INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

NPTEL

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

COURSE TITLE
ELECTROMAGNETIC WAVESIN GUIDED AND WIRELESS

LECTURE-37
DIFFRACTION-I

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Hello and welcome to Electromagnetic Wavesin Guided and Wireless Media MOOC course. In the previous modules we were discussing channel models, we restricted ourselves to very simple channel models as the course is not sufficient as one or two lectures is not sufficient to cover the various aspects of wave propagation in different type of channels, and especially the wireless channels, there are lot of secondary effects such as multipath fading, that we have not touched upon, and those things take a slightly out of the electromagnetic wave nature and to consider the detailed environment and other things, therefore we have avoided that.

However we did see a simple two ray model, and then we calculated what is called as break point, you know as you will monitor the power that you measure as you move away from the base station antenna you see that the power of course drops and this power drop would initially be around 20 DB per decade as predicted from friis transmission formula that we have seen earlier.

However in reality in urban centers this drop will be instead of -20 DB per decade it would be more than that, it would be -30 DB per decade on an average, and then after the break point is reached, beyond the break point the slope actually changes over to -40 DB per decade, so clearly this on an average this you know, this type of power changes or power variations as you move away from the base station antenna can clearly limit the range of the, or the useful service area of the base station antenna, so if you go beyond that then you need to have one more base station antenna, so in fact this is the basic idea of dividing the geographical area into number of cells, so that each cell is served by a base station antenna, and as you move from one cell, of course it cells are not marked in border areas or something like that, this are mostly dependent on the, or this are mostly defined on based on the you know abstract notion of a cell that divides up the region of cell, you know mobile services that we are talking about, so there is no fences around the cells, but it’s rather the conceptual idea that as you move away from the base station antenna the power is going to drop, and once the drop becomes so much, that is the power drop becomes so much or the attenuation becomes so much than that are receivers will not be able to detect any more power, which of course is necessary for us to demodulate the information that is being transmitted from the base station antenna to the mobile antennas or mobile receivers, so when that cannot be done, that is when the power drops below the minimum specified power levels or the sensitivity of the receivers, then that would be considered to be the edge of the servicing part, right.

So we have seen that this long term average thing is, or long range you know power loss or attenuation is one phenomenon of that one, and then we saw that around this long range you know on an average power variation, there could also be variations as you move from one part of the center or you know urban area or semi-urban area to another part of the center and this happens more frequently in the urban areas because of the trees, buildings, poles, and any other you know big obstacles that are present, okay, so the effect of this is sometimes the signal would gain, sometime the signal would be lost and so in some sense there would be variations around this long range, or a long range attenuation that we have seen.

On top of that diffraction effects or what is called as shadowing losses, there could be minor changes as you move over a wavelength range or small, couple of wavelengths range and that minor changes in the or rather those are major, but those dynamic changes which actually happen as you move very closely away from the, in terms of certain wavelengths from any given position is called as multipath fading, so we have seen all that, and we have seen that break point analysis can also be developed.

What we have not seen and what we have just mentioned is this phenomenon of shadowing, right, shadowing is something that is very familiar to you, so you are talking to your friend, or you go behind them you know exit a building or just go behind the building, immediately the signal drops, right, and in some extreme cases you can keep saying hello, but the other person doesn’t receive anything, clearly indicating that the communication path has been broken, right, so this is called a shadowing, and this is one of the most prominent effects of urban mobile communication systems, right, so in order to overcome this shadowing either one has to place more base station antenna, so that the coverage is more which of course has its own problems in terms of health hazards, in terms of regulations, and in terms of simply people refusing to let their areas rent out, you know to put more towers, so that is the problem that is associated with that.

The other way is to really understand why this loss is occurring, and possibly can we do something about that, okay, so maybe one can actually have you know region of big buildings or in a group of labs or an institute, the smaller antennas which kind of interface with the main or the base station antenna catch a portion of the signal and then serve as hot spots, right, so like not exactly the hot spot that you are familiar with, you know where you put the wireless routers and other things, but these hot spots are made out of small miniature antennas which serve no more than a few feet, right, so this way one can, I mean you know to even do that and where to place this antenna and other things we need to understand this phenomenon of diffraction which is actually causing the gain variations to occur, right.

So in order to understand that what we are going to do in this and possibly in the next module as well is to study this phenomenon called diffraction, now please do not infer from whatever we have said that diffraction is only limited to understanding of wireless channels, absolutely no, diffraction shows a pin places in everywhere wherever there is a electromagnetic wave involved, in fact it shows up even in places were acoustic waves are involved, okay, these goes under the general name that electromagnetic waves or light waves bend around the corners, right, I'll come to that one in a moment or so, but this is a phenomenon that you must have you know, you have seen, and if you haven't seen what you can actually do is squint your eyes towards any light source, okay, so just try to close your eyelids, you know, not completely of course when you close it completely you won't see anything, but you just close it such that you are seeing some amount of light, so as you close what you have created you know this small area that you are seeing is what is called as an aperture.

Aperture is a fancy term for an opening, okay, so what you have done by closing your eyes to or bringing your eyes closer to know each other the upper, this thing, is that you have just letting light pass through this small aperture.

Now from high school physics you have been taught that light is you know ray, and rays pass through in a straight-line, and if there is an opening then light will actually pass through that opening and everywhere else it would be black, right, so if you, you know imagine that you have some sort of an aperture here, okay, and then light which is light source which is kept far away from this, then what you expect on the screen that you would place at the back would be that, please note that this is the aperture, so outside this aperture whatever we have is all opaque screens, right, so of course you won't see it in this way, you would see it in this particular manner, so you may imagine that you have an opaque screen or an opaque block or something, and then there is a small opening here, I have shown the opening to be quite large, but this opening should be just about few wavelengths of order, okay, it should not be too high, rather it should not be too much, too wide, then you won't be able to observe this effect, right.

So as you squint your eyes and then look at the light source, this is what you are actually doing, so there is a squinted eye, light source is somewhere kept on the wall, quite far away, a few feet away and then that light rays would travel through this aperture, right, so from whatever we have learnt in high school physics if that was true, then let's assume that the light rays are essentially uniform plane waves that are coming through us, and these rays as they, you know the central ray would of course pass through as it is because it is not seeing any obstacle on this side, however what you would expect is that the edge rays would simply transmit through this one, right, in fact this region is what we call as shadow region, right, because no light is supposed to pass here, and if you measure the width of this aperture or rather if you know the aperture width to be A, then the light width would also be exactly equal to A is what the geometric optics from high school physics would tell us, okay.

However in practice what you would see is that these rays do not essentially remind in that manner, but they kind of spread out from this, so where you did not expect any light, so this was no expectation of light, right, there was no expectation of light because we assume naively that the rays would actually, the edge rays would actually propagate in a straight line, and they would hit this particular observation screen and whatever this one would also be, I mean all these things would have been the shadow region, right, earlier this would have been the shadow region, but unfortunately that is not true, there will be light in this region, in fact if you can effort an extremely sensitive light then you can also observe some amount of light here, because this width and of course if you move this away, not at the given position, so if you move this position away to a new screen that you can put somewhere at a further distance from this, then you would actually see that light spreads out even more, okay.

So in some sense if you consider this as earth and this as moon, right,
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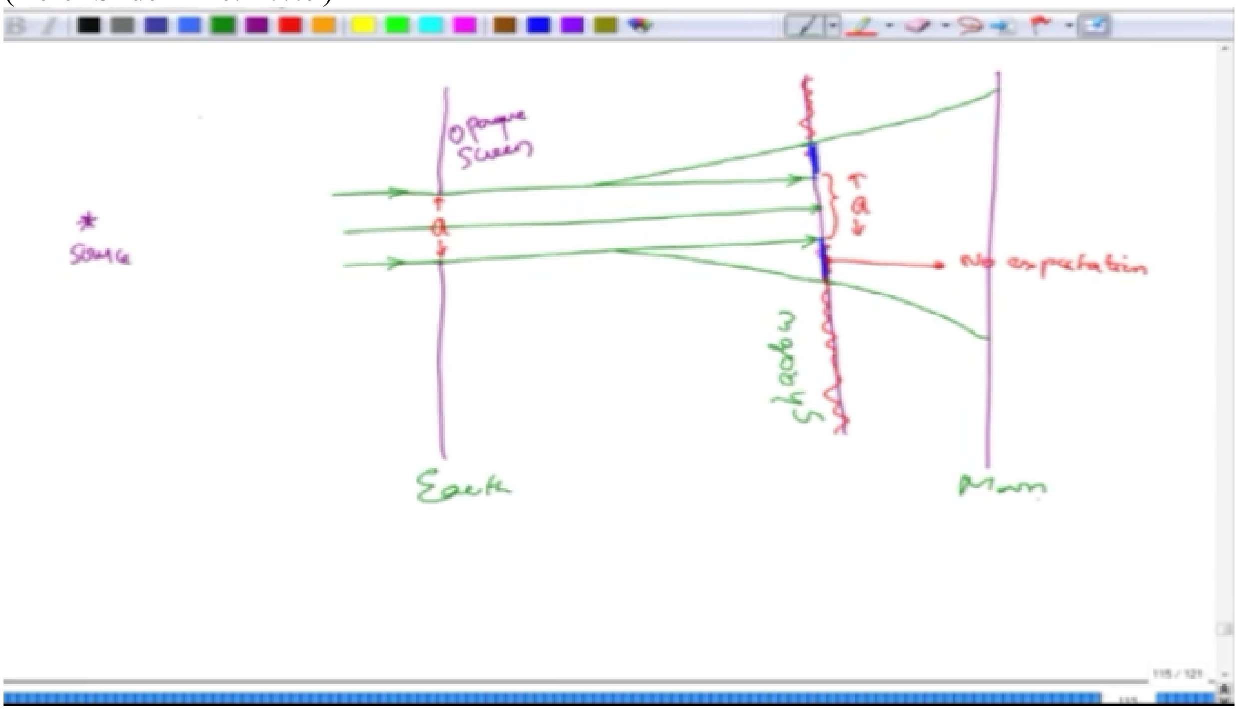

and you want to observe moon by sending light, then if you’re sending this kind of an ordinary light, the diffraction effect, because the distance between earth and moon is so large the diffraction effect would actually move you know or would create a spot of light which is very wide, perhaps as made as moon and as the you know beam size increases the energy that you would see at a, on a particular you know patch of intercepting this particular you know electric field or the light wave would be very limited, this is just like an antenna right, so you have an antenna whose radiation pattern you can imagine, keeps going wide and wide, and if you have a receiving antenna that is only so much, then it will impact or it will you know perceive only part of this light source, and because light as it spreads out it has spread out its energy in a wider range, so when you probe only a small area you will actually see very, very little amount of light being intercepted, this is in fact the concept of effective aperture that we have already talked about in the context of an antenna, okay, so there is also a very close relationship between diffraction phenomena and antenna radiation pattern, perhaps if you take up a course on antennas you will discover this relationship, the basic idea is some antenna patterns can be considered to be diffraction patterns, okay, not all antennas, some antennas, those antennas are called as aperture antennas.

Anyway coming back to our discussion, the basic idea is dispersion causes the beam to actually spread out, okay, so this spreading out phenomena is you know creates light where there was no light to begin with, right, so this phenomenon is called diffraction, and we want to understand this diffraction, so we will turn to some simple models of diffraction because the subject

although originated mainly in the context of light, and it was used as kind of a precursor to tell us about light phenomena that cannot be explained by the usual corpuscular theory which was quite popular during Newton's time, so when they tried to explain diffraction they just could not explained diffraction based on the corpuscular theory, however wave theory was you know invoked and wave theory was actually you know useful in explaining this, you must probably have heard off what is called as you know this Huygen principle which says that light incident at every point on this aperture would actually be equivalent of creating small secondary waves, and these waves would then go away in this particular manner and these waves when they intersect at a screen will create the complicated diffraction or rather they will complicated interference they will create the diffraction pattern, so this phenomenon is called as Huygen's principle, sometime Huygen, sorry Huygen, Huygen is also spelt out as Huygen,
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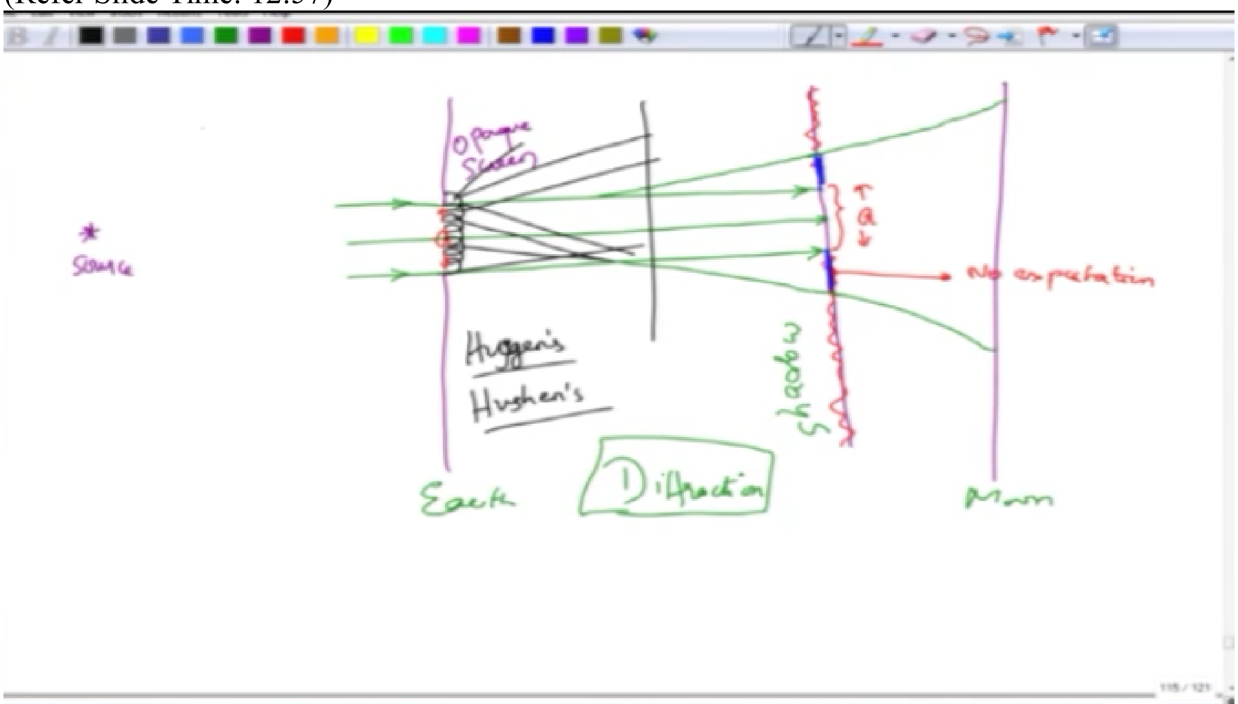

so either way the person was actually responsible for lot of interesting work in during the early optics area, and this thought of or this ideas of spherical wavelets and secondary waves producing you know further waves was basically because of this person.

However Huygen's ideas were put on a firm mathematical or reasonably firm mathematical foundation by this well-known scientist whom you have heard in terms of circuit laws as well called Kirchhoff and this Kirchhoff Huygens theory of diffraction was kind of quite useful or this one, of course the first person to do this was Fresnel and then Fresnel Kirchhoff or Fresnel Kirchhoff ideas were you know kind of the first mathematical steps to put this Huygens principle on proper mathematical basis, and later this was amended by a person called Sommerfeld, okay,
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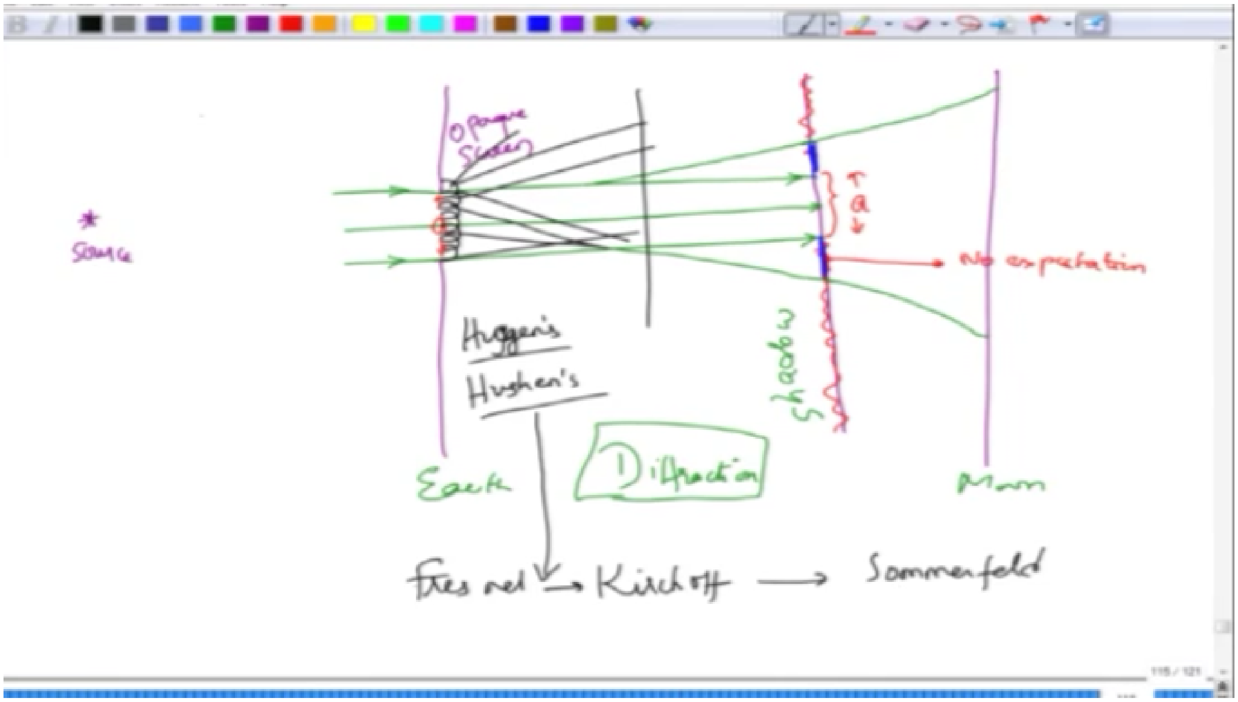

so today we use this Sommerfeld in diffraction integral to understand the effects of diffraction, okay.

Anyway the goal is not to derive this because this theory is very, very complicated, and although as I said it originated in the context of light, and understanding the effects of this light phenomena, it turned out that it is very difficult to solve diffraction problems when light is involved and apertures are involved, but rather comparatively, comparatively it is easier to you know understand this phenomena in the acoustic waves, and a person called Lord Rayleigh, a scientist called Lord Rayleigh who also won noble prize did lot of work in acoustic wave diffractions, okay, so he published a very good text book called as The Theory of Sound, and if you get an opportunity to read that one you will read how sound waves can be analyzed and how sound waves diffract and all those curious phenomenon of sound consider as a wave was actually derived, most of it was you know derived by Rayleigh, Sommerfeld, and these guys, and Rayleigh's work is specially I would recommend it, if you get an opportunity to read that one, okay.

So what we are going to do is to kind of get to Sommerfeld integral or Kirchhoff Sommerfeld diffraction integral, but without going through to many mathematical steps, okay, so yeah, before we go to that I'd like to mention this point, which of course I have mentioned, so if this aperture opening was A, or the radius of this one was A, or the width of this one was 2A, then this spot from geometric optics would predict that it would be A, but wave would predict that it would be actually greater than A, right, so because that's the whole point of this diffraction, so while we would have thought that in the shadow region there should be no light, but that strictly speaking it is not really so.

And coming to Huygens principle or Huygens wavelet, what he have assume was that everywhere you know light is incident on this aperture, it would actually create a spherical wave, okay, or a spherical wavelet, and spherical waves are given by, they are dependent only on the radial direction that is as you move away the amplitude decays and we call this as spherical wave because with some amplitude let's say size 0, they exhibit two phenomena, one would be the amplitude decay in terms of 1 over R, and the other one would be the phase change which actually occurs as E power –JKR, right, so the KR is the phase term, of course I'm using the phasor notation, the correct expression would perhaps be sai (RT) real part of sai 0/R E power J omega T-KR, correct, so this is what the correct expression for the electric field would be or the magnetic field.

See we have made one major assumption here, we have discarded the vector nature of the light, we do know of course that light consists of electric field and magnetic field and the proper way to address light, especially light propagating free space would be to actually think of light as a vector field, right, however the vector diffraction theory is much more complicated than the corresponding scalar diffraction theory, okay, so we will not consider the vector diffraction theory, we will assume that we can represent light that is falling on the aperture or light propagating or the base station antenna transmitting a certain radiation, all these electromagnetic waves can be described in the context of diffraction as some simple scalar fields, okay, so we are ignoring the vector nature of this, but the assumptions or the approximations that we make are not so strict, most of the diffraction phenomena can still be explained in the context of scalar diffraction integrals, so that is what, that is why we have chosen the scalar.

And we have chosen a new variable sai, because the sai could be EZ, XZ, EX, HE, HZ, any of those components, it could also be ER, HR, you know all those components it could be, so I'm representing everything collectively as a scalar sai (R).

And as I have said this is in the phasor domain, this is the real time domain, time or rather the time domain wave forms that you are actually looking at, or the fields that you have, we will work with phasor domain so that I don't have to write this real part as well as this E power J omega T part, okay.

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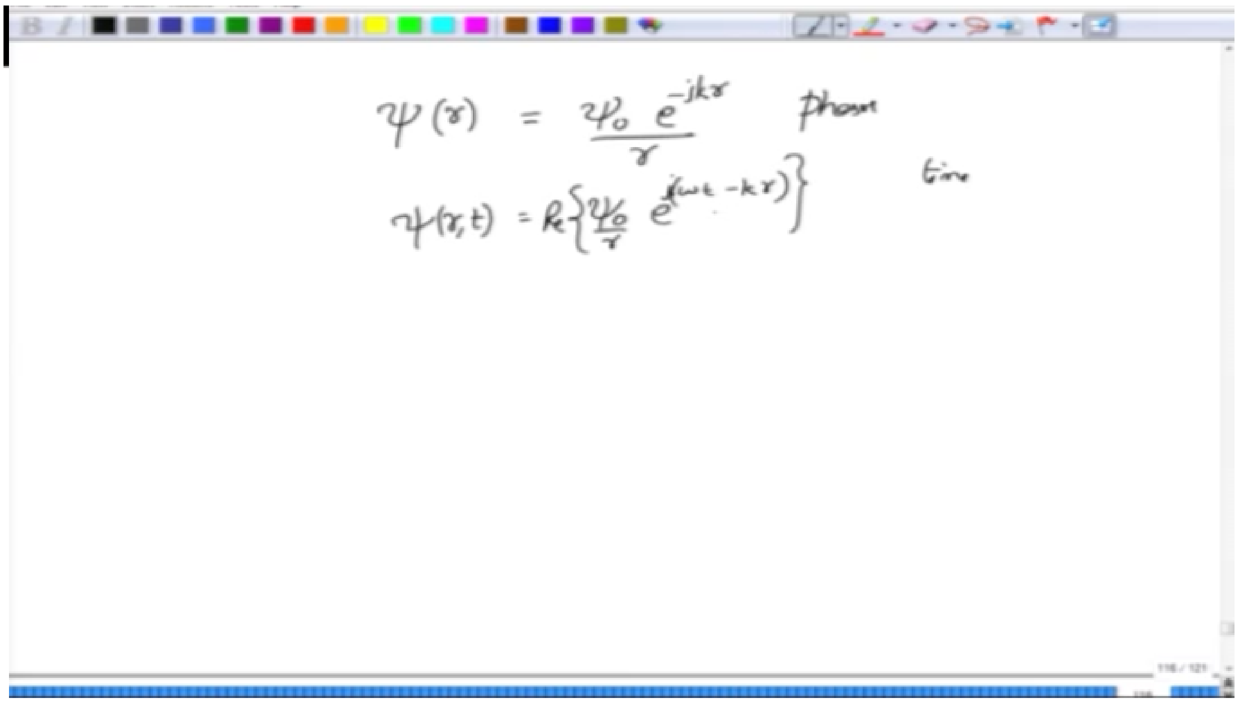

So this is called as a spherical wave, why is this called as a spherical wave? Because if you interpret this as electric field, then the power density of this one would be proportional to magnitude of the electric field square, right, so it would of course be E x H, however in the scalar approximation H is kind of sai(R), E is also sai (R), so therefore and because sai (I) is a complex number, and energy has to be a positive quantity, so we will just take the magnitude square, so when you take the magnitude square the phase term will of course drop in, but the sai(R) magnitude square would actually be sai 0 magnitude square divided by R square, right.

So now if this is the wave which is actually falling, the power is falling as 1 over R square, the overall power will be what? Power will actually be constant, remember power will be obtained by integrating this one over the solid angle D omega right, and when you integrate this one R square and R square here will cancel, so there is some sine theta and D omega is basically D theta, D phi, actually D omega is R square D theta D phi, I have just kind of written this R square explicitly show that this R square can cancel out, and this quantity assuming that sai 0 is independent of sine theta,

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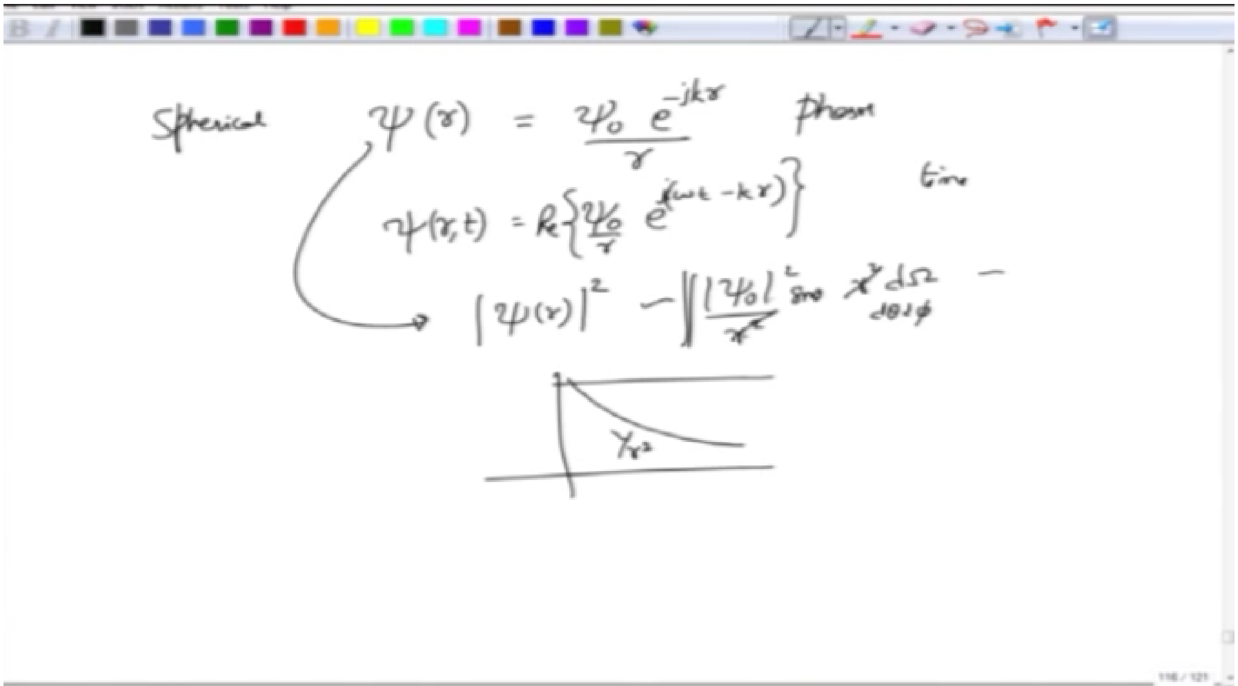

then this would actually be a finite value, right, so remember theta goes from 0 to 90 degrees, not 0 to 80 degrees, and phi goes from 0 to 2 pi, right, so the power density goes down as 1 over R square, but the power itself stays constant, right, and this is the first assumption of Huygen.

So we are going to erase all these, what I'm interested is to let you know that this is a spherical wave, and once we understand that this is the spherical wave we can go on to the next step, okay, we will assume first of all that the source itself which is placed at some point, itself emits a spherical wave, okay, so I mean this also okay, because we have consider the source to be very far away and we assume that the source itself is going to emit a spherical wave, so in effect what we are assuming is a point source, okay, so we are assuming a point source.

If you don't assume a point source, but then you assume that the electric field or the light wave could actually be given by a certain point P, right in the XY plane, so this is the XY plane that I am considering, so this is on the XY plane, and this point we will consider the source, I'm not assuming source to be a point source, we will later on make that assumption, but the development of the theory will be slightly easier if I don't make this assumption, okay.

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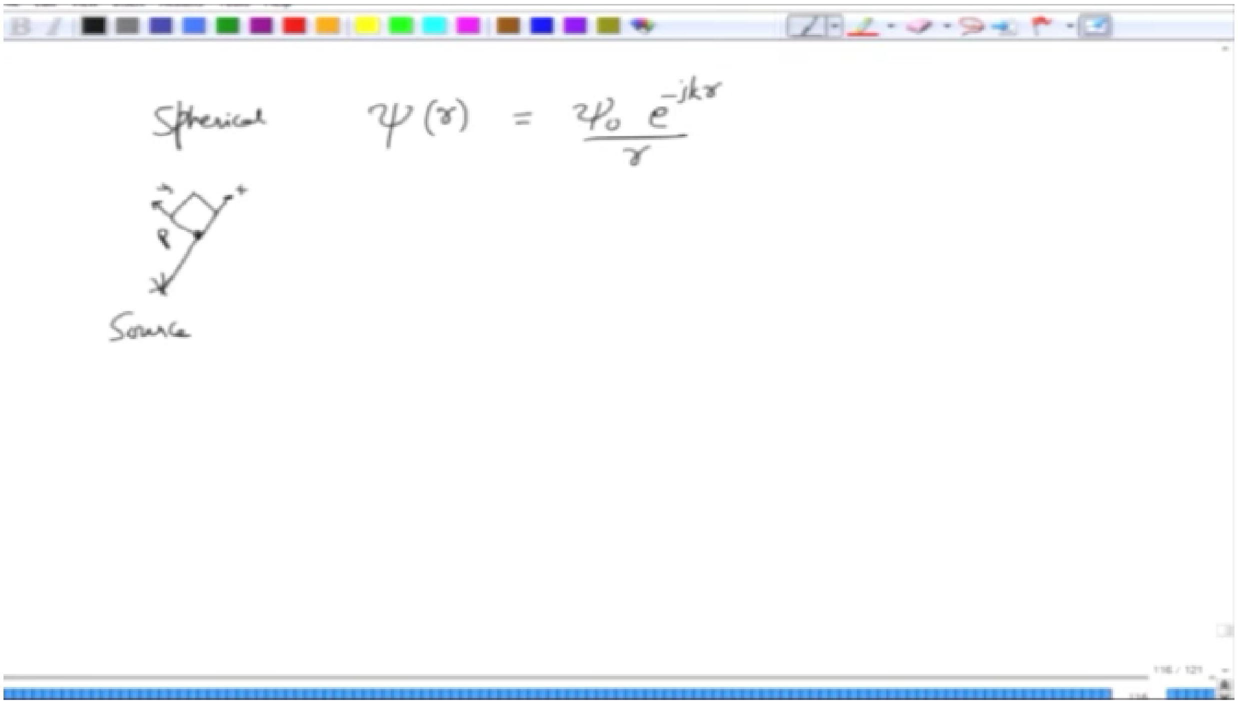

So this is a source and this are all the points, right, the source actually has a certain finite extent, so the source has a finite extent and I'm considering the source, the part of the light that is being emitted at this point which I have labeled as point P, okay.

Far away here is where my aperture is located, okay, so I have this aperture here which I will draw it, I have only indicated, I mean I'm indicating a circular aperture but this is clearly not limited to a circular aperture, okay, so this is my aperture which I will call as A, and there is a straight line which you can draw, which goes from point or rather there is an origin for this aperture in that particular Z plane, so this is a constant Z plane, so clearly this would be some X axis, this would be some Y axis, and the plane that I have plotted here is also similar, so it's an X axis, Y axis plane, but the distance between this origin to this origin would be different,
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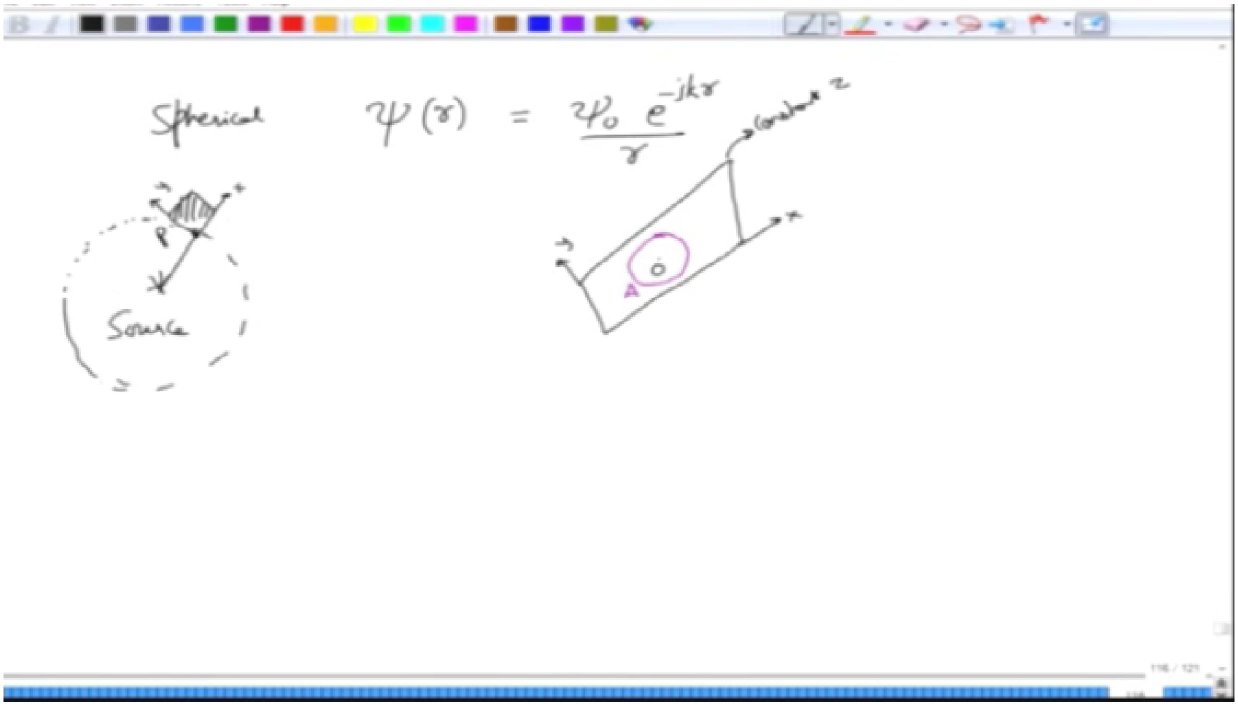

and from the distance between the source point on to the origin can be given and indicated as R, okay, so by indicating this as R, or rather R0 I will indicate this as the distance from the point of the origin and the aperture plane on to the point of the source which I'm considering here, so this is the part of the source.

Of course there will be some portion of source here, portion of source here, here, these are all potentially points that I have to consider, I've made it slightly generalize to the sense that I'm only considering the point at this particular point, okay, so at this point there will be a spherical wave, so you have to imagine that there is a spherical wave and if I consider a point Q here on the aperture and indicate that Q has a you know the coordinates X0, Y0 and 0, remember I'm considering this as a constant Z plane, so I can take this plane to be at Z = 0, okay.

I will also further assume that the radius of the aperture A is very small compared to the distances that are involved, so initially the distance that from the point P on to the Z plane where the aperture is located we have indicated this to be R0, so we will assume that this A to be much smaller than R0,

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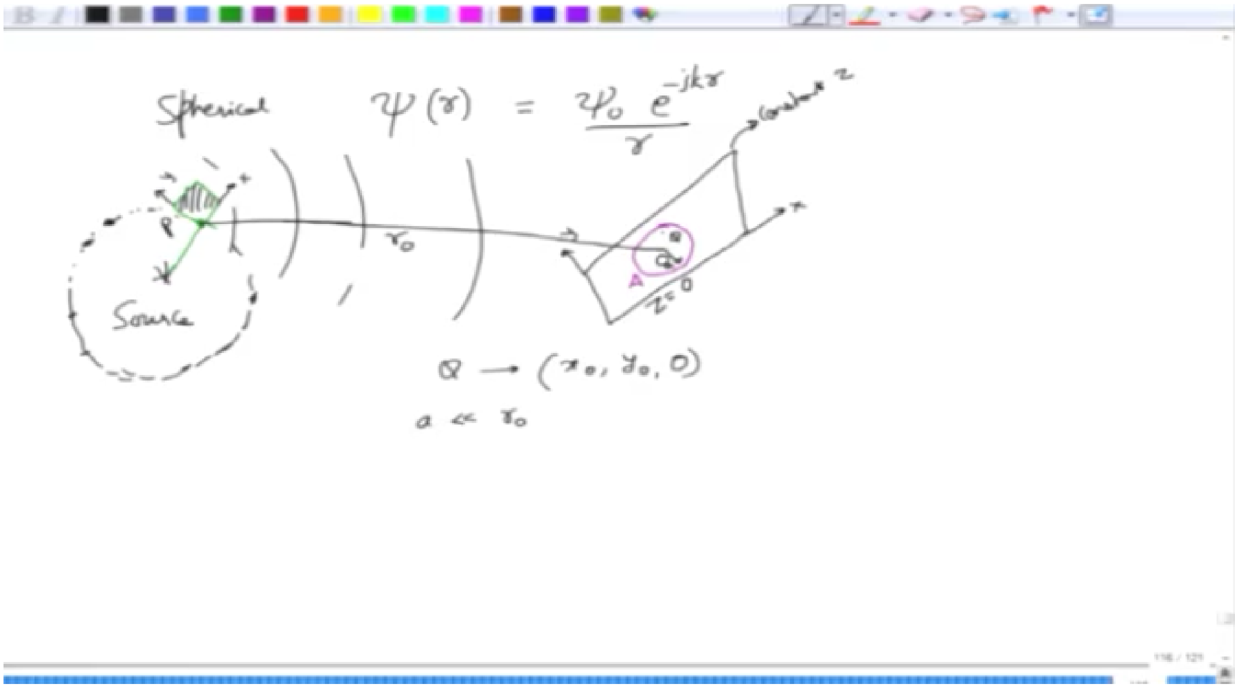

so essentially looking from the position of the source and on to the aperture, the aperture should look very, very small, okay, it should look like a very small opening, it should not look like a big cave and you know you have a source which is very small like a laser pointer here, no that is not what we are looking at, what we are looking at is in the other way around, so you have a light source which is descent enough, but the aperture is very, very small, okay, so that's what we are looking at, and I have indicated the point Q on the aperture because the light from point P would reach point Q here, okay, this distance of course is R, you know that this is the R value is the one that actually matters because the spherical wave may actually begin to you know emanated from point P,

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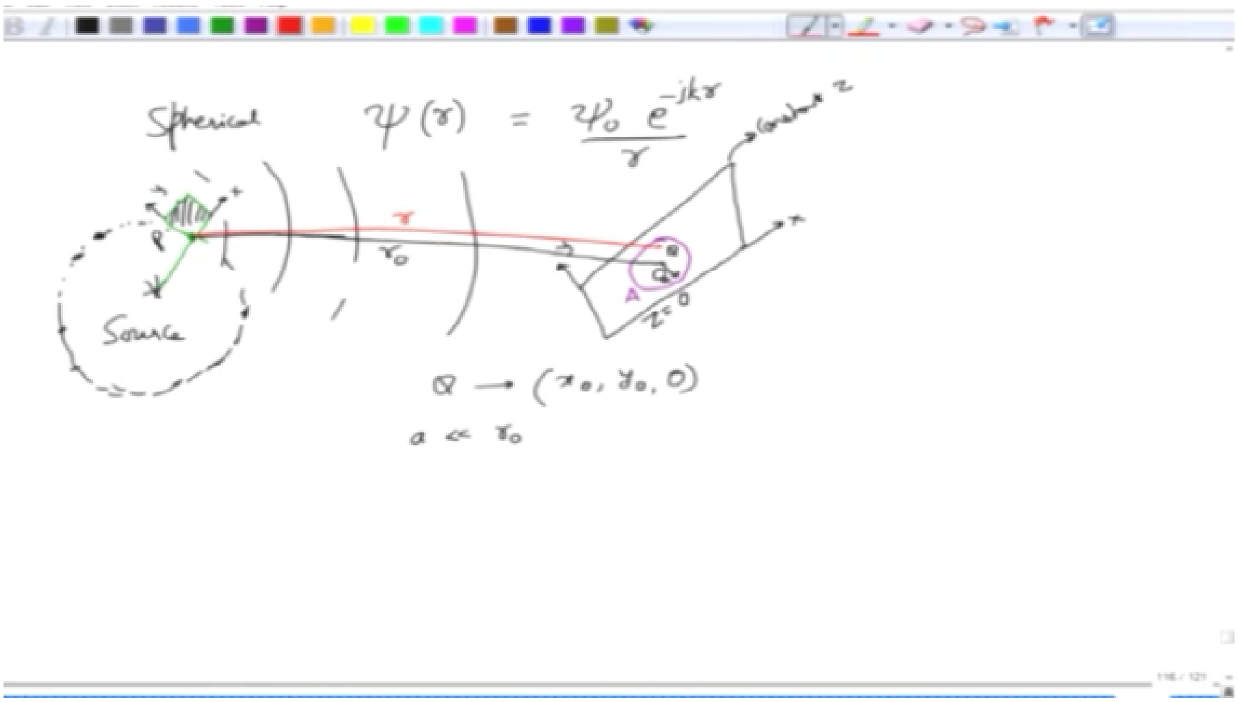
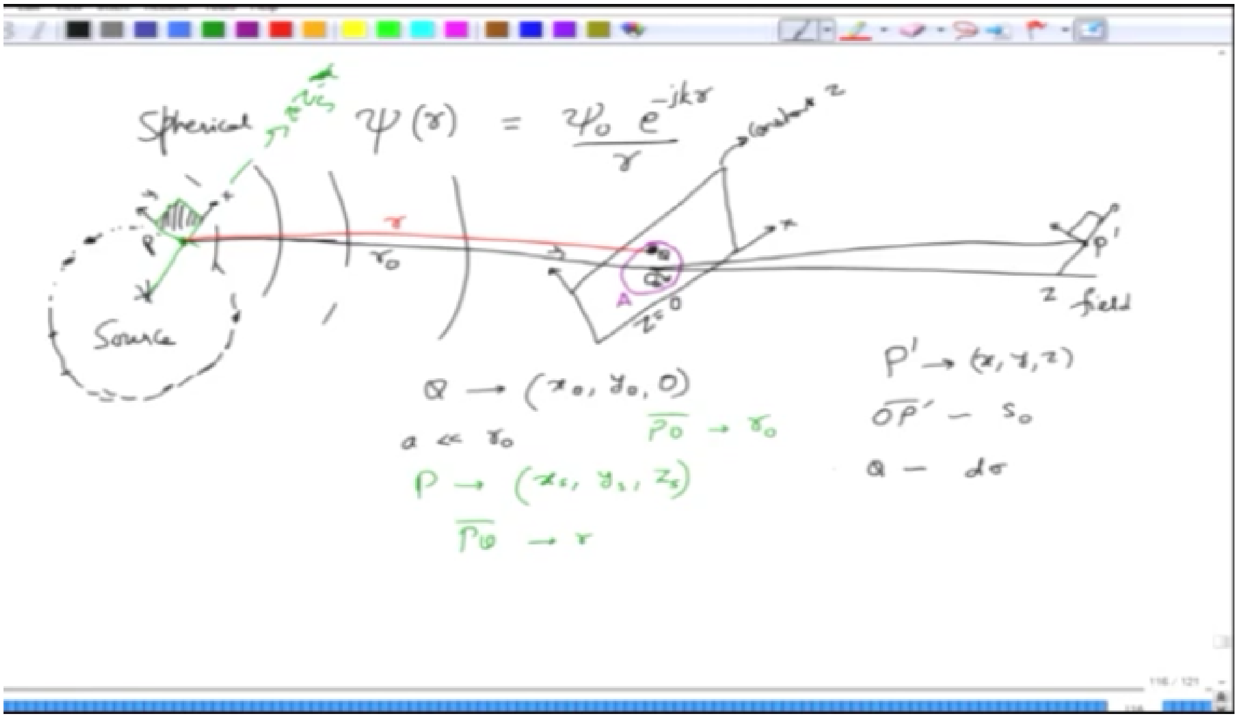


the amount of attenuation that the wave would undergo would actually be given by this distance PQ, which we have indicated by R, okay.

Incidentally P would be identified by the coefficients which we will call us I mean coordinates, which we will call us XS, YS and perhaps ZS, okay, meaning that this is located at the, that is this plane is located at ZS = constant, or Z = ZS, okay, so the distance PQ is R, and the distance PO is R0, so these are all up to the source point, okay.

Now we will imagine that I'm going to go to another plane which of course would still be the XZ plane, but at this plane I'm going to consider the field point and label it as P prime, and then I will have the same you know X and Y directions that I have indicated, so this plane would be located at plane of Z, and the coordinates of this one, this point P prime will be X, Y and Z, okay, from the origin O of the aperture if you go to a point P, so the distance OP prime we will call it as S0, and because light has been now incident on point Q you can imagine that there is a small area around point Q, which has the area which we will designated as D sigma, okay, and this D sigma corresponds to the infinity symbol area from which the part of the spherical wave (Refer Slide Time: 25:15)



that has been incident on Q would then begin to propagate under each the point R, okay, so I'm going to write down this point, from the point where the light source has been incident Q, so please note that they should have been you know straight lines but on this board I'm not able to draw them correctly, okay, so we will call this distance as S0, and we will call this distance as S.

Again we ask that this A, B very far away from this S0 or equivalently S, okay, this assumption that the aperture looks very small, located from the source as well as from the object or the field where we are looking at is what is called as Fraunhofer limit,

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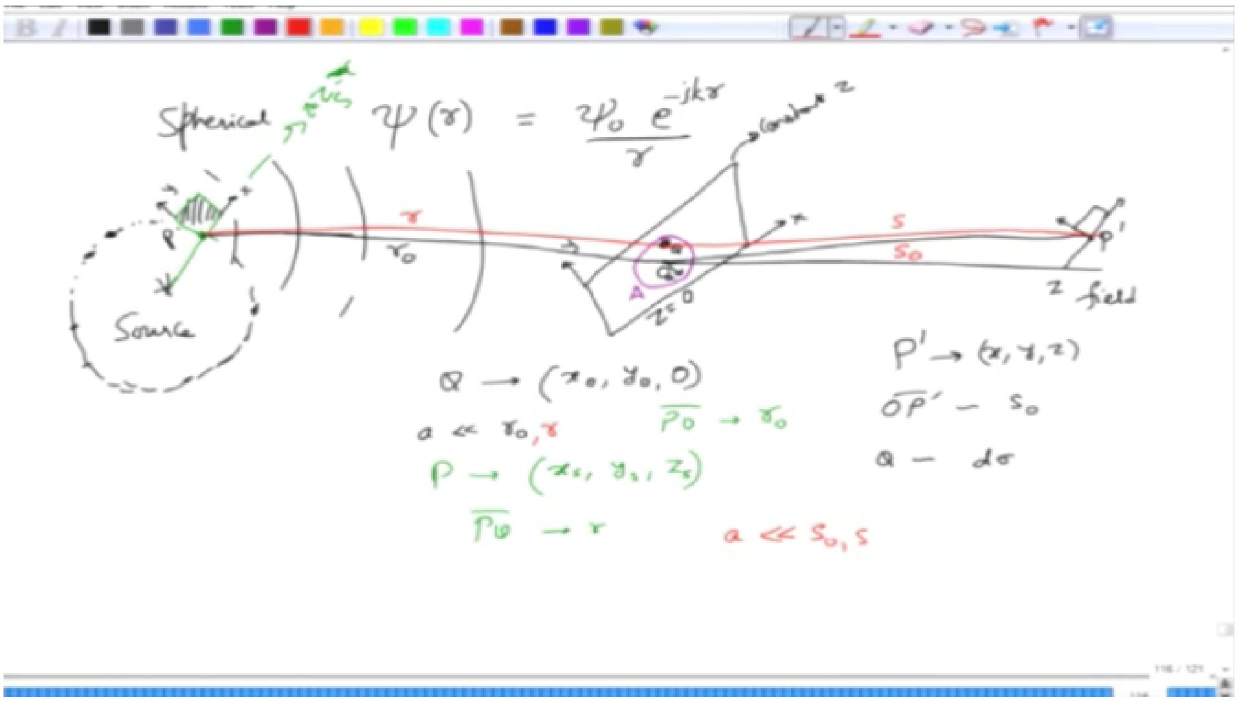

okay, and the corresponding diffraction is called as Fraunhofer diffraction, there is a different formula which we will develop shortly, which actually gives the limits of where the Fraunhofer diffraction typically begins, okay, but for now this is our geometry, and the basic idea of developing this Fraunhofer diffraction integral, simply the diffraction integral is to recognize three things, right, first the source at P prime would actually generate spherical wave which would propagate and land at sai(Q) okay, from sai(Q) there will be a secondary wave which is also spherical wave, so this is also spherical wave, and sai(Q) would also be a spherical wave, with this spherical wave as the kind of an input in some sense you will then land up with the field at the point P prime, okay, sai(P) was the source point, and then sai(P prime) is the field point, okay, so the source point which is kept at XS, YS,

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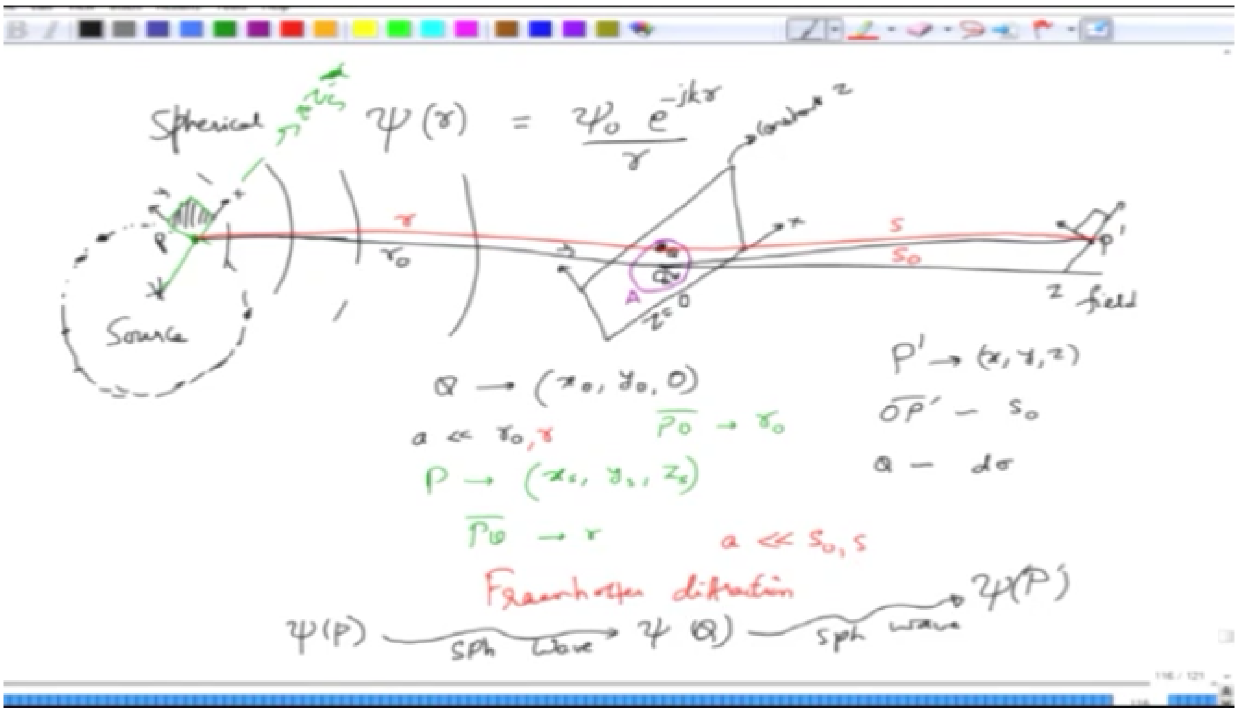

and ZS right in that particular point would first emanate a spherical wave and then you would land up with that spherical wave at point Q, from point Q you would again have another spherical wave because of Huygens principle and then it would land up the amount of certain thing would I mean the spherical wave would have, part of the spherical wave would land up at point P prime, which is defined or which is given by the coordinates X, Y and Z, okay.

You could of course observe two things right, one we have neglected the obliqueness factor from P prime to Q, that is we haven't yet written that but we are going to neglect it, in the sense that whenever I write this P prime to Q, the direction of the P prime Q vector or the line must also be taken into account, because P to Q and of course Q to P prime this is also second same observation, they will be different if you go to a different point, for example if this is my other source then from P to Q the direction of this line will be different from the direction of this one, right, so you can clearly see that this directions are different and this factor in both source to the aperture and aperture to the observation plane has been neglected and this is called as the neglecting the obliquity factor, okay, the fact that source and fields could be oriented in different directions is being neglected.

Since this is coming to an end this module, we will of course continue this work in the next module, I will then tell you another assumption that we are going to make which would simplify on our analysis. Thank you very much.

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