Lecture – 04
Sinusoidal Excitation of Transmission Line
(Propagation constant, Characteristic Impedance)

Hello friends, welcome to NPTEL MOOC, on Electromagnetic waves, in guided and wireless media. We will begging module four by recapitulating, important equation from module three. So, as you can see hear on the transmission line if we represent the voltages and current as the phases,

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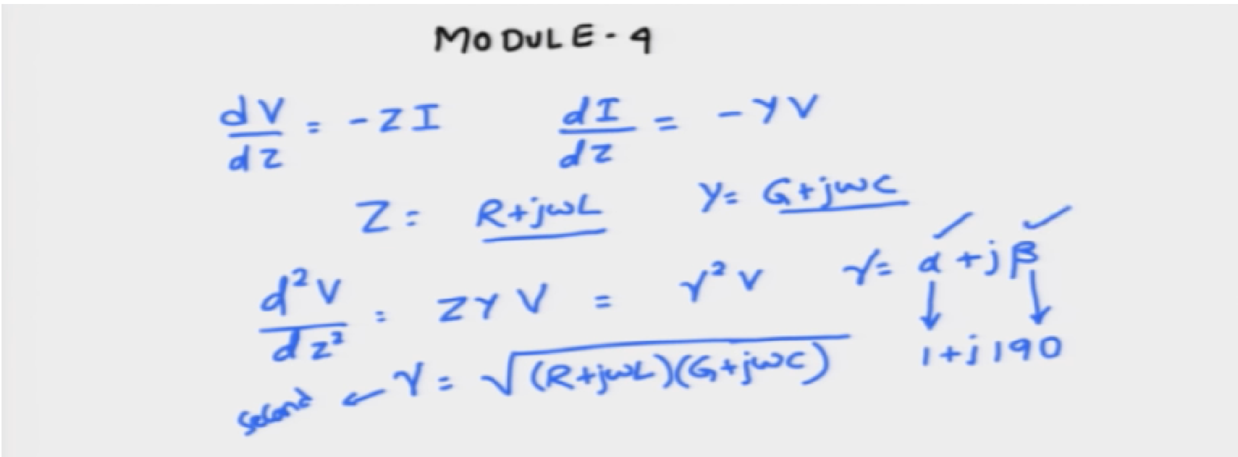

because these are time harmonic waves now, having a frequency omega radios per second, then the telegraphic equation ,that we describe the next module, actually becomes rather simplified into ordinary differential equations. So, DV by DZ equal to minus Z times I, Z is R plus J omega L ,you can think this series to total impedance seen by the line per unit length of course and then DI by DZ, will be related to ,minus Y are is equal to minus Y times V where Y is the parallel admittance, that is given by G plus J omega C, where these quantys are again per miter quantys and because our goal is to uptime the solution for the voltages we can actually differentiate the voltages equation and use the current equation in order to uptime equation in which left hand side and right hand side or both, are functions of voltage alone. And that equation D square V by D Z square which is equal to Z, times Y, times V. Okay? Z is, the series impedance, that we have described, which is R plus J omega L, Y is G plus J omega C and the product of these two is called Gamma square, and Gamma will as you can clearly see is given by square root of Z into Y and that will actually be a complex number, because these are a compels number as a such. Right? And this compels number, can always be written in terms of, its real part and imaginary part ,we have to decide, I mean we have to determine, what is the real part alpha? What is the part beta, from solving this expression? Okay? If gamma terms out to be something like ,one plus J 190, in that case you can associate alpha 2 one and find out the beta equal to 190, but this value actually depends on the frequency and in addition depends on R,L,G and C. So, because its depends on the primary constancy of the transmission line R,L,G and C this is called as secondary constant and this two also called as, secondary constant, that is alpha and beta, are called a secondary constants. We will very shortly see the significations of this alpha and beta, because you go back to the expression, here but we have,

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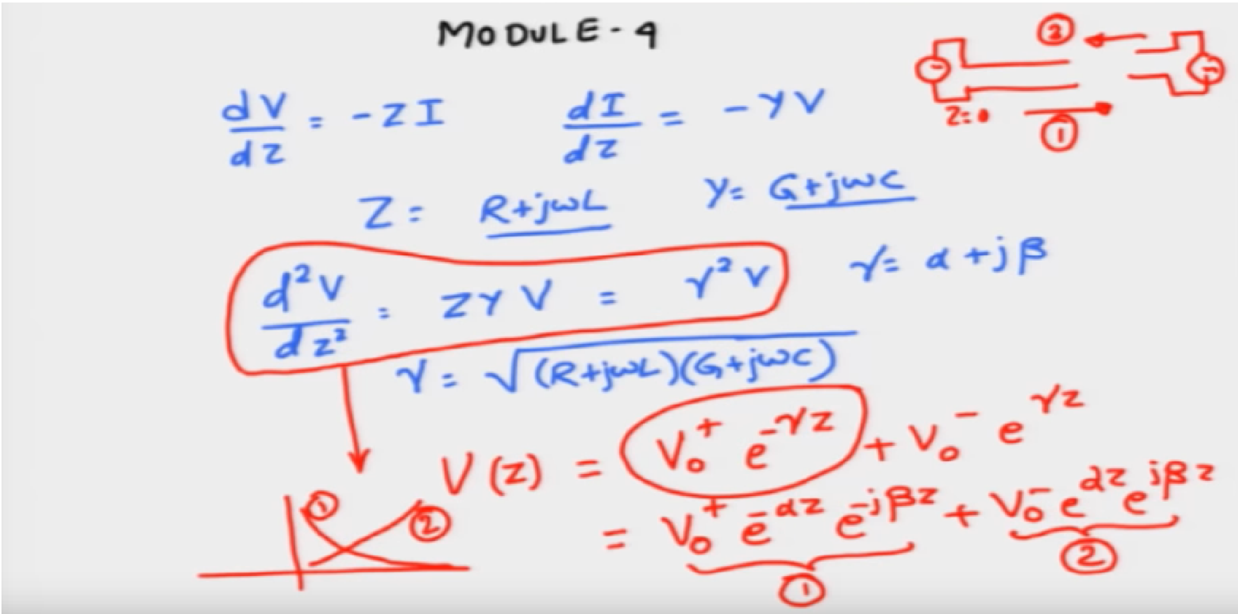

D square V by D Z square equal gamma square V and ask what would be the solution of this differential equation, from your studies in differential equation in aliyar courses, you will say that well solution have to be exponential. So, let as, write down the solution as, V of Z equals V some constant, which we called as V0 plus, and then have E bar minus gamma Z plus V0 minus E bar plus gamma Z. Okay? Where gamma of course is this quantys, so substituting gamma into this, we will see that, there will be V0 plus E bar minus alpha Z and then you have E bar minus J beta Z plus V0 minus E bar alpha Z E bar J beta Z. If alpha and are beta positive constants, as they are actually in the particular case, in many cases, then this first team, you can see that as Z increases, E bar minus alpha Z decreases. Because, E bar min us alpha Z is decaying exponential. So which means that they waves ,that are launched at the sources end ,as the travel along the transmission line, then they amplitude of those waves are DK, however you may think that this alpha being a positive quantity ,the second term should actually you know, increases, it will increases ,not in that since, but there is any reflected wave or that is any voltage, that is very large from the Z you know , on the right hand side, so to speak ,then that voltage actually drop , as it approach to this end. Right? So, this would correspond to, the second term , this correspond. So this is a second term, this is the first term, they just tell you the direction in which voltages or propagating so, this E bar minus J beta Z is a positively traveling or forward going voltage, because usually ,the you know transmission line is assume to start Z equal to 0 , an then continue to this side and the voltages that are actually you know, propagating along the positive Z direction or given by this first term, which is V0 plus E bar minus gamma Z and if ,inside of launching voltage Z equal to 0, you decide launching voltage at Z equal to infinity and then you have a transmission line, in this reveres you know or in this particular direction, then this corresponding to the second term her, because the voltages are propagating in the back word direction, it does not mean the voltages are actually getting amplified known it's only means that Z is negative for the second term hear, Z is positive for the first term hear. Okay? So, they both essancily describe decaying electric fields or sorry, decaying voltages, expect that the location and then direction of they are waves actually, flighty different .Okay? So, with this in hand let as consider this situation,

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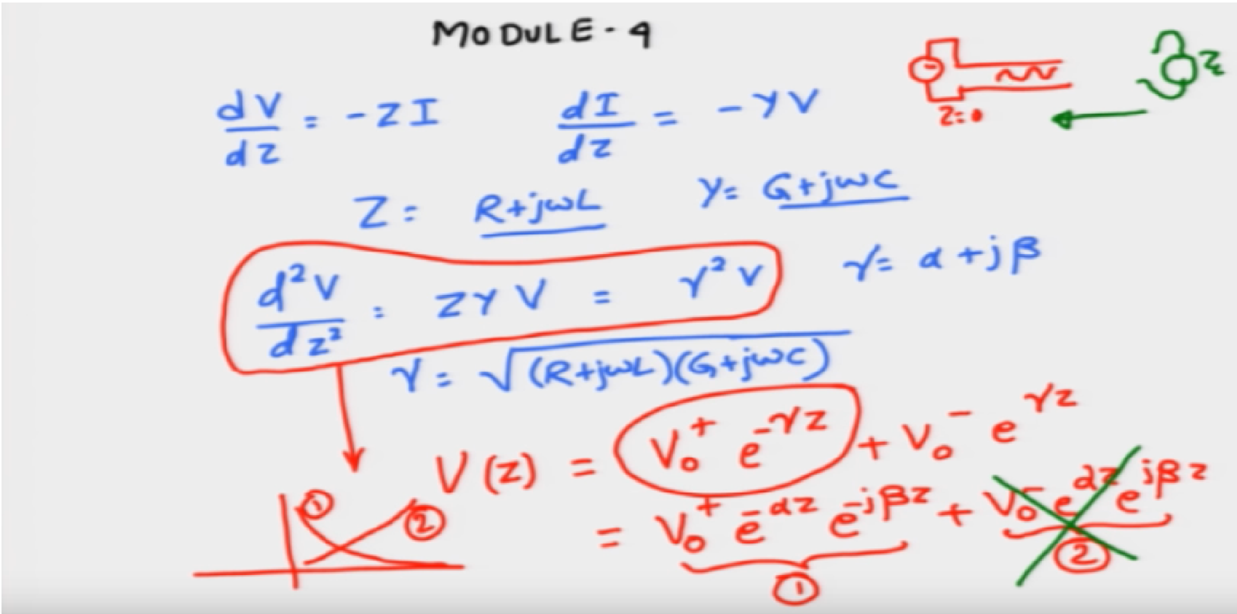

I will assume ,that there are known loads or at list I will move the load all the way to infinity and then I ask, what kind of voltage wave can existed on this transmission line? It's obverse that, when you have, they loaded moved all the way to infinity; the second term will not existed. Because, there is no source for this second term. The second term which representing the back word travelling wave, that is wave, that is propagating along the minus Z axis ,right or the propagating along in the opposite direction to the forward wave, can come into physical reasons, one you actually have source hear, so that they should be a source. But, because we have a set no such source, this reason is eliminated, so there is no need for this second term to appear or if the line is finite, but you terminate the line in a impedance Z L, which is different from, what is called as characteristics impedance of the line? So which means that, the forward going voltage first travel? Encounter the load and be reflected and then becomes no will travel in the backward direction, however we have eliminated this situation all so by saying that the line excitant all the way to infinity. So therefore the only wave that you can have is the forward going wave. Okay? So, we will take the forward going wave, and then ask, this situation, what would be the current wave on this line. Okay? To obtain the current wave all we have to do is to use to these two equations, I am erasing this hopefully, you would have already noted down, what are the corresponding values? And therefore I am no erasing this one, this one is going to new page, because I want to use the equations show you, what is the relationship between the voltages and current. Okay?

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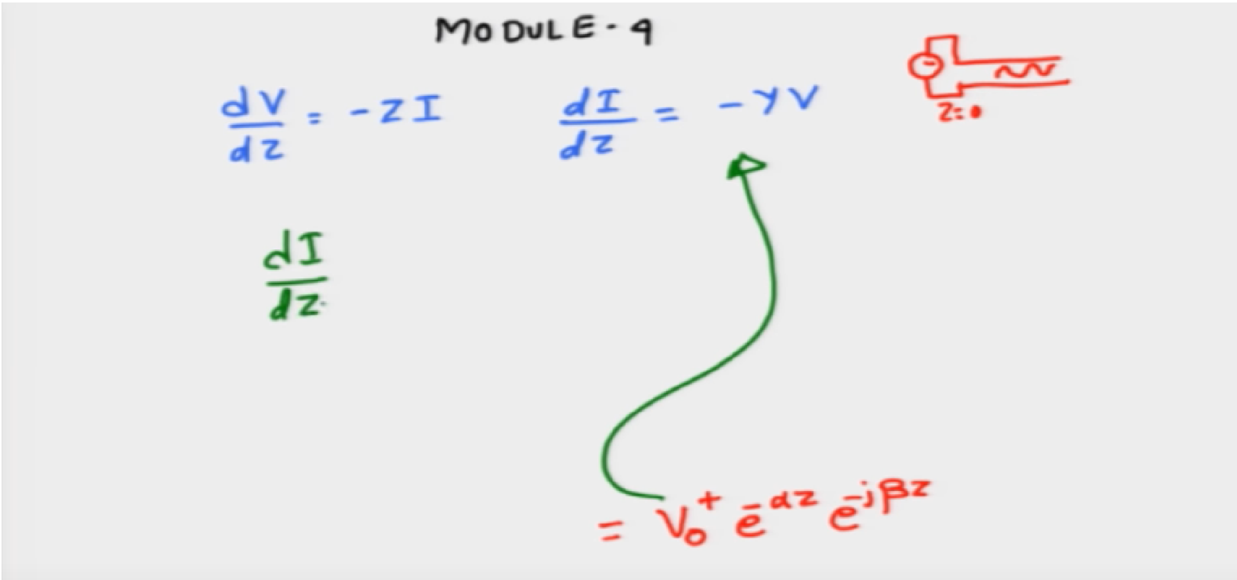

Substitute this voltage expression to this equation and then obtain, what is DI by DZ. Okay? You can actually substitute this one into, these first equations and then find out what could be the relationship with I, N, V.

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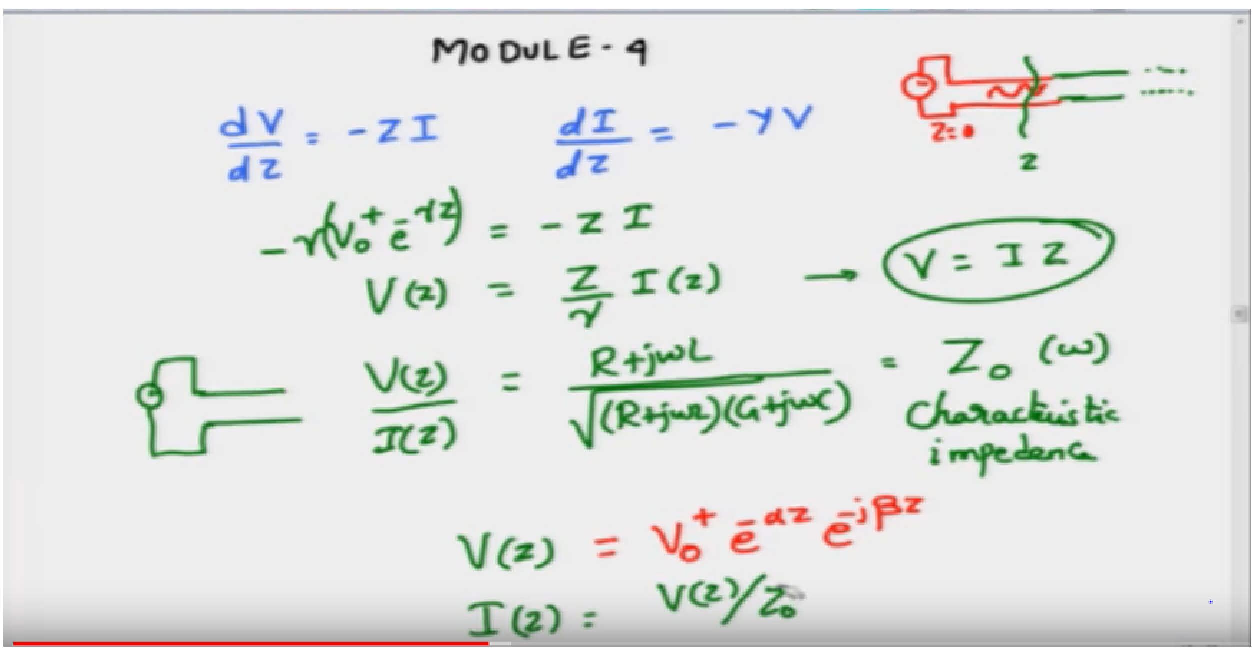

So, substituting this V of Z, which now only has forward going voltage, into this first expression or into the expression for the voltage you will see that this could be minus gamma V0 plus E bar minus gamma Z, which equal to minus L. Okay? Please note that, this z here on the hand side corresponce impedance, but as the small z actually corresponding to the access the have taken and what is this V0 plus E bar minus gamma z? This is basically, voltage phaser at any length, on the transmission length. Right? So, this playing Z, away from o, the voltage phase the as the value of v of z and this will be equal to Z gamma, times I is the corresponding current phase value okay? And we know letters but that how it's normal it lecture so actually using them of fully should be the there no confection among the letter and symbol any Z. okay? So you have at any Z .okay? On the on the physical situation that the line existence all way to infinity so for this line at any Z if I look at the ratio of forward going voltage V of Z to I of Z. okay? In this case this is the total voltage because, there is only one type of a wave and that ratio will be equal to Z by gamma but I also know what is Z is R plus J omega L gamma is square root of R plus J omega L, types G plus J omega C there four this ratio which is V of Z to I to Z for the physical situation that the transmission line extended all the way to infinite the ratio of the ratio the total voltage phaser to the current phaser is denoted by a special symbol called as Z naught and the Z naught is called as, 'characteristics' impedance of the line, its call characteristics impedance because the value of this Z naught depends only on the primary consistent of the transmission line it depends only on R,L ,G and C of course this function of sequence of omega. But, usually you know when you specify Z naught you specify this at a given, I mean at a assume frequency omega. Okay? If you change omega, of course this will also change in general. Okay? They are situation where Z naught becomes independent of frequency, we will consisted those cases shortly. So, if this is a voltage the current phaser of course will be given by I Z is equal to V of Z divided by Z naught and V of Z given by V0 plus E bar minus alpha Z , E bar minus j beta Z . what would be the situation, if you turn the location of the source, from Z equal to 0,from Z equal to infinity, in that case the only wave that you have a backward travelling wave .okay? And ratio of the total voltage, which will be just the backward travelling voltage and then the current, which would be the backward travelling current, will be also denoted by characteristics impedance Z naught. But, you have to you know, change that, science of that slightly ,because of the positive Z ,negative Z kind of the think, so in that case again you will still have the same concept characteristics impedance.

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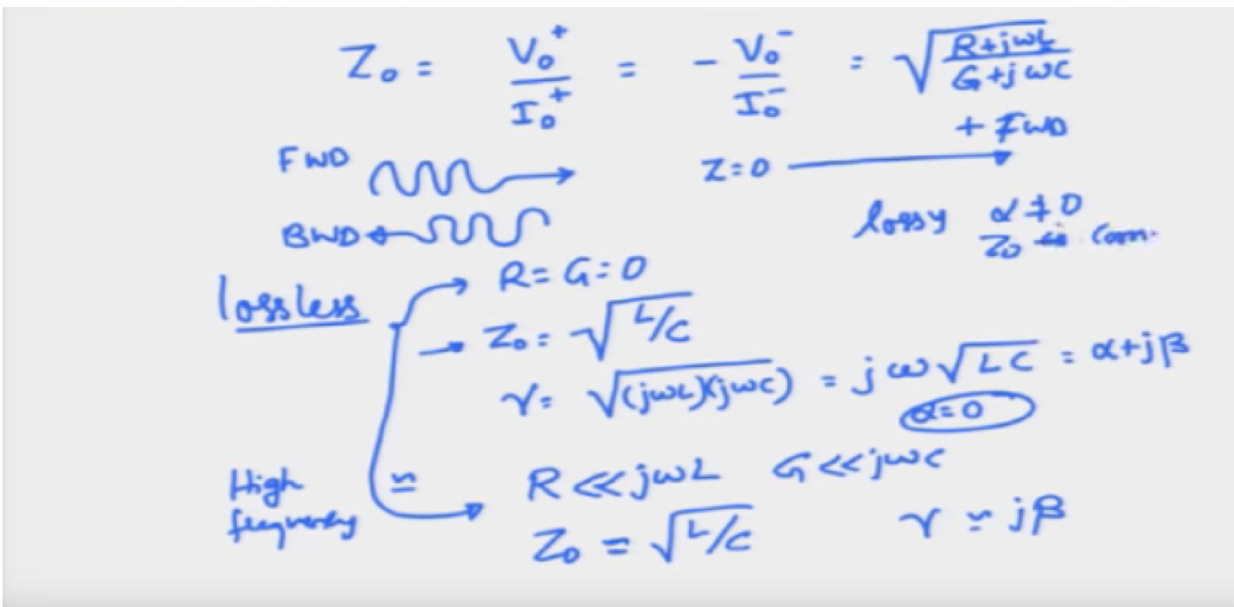

So, whether it is forward going voltage or backward going voltage, waves are current whether where considering, Z naught can be defend as, the voltage phaser amplitude. Okay? Of the is forward going voltage, to the is forward going current. Please, note this one. So, this is the forward going voltage along plus Z direction and this is the backward travelling wave along the you know, along this one from the right to left as such right? This would be also equal to minus V0 minus divided by I 0 minus, the minus science simply comes because ,these waves are going as E bar minus gamma Z , where are this waves are going E bar plus gamma Z and this minus side the differential equation. Okay? So, this is what we called as forward wave and this is called as backward wave, so this is a just the notation , which one is forward, which one is backward is the most likely convention and we take the convention that Z equal to 0 and propagation beyond that is consider to be the forward waves. Okay? So, this the characteristics impedance again, the ratio of characteristics impedance at that particular frequency is given by or the characteristics impedance expressly given by , square root of R plus G , R plus G omega L divide by G plus J omega C. Okay? Immediately we can ask, if they Z naught is present, it is possible for as to actually have Z naught be independent of frequency omega .what is that advantages have a Z naught independent of frequency omega? First you don't have to calculate Z naught for a different frequency that is a big advantage and second a most important advantage is that, when Z naught is a purely real then the voltages on, the line and current on the line or going to be in phase. Okay? I will leave this exercise to show you that, when Z naught is a complex as, it is in general give by square root of R plus J omega L plus G plus J omega C , in that case voltages and currents or not going to be in phase ,which impacts there delivery of power. So, if you're using this transmission lines to look at this how much of power delivered to the load, from the source, then, the maximum power will be delivered when Z naught is, you know, real, when Z naught does not depend on the frequency. Because in that case, voltages and currents are going to be in phase and they can deliver, as much power is permitted by the equations, as much power is possible. That would be the maximum power they can, deliver when Z naught is, real. Okay? And in that case we call the line to be loss less, because the reason or one of the conditions when Z naught is real, is that R= 0 and G = 0, in this case even Gamma will be purely imaginary and Alpha will be equal to 0. Okay? So for that case we call it as a loss less case or maybe low loss case, when R and G are very, very small, compared to geomega L and geomega C. However if R and G are not small or not zero, then you have to deal with the fact that the waves are going to be attenuated along the line, that is, they will lose, the amplitude along the propogate along the line, plus that they will not be able to deliver maximum power to the load. Okay? So lossless transmission line is, the simplest, mathematically ideal line. But it will not be realized in practice, in many cases. This situation can occur into two different varieties. One is, when R = G = 0, which case, we know that Z naught will be real. Because Z naught will then be equal to square root of L by C. Okay? And then Gamma, which was square root of, J omega L into, J omega C, will be equal to Omega square root LC, of course there is a J out there. And this is equal to, Alpha, plus J Beta. Clearly Alpha is equal to zero. So these are equivalent conditions, for a lossless transmission line.

So either from the propagation constant gamma, finding that Alpha equal to zero, leads to the lossless transmission line or showing that Z naught is real, will lead to a lossless transmission line. Okay? The second way, in which they can occur, but this is not exact, this is an approximation, but a very good approximation. Is when R is, is much, much smaller than J Omega L and G is much smaller than J omega C. This region of operation or this regime of operation is called as, 'High frequency operation of the transmission lines'. Wherein, the inductance and capacitance affects are much prominent, compared to the resistance and conductance affects. Okay? In this case also, Z naught will be real; it will actually be approximately real. So it would be square root of L by C, no change there. And gamma is also approximately equal to, J beta. Okay? These approximations tell you that, when you operate a nominal transmission line at very high frequencies, then the line can be almost considered to be, lossless. Okay? Of course there is a certain bandwidth for the line, for operation. Beyond that frequency if you operate, then the line will be completely lossy. But for various other reasons, which will become clear later on in the course. But for the operation that we are normally interested, the line frequency is usually in the giga hertz range and there you can assume that the losses are kind of, you know, small. Okay? At least we make this assumption, so that we can work out the equations and get some back of the envelope calculations done, before we actually get the quantitative results. Okay, this simplifies our mathematics, this simplifies our understanding. So we will continue for some time, assuming lossless scenario. Okay? However a line would be lossy, when alpha is non-zero and Z naught is a complex number. Okay?

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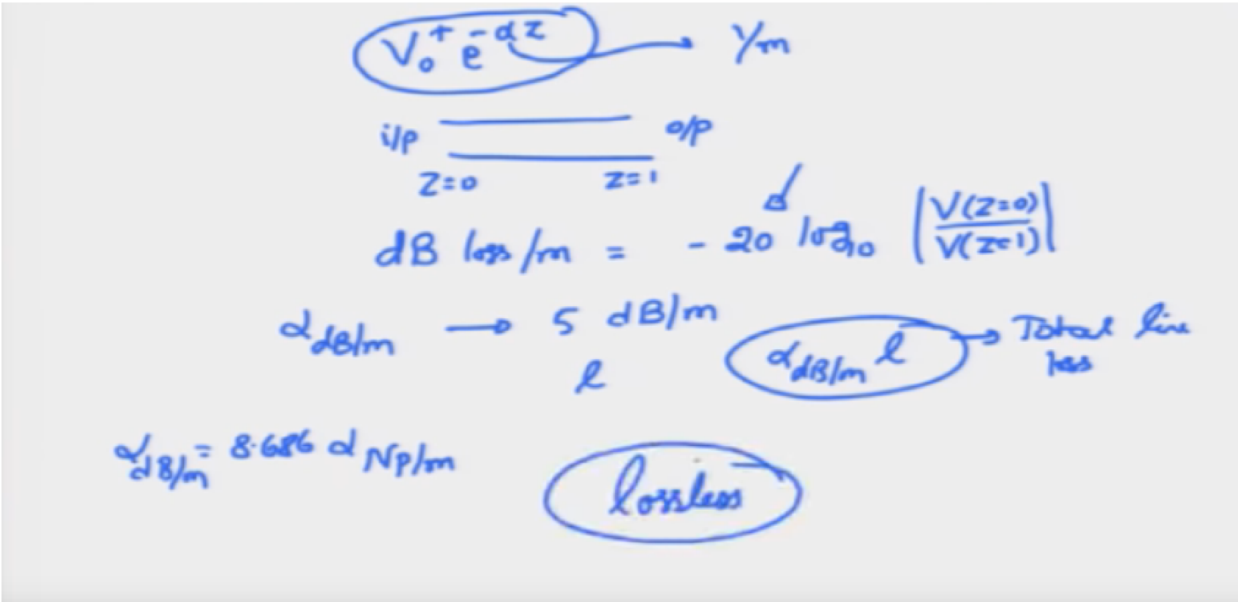

So the lossy lines actually have their amplitudes, at least for the forward going wave, only if you consider. The amplitude will decay as, V zero, e power minus alpha, Z. Okay? So if you take one meter of this line, so say, Z = 0 to Z = 1 and then evaluate what would be the ratio of the voltages at the input and the voltage at Z = 1. Then you can actually define, what is called as, ‘Decibel loss per meter, as, minus 20, log, always with a base 10 here, V at Z = 0 divided by V at Z = 1. Okay? That is, the magnitude of the voltages, that you are going to find out at Z= 0 to Z = 1 of a transmission line, where only the forward going voltage, at least as I’ve assumed here. This ratio if you take and then take a minus sign here, because V of Z = 1 for a lossy transmission line will always be less than V at Z = 0. So that this log will be negative, negative and negative will make it positive. So that you can specify, ‘Hey this transmission line has a loss of about 5 dB per meter. Okay? And this is happening because of this, attenuation constant, Alpha. Okay? And this fellow is called as, ‘Alpha dB per meter. Okay? It is the way that we normally represent. And we take this 20, of course, because we are looking at, voltage attenuation. If we were looking at power attenuation, power is voltage square and therefore you would simply have used a 10 factor here, instead of a factor of, 20 there. But this characterization of alpha in terms of dB per meter is very important, because that will tell you, how much loss you can expect, that is contributed from the line alone. Okay? Well, we would expect that or we would, or you know, we would be very happy, if that line would contribute, Zero loss. But in practice it doesn’t just happen that way.

So this alpha in dB per meter tells you, how by an amount of dB, the loss is contributed by the transmission line. Of course if the line has a total length of, say some, L, then you have to multiply this alpha, in dB per meter to L, inorder to obtain the total line loss. Right? So the contributed loss from the transmission line can be obtained my multiplying the physical with the attenuation parameters. In some cases, you will also find that alpha is, you know, given in what is called as, ‘Neper per meter’, which is, called as a natural unit here. Because in this expression of, e power minus alpha z, alpha should be, having a units or dimensions of 1 by meter and instead of writing this as 1 by meter, we decided to honor an engineer called Neper, who worked with all these losses and decibels. So we call this as, ‘Alpha in Neper per meter’. And this can be related to Alpha in dB per meter as, this expression, perhaps is well known to you. This is 8.686, alpha in neper per meter. So there is no log here, but this conversion factor of 8.686 comes from, converting the log to the base 10 to the log to the, I mean, to the base E, which is called as the, ‘Natural Log Units’. Okay? So this is for the lossless case, for now. We will not talk about, sorry; this is for the lossy case now. We will not talk about lossy cases for some time; we will continue our development of further transmission theory, in terms of loss less transmission line. And as I have told you earlier, in this module, this is a very good approximation for most practical transmission lines. When the transmission line is lossy, anyway you have to invoke some numerical methods or computer stimulation packages, to really get quantitative numbers. Okay?

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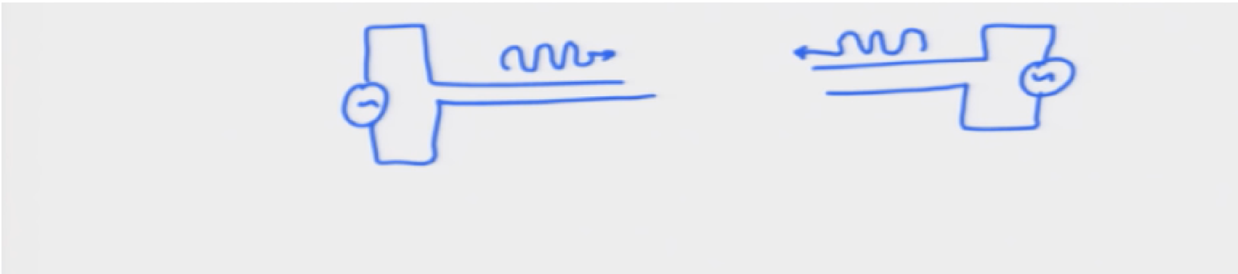

But to get a good understanding, we will assume the line to be lossless. So far we have considered two kind of situation. I had a source here and then I had this line, which was going all the way to infinity or I had my source here and then the line, which was again, going all the way to, I mean, coming all the way from, infinity. Right? So these two situations, gave rise to either, only a forward going voltage or only a backward travelling wave. Right? So this was what we actually had. Now in practice, of course, you don't really get infinity length lines or infinite length lines. You actually get finite length lines. Right? So when the line is finite, what really happens? Let us think physically first. I have this arrangement, where I have this transmission line, maybe we will also have to put in some,

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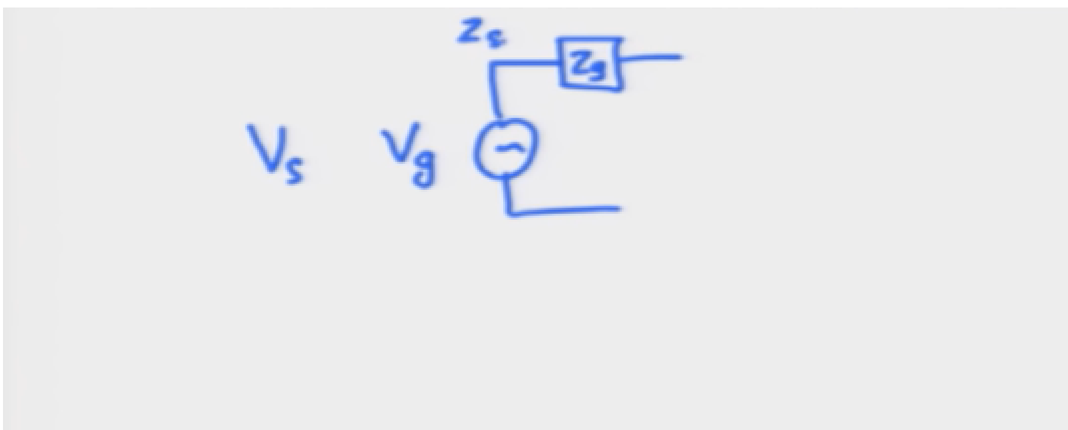

you know, load, impedance to this one. So we will call this load as, say, Zg, g stands for generator and this word generator is a relic from older notation and this V is called as, Voltage generator, Vg. Because most transmission line theory, was developed, to describe, you know, the power transmission, which happens at low frequencies, where the generator was this, hydraulic power generators and other kinds of generators. So the term or the terminology of Vg and Z g had, has still persisted in the, modern transmission line as well, so we will continue to use this. Sometimes I will use Vf and Zs, to indicate the, source voltage and the load, sorry, it is the source impedance. Okay?

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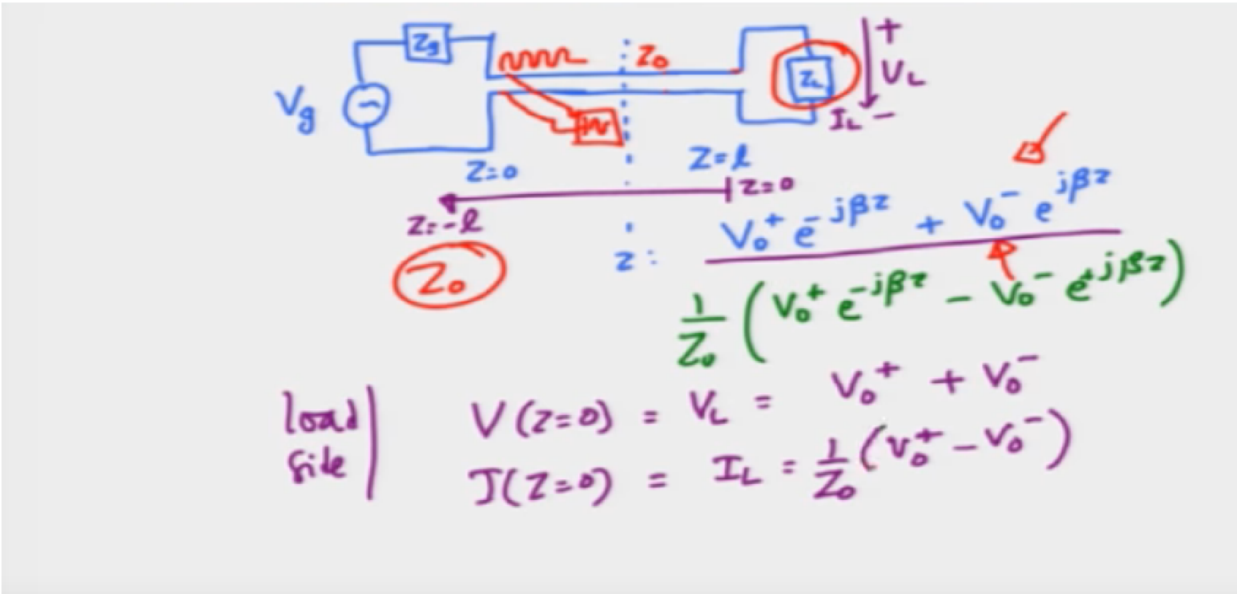

At this operating Omega, we are talking about impedances, because I'm allowing for the fact that or I'm allowing for the possibility that, the load, sorry, the source impedance, as well as the load impedance, may actually be complex. That is, they may contain some; reactive elements or at least they will be modeled, as containing some reactive elements. Okay? So, this is the condition. So you have this transmission line here, which begins at whatever Z=0 and then this continues all the way and gets terminated. Meaning, that we will attach a load, at the plane, Z=L. Okay? So I'm going to attach a load here and this load is given by or denoted by ZL and as I have told you, the center portion is the transmission line, these wires which I have drawn in terms of a square kind of a thing are actually are zero delay ideal wires. Okay? At any plane Z, what is the total voltage and current? The total voltage and current will be, in general V zero plus, e power, minus J beta Z, because I don't have any losses here, I just have V zero plus, e power, minus J beta Z. And then, V zero minus, e power, J beta Z. Where is the second term coming from? Physically think. I have this forward going voltage. Okay? This forward going voltage keeps on travelling. So you may actually start looking at this, in terms of an oscilloscope.

So if you connect a hypothetical oscilloscope here and launch the voltage, the voltage will start to appear, but with a certain amount of, faceshift. The voltage comes and you know, it continues in this way. What is crucial is to see that the ratio of the voltage to current, on this one, is this load were to be at infinity or the line were to be at, you know, all the way connected to infinity, the ratio of this voltage to current phaser is given by, Z naught, which is the characteristic impedance. So the voltage phaser, at least the forward going voltage phaser actually sees Z naught, but then suddenly, at Z = L, it encounters a load, whose value is ZL. Now just before the Z = L, the load is Z naught, just at Z = L the load, or the ratio of the voltages and currents has to ZL. The only way, you can, you know, connect these two inconsistent relations, is to, know, propose that there is an additional voltage, which we will call as the Return voltage or the reflected voltage, that will, you know, make the balance of the ratios of voltages and currents to be, equal to whatever Z naught and ZL correctly. Okay? So that is the reason you get this reflected voltage. In fact, when you look at electromagnetic wave propagation in free space, we will realize that, if a wave goes from one medium to another medium and whose properties are different, then there will always be a reflected electromagnetic wave. In much of the same manner, you have a reflected elect, voltage here and this reflected voltage will propogate, in the direction opposite to forward or incident voltage. Incident voltage is the voltage that is launched on the line, which moves along the plus Z direction. Okay? So, coming back. Now how do we find out all the relationships between this and it is possible for us to actually determine, the how much, you know, reflection can happen, when you terminate the transmission line, with a given load impedance ZL? Yes, it is possible for us to do that. You know, and the calculation is rather simple.

So we start with the total voltage, which was, you know, V zero, plus, e power, minus, J beta Z, plus, V zero minus e power, plus, J beta Z and the current of course if given by 1 by Z naught, V zero plus, e power, minus J beta Z, but there is a minus sign, please remember. Right? For the reflected voltages, there would be a minus sign here that would relate the current. So I have a voltage relationship and I have a current relationship. What will be the situation at the load side? Okay? So at the load side, before I go to the load side, I want to also now introduce you, to a slightly different terminology or the co-ordinate system. In this co-ordinate system, we actually locate the load at Z = 0 and then let the voltage, you know, travel in this axis. So this, again this comes from, mostly from antenna related work, wherein, people knew where the antenna was located, that was their, you know, load, as such. But then they wanted to connect a transmission line. They wanted to know how much the transmission line should be connected to the source, such that, I know, make the, you know, reflection go away or some kind of an additional thing, that I'm interested in the antenna part. Okay? So for those people, it was natural to consider the load to be at Z=0, and then you know, make this negative axis. So still it doesn't mean that, backward and forward waves have changed their directions. It only means that, I have changed my co-ordinate system. Okay? When I change my co-ordinate system, the load now will be at Z = 0 and the source, if at all, has, the transmission line has a length L, then will be located at Z = minus L.

You could, of course have used a different notation, I mean, I could have used a different notation, instead of Z, I could have used Z Prime. But you know, I decided to use this notation of Z itself. We will also switch from this notation to a different notation, but I will always tell you, where I'm switching this co-ordinate systems. Okay? It may be little confusing, but for any transmission line engineer, it is necessary to know and work with both co-ordinate systems. Okay? So with Z = 0, located at the load. Right? With a new co-ordinate system, what would be voltage at Z = 0? That will actually be the load voltage, which we will call as, VL. VL will appear here and iL will be. Through this load, iL, I mean, through the load ZL. So the load voltage at Z=0, so this is at the load side. Right? Will be equal to VL, which is actually equal to, V zero plus, plus, V zero minus. Because in these expressions for voltages, Z is now equal to 0. Okay? Similarly for the current i, Z=0, we will have iL, which is equal to, 1 by Z naught, V0 plus, minus, V0 minus. Okay? Now, we can manipulate this equations and then talk about, how much of the voltage that has been incident, which will be reflected back. And we can also tell, how much of the power that we have incident is reflected back. We can also, you know, look at very special cases of termination. We will do all this, in the next module. Thank you very much.