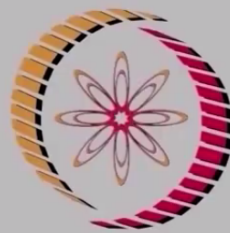




**Indian Institute of Technology Kanpur**



**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**  
**Electromagnetic Waves in Guided and Wireless**

**Lecture-40**  
**Interference (Double slit experiment, Fabry Perot Interferometer)**

by  
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Hello and welcome to NPTEL MOOC on electromagnetic waves in guided and wireless media. This is our last module. In this module we will continue to look at interference, describe another optical element and finally return very briefly to the wireless channel model that we were considering in terms of the diffraction thing.

The diagram shows a diffraction grating with slit width  $w_x$  and period  $a$ . The observation point  $P(x, y, z)$  is at a distance  $z$  along the  $z$ -axis. The path difference between rays is  $ax$ . The field  $\psi(x, z)$  is derived as:

$$\psi(x, z) = \cos(2\pi f_x a) \operatorname{sinc}(f_x w_x)$$

where  $f_x = \frac{x}{\lambda z}$ . The intensity  $I$  is proportional to  $|\psi|^2$ :

$$I \propto |\psi|^2 \rightarrow \cos^2 \frac{2\pi f_x a}{\lambda z} \operatorname{sinc}^2 \frac{x w_x}{\lambda z}$$

Additional notes include the phase difference  $\omega x = \frac{\pi}{2} x = \frac{\pi}{2a} = \frac{\pi \lambda z}{2 a \lambda z}$  and the condition  $\operatorname{sinc}(f_x w_x) \approx 1$ .

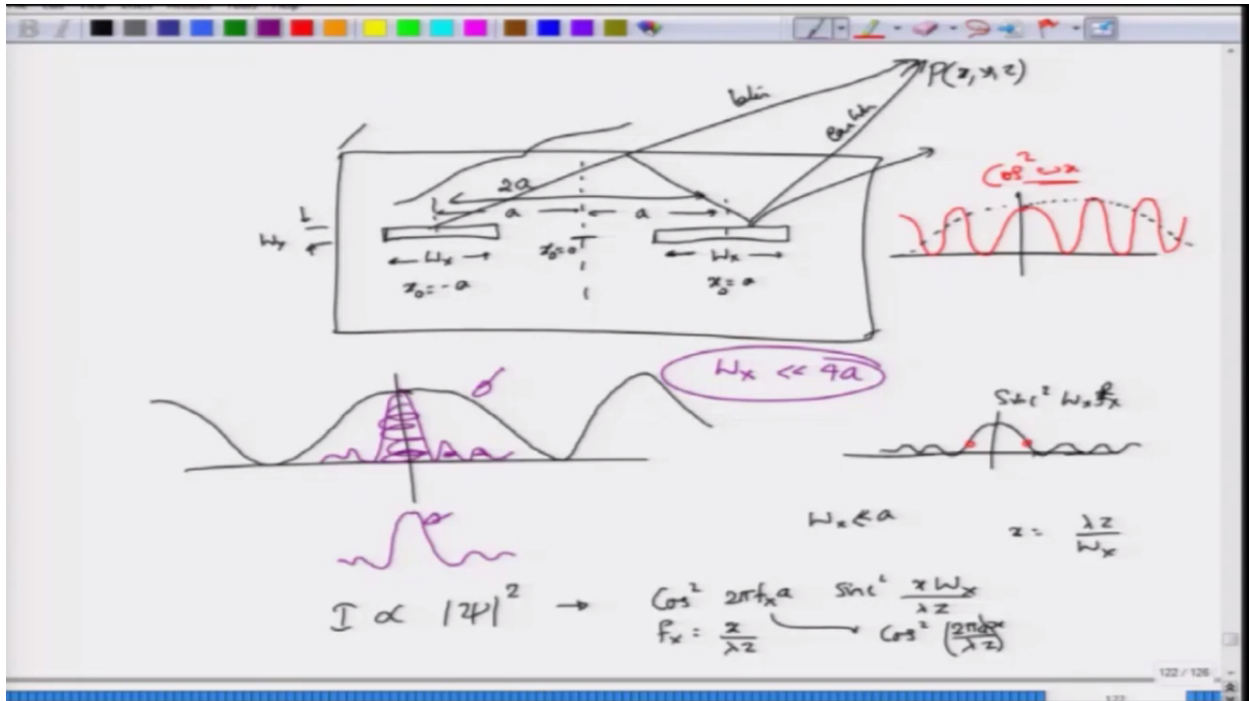
So continuing from where we left off in the last module we have this arrangement which is the typical arrangement for a double slit experiment. You have one slit here and you have another slit. The separation between these two slits or the centers of these two slits is  $2a$  and I have arbitrarily taken the slit to be narrow in the X direction with very small width along the Y direction. Of course in an experiment if you wish you can also take this slit in the Y direction but what would change is what is the plane in which you are going to or along the axis in which you are going to get the interference. That's all that is going to change. So with that in mind we already have said that individually the fields, the diffracted fields because of the aperture are known and what is that needs to be done is to essentially I mean recognize the fact that we are dealing with far field condition in this one and we are dealing with two slits and these two slits are not centered at the origin but they are centered at  $a$  and  $-a$ . So in terms of pure physics of the waves the fields which are nearer to this – so if point P is located in this manner which I have shown then the fields from this slit which is the right-hand slit for me would actually reach earlier.

So this is the slit in which the fields would reach earlier and here the fields would reach later. And this path length difference is essentially what's going to constitute the extra phase factor which will determine whether we are going to have constructive interference or a destructive interference. Of course you have this point P here as you move the point P along this X direction right in one of the axes then you see that the path length difference will also change. So at the center you will have zero path length difference because fields from both sides are reaching at the same time but if you go to the other extremes there would be situations where one of them would be reaching earlier and the other one will be reaching later and if this path length difference is multiple of  $\lambda$  then you will get 180 degree or  $2\pi$  phase shift if there  $\lambda$  by 2 then you will get 180 degree phase shift and therefore there won't be any I mean there will be destructive interference so on.

So we have seen this in the context of antenna arrays as well. Of course we can see only two element array. The same ideas work here as well. However, now what I am considering is in terms of the diffraction part of it. So the expression remains I mean can be easily obtained as we have shown in the last class. So this would be the total field that you are going to see at any distance  $Z$  and at any point  $X$ . Of course you are seeing this diffraction along the X direction, sorry interference along the x direction because we said  $WY$  is equal to 0 and or rather very close to 0 so that the sinc function will have very-very broad band width and therefore that value within that bandwidth will be almost 1. So we can neglect this Y directed or the Y dependence and just have the X dependence here. So this would be  $\cos 2\pi FX$  say something like that and then the intensity of the beam or intensity of the waves that are interfering would be proportional to magnitude square of this total field. So this would essentially give you a cosine square  $2\pi FXa$  times a sinc square  $FXWx$ . Now we know the properties of sinc square  $FXWx$  and we also recognize that  $FX$  is basically  $X$  by  $\lambda Z$  so you can substitute for  $FX$  in both cases  $X$  by  $\lambda Z$  and we know that for the sinc square function if you write this as  $X$  by  $\lambda Z$  then if you fix  $Z$  which is far away from the upper set but we fix  $Z$  then at certain  $X$  value which is equal to  $\lambda Z$  by  $WX$  you are going to see that this function would actually go to zero. So that would be the first time it goes to zero and other times also it starts to go to 0. This is the sinc pulse that we already know.

What about this cosine square function? Writing the cosine square function also as  $\cos^2 2\pi X$  by  $\lambda Z$  times  $a$ . So we'll write it in this manner and then recognize that this cosine

square is basically a cosine square of some kind of an  $\Omega X$  where  $\Omega$  is the frequency of this wave which is given by  $2\pi a$  by  $\lambda Z$  and of course if you are interested in the frequency  $F$  similar to frequency  $F$  that'd be  $a$  by  $\lambda Z$ . So when this argument of the cosine function goes to  $\pi$  by 2 that is when the first time it will actually go to 0. So when this goes to, argument goes to  $\pi$  by 2 meaning that when this  $\Omega$  is equal to or rather  $\Omega X$  will be equal to  $\pi$  by 2 first time then  $X$  will be at  $\pi$  by 2  $\Omega$   $\pi$  I mean  $\Omega$  you know which is basically  $2\pi a$  by  $\lambda Z$  so  $\lambda Z$  it goes on to the top and then  $\pi$  and  $\pi$  will cancel.

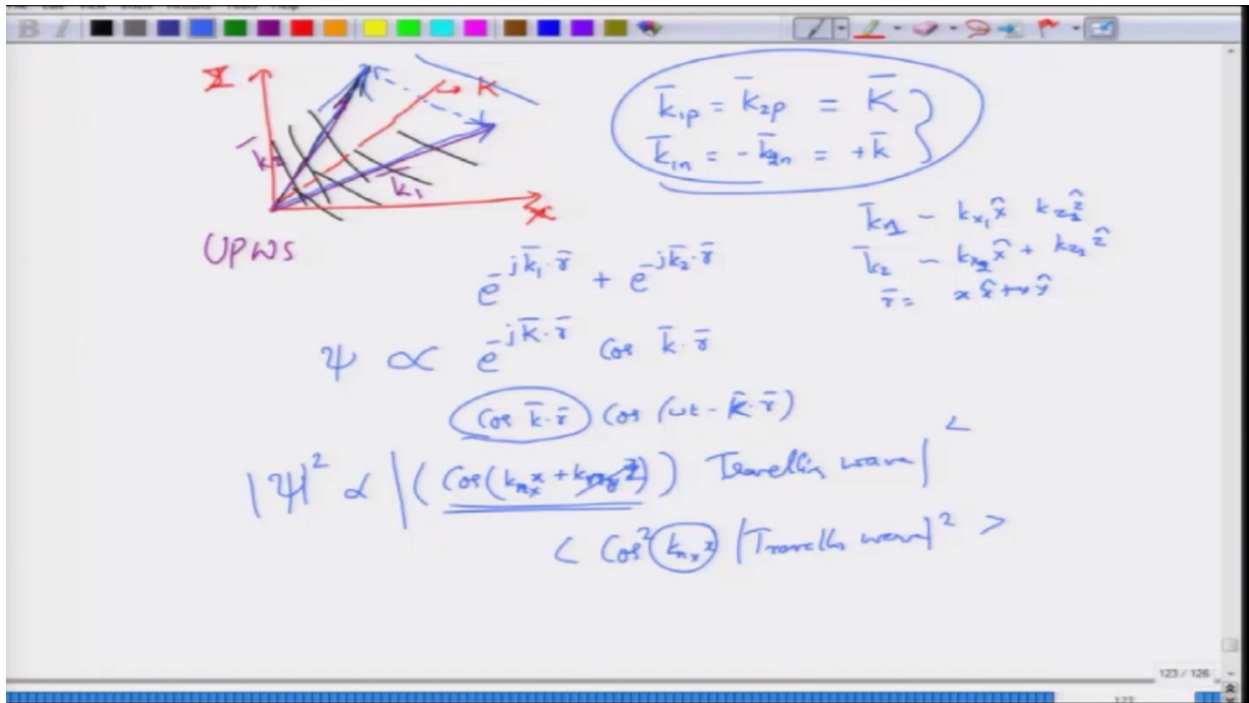


So basically what you will get is  $\lambda Z$  by  $4a$ . Let me raise this so that you can see this clearly. So what we have is this case where you have a cosine square function reaching its minima at  $X$  is equal to  $\lambda Z$  by  $4a$  for the first time of course on symmetric condition and then it goes on beyond that.

now you look at this. This is  $\lambda Z$  by  $4a$  is when the cosine square function is going through 0 and then it will again go through 0 after certain distance as you can clearly see. Sorry this should have been a cosine square function so please note that this is  $\cos$  square function  $\Omega X$  is what I am plotting. Now if this bandwidth of the sinc function is made quite large that is to say if I take  $W_x$  to be smaller than  $a$ . So if I take  $W_x$  to be much smaller than  $a$  then what will happen because the beam width of the sinc square function is inversely proportional to this  $W_x$  so then what will happen is this field would actually be modulated by the sinc square function. So this would be modulated by the sinc square function meaning that you have a situation that would look like this. So you have an envelope of sinc function or sinc square function and within that envelope you actually have a cosine square assuming that this distance is larger so you have a cosine square function. So this is the interference that you are going to see what you will see in the two dimensional case is a bright band followed by a dark band but the

intensities of this bright and dark bands actually starts to diminish as you move away from the center of the interference plane and this diminishing is because of the finite slit width and that comes in because of the  $WX$  or the sinc square functional dependence on the aperture width. So yes you can minimize this the diminishing thing provided you actually pull these things far apart such that this sinc square modulation function becomes kind of – it's kind of made smaller as possible. So the bandwidth of this sinc function could be increased by keeping  $WX$  to be quite small. That is why we actually say that it's a narrow slit and the narrow slit must be spaced sufficiently apart at least the basic condition that you have to recognize is that  $WX \cos^2$  of  $\lambda$  by  $Z$  whatever that we had was also this thing so  $WX$  has to be certainly much less than 4 times  $\lambda$  for you to observe this type of a behavior. If you don't then what will happen. Then you have the other way around. So then what you have is the sinc square function being modulated by the cosine square. So if you had then this is the sinc square function that you have and this sinc square function will be modulated by the cosine square, sorry, let me write down this way. So I will have so right now what we had was a sinc having a broad bandwidth. Now we will have a cosine function having a broad bandwidth and then this is the cosine function I am just writing one half of this thing and then within this you have a sinc function. So basically meaning that you will see only almost a bright spot the rest of the spots will have very small amplitude and the essential thing about diffraction or the interference would be modulated. So this is the kind of behavior that you are going to get. So by varying the separation and by varying this one you can actually switch from one extreme of the interference to the other type of things interference. So one varying you will see multiple bright and dark patches with amplitude being very slow and then in the other case you will see essentially a single bright pattern and then very very small other patterns as well. But this will not go beyond the spatial extent of the interference region will not be very high. And this is not what we normally call as interference as well. We expect the interference to be something in the cosine square kind of a thing.

So this was about interference but if you exchange these general ideas I want to explore that extending this into slightly general ideas because this is interesting and important. So you may actually have two waves which are propagating at two different directions. So we have their  $K$  vectors to be  $K_1$  and  $K_2$ .



So these waves could be emanated from some place and they are essentially uniform plane waves for us. So meaning that they have a face front that would look something like this. So these face fronts are of course always normal to each other and we have this kind of a behavior. So wherever these face fronts or the wave fronts cross that is the interference region. We have taken the, sorry I should take the lengths of these two to be same in the sense that their magnitudes  $K_1$  and  $K_2$  we will have to assume it in this particular manner in such a way that for the horizontal component. So imagine that this is some kind of an XY plane. So this is the XY plane or maybe this is a Z and X plane that we looking at and the wave is propagating in this plane. So clearly this central red line would be the from the origin it would be drawn and we will call this as  $K$ . What you can see is that if you project this  $K_1$  onto this horizontal line and you project the same thing onto this one you see that these two  $K_1$  and  $K_2$  wave vectors have the same parallel part of this that is they have the same value of  $K$  along the central  $K$  line. So we call this as  $K_{1p}$  that is the projection of  $K_1$  vector onto the  $K$  axis we call this as  $K_{1p}$  and you can clearly see that this would actually be equal to each other. What would not be equal to each other is  $K_{1n}$  being equal to minus  $K_{2n}$  so because this is clearly the direction that is going to be giving you. So one of them will be directed here the other one will be directed on this side for the normal components and for this normal components  $K_{1n}$  will be minus  $K_{2n}$  which we will call as minus  $K$ , sorry plus  $K$  and please note the difference is this is a capital  $K$  this is small  $k$ .

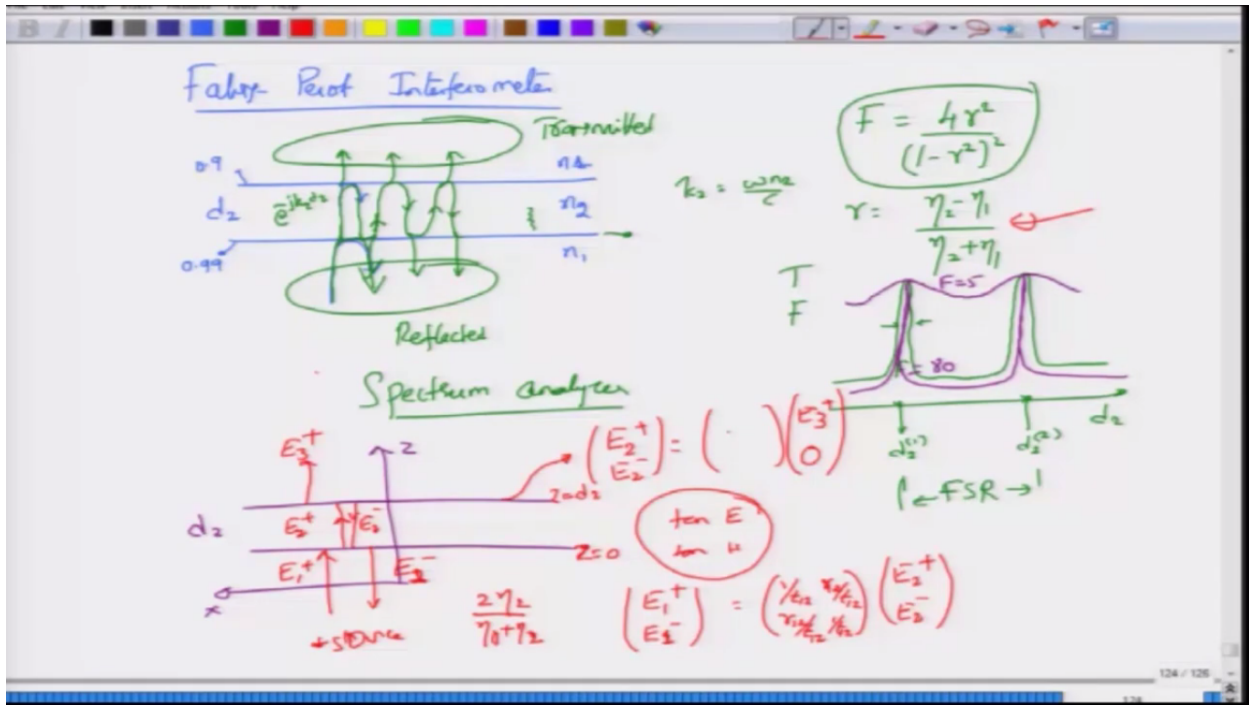
So the total field at any plane that you are looking at will be the sum of these two. So it would be  $e$  power minus  $j k_1 \cdot r$  plus  $e$  power minus  $j k_2 \cdot r$ . I have of course assumed that these two waves have the same amplitude. If not then you will have to worry about it slightly and we have also assumed that these two components have the same frequency.

So these are waves which are being guided by the free space and because of the fact that they actually have different directions  $K_1$  and  $K_2$  but their uniform plane waves. There will be

situations where their wave fronts are interfering with each other and that is what is giving rise to this interference thing. So you have  $e^{j(k_1 r - \omega t)}$  plus  $e^{j(k_2 r - \omega t)}$  this identification with  $K$  and  $\bar{K}$  or with  $K$  and  $k$  you can rewrite this as  $e^{jK r} \cos k r$ . So you can clearly show that this is correct because you are going to write down  $K_1 n$  as  $K_1 x$  and  $K_2 n$  as  $-K_1 x$ . So you are going to get the field total field. So the total field is proportional to this  $\cos K r$  times  $e^{jK r}$ . Okay and the real field would of course be equal to  $\cos K r \cos(\omega t - K r)$  where this capital  $K$  so this is capital  $K$  and that is a small  $k$  that we have considered. So in our example we considered  $K_1$  or rather  $K_1$  to have  $K_x$  component and the  $K_z$  component that is what made up our  $K_1$ . Similarly  $K_2$  also had  $K_x$  so we'll put this 1 and 2 so 1 or rather  $2x$  plus  $K_z$ , sorry this is 1 so  $K_z$  2 times  $Z$  and  $r$  therefore will be equal to  $X \hat{x} + Y \hat{y}$ . So you can go back and put this  $r$  and of course this  $K$  will also be having certain  $K_x$  that is capital  $K_x$  and capital  $K_y$  and so on and this would be the other case. So if you think of this then what is this fellow telling you this is telling you that this is basically going to be some  $K_1 n_x + k_1 y$  or rather  $K_n x$  and  $K_n$  or I should write this as  $K_n x$   $K_n y$  times  $Y$  where you have to determine what is  $K_n x$  and  $K_n y$  based on this decomposition that you need to do.

So when you do this one and you realize that, sorry this is not  $Y$  this is  $Z$ . So this times a traveling wave. So there is a traveling wave which is of the form  $\cos(\omega t - K r)$  whereas for this part which is the amplitude that is getting multiplied to this traveling wave the amplitude actually depends on what position  $X$  and  $Z$  you are in. So this is of course inside the bracket this is basically  $\cos(k_n x + K_n x + K_n y$  or  $K_n z$  but the total field will actually be magnitude square of this so this would be traveling wave and the corresponding amplitude changes that are happening at every  $X$  and  $Z$ . So if you further make an approximation that I am going to consider the normal components or the amount to consider that the waves are along certain direction so as to make this  $Z$  equal to zero then what you see is that  $\cos^2 K_n x$  times the traveling wave would be the wave intensity and the average value if you look at it the average now depends on what value of  $X$  you have. So it is something like fading that we considered in wireless media where in the amplitude kind of changes depending on time there. However, here depending on the position of the space the amplitude can change and therefore the average power at each position will be different and this is simply happening because intensity is always the sum of amplitude square. If it was the other way around that is intensity sum of intensity then it would have been a different thing but because the phase factor is present and because this intensity is the magnitude square of the sum of the amplitudes that product terms is what is going to give you this interferences. I mean and the classic double-slit experiment that we considered also fell into this kind of a nature. So in that case you had two waves coming in and if depending on the path length difference there was a phase difference which translated into amplitude variations at different parts of the screen.





So that was about interference. We could say lot more about interference but we will not go to that thing. Rather I want to very briefly consider an optical element called Fabry Perot Interferometer.

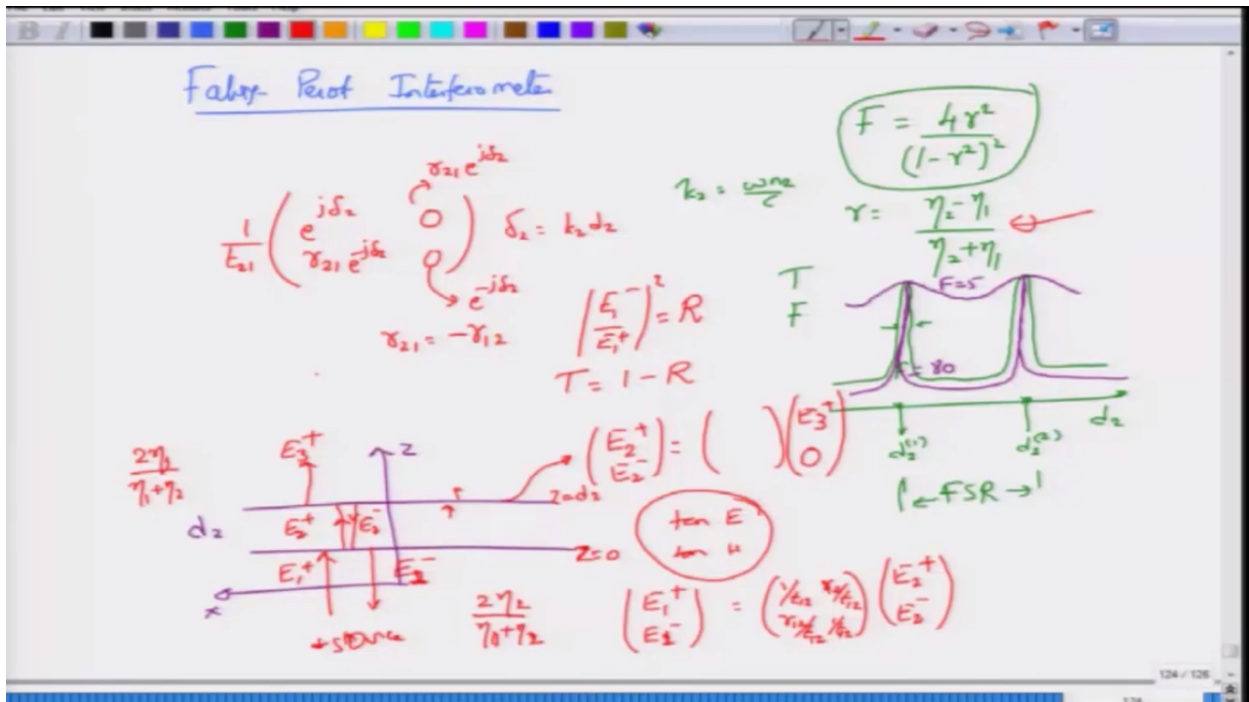
I will not go through the math here because let's face it we are at the end of this module and probably you've seen enough math here. I'll give you the basic idea and leave you with the expressions which you can fill in later on and if you can actually see that with two highly reflective mirrors which are spaced a certain distance  $D$  two apart covered by refractive indexes  $n_1$ ,  $n_2$  sorry,  $n_1$   $n_1$  outside and  $n_2$  here you can actually get pretty much complete transmission. So if for example this fellow has a reflectivity of about 0.99 whereas this fellow has a reflectivity of about 0.9 so what you would actually expect is that when you place them apart in this manner no light should be reflected because the light which is incident 99% of this light is coming back and if you incident light on this mirror about 90% of the light would come back. So what you would expect is when you put them parallel to each other with a slight distance you would actually expect that most of the light to come back. Yes that also happens but what is more surprising is most of the light can actually pass through this mirror. So how is this magic happening. The magic is happening because of this reflection or rather interference. You see light is incident here. Some portion of the light or rather more major portion of the light seems to be coming back. However, light is also moving or some portion of the light also goes into the second medium, travels picks up a phase which is say  $e^{-jk_2 d_2}$  where  $k_2$  is given by  $\Omega n_2 / c$  and  $d_2$  is of course the distance so it picks up a certain phase and then it gets reflected at this point. Then again it some portion of the light will travel out. The reflected part will come here and again it will be reflected back because this is also reflecting mirror. It acts as a mirror to both sides of incidence but some portion of the light will actually come back. So this fellow goes back here, some portion comes back, and some portion is transmitted, some portion comes back and reflection further further and there would actually be an infinite number of reflections and when you combine all these beams here I am showing them in this manner that

different beams are coming out but usually the spacing is very small and the finite extent of the beam is I mean, the beam is doesn't have an infinite extra it has a finite extent and when you look at the over all power here this would be the total reflected power and this would be the transmitted power. And there are very interesting or there are very simple ways in order to calculate how much of the fraction of the incident light is transmitted and how much is reflected. It turns out that it depends on a parameter called as fineness of the cavity whose expression is given by  $4r^2 \text{ square minus } 1 \text{ minus } r^2 \text{ square } r$  where  $r$  is a reflection coefficient which is  $\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ . This is the reflection coefficient at this plane. Of course the reflection coefficient for the waves incident from this side the reflection coefficient will be minus sign for that but for our case we will simply consider this to be this one. So this expression can be easily derived with the method that I will outline to you but I won't solve it in detail. What is the importance of this  $F$  is the larger the  $F$  this is something like the  $Q$  factor of a cavity. So the larger the value of  $F$  the smaller will be the peak of the transmission. So if you actually plot as a function of this separation  $d$  for a given wavelength that is fixed then there will be certain values of  $d$  over certain value of  $d$  to vary in the phase will constructively interfere and the total transmission can actually reach maximum and from there it drops off. So the width of this peak is given by this or is controlled by this fineness of the cavity and after this it will again go through for some other distance. So let us say this is happening for  $d$  of 1 then at another distance  $d$  of 2 again the transmission of the peak will go to its maximum. And the distance between these two is sometimes called as FSR. Usually what you do is you keep  $d$  fixed and then vary  $\lambda$  in that case you call this as FSR but the basic idea is the same. So either you can keep  $d$  moving while keeping  $\lambda$  fixed or in the usual case you keep  $d$  fixed and then move  $\lambda$ . Of course when you move both then you can resolve the spectrum of the light and therefore this interferometer which is a way of combining beams can also use to analyze spectrum of the light. As I said the fineness controls the kind of transmission peaks. So when you use a small fineness so let us say this is  $f$  equal to 5 whereas this one is  $f$  equal to 80 then you can see that the peaks don't really go deep into it. When you increase  $f$  to be even larger then you get even smaller peaks and that is what you are essentially going to get.

So all this is happening because of interference and you can understand this one by this simple model. You have uniform plane waves we will assume that and then this is my  $Z$  axis. So this is my  $X$  axis let us say. There is a incident electric field which we will call as  $E_1$  plus and then there is a reflected field because of this which we will call as  $E_1$  minus then inside the field there would be  $E_2$  plus plus meaning positive  $Z$  traveling waves and there would be  $E_2$  minus outside you have only one which is  $E_3$  plus because I am putting my source here. If I put the source at this end then  $E_3$  plus and  $E_3$  minus would be present and even plus or even minus would be the only wave that you are going to get.

Now what you have to do is apply the boundary conditions one at this  $Z$  equal to zero and the other one at  $Z$  equal to  $d$ . You have two boundary conditions one is tangential electric field and the tangential magnetic field. So when you apply the boundary conditions what you will see is that at this boundary at least you are going to be able to write down even plus and even minus in terms of some matrix  $E_2$  plus and  $E_2$  minus. So you can write this in terms of some matrix and I will give you the matrix and I will ask you to determine what the matrix should be or rather the expression of derivation I will leave it to you. The matrix is going to be  $\begin{bmatrix} 1 & t_{12} \\ r_{12} & 1 \end{bmatrix}$  this is  $\begin{bmatrix} r_{12} & t_{12} \\ 1 & t_{12} \end{bmatrix}$  where  $t_{12}$  is the transmission coefficient which is given by  $\frac{2\eta_2}{\eta_1 + \eta_2}$ . Please note that  $\eta$  is the impedance and not the refractive index. Of course you can also write down  $t$  and  $r$  in terms of reflection – I mean refractive indices but it's quite easier

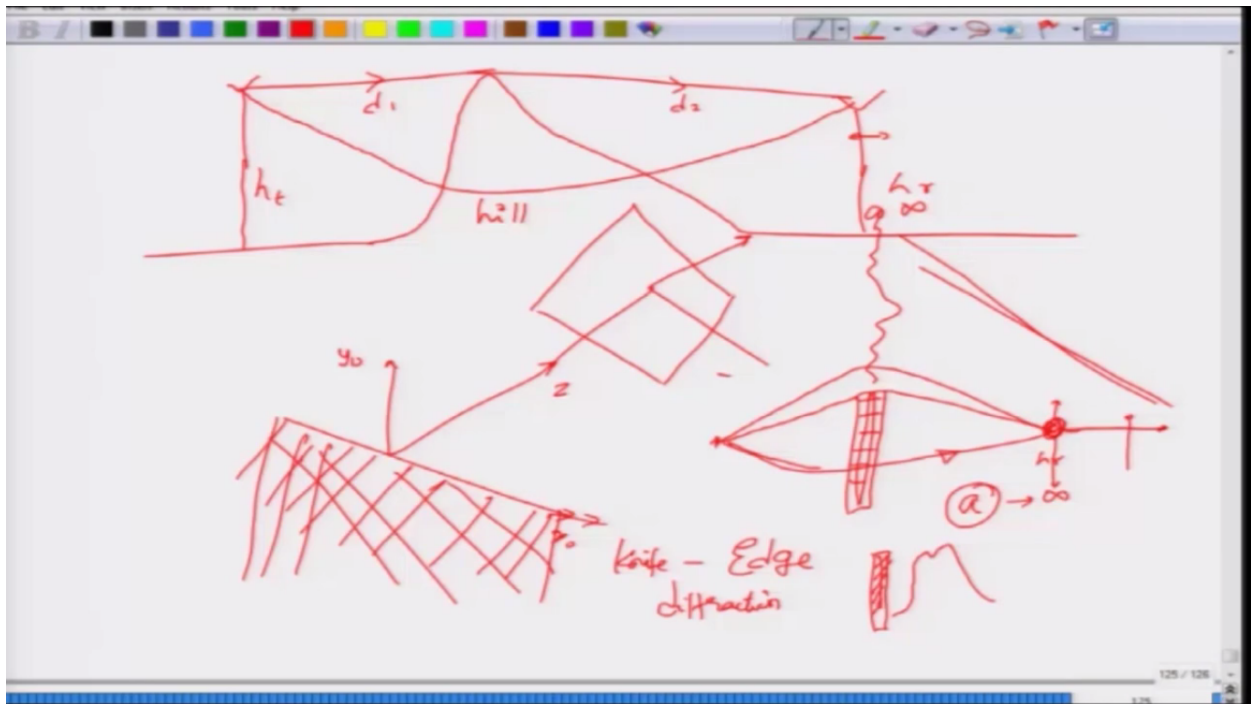
for you to remember the formula in terms of wave impedance that's why I have given you in terms of wave impedance here. So  $r_{12}$  is similarly we have already written in  $\eta_2 \sin \delta_2$  by  $\eta_2 \cos \delta_2$  plus  $\eta_1$ . So this would be the matrix relationship. Now when you apply boundary conditions at this part you are going to get another relationship which can also be put in the form of a matrix times  $E_3$  plus if there was  $E_3$  minus this would have been present but because there is no  $E_3$  minus you can actually put this equal to zero. And this matrix is given by, I will have to erase a few diagrams here. Hopefully you can go back to the video and then look at these diagrams again if required.



So this matrix is actually given by slightly complicated. It is given by  $1$  by  $t_{21}$  where  $t_{21}$  would be the transmission coefficient with second medium incidence first medium as the transmitter. So that would be  $2\eta_1$  by  $\eta_1 + \eta_2$  and then you have  $e^{j\delta_2}$  this would be  $0$  and then you have  $r_{21} e^{-j\delta_2}$  that would be  $0$  where  $\delta_2$  is  $k_2 d_2$  which is the overall phase factor that you are going to get. So this  $0,0$  is only for this special case otherwise there will be similar  $r_{21} e^{j\delta_2}$  and this term would actually be  $e^{-j\delta_2}$  as you can clearly derive here. What is  $r_{21}$ ?  $R_{21}$  is basically minus  $r_{12}$  because you have simply interchanged the incident and the reflected media. So when you do this and then write down the ratio of  $E_3$  plus to  $E_1$  plus which is the transmitted to the incident wave we are going to get or you actually do this first. You take  $E_1^-$  to  $E_1^+$  and then find out  $E_1^-$  to  $E_1^+$  ratio and take the magnitude square this will give you what is called as the reflection coefficient or reflectance because it is a power relationship and then you can find out the transmittance of this device by taking  $1 - r$  assuming that this device is lossless if there is loss or gain in the

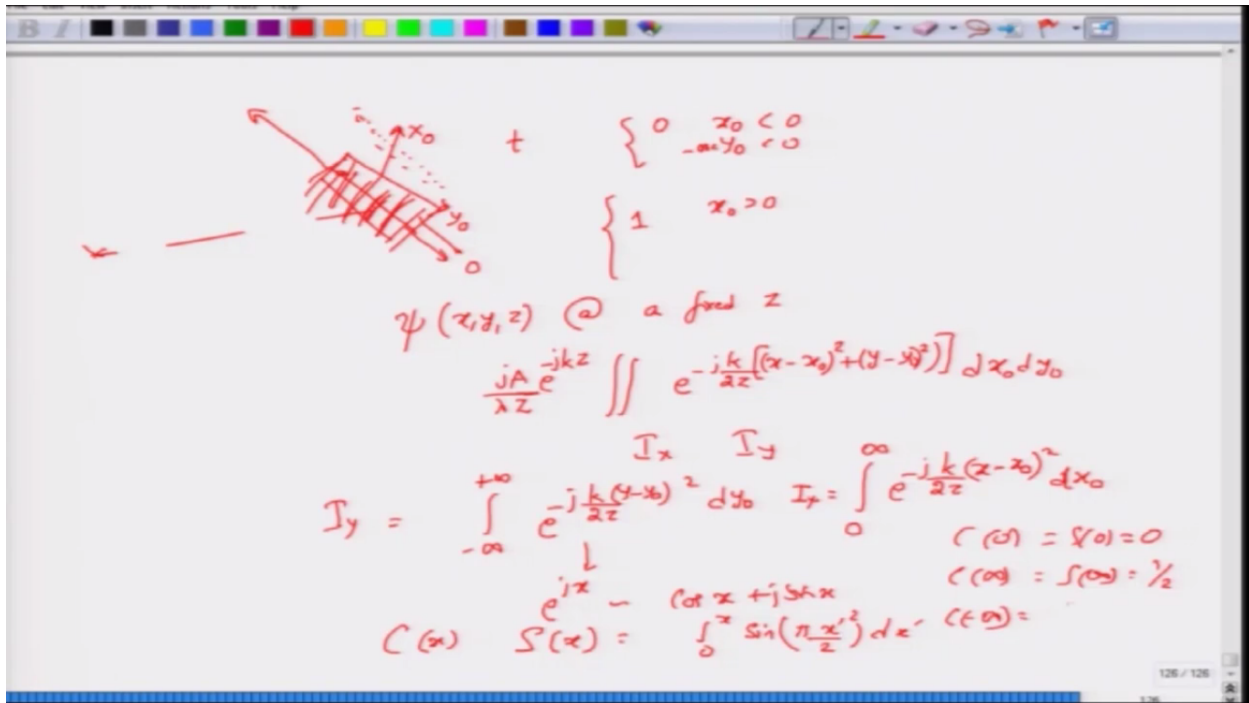
medium  $d_2$  or in that middle layer then there will be changes in the finesse and other things which we will not explore here.

The point I wanted to mention here was that with the simple understanding of the plane waves which are waves provided by the in the free space it is possible even when you put a slab it is possible for you to understand what happens to the waves that are propagating and how this interference, multiple interference can lead to different properties than what you can expect. So this is what I wanted to tell you about the Fabry Perot Interferometer. There's lot of interesting things that you can do with Fabry Perot Interferometer that would be something for a different course.



Finally I would like to come back to this diffraction problem. So we had seen this scenario. So you had a hill here or a tree or a building and then over a transmitter and a receiver antenna you had waves which are propagating. So there could be a line-of-sight propagation and then light or electromagnetic wave could bend and then after bending so you had this hill and you had this transmitter and the receiver antennas. Of course I have to put the ground level somewhere. So on this ground level we mark  $h_t$  and  $h_r$  and what we wanted to consider what would happen when such a hill or a building or some kind of a tree would be appearing in between. Of course it will lead to losses because this would be the direct path and this bending over would actually bending in the region of the shadow. So this would be the similar situation to the case that we already considered when we introduced diffraction. So what we want to do is to just give you very brief formalism and then tell you what will happen as you move away from this position. So as you move slightly into this as you displace your receiving antenna then what will happen to the field strength here. The problem that you would like consider would be something like this. So you

have your Z axis propagating and then you have this Y0 and X0. We will assume that this would be the edge. So this is like the building that you have and the waves that are present. So I should have probably written it nicely here. So this would be the edge that you are considering and of course at very far away distance you actually keep a plane here. So you can keep a plane and then you want to find out what would be the field at any of this particular path. So this would be the Y0 I mean this would be the X0 axis and this would be the y0 axis or the other way around. It doesn't really matter. And similarly you will have two axis up there. So this mimics the building kind of a thing. So you can imagine that the building is present and the source is kept here so there will be like from the source the source is actually seeing complete a tall building here with extent which is wide on both sides and about that building this is completely open air. So one thing that would happen is that I cannot use Fraunhofer diffraction in this case because I have a building here. My diagrams may not look very nice but I have a building here and our source here. The aperture is kind of open all the way to infinity at least to the ionospheric case which we can thought kind of think about to be very very large from the building height few kilometers. Therefore, we will put this as infinity. So what you are trying to find out is the field at this point and as I've said there is a straight path and then there is a bending over shadow path. The straight path of course would just touch here and the shadow path is the one that would be followed because of the diffraction. So no matter what position you keep your receiving antenna you can move this receiving antenna wherever distance you want. As you move you are going to see more and more of this free space. So it's that you will never reach the Fraunhofer limit here because the aperture area  $a$  is actually going off to infinity. So as you can see we have in this particular case you have a building which is very tall and obstructing the view here. So this is the problem. So as the aperture area is or the as the aperture radius is very large no matter how far away you take your receiving antenna you will never be able to get to the Fraunhofer limit. So what we want to understand is what happens as I reduce the antenna height or increase the antenna height and maybe I do it at different distances away from the building will my reception improve or reduce. What you will see is that as the height is increase or decrease as you move deeper into the shadow region or you come out of the shadow region the electric field or the other the total electric field that is received or the intensity of the electric field or the power of the field that is received will actually oscillate in a certain manner. This is a very well-known problem in optics called as edge diffraction. Sometimes called as knife edge diffraction meaning that the edge is very sharp. You could of course have an edge that looks something like this. This is a very – this is a complicated edge however what we have considered is sharp edge so it is – it kind of cuts off directly in a in a flat-top manner. So this is called as an edge diffraction problem and this was solved of 100 years ago by a person called Sommerfeld and we are using those ideas here in order to understand what is the field thing. The catch here is that unfortunately you cannot use Fraunhofer diffraction integral. For that you have to go back to the full, you have to include the quadratic phase part as well.



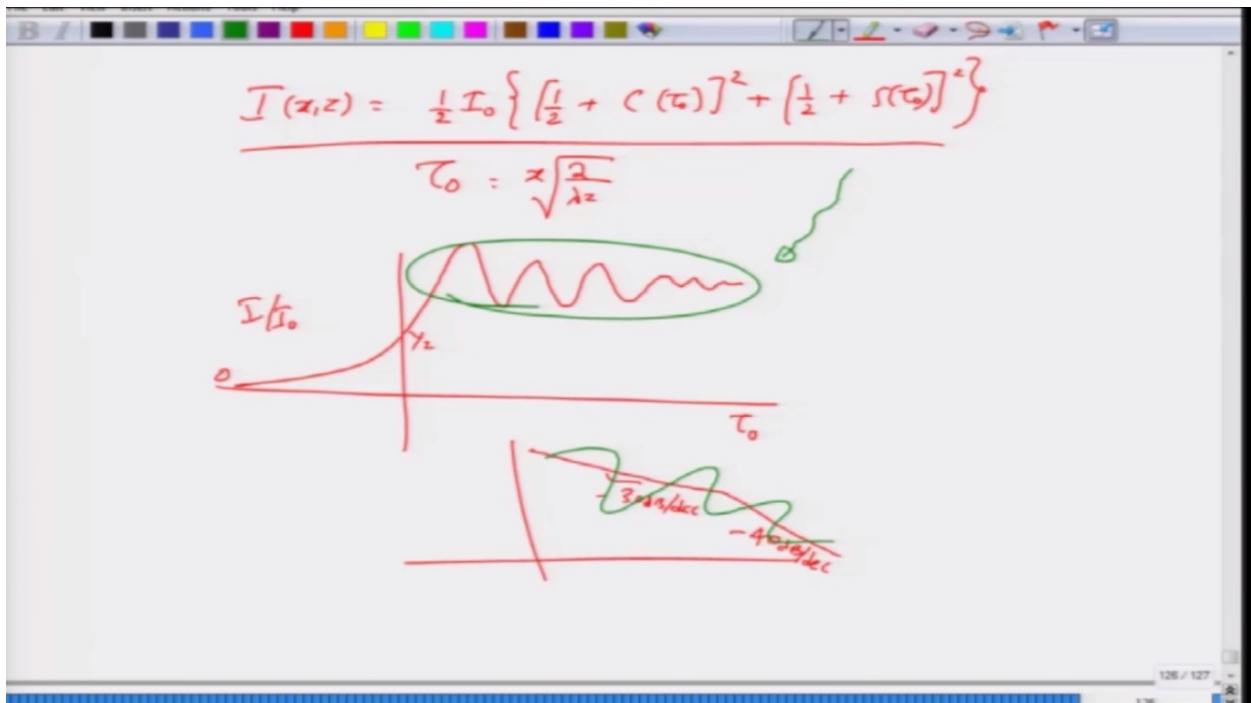
So we will set up the problem in such a way that the edge is this is I will call as the X0 axis this I will call as the Y0 axis not exactly the Y0, X0 we have just switched it over so that the things becomes like I mean you know I have solved this in my notebook in this coordinate system. So I am just continue to that one.

So clearly if I have the source then the source will actually see this edge. So the source will see this edge in such a way that the transmission function here from the source to the point here the transmission function will actually zero here everywhere. It would be zero for X0 less than zero and for Y0 anywhere from minus infinity to infinity and it would be equal to one that is the transmission function T would be actually equal to 1 when X0 is greater than zero meaning that if you stand above the building the light or the source will be directly visible to you. So there is no stopping here. The opaqueness of air is taken to be zero whereas the building is assumed to be completely opaque in not letting any light pass through it. So this would be the situation that you have for X0 less than 0 you have this case and for X0 greater than zero you have this particular thing.

What we will assume is this psi which of course will now be function of X, Y and Z at a fixed set. At a fixed distance from the building this integral should then involve certain amplitude so e power minus jkz will be present. I am still making some assumption that Z is larger but I cannot ignore the quadratic phase. That's the problem that I have. So I have this ja which is the amplitude of this wave that I have taken divided by lambda times Z. So this portion of falling off is still present but in terms of the integral that I have, I have to go to the integral which says e power minus jk by 2Z X minus X0 square plus Y minus Y0 square, sorry this is Y minus Y0 square. This would be the integral which I will have to integrate over the aperture dx0 and dy0 . Now clearly just as the Gaussian case we considered you can remove this integral and make it into product of two integrals one along X the other along Y. the integral along Y will be given by



minus infinity to plus infinity. That is the range of this Y integral because you see the edge is going all the way here. At least we will assume that this is so wide that you can replace that with minus infinity to plus infinity. So you have  $e^{-jkY}$  by  $2 \int_{-\infty}^{\infty} Y \sin^2 Y \, dy$ . What is the integral over X? The integral over X will go only from 0 to infinity. It won't go from minus infinity to plus infinity and you have  $e^{-jkX}$  by  $2 \int_0^{\infty} X \sin^2 X \, dx$ . So this is the expression that you will have and it is quite difficult for us to actually carry out these integrations even when you look at the tables and in fact about 100 years ago people did not have cheap access to or access to cheap computing facilities. Cheap meaning it's very economical computers that you can have and you can simply write down a MATLAB script in order to solve or obtain the values of these integrals using well-known numerical integration formulas but a few years ago or 100 years ago people did not have access to those. So they actually split these integrals into two. So the exponential function  $e^{jx}$  can be split into  $\cos x$  plus  $j \sin x$ . So they split these integrals into two after making some change of variables in this one and they called one of the integral as the sine integral  $\text{Si}(x)$  one of them as the cosine integral and the other one as sine integral. These are specifically defined as so  $\text{Ci}(x)$  or  $\text{Si}(x)$  is defined. Let me write down this  $\text{Si}(x)$  is defined as  $\int_0^x \frac{\sin t}{t} \, dt$  which is a dummy variable divided by  $t$  times  $dt$ . Similarly you will have  $\text{Ci}(x)$  also again given by this replace sine by a cosine function. And there are certain properties of this cosine and sine functions. So  $\text{Ci}(0)$  will be equal to  $\text{Si}(0)$  will be equal to 0 you can verify that this would be the case and  $\text{Ci}(\infty)$  and  $\text{Si}(\infty)$  that is you let the upper limit go all the way to infinity, this sine and cosine integrals will actually be equal to  $1/2$ . There won't be this one 0 they would be  $1/2$ . Similarly  $\text{Ci}(-\infty)$  and  $\text{Si}(-\infty)$  integrals will be minus  $1/2$ . So the overall integral that you will have and the overall expression, so the overall electric field that you are going to get can be written in terms of its power you can verify this. This is I'll give you a reference for this one because the intervening mathematics is quite difficult.



We don't want to get into that. The intensity can be written as about half  $I_0$  half plus  $C$  of some  $\tau_0$  square. So  $\tau_0$  square plus half plus  $S$  of  $\tau_0$  square where  $\tau_0$  is a variable that has been defined as  $2 \sqrt{\lambda Z}$  times  $X$ . This is sometimes called as Fresnel Kirchhoff parameter. This is a complicated expression but if you plot the intensity pattern normalized to  $I_0$  what you will find is that and you plot this one as a function of  $\tau_0$  which is basically this  $X$  into  $2 \sqrt{\lambda Z}$  you will find that exactly at 0 this fellow will be 0 and I mean half and then after that it would actually start to oscillate. So this would be  $1/2$  and this would almost go to 0 when you move into the negative regions up there and what you see here is that it would actually be oscillating.

So this is a very interesting phenomena. What it means is that at some point the field is reinforcing, at some point the field is actually not reinforcing. Please remember our or please recall our big point analysis that we did there was a minus 30 DB per decade or 20 DB per decade and then there was a minus 40 DB per decade as the long range attenuation. However, around that we saw that there was shadowing effect and this is exactly the shadowing effect that is as you move up and down onto the building side or at a given  $Z$  as you move up and down the the receiver antennas or your mobile phones then you will see that the total amplitude decreases and increases. So there is actually a variation in the amplitude and this variation continues with smaller amplitude as you move away from the building. That is why certain areas of the building you suddenly see that you can hear the other person speak very well and at certain areas you cannot hear just behind the building and as you move away slightly you can see the effect of shadowing.

So I admit that this was not a very lengthy mathematical justification for any of this. As I mentioned that would be quite difficult for us to go into the details in this course but what you have to understand is the shadowing thing is not always bad. Sometimes it can be good. On an average it is bad but sometimes it may be good and therefore best case would be that certain areas of building will be good for you. You can have good communication. Certain areas will be very bad for you but determining which worries which is a problem and on top of it we have multipath fading which can also change your signal strength very rapidly.

So this concludes our course and before we conclude I just like to point out the different ways in which we have considered electromagnetic waves and the corresponding guided structure.

Thank you very much.