, but now at a certain distance on the transmission line I will attach another transmission line, so let’s say this length I will call as L1, I will attach another transmission line and terminate this transmission line with a short circuit, okay, I can terminate it with open circuit or we can terminate it with short circuit,

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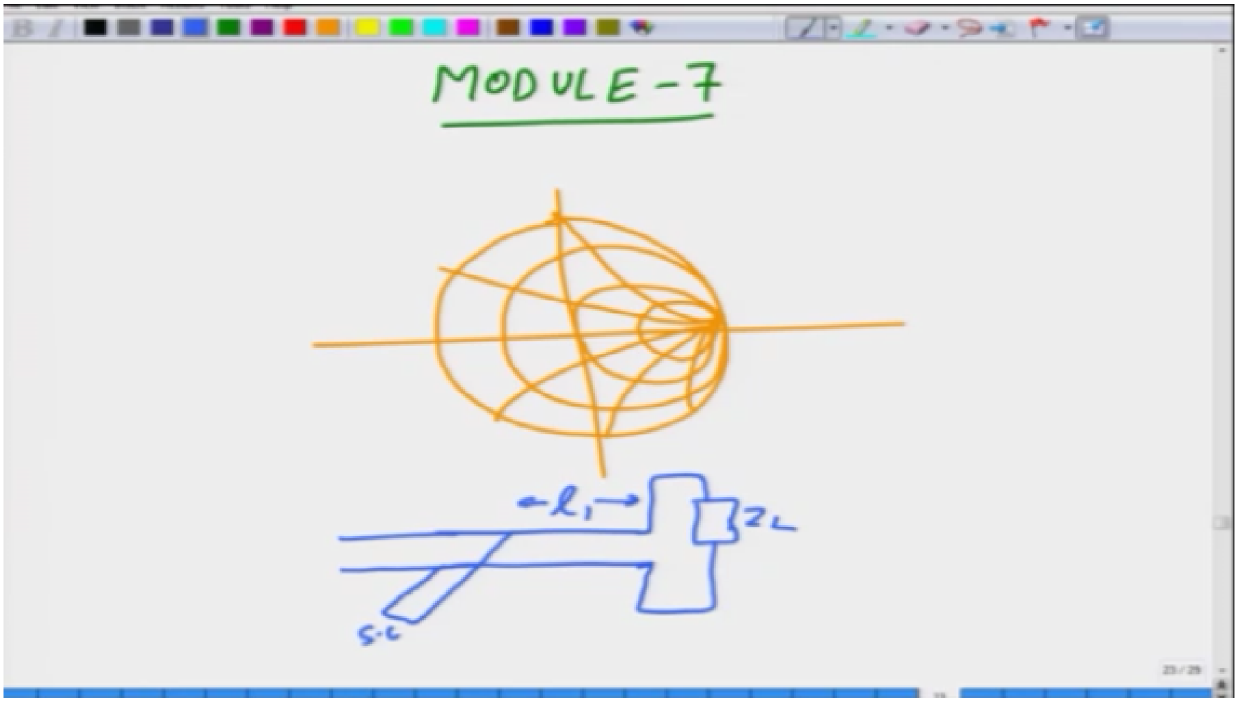

but I have simply decided to terminate it with short circuit, okay.

And let's say the length of this line is L2, and what I want is to find out what is the equivalent admittance or the impedance at this point, okay, I'm interested in finding the equivalent admittance or the impedance of this particular, you know at this particular point okay or the plane, how do I go about it?

Well because you are looking at connection of two transmission lines in parallel, right, it would be wise to work in admittance coordinates and you can convert admittance coordinates by simply treating the Smith chart as an admittance chart after you started off with impedance for some specific numbers, let me take ZL normalized to be say 0.4 + J point to, please note that normalized impedances do not have an ohm referred out there because it's normalized, they don't have any units and I will take L1 to be equal to say 1.537 centimeter and I will take L2 to be equal to 11.25 centimeter and I will say that lambda on this line is about 3 centimeters, okay, so this is the problem that I have, I have been given lambda, I have been given L2,
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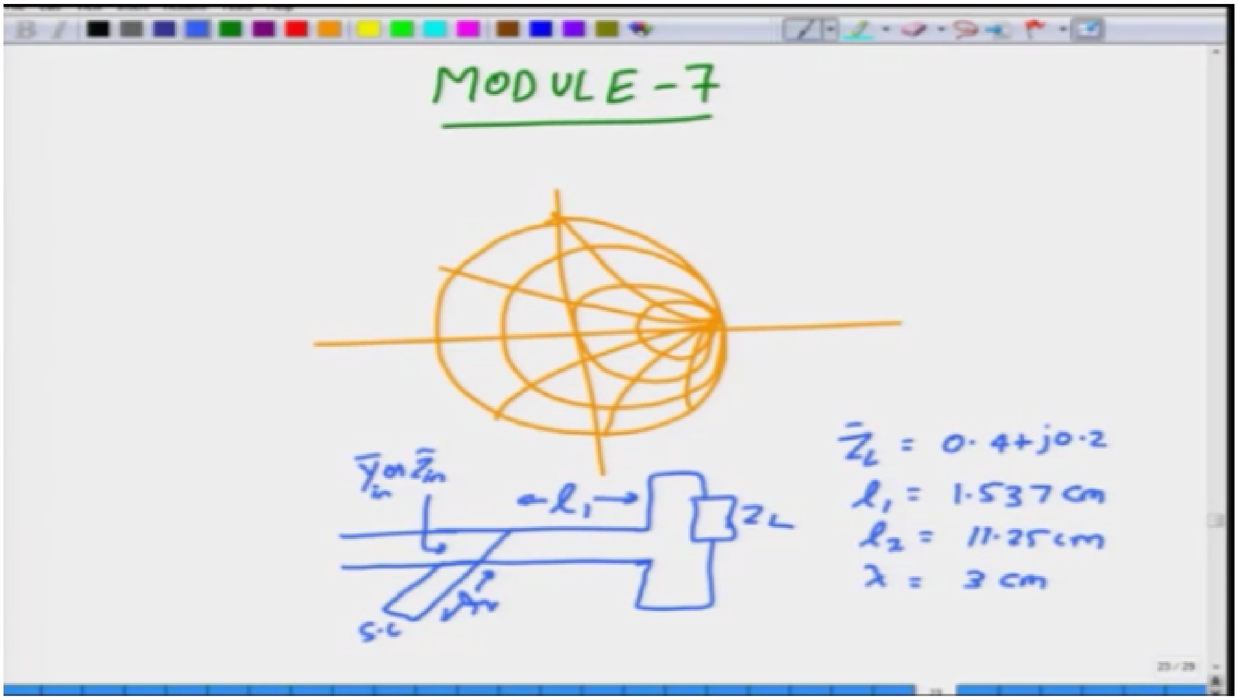


I have been given L2, I want to find out what is the equivalent impedance at this point, okay.

Please carry out this problem parallelly or after listening to this one, redo this problem again in your notebook, okay. What's the first step? Well, you start with ZL bar, it would normally have calculated but luckily they have calculated it for you, so locate ZL bar, okay, ZL bar would be located on some point 4 which is in this region, plus point 2 again which is in this region, so let's say somewhere here we locate point A which corresponds to this ZL bar, okay, so I have located this at point A, then what I have to do is that I actually draw a line okay, such that it passes through this chart and on the outer scale WTG scale note down the value here, okay, so note down what would be the WTG scale here, and once you have noted down then you move on this transmission line, okay, we'll assume that both transmission lines have the same propagation, sorry have the same characteristics impedance which anyway you don't even need to know at this point, because we have been working with normalized impedances, but if you want you can take Z naught to be equal to 50 ohms, okay.

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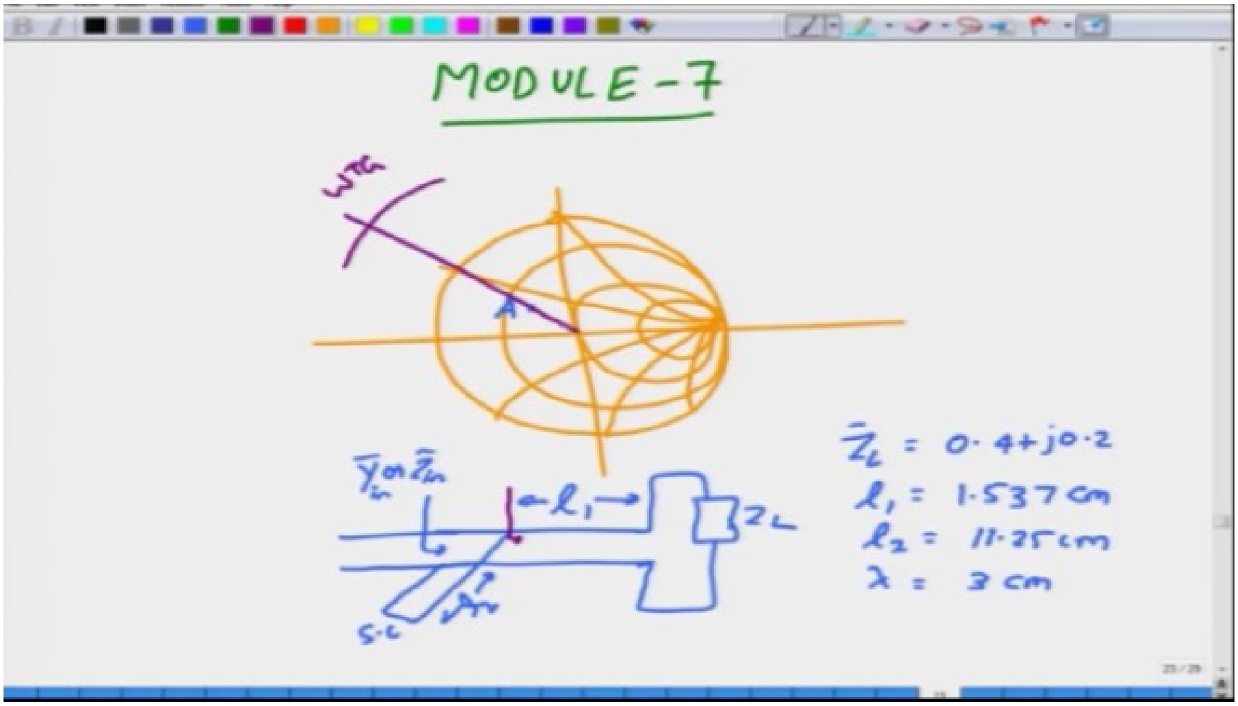


So after locating here and after drawing a line and extending it on to WTG scale and noting down the points over there, now I also want to draw a constant SWR circle, okay which is obvious, so specify this constant R equal to 1 circle with a different colour, because this circle will be very special for us, so always draw slightly thicker circle on this R = 1, so that you are now working with that.

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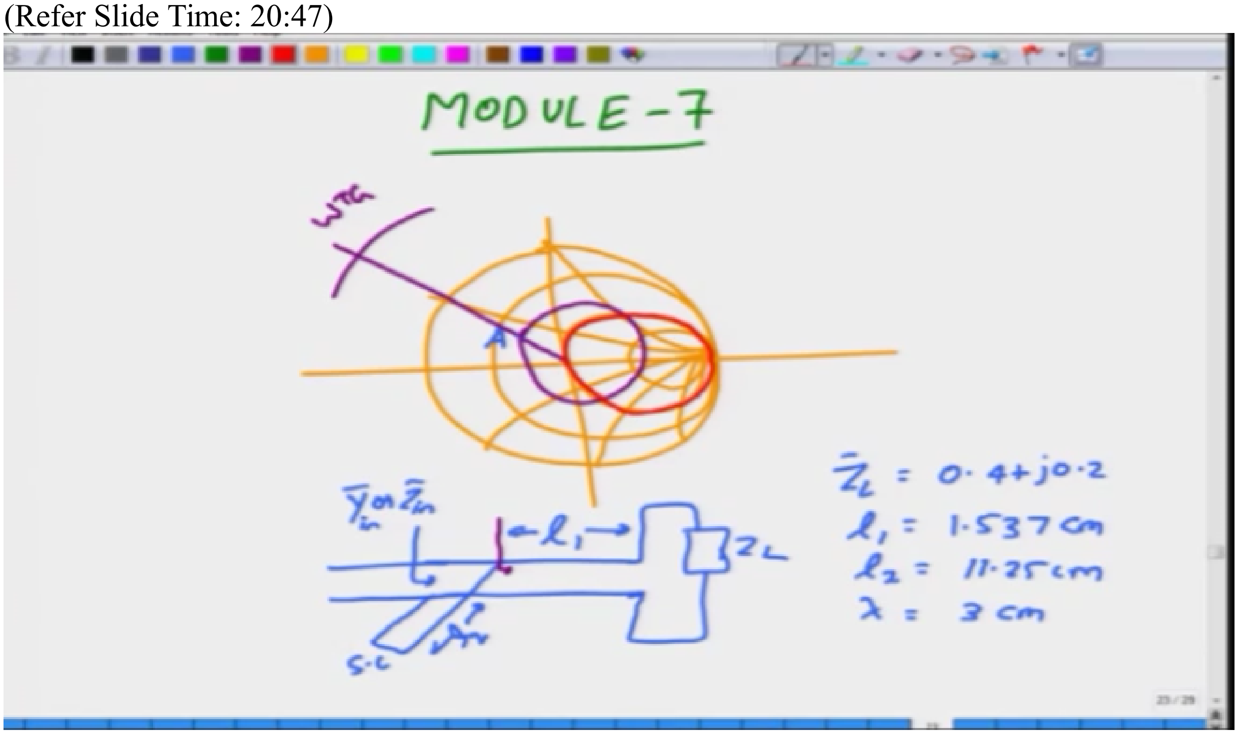


Now what you have to do is to move from this load point all the way up to L1, right, I mean a distance of L1, but because of the transmission line you are going to move distances relative to lambda, you simply calculate what is L1/lambda, this would be 1.537 divided by 3 centimeter, so this is slightly how much? This is greater than, this is about lambda/2, lambda/2 would be 1.5, so therefore this is slightly greater than 1.5, right, in fact the actual distance is just 0.037 lambda, because that is the excess one that would decide what is this one, right.

Now before you can do that because you want admittances you have to first convert the impedance point A on to admittance,
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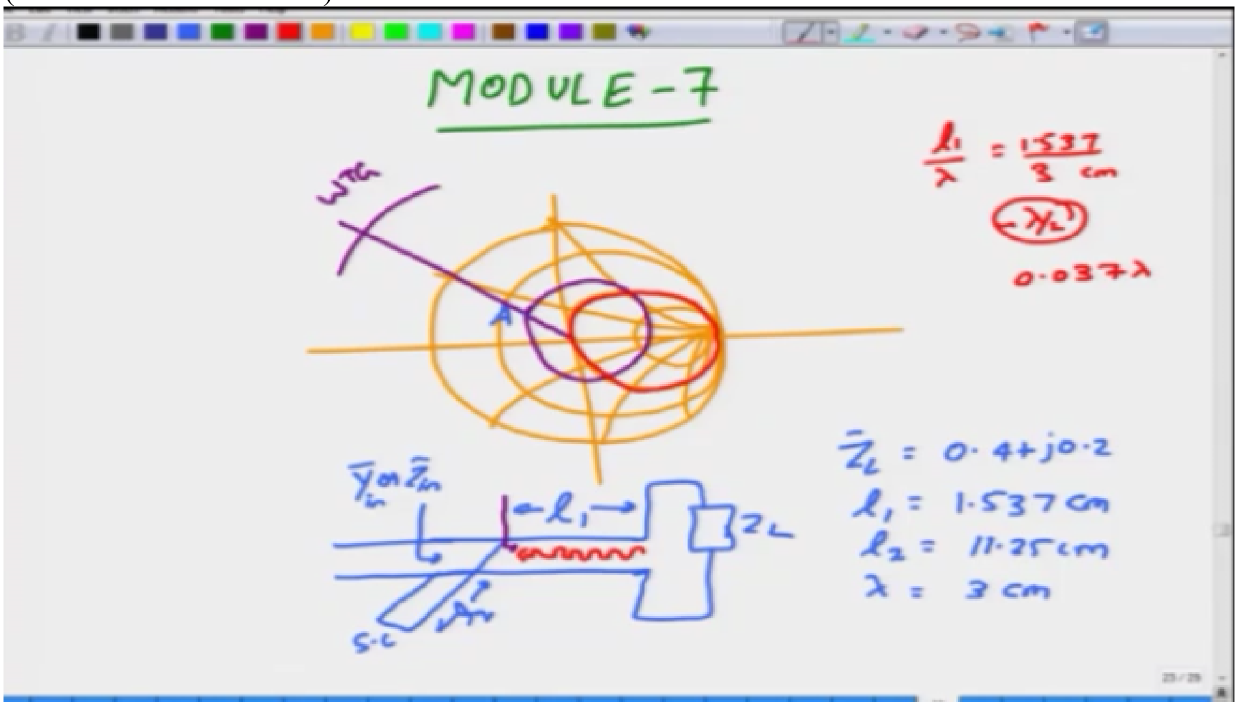


and you do that by simply moving on to the opposite side, so I’ll use this one so point B corresponds to admittance, you can read the admittance value B as 2 – J, and after you have read the admittance on the same SWR circle or rather first you extent this, note down what is WTG coordinates, and then you have to move along the circle, okay, and how much you have to move? You have to move only a distance of 0.037 lambda, meaning? To this WTG coordinate you add 0.037 lambda and then move, okay, you will see later on that it will actually lead you on the outer WTG scale up to this point which we will call us, so this we will call us B prime, we will call this as C prime that is any numbers on the WTG scale I’m denoting it by primes,
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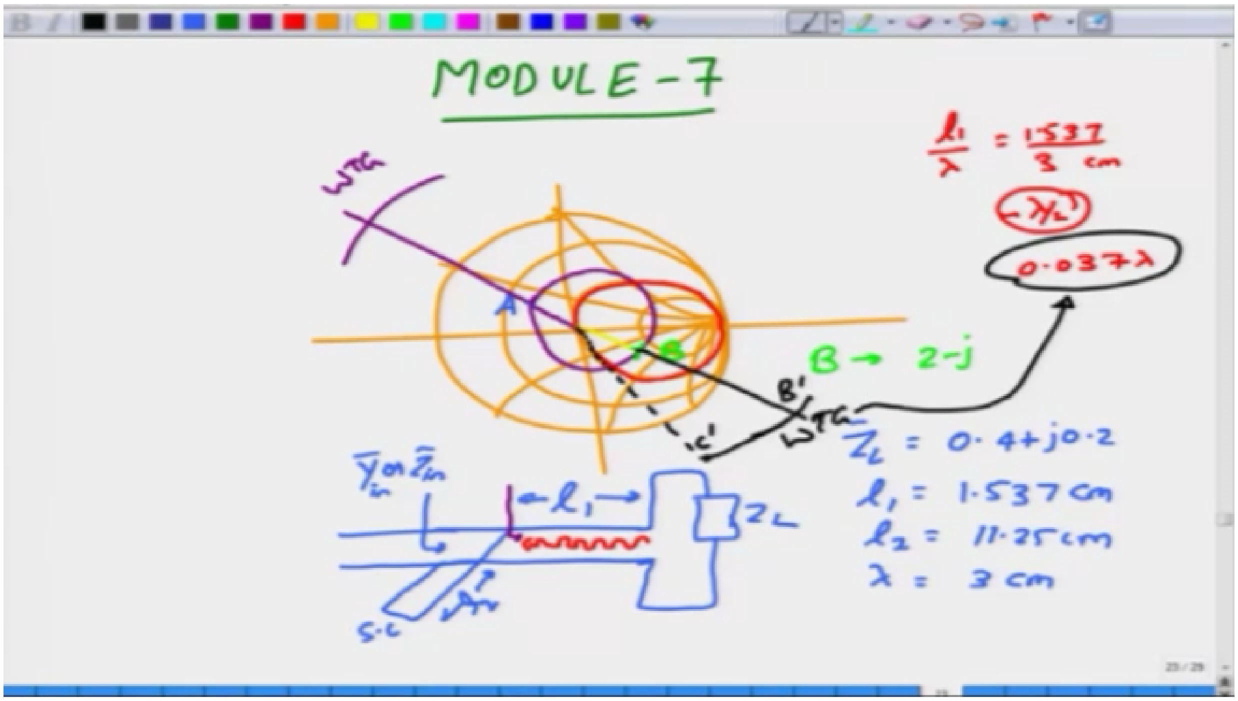


and then moving along the line to see that it actually is cutting the point here, this is interesting.

You know what is the coordinates at this point? I know that the coordinate of the real part of this one is or equal to 1 because it is actually intersecting this red circle, it turns out that this value is actually 1 – J, okay, so the normalized admittance that you have found is actually equal to 1 – J, this point is what we would call us point C, okay.

So doing this calculation from ZL all the way to moving the distance of L1 you have reached an admittance of 1 – J, okay. Now what do you do? Well, you have a short circuit here and the length L2 that we have taken which is 11.25 centimeter, so this length is actually quite large, but it's also short circuited the load end, and now where is the short circuit admittance? Well, this is the open circuit impedance, but this is the short circuit admittance, right, so this is short circuit admittance at this point, and on the WTG scale you can clearly see that this would be 0.25, and when you move a distance of L2, you are going to move a distance of L2/lambda which is relative to wavelength, and it will see, I mean it will be something like 0.25 lambda, okay, you can figure out if this is true or if this is not 0.25, this is 0.0, 0.125 lambda I think, yeah, so this would be 0.125 lambda so after subtracting the full wavelengths from this one so, 11.25 – 3 – 3 – 3,

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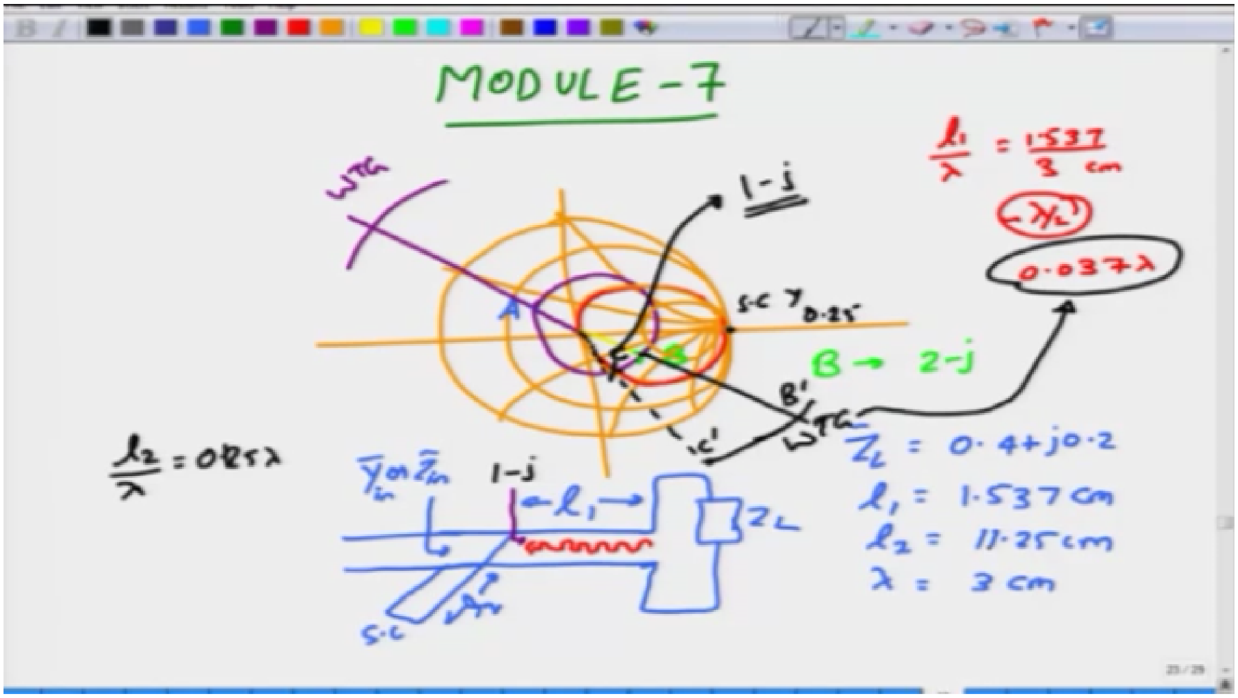


right, so you move, so remove 9 so you will get 2.25, and even from 2.25 you can actually remove I mean make this one divided by 3 on to the lambda and then you will get some number here which I hope it is equal to 0.125 lambda, so if it is equal to 0.125 lambda, then no problem or is this is more than that? Anyway, so don't worry about it, because this is the open circuit point, this is the short circuit admittance, right, so you move a distance of about 0.9, okay, this is the admittance that you have moved.

And how much is the admittance that you have actually moved? Remember in the impedance scale this was negative, whereas this region was, sorry this impedance was positive, whereas here the reactance as far negative, but because you have converted the chart into an admittance chart, this region is positive admittances or susceptance and this region is negative susceptance, okay, so what you have done is that as you have moved you've obtained admittance of about +J,

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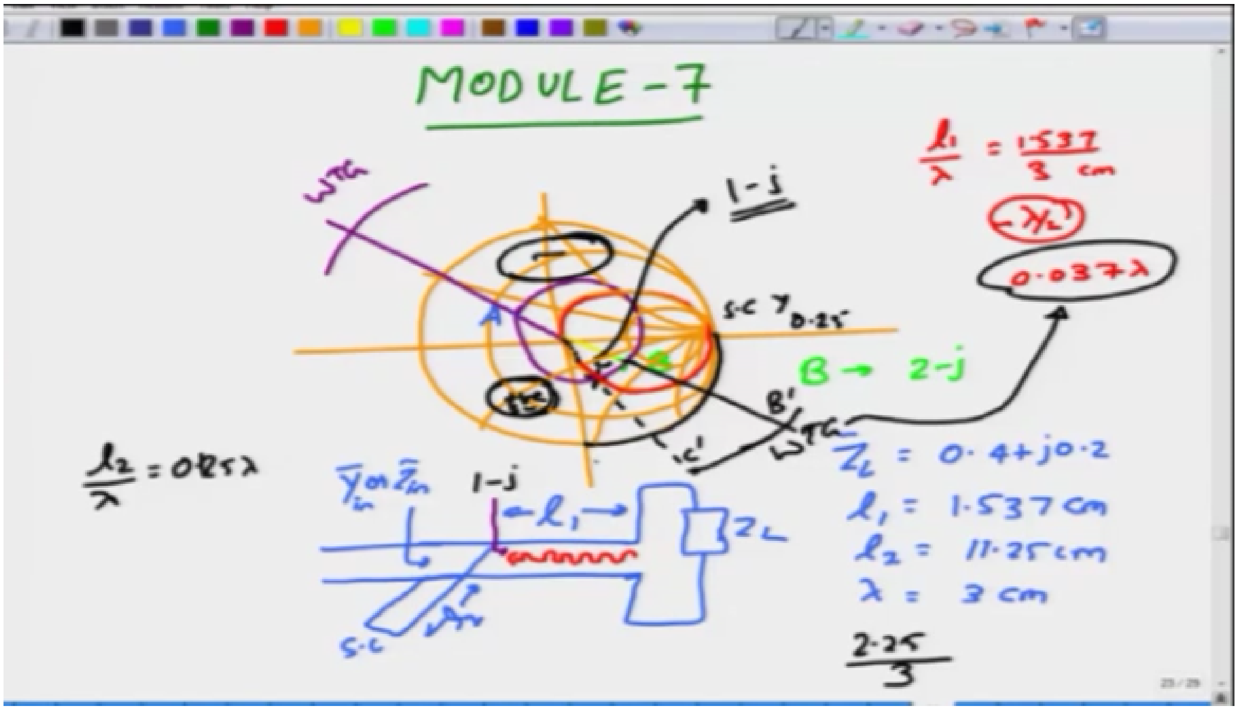

okay that is interesting you have obtained an admittance of about +J, and now the total admittance that you are going to see will be the sum of the admittances that you see here, so this admittance is +J, so the sum of the admittances that you see here why in, will actually be 1 – J + J which is equal to 1, right.

Now what is the meaning of admittance equal to 1 or normalized impedance equal to 1 that it means that for a wave that propagates here, the equivalent admittance it sees in this plane is actually equal to Z naught, right impedance equal to Z naught so the admittance is equal to 1/Z naught. The implication of this is that if the wave sees the characteristics impedance at that particular operating frequency, it means the reflection coefficient gamma is actually equal to 0, so there is no reflection anything that is coming from this plane, the entire wave is propagated on to the other side, so you have eliminated reflections on this main transmission line by connecting across another transmission line and this connection of a transmission line is called as you know the stub, in this case the stub was actually quite long, but usually the stub is made quite small by choosing appropriately the termination, so here we took short circuit termination, but you can take open circuit termination and then you can see that we'll be able to again obtain this matching condition and the beauty of this matching condition is that, you know is that the impedance that you see after connecting this stub will be equal to the characteristics impedance there by eliminating any reflections on this mainline, okay.

However there are still reflections at these points, so for example in this side of the transmission line or in this region of the transmission line there is reflection because the impedance here is not the, not equal to characteristics impedance, so there is a mix match in this region, the VSWR will be nonzero, or rather greater than 1 here, the VSWR here will be equal to infinity on the stub because on an ideal lossless line or an ideal lossless stub terminated in short circuit or open circuit will have gamma L magnitude equal to 1 and therefore SWR value will be equal to 0, so this problem which we actually went in the reverse way okay is called you know designing or impedance matching technique, so what we have done is to actually design a transmission line circuit or rather we have taken the design transmission and then showed that

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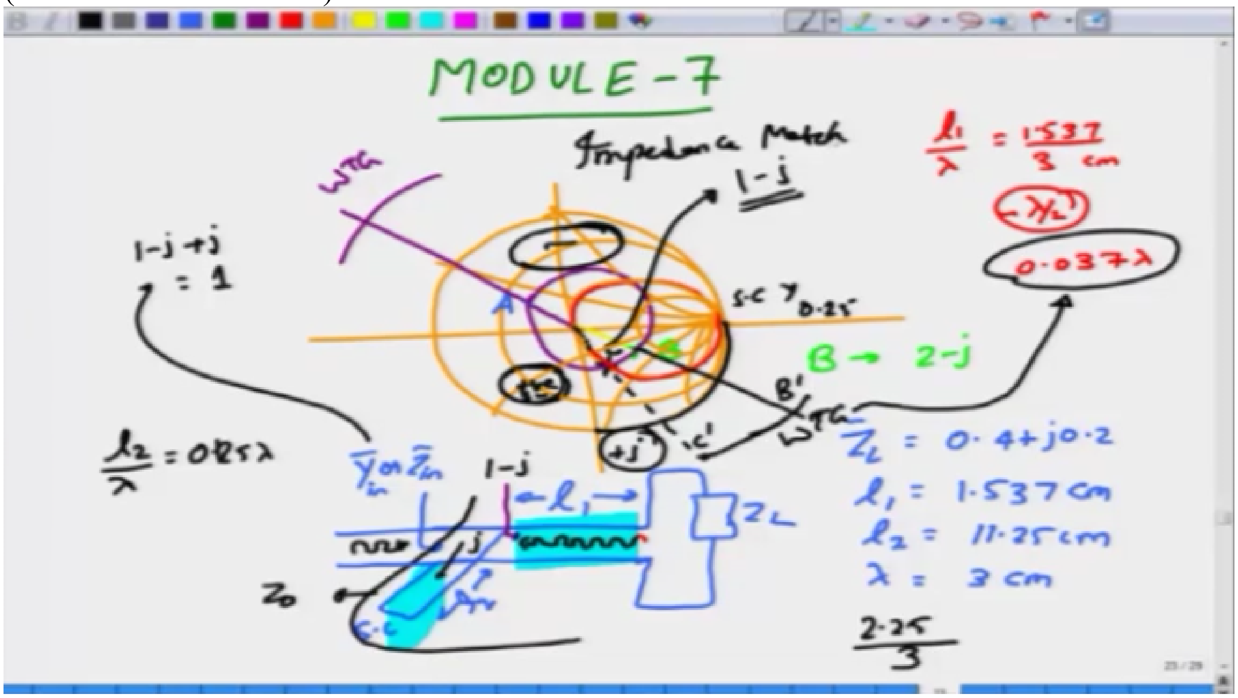


impedance matching is happening because someone is actually calculated this length L1 and length L2 appropriately with a given termination whether it is short circuit or open circuit, usually what happens is you are given a transmission line, you are given a load, you are given the operating frequency or wavelength and then ask to design a transmission line matching circuit, this is called as a stub matching circuit and this is a shunt matching network, shunt being another word for admittances, so because it can then parallel, this is you know parallel admittance thing and you have actually managed to match the transmission line, the main transmission line such that you have eliminated reflections on that mainline by appropriately taking the distance as well as the length of this stub, so where you put the stub and what would be the length of the stub are the two design parameters.

And given these two parameters can be varied depending on the values of ZL and Z naught that you have, you can always match any network, of course this has other draw backs which are overcome by increasing the number of stubs, so you go from one stub, two stub, three stub, and multiple stubs, those things are best left a different course.


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