## **INDIAN INSTITUTE OF TECHNOLOGY, KANPUR**

## **NPTEL**

## **NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING**

# **COURSE TITLE ELECTROMAGNETIC WAVESIN GUIDED AND WIRELESS**

# **LECTURE-36 FURTHER DISCUSSION ON WIRELESS CHANNEL MODELLING**

**BY DR. K PRADEEP KUMAR DEPARTMENT OF ELECTRICAL ENGINEERING IIT KANPUR**

Hello and welcome to NPTEL MOOC on Electromagnetic Wavesin Guided and Wireless Media. We were discussing the two ray model of simple wireless propagation, well I should say that simple two ray model of wireless propagation of electromagnetic waves, and we arrived at a very interesting point where we had electric field from the direct path as well as electric field from the non-direct path expression for these two, and we had just begun to combine them and I hope you remember this particular picture





which was the two ray model picture that we were taking, and then I told you that because the angle theta that comes you know that would be here denoted here are essentially equal, then

their cosines are also equal which allows you to express this horizontal distance D prime in terms of the range R as well as H1 and H2, right.

Similarly you have D double prime, so obviously D1 if you now look at what is D1, D1 will be given by this triangle, okay, so I will mark the triangle here, so this is the triangle that you would actually use to express, I mean obtained an expression for D1, and you can use this triangle with D1 as the hypotenuse, D prime and H1 has the 2 sides of the you know, of the triangle, so D1 will be equal to square root of D prime square which is the horizontal distance + H1 square.

You can similarly show that D2 you know from this triangle that I have that I can write D2 as square root of D double prime square  $+ H2$  square, okay, and then you can substitute for D prime here and you can substitute for D double prime into this expressions, and then simplify the two expressions,



(Refer Slide Time: 02:04)

I will leave this as an exercise for you to do, this would be  $D1 + D2$  = square root of R square + H1 + H2 whole square, okay, because when you add them up this H1/H1+H2 and the other thing will actually be removed from that, okay, so you can actually express all this in terms of D1 and D2 as the sum of this indirect path propagation as a simple formula here.

Now you also need to relate D, H2 and H1 and that relationship can be obtained by simply projecting you know, right, drawing a line from H2 on to the base station antenna tower, right, so this will intersect, so this height is H2, obviously this height will be H1-H2, now you look at a triangle that is this one, okay, and then write D as square root of R square, because the length is about R and so this D will be equal to R square  $+ H1 - H2$  whole square, okay, so this is D1 + D2 and this is D, (Refer Slide Time: 03:19)



these expressions are important in the sense that you are going to combine this two expressions in order to obtain what is delta D, remember delta D is the path difference between direct and indirect path, so you have  $D1 + D2 - D$  as delta D, okay. (Refer Slide Time: 03:34)



Now because  $D1 + D2$  we had already written here, so once again so we have already written  $D1 + D2$  here, and you can then show that this would be, so you can add them  $D1 + D2 - D$ will be square root of R square + H1 + H2 whole square, minus square root of R square + H1 – H2 whole square, okay, you can check my calculations whether I've done this expressions for

D1 and D prime correctly, so if I have done that one then this is alright, so you can show that these expressions are fine,



and once you have these expressions with you then you can simplify this, okay, what simplification you can do is to take these  $D1 + D2$  expression, I'm going to erase this you know picture here, I hope you have noted down the picture, you have recreated the picture on your notebook so that you don't have to rely on my pictures here.

So what I'm going to assume is that R is much larger than  $H1 + H2$ , we have already made this assumption because we have individually asked for R to be much, much higher or larger than H1 and H2, we are now also going to say that R is much larger than  $H1 + H2$ , so which allows you to remove this R and then write this as square root of  $1 + H1 + H2$  square divided by R square, which you can further approximate using binominal theorem as  $1 + H1$  H2 square divided by 2R square, okay, so this is what you are going to get.

Next you have R into square root of, sorry you have this term which is R square  $+ H1 - H2$ square, so following the same ideas here you can write this as  $1 + H2 -$ , I mean  $H1 - H2$  square divided by R square employ binomials theorem again to write this as  $1 + H1 - H2$  square divided by 2R square, okay.

And delta D is basically the difference between this expression and this expression, clearly R will cancel or rather 1 and 1 will cancel from each of these expressions and you pull R into it, so that this R in the denominator will become instead of R square it will become R, so this delta D will be given by  $H1 + H2$  square divided by  $2R + H1 - H2$  square divided by  $2R$ , you can you know use your usual A+B whole square formulas, and find this thing out that it would be 4 H1 H2 divided by 2R which is basically you know 2 H1 H2 divided by R, okay, (Refer Slide Time: 06:21)

$$
\frac{B}{\gamma} \sqrt{1 + \frac{(b_1 + b_0)^2}{\gamma}} = \frac{\gamma}{\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] - \frac{\gamma}{\gamma}
$$
  
\n
$$
\gamma \sqrt{1 + \frac{(b_1 + b_0)^2}{\gamma}} = \gamma \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] - \frac{\gamma}{\gamma}
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] - \frac{\gamma}{\gamma}
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] = \frac{\gamma}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] - \frac{\gamma}{\gamma}
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] = \frac{\gamma}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] = \frac{\gamma}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right] = \frac{\gamma}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\n
$$
\frac{1}{2\gamma} \left[1 + \frac{(b_1 + b_0)^2}{2 \gamma^2}\right]
$$
  
\

so this is the expression that you are going to get, so D1+D2 and we wrote D as this thing, so in here what we have actually done is to write an expression for delta D which you can push into the expressions for the electric field and then obtain the simplification there, but then we still have to leave with one additional factor here which is basically D/D1+D2, okay, so because we don't have that relationship we did not develop it, again that's not too difficult to develop from that one, so you already have D1 in this particular expression, so  $D1+D2$  square root of R square + H1+H2 square, okay, and in place of R square you can write D square – H1-H2 whole square, so we'll come back to this approximation shortly, okay, (Refer Slide Time: 07:18)

$$
\frac{d}{dt} = \frac{h_1 \times 2 \times 3 \times 5 \times 7 \times 8}{\sqrt{1 + (\frac{h_1 + h_2}{\sqrt{x}})^2}} = \frac{1}{\sqrt{1 + (\frac{h_1 + h_2}{2x})^2}} - \
$$

this approximation shortly but we also need because of the electric field that we have written in the previous module, we have this term D/D1+D2, we need that ratio, so I'm going to look at that ratio by substituting for R square into this expression, so substitute R square into this expression and then write, okay.

So I'm going to write another expression here, sorry for all this math, but you can recreate all of this math, this is just some simple trigonometry and some algebra that we are putting in, so D1+ D2 on to the left hand side will be equal to square root of R is basically D square, then you have H1+H2 square – H1-H2 square, okay.

So clearly you can simplify this further, this  $H1+H2$  square  $-H1-H2$  square we have already written, in this case delta D that we had written I should have put in the minus sign, so otherwise they won't really add up, I'll not get the correct sign, so sorry about that, but in here you can see that this approximation that is  $A+B$  whole square  $-A-B$  whole square is essentially the same approximation that we have already or rather same expression that we have already written, that would be 4 H1 H2, okay.





Now I can take D as a common factor out, and then have  $1 + 4$  H1 H2 divided by D square which again I will rewrite using binomial expression as  $D1+D2$  on to the left hand side, D into 1+ 4 H1 H2 divided by D square divided by 2, okay, so this is approximately, well this is not approximate this one was the approximate thing, so this is equal to D into  $1 + 2$  H1 H2 divided by D square, okay, so we get an expression for D1+D2, we also have an expression for delta D, the expression for delta D will have the horizontal distance R which is what the meaningful distance for us, right, why? Because imagine that there is a base station antenna up here, okay, and then you have you know this is an actual environment that you have, so maybe there is a building here, then there is a tree here, there is another tree here, then there is a building, and let's say this is the path in which the vehicle is moving, so that this is the path the vehicle has

moved, so what we are of course interested is to find out how the received signal would vary at each of these paths,



but we are also interested because it's very difficult to navigate and find out every point what would be the electric field, we are most often interested in this horizontal distance which will tell us as I'm away from this base station with the certain range R, what is my received power, okay, so this is the true path, the black one that we have written is a true path and that is what we should have used in the evaluation, but in many cases because of the problems you know you have to use the traffic, I mean there is traffic going on, you have to isolate the traffic all those things are not going to happen, people will not stop their cars, so stop their scooters and let you do all the measurements, right, unless the government mandates it, so it is not possible for you to think of the true path and then look at the electric field at every path, right, rather you will use some kind of a you know to some experiments but then you also interested in simplifying your assumptions or rather simplifying your scenario and one such simplifications is to simply calculate the distance from the base station antenna, and that is what this expression will tell you 2 H1/H2 by R is the path length difference delta D which goes into the expressions later on, right.

So this expression will come back to it, but right now we have a nice interesting you know expression which relates  $D1+D2$  and D, so in fact you can write the ratio  $D/D1+D2$  as  $1+2$  H1 H<sub>2</sub> divided by D square, okay, so you can write this and then you have  $1-1+2$  H<sub>1</sub> H<sub>2</sub> divided by D square, and then you have E power –J delta or rather B, delta D, right, so now delta D is basically  $D1+D2 - D$  and that is going to be as we have already written 2 H1 H2 divided by D square, okay, so why this relationship? Because we are going to assume that you know these are kind of similar quantities, so R will be approximately D, and you know  $D1 + D2$  is approximately in the order of D, so all this approximations will eventually tell us that you can use you know you can use these expressions and you can then again assume in the amplitude (Refer Slide Time: 12:03)

 $d_1 + d_2 \leq d \left(1 + \frac{h_1 h_1 h_2}{\frac{d}{d} \delta}\right) \leq d \left(1 + \frac{2 h_1 h_2}{\frac{d}{d} \delta}\right)$ <br>  $\frac{d}{d_1 + d_2} = \left(1 + \frac{2 h_1 h_2}{\delta}\right)$ <br>  $d_2 + d_1 + d_2 = d$ <br>  $\left(1 + \frac{2 h_1 h_2}{\delta}\right) \tilde{e}^{i \frac{d}{d} \beta} \frac{d_1 h_2}{\delta}$ <br>  $\delta \leq d_1 + d_2 = d$ 

part that this component 2 H1 H2/D square can be neglected so that you can rewrite this as 1-E power – J2 beta H1 H2 divided by D square to the magnitude square, okay, granted we have made lot of assumptions here, but as our goal was to find out an expression, if you don't want to make this assumptions you don't have to make this assumptions, you can go back the equations are valid, the expressions are valid with those approximation that we have already made, right, but if you don't want to make further approximations, fine, you don't have to make, you can use a numerical package such as Matlab or something to just plot the received power as a function of the distance D and then show what would be the functional dependence of the power on D, right, but for now we will write down this, so this fellow can be written as, because it's a 1-E power – something, you can show that this is essentially going to be sine square of beta H1 H2 divided by D square, and because this H1 H2 into beta is assume to be quite small compared to D square, you can further approximate this as beta square H1 square H2 square divided by D power 4, somewhere I probably made a mistake, so when I took the square root I think I did not take the square root properly, so this should have been D, this should have been D rather than D square, oh because there was a D on the top.

You revise this expressions so this should be D, not exactly D square, okay, so sorry about that, again this would be D square so this is alright, so this is not D square, this is all just D, okay, I'm sorry about this small confusion in the binomial thing I kind of hurried up here, so I did not look at this correctly, so you get beta square, H<sub>1</sub> square/H<sub>2</sub> square divided by D square as the term that is proportional to this, so please remember that while the electric field is proportional to this, right, there is also an E power – J beta D/D factor that we had already pulled out as a common thing, right, so with that already taken out as a common thing you will have to reinsert the magnitudes square of it which luckily will simply be 1 over D square, okay.

And then when you stick PT GT GR on to the expression here in addition to the electric field this would also be the thing, then you will actually get this one as beta square H1 square, H2 square divided by D to the power 4, okay, so this is the origin of a D to the power 4 dependence of the electric field, and this would actually happen in the assumption that we have made in terms of H1 and H2, so we had this 2 H1 H2/R and we neglected this 2 H1 H2/R or 2 H1 H2/D both are essentially of the same order, we neglected this, we said that this is much much less than 1, so which actually means that R must be much much higher than or larger than 2 H1 H2, and the value at which  $R = RB$  which is 2 H1 H2, we neglected the phase here, so we include the phase part here so we said 2 beta H1 H2/R is much much less than 1 so which actually



implies that R is much larger than 2 H1 H2 beta, and we know that beta is basically 2 pi/lambda, so this actually becomes 4 H1 H2/lambda, okay, so for the value of R at which 4 H1 H2/lambda and beyond, right, then the propagation will be of 1/D4.

But for R that is horizontal distance less than this capital RB, the propagation will actually be 1 over D square, as you can show by you know the appropriate expressions, okay, so it seems that the received power is not going as a single exponent, right, so there is some value  $R = RB$ , okay, this is the horizontal distance R that I am plotting, so this RB is called as the break point of the you know slopes, and if you look at the average received power or the power received, you will see that this was going in at a slope of 20 DB per decade you know or an exponent of 2, whereas right after this breakpoint or approximately after this breakpoint the slope would actually go as -40 DB per decade that is the path loss exponent has become 4 here, whereas the path loss exponent was just 2 in the power calculations for the range which is below the breakpoint.

(Refer Slide Time: 16:39)



because of the finite conductivity of the earth that we have so far neglected, okay, finite conductivity, right, so finite conductivity causes this slope to actually change over slightly to -30 DB per decade, okay, and from thereon you will have -40 DB per decade, so you have a slightly sharper follow off even in the range for less than break point, it won't be the friis 20DB formula or friis path loss exponent of 2, simply because of the fact that we cannot consider the earth to be a perfect electric conductor, there is always some amount of finite conductivity or lossy material that one needs to consider, okay.

So to sum up you have before this range either 20 DB per decade or -20 DB per decade as per you know the conductivity of the earth, but more realistically this is about -30 DB per decade, and beyond this breakpoint one can well approximate the loss as -40 DB per decade or rather the receive power as -40 DB per decade, decade means to change from say 1 kilometer to 10 kilometer or from 1 meter to 10 meters, okay, so that's 10 fold change in the R value that you are looking at.

(Refer Slide Time: 18:00)



So this is your received power, now if you actually look at the experimental data, okay, of people who have measured this you know received power you actually don't find, so you expected here in this particular manner, unfortunately the data that you will get you can show that there is going to be a large variation here, so the actual measurement point will show that the overall you know the actual received power will have in addition to this straight line you know path which would be an empirical fit, there would also be variations around it, okay, so the power actually varies around this one, okay, and then there are this small scale variations which actually come up, when you zoom in closely you will see the small scale variations come up everywhere, okay.

So the green path is the straight path loss or what is called as average path loss which goes as 1 over D cube or 1 over D 4 depending on what the conductivity as we have talked about, so this is the best empirical fit that we have, (Refer Slide Time: 19:09)



and in the long range approximation the power actually goes as 1 over D cube or 1 D4 is called as the long term or long range power, and this power is what is you know the, what will give us the average power, okay.

But there are power variations around it, and these come from the obstacles that are actually present which we haven't consider, for example in the case of the two ray model that we considered you had H1 here, and then you had H2, we said that it was reflected right off from the earth, right, but it need not be the case, suppose there is a small obstacle, maybe there is a road hump, okay, light would also be reflected or electromagnetic waves would also be reflected from that hump, now we haven't accounted for this obstacle, maybe it's not just one this one, there would be multiple heights, okay, there may be some additional humps and then there could be road you know cavings or there could be buildings, there could be trees, (Refer Slide Time: 20:05)



so you have lots of different varieties where the electromagnetic waves could get scattered and arrive at the receiver, and one such effect is this shadow and effect that we have talked about, I mean that we are talking about, so this is called as the shadow effect, okay.

To put it simply imagine that this is where you have you know transmit antenna, but right of the transmit antenna let's say there is a building here, okay, so this is the building that is present, and this is where your receive antenna is, so in fact there seems to be no direct path to this at all and this is correct, there could be reflected path, there could be additional reflections that can come off, these are multiple path that are possible, but there is also this path which has to go through the building.

Now EM waves will, what they will do when they encounter the building, is that they try to diffract from the building, okay, so this height maybe we wrote it too high, but then light, I mean rays will try to bend around and there would actually be a non-line of side path that is possible, okay,

(Refer Slide Time: 21:02)



so this kind of effect which come from tall buildings or white buildings or trees or some leaves behind the trees, I mean so many of manmade obstacles will actually cause the local power to fluctuate from it, however this local fluctuations is also of the order not quite small radius, I mean if you, it's not of the order of wave length, it's the order of wave length, I mean it's off the order much much longer than wave lengths, okay, so you have an average power, on top of the average power, the power will actually fluctuate because of this buildings which can dynamically come in between trees and other kind of manmade obstacles and they will you know be called as the shadow effects, sometime this can also be because the mobile user himself or herself will go around the you know the building and therefore electromagnetic waves would not directly be able to reach it, okay, so this is the shadow effect.

Unfortunately to completely include the shadow effect we need to know lot more about this particular phenomena called as diffraction which is what we are going to consider in the next module, okay, to complete this shadow effect analysis, but the effect because of this multiple rays which I have written you know some rays coming directly, but some rays being you know reflected off from various obstacles, some rays being scattered by the building, these effects are included under what is called as multiple path or simply multipath effect, okay, so this would be or this would be multipath effect which will show variations of the order of wavelength, so you can just move about corresponding to say 900 megahertz, what is the corresponding wavelength you can calculate and over that wavelength or couple of wavelengths if you move, then the received signal power could actually fluctuate from that, okay. (Refer Slide Time: 22:49)



And what is meant by fluctuation means? The received power could actually go down usually and this multipath will lead to what is called as fading of the signal, so the term fading simply indicates that the signal amplitude is kind of reducing and the amount of reduction as well as the rate at which the power is reducing, so for example you may be carrying your phone and then walking along, some point the reception is bad, meaning that fading is very high there, but as you move away slightly, you know some wavelengths away the reception could suddenly be better, okay, and or in other direction it could be better, so this rate at which you are going to have your poor reception then come back to the good reception then again back to the poor reception, this phenomenon is called as fading and then the rate at which this happens is called as fade rate, okay.

You can see that this is not only because of the multiple paths, this multiple paths could also be dynamically induced, meaning that you know you may decide as you are moving, you may decide to go in a certain direction of the different velocity and suddenly your theta and phi coordinates are different, okay, and because they are different there could be, and the you know obstacles diffracted waves could also be in the different orientations, sometimes they may add up, sometimes they may destroy each other, so you will get power fluctuations of that particular order, so remember there are three different mechanism wherein you are going to get power changes or power losses, one is the long term variation or the long range variation which comes you know because of the path loss, this is the average power that you would receive.

And then there is this shadowing effect which is also larger order, but not to that order as the path loss average path loss thing, and then there are this minor variations or the fading which occurs because of this multiple paths, okay.

Technically we assume that each path is independent, each path has a certain amplitude under phase, okay, and we give certain probability distributions based on measurements and based on some theoretical simplifications, and you can show that this fading parameter which we haven't defined here, we will do it later on in other module if we have time, but this fading parameter or the fading phenomena can be expressed as a random variable, okay, at any given point your total input power will have the average power + the fluctuations, and these fluctuations because of the fading can actually be approximated by what is called as Rayleigh fading, okay.

And in some cases and what is called as Rician fading, okay, so these are the topics that you know course and wireless communications as in other NPTEL courses probably, can tell you more about it and you can also do nice models, beyond what we have done in the simple models, but our story is not complete yet, because we have seen that when waves interact with buildings, meaning that there wavelengths are essentially of the same order or slightly higher order than the you know apertures that you are thinking off, you know you can think of the building as an aperture, as an opening for the electromagnetic waves and then these wave would simply diffract from it, okay, so this phenomenon of diffraction or diffraction, we will study it because it is very important, not only for this channel modeling, but it is also important as a example of the propagation of electromagnetic waves through certain optical materials called mirrors, lenses and apertures, okay. Thank you very much.

## **Acknowledgement**

#### **Ministry of Human Resource & Development**

#### **Prof. Satyaki Roy**

#### **Co-coordinator, IIT Kanpur**

#### **NPTEL Team**

**Sanjay Pal Bharat Lal Ashish Singh Badal Pradhan Tapobrata Das K. K. Mishra Ashutosh Gairola Puneet Kumar Bajpai Bhadro Rao Shikha Gupta Aradhana Singh Rajawat Sweta Nipa Sikdar Anupam Mishra Ram Chandra Manoj Shrivastava Dilip Tripathi**

**Padam Shukla Sharwan K Verma Sanjay Mishra Shubham Rawat Santosh Nayak Praduyman Singh Chauhan Mahendra Singh Rawat Tushar Srivastava Uzair Siddiqui Lalty Dutta Murali Krishnan Ganesh Rana Ajay Kanaujia Ashwani Srivastava M.L. Benerjee**

# **an IIT Kanpur Production**

**© Copyright Reserved**