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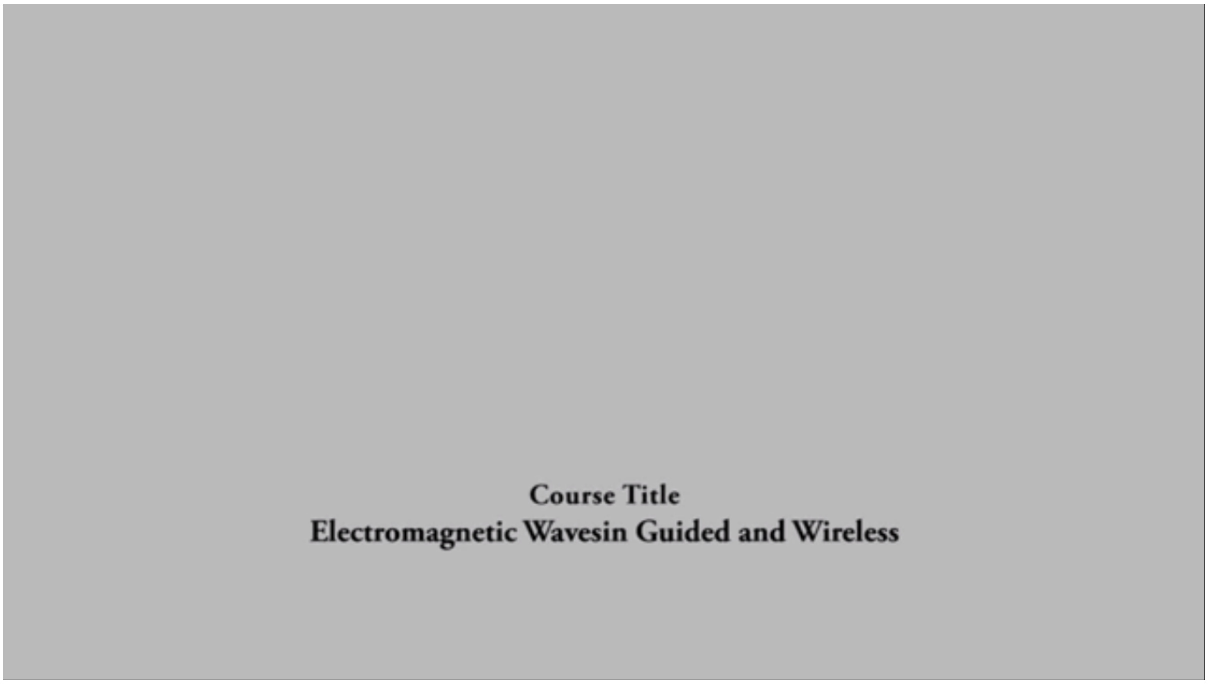

Course Title
Electromagnetic Waves in Guided and Wireless

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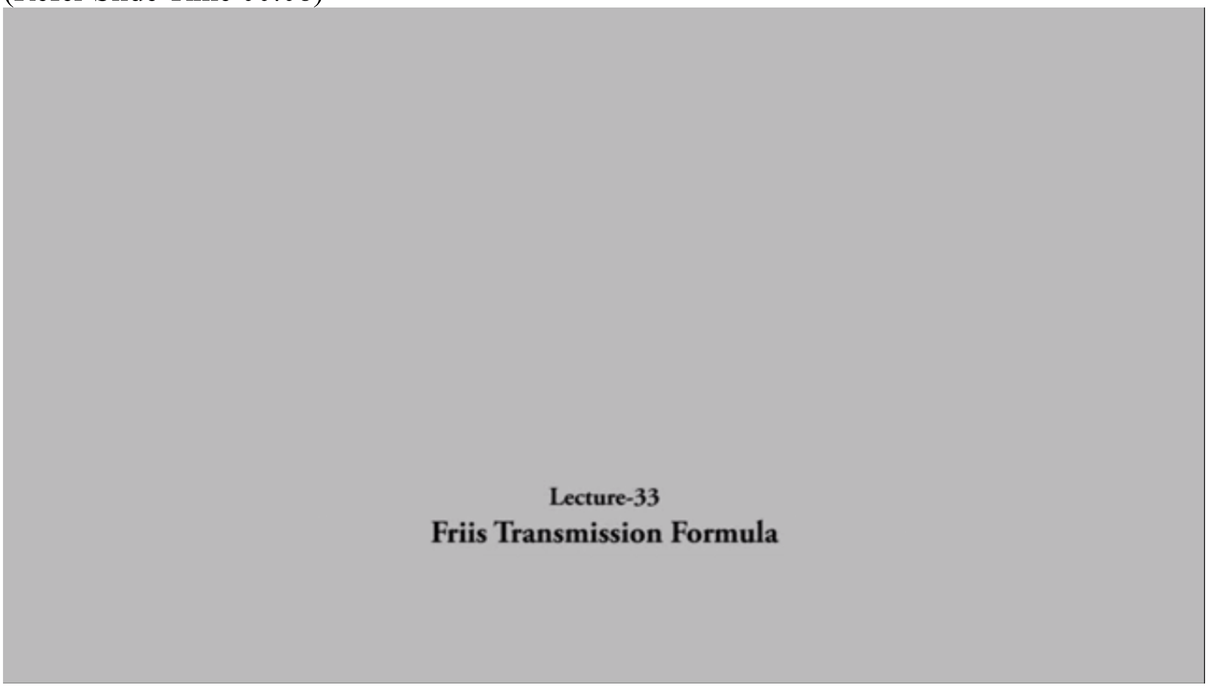

Lecture - 33
Friis Transmission Formula

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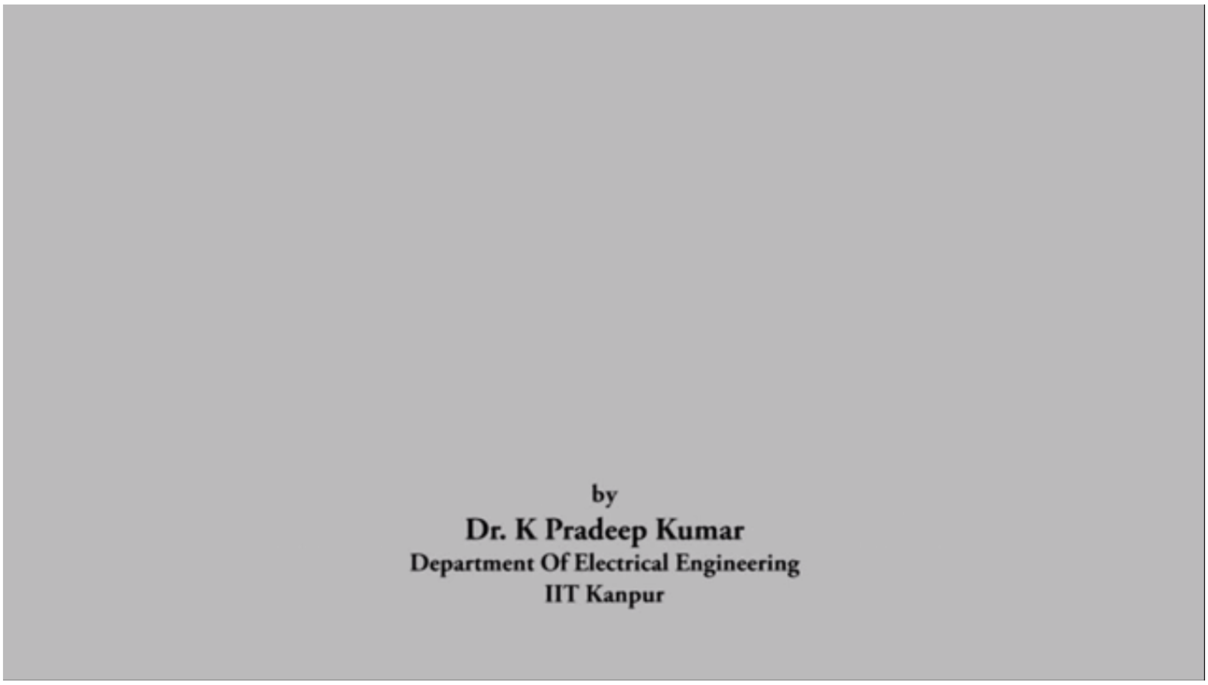

by
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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. In this module, we will look at what is called as Friis Transmission Formula, which tells us how the power that you receive at the receiver will vary and how does it depend on the transmit and receive antennas. Okay.

So we have already set up the basic ideas about the received power. So when there is an electromagnetic wave impinging on the antenna, what you have to do is to look at the pointing power density and then multiply that pointing power density with the effective aperture.

Now, obviously, the effective aperture may be oriented in a different direction than the actual aperture meaning that you have take into account the possibility that the field and the aperture, if they are perpendicular to each other, then there won't be any power that is intercepted by the receiving antenna.

On the other hand, if the orientation is complete meaning that if the EM wave is propagating in this direction and the aperture is also in the same, you know, orientation so that you can grab the maximum amount of power, then you will essentially obtain maximum power.

So this concept is also related to the effective length meaning that the orientation, the vector nature of the electromagnetic wave means that unless the effective length or the effective aperture are oriented appropriately parallel to the direction of propagation or rather perpendicular to the direction of propagation and that you won't have maximum or you won't have the maximum possible power to be grabbed by the receiving antenna.

And we expressed this effective aperture in terms of the effective length in the previous module, and we also related what would be the relationship between effective aperture and directivity. Although I did not derive it, I gave you the equation because that is very important for us in the next step of development and you can see the important relationships that we have developed in the last module being summarised here.

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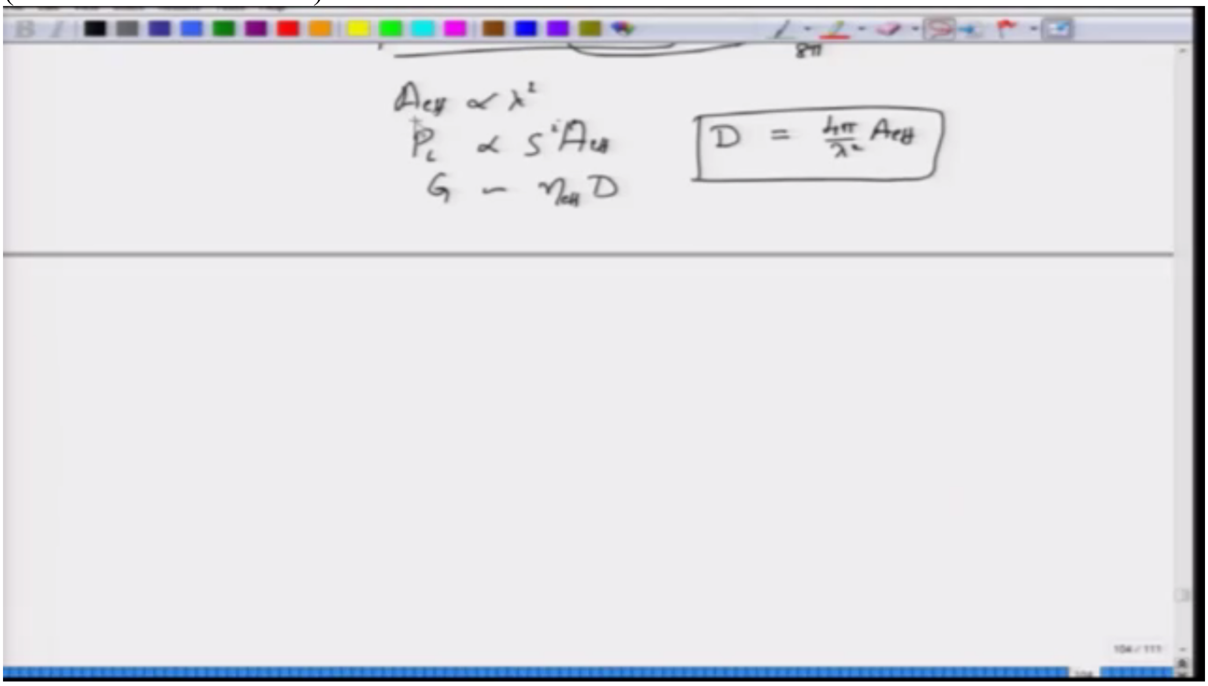

Effective aperture scales as λ^2 at lease for the electric short dipole antenna. It may not be true for other antennas. So you have to be careful as to what antenna is being used and you know use the effective aperture of that particular antenna. And then the power that you receive will be proportional to the incident power density that would be there on the antenna at the receiver antenna times the effective aperture that you can have. Okay.

And then you have this gain of the antenna being related to the directivity in this particular manner that is efficiency times D. Usually, you take efficiency to be equal to 1 in order to simplify the calculations, but in practice efficiency is something that you have to experimentally calculate. And of course, when I write G and D, you have to remember that G and D are dependent on θ and φ. These are the patterns that you are actually looking at and the pattern is dependent on θ and φ.

However, many times we only use the maximum value of D because we do want to orient our receive antennas in such a way that we are obtaining the maximum power, right? So for that you will have to orient it in the direction where the beam energy is maximum. Okay.

But please note here one very important thing. When I say maximum power, I only mean the maximum power that one can extract from the freely propagating electromagnetic wave by putting up an antenna. Okay. So the antenna in the near, in the, in the vicinity of the antenna, there is an electromagnetic wave propagating and you put your antenna in order to grab as much power from that as possible, but that is not the whole story because that power would effectively show up as a voltage across the antenna terminals as we have seen and that voltage along with the antenna impedance which we called as Z_A, right, will then determine how much voltage is actually transmitted to the load that gets connected to the receiving antenna.

So, if, for example, you have an RF block, right, the RF receiver chain connected to the antenna and antenna is the front end, usually, the next block would be a low noise amplifier followed by a filter and so on. The input impedance of the low noise amplifier is the one that will interact with the antenna impedance and the antenna voltage that would appear at the antenna output terminals. Okay.

So I am not talking about the power, maximising power transmission there. I am maximising the power transmission in here, right? I am orienting my antenna in such a way that I grab as much as energy from the electromagnetic wave in order to maximise the antenna terminal voltage V_A. From there I have go, you know, like I can do whatever I want to do in order to maximise the power transfer. That is a separate thing and of course when I do a conjugate matching, you know, with Z_L = Z_A conjugate, then I can extract maximum power from the equivalent circuit of the antenna that we have already seen, and that is another maximised power.

And we have already seen that, sorry, we have already seen that that power that you are going to obtain at the load will always be less than that maximum power that you can actually obtain and the maximum load power that can be delivered by the antenna to the load that is connected is given by this expression, right?

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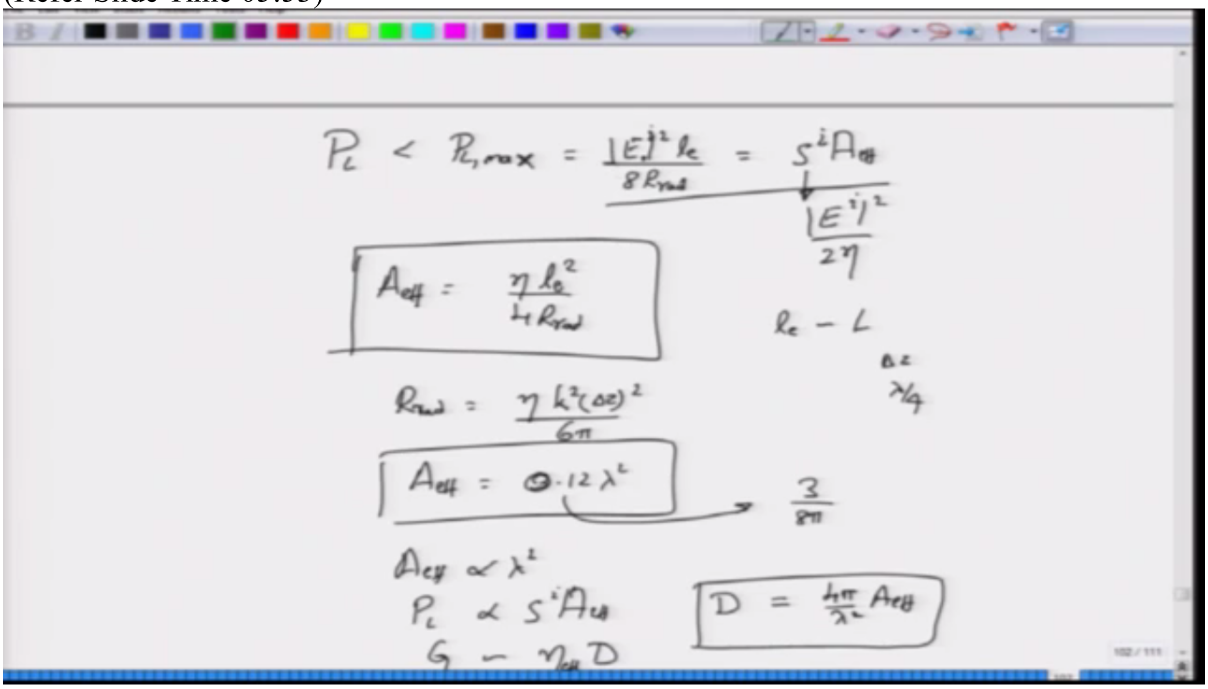

So you have this expression that we derived in the last class. In fact, from this expression is what we derived the effective aperture and utilising the value of radiation resistance, we found out for the short dipole or a dipole with, sorry, $\Delta z(\lambda/4)$, we found that $A_{effective}$ was roughly .12 times λ^2. Okay. So these relationships and where I am maximising the powers are actually very important for us to know.

Now that we have these relationships, we can complete the derivation that we were looking for and understand how much is the power that you would obtain at the antenna terminals when you consider both the transmit antenna as well as the receiving antenna. Okay. So that is what we are going to do.

So we have a transmit antenna. Okay. I am just representing this transmit antenna in the form of a, you know, this horn, but, of course, that is not the antenna that we are considering. It is just a pictorial representation of the antenna, right?

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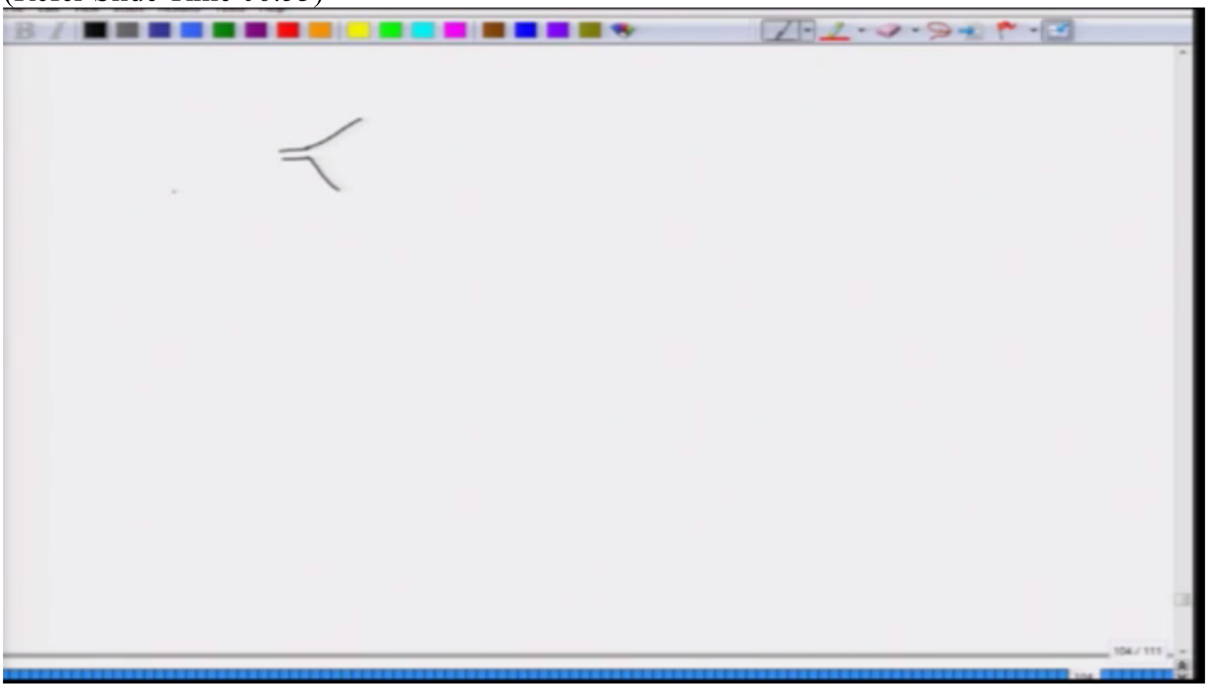

Now suppose I connect this antenna to a source, which delivers to the transmit antenna the amount of power P_T. Okay. So I am delivering this power P_T to the antenna and if the antenna has a gain of G_T, which, of course, would be proportional to θ and φ, remember this would be a function of, so not proportional, it will be a function of θ and φ.

So you have this gain, which is the pattern, right, the power pattern of the antenna. The actual power that is radiated onto the free space or the connection between transmitter and antenna, medium of connection between the antenna and transmitter antenna would be the radiated power will be P_T times G_T. In the given direction of θ and φ, it would be P_T times G_T.

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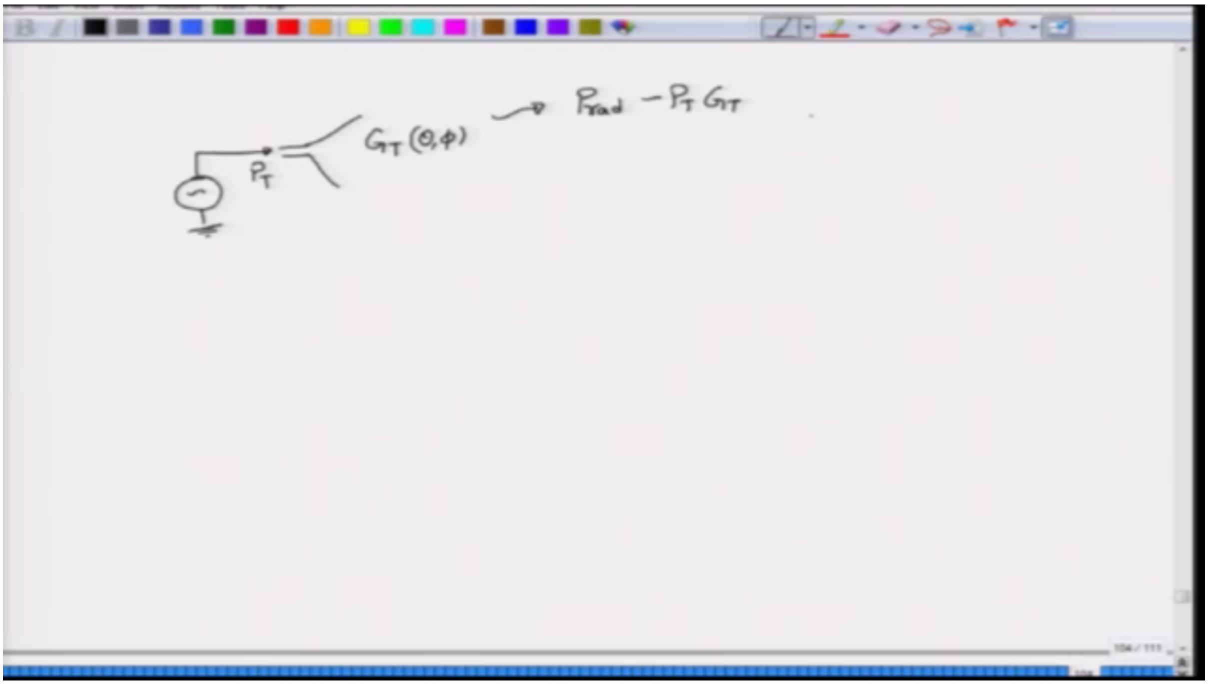

I will suppress the notation of θ and φ in these expressions just for simplification, but you should remember that, technically, if I change θ and φ, that is if I move from one position to another position, the receiving antenna or the transmit antenna, then the power that is received at the receiver will also change. Okay.

Anyway, at this point, we don't really have any receiving antenna. We have just taken the transmit antenna with its gain pattern and then supplied an amount of power P_T so that power will be distributed by the antenna in space depending on the direction pattern or the antenna pattern, right, whatever the power pattern that you have, that energy will be distributed in that way.

So, if, for example, the antenna power pattern is in this manner, okay, this is a 3D picture that I am showing. So with two additional side lobes what it means is that the power is distributed in this direction. Okay. In this way the power is distributed in the maximal way and some amount of power is also distributed here, which if you don't keep an antenna, you are only going to lose out that power. Okay.

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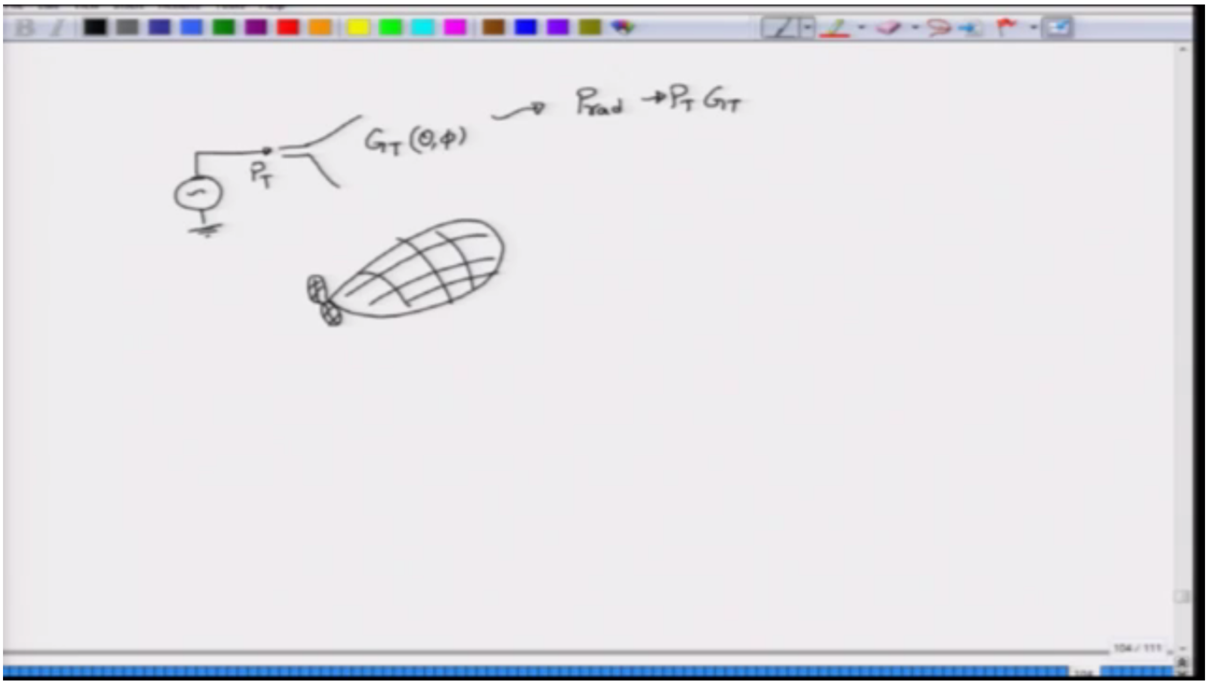

Anyway, so that was just a pictorial way of representing this radiated power.

Now that radiated power is okay. If you imagine that you are at a distance r from the transmit antenna, you can find out what would be the power density. Okay. The power density will be the power being radiated or power being carried by the antenna divided by $4\pi r^2$ where r is the distance from the transmit antenna. Okay. I am using small case r. Customarily, this is actually written as a capital R indicating the radius of the sphere that, of course, there is no actual sphere. It's an hypothetical sphere around the antenna whose radius is R and this is the power density, right?

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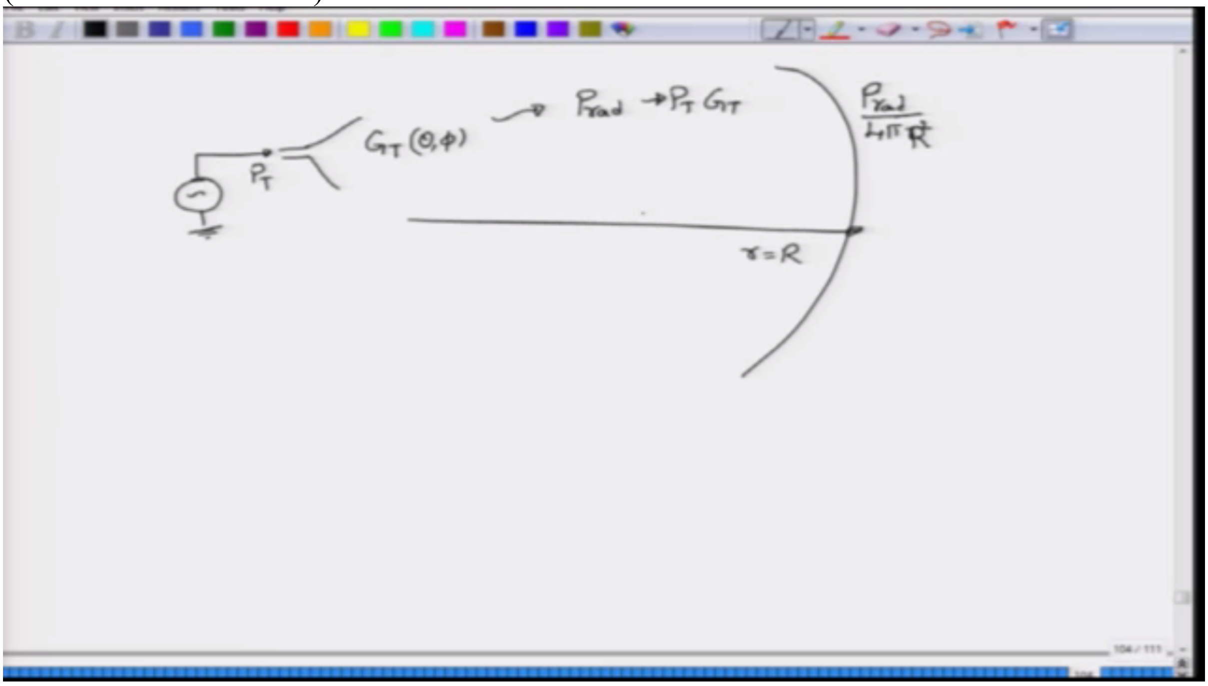

Now if this power density were to appear at the input terminals of the antenna, okay, if this were to appear at the input terminals of an antenna, then what would be the power that the antenna would receive? The power that the antenna would receive will be whatever the effective aperture of the receiver is, right, times the power density.

So if I call power of the receiver P_{Rx}, that would be the radiated power density that is present or its value at the antenna input terminals times the effective aperture, but I know that effective aperture can be rewritten in terms of D. So I can rewrite in, also I can substitute for P_{rad} or the radiated power as product P_T G_T.

So I am going to do that. P_T $G_T/4\pi r^2$ and effective aperture is basically $(\lambda_2/4\pi)$ times D of the receiver. So I am going to write this as D_R. Okay. And what is D_R and G_R related to? $D_R = G_R$ divided by efficiency of the antenna. Assuming that this efficiency is maximum, that is it is taken to be unity, then I can simply replace D_R by G_R. Okay.

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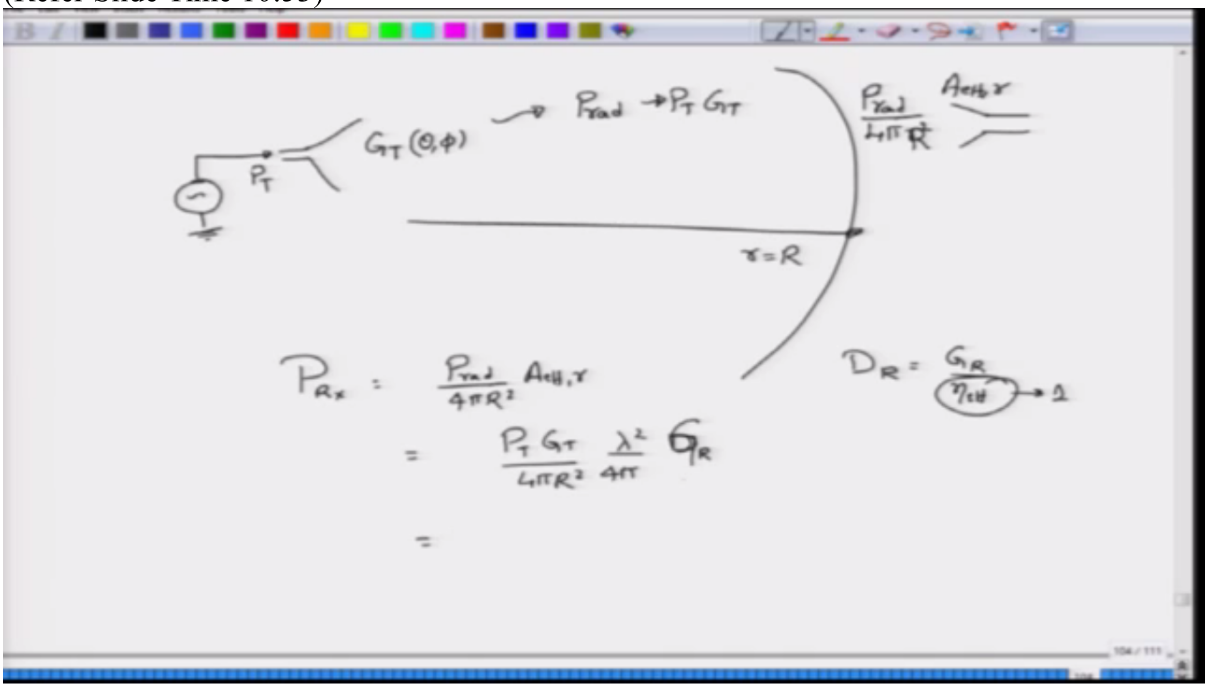

So let's collect all these terms together. So I have P_TG_T, which, of course, is oriented with respect to θ and φ and G_R is also a quantity that is dependent on θ and φ. Therefore, you can see that it's not only that receiver antenna, you know, maximises the power that is delivered to it. It's both the transmit antenna as well as the receive antenna.

So if the transmit antenna is radiating this way and the receive antenna is kept here, then you will only intercept a part of it. So if the receive antenna is kept here, you won't receive any power because the power is kind of travelling in a, you know, different direction. So it's important to match the beam directions of both the transmit as well as the receiving antenna. This perhaps is a very commonsensical thing, but that is kind of mathematically included in these expressions, right?

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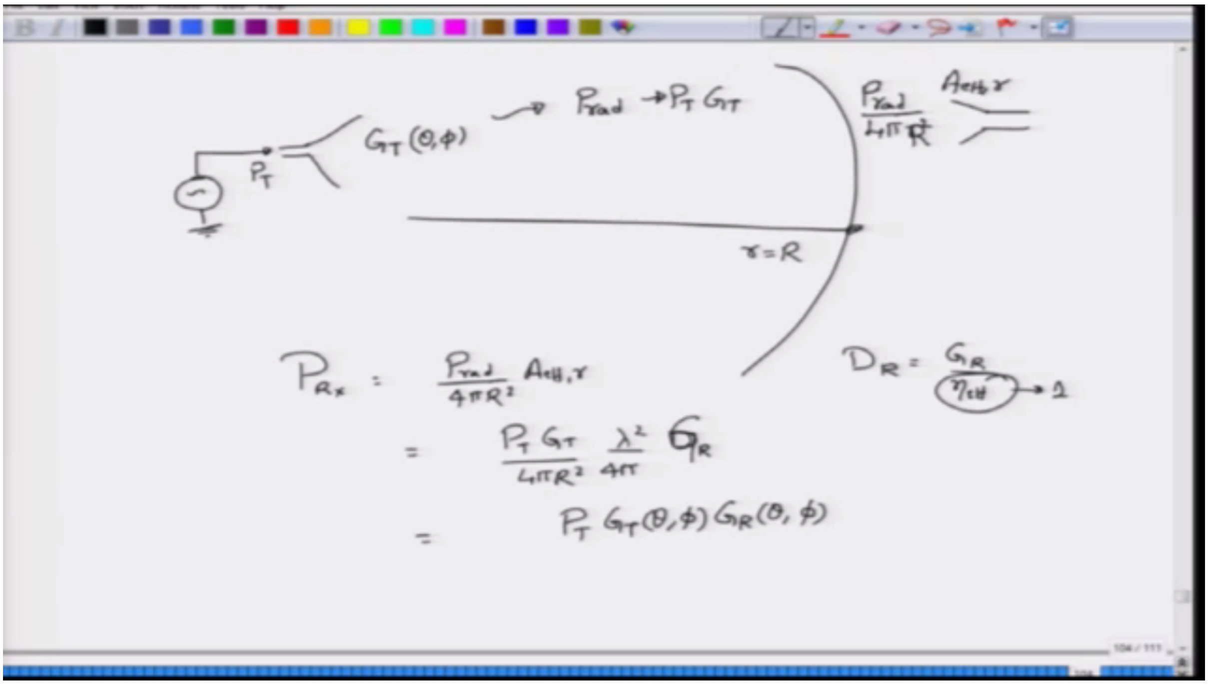

So you have $P_T G_T G_R$ divided by so there is a 4π here and another 4π here. There is a λ in the numerator. So I can rewrite this one as $(\lambda/4\pi R)^2$. Okay.

So this is the power that I am going to receive at the input terminals of the antenna, and this power would essentially generate certain antenna, certain voltage at the antenna output terminals, which when you connect through the antenna impedance Z_A, which is usually complex to the load impedance Z_L and conjugate match, right, so if you conjugate match Z_L to Z_A, then you will extract maximum power at the load. Okay.

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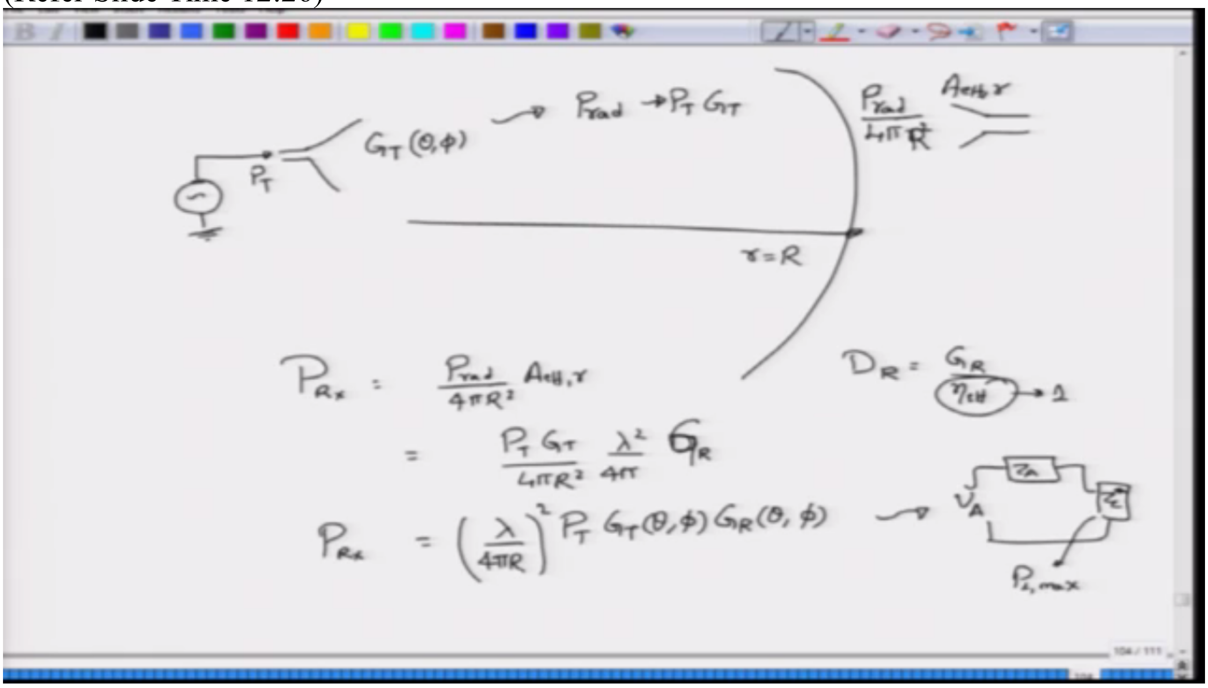

Of course, in practice, none of these conditions can be met ideally meaning that you can extract maximum possible. The overall efficiency of the link will always be less in the sense that it will be less than what one can achieve in a maximum way. Okay. But if you take care as much as possible to orient the antenna in the direction of the same beam, maximise beam direction of the receiving antenna or rather receiving antenna oriented in the transmitting antenna direction and maximise the, I mean, maximize the power transfer to the load by conjugate matching, then you can increase the efficiency of the system to a very high level.

Of course, please also note that it is not the power that delivers that finally tells you how much is the information content. Information content depends on the voltages because you are not coding them in terms of power.

Usually, the, if you are following some kind of a coherent coding, coherent modulation scheme, then information may be sitting in the phase and getting the phase information is more crucial than getting the power information because if you are only, you know, modulating the phase information is in phase and you have modulated the phase of the carrier, then it becomes important to extract the phase so at which point you may find maximising power by conjugate matching to be useful or not, but that depends on the problem at hand. Okay.

I am just telling you that only maximising the power is not going to give you the full benefit of the link. You have to understand the problem and see where information resides. Of course, if the information resides in the form of power, like, you know, on, off power keying systems, then maximise the power by all means. Okay.

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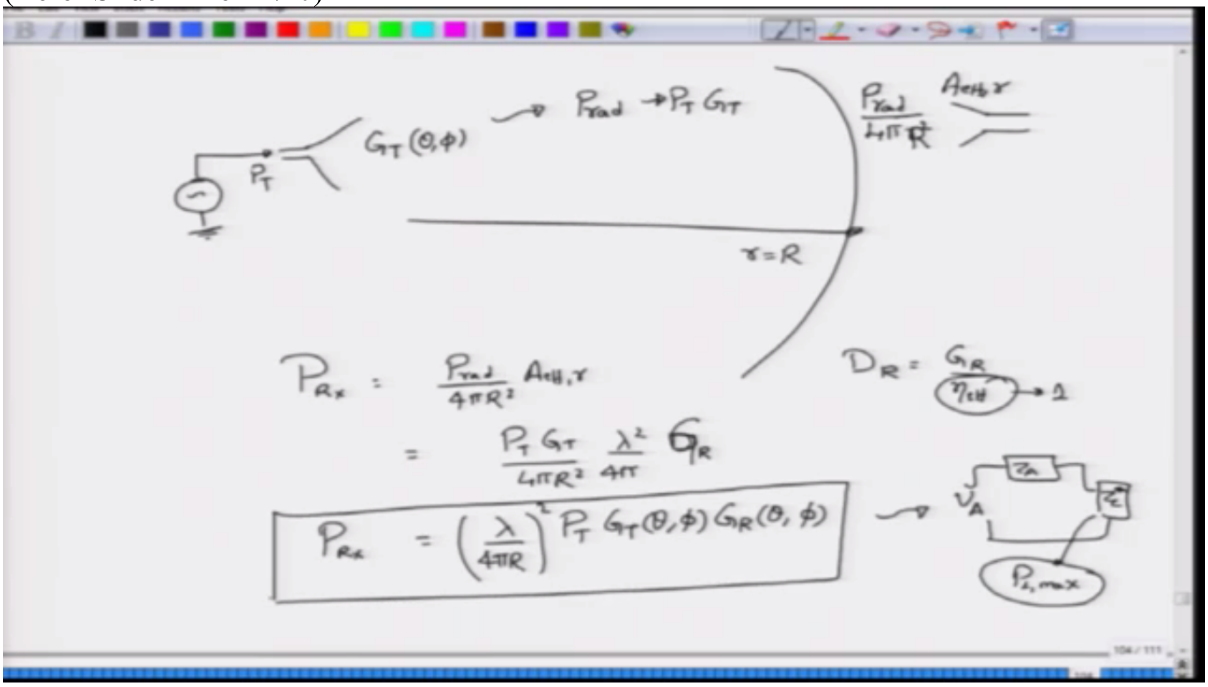

So, anyway, this formula that we have written for the received power in terms of the transmit antenna, in terms of the input power to the transmit antenna and the antenna pattern is actually very important relationship. Okay.

In fact, we call this term, which is getting multiplied to $P_T G_T G_R$, this, you know, term that is there in the numerator, we can rearrange this term to show up in the denominator. We can write this as $P_T G_T G_R$ suppressing θ and φ divided by $(4\pi R/\lambda)^2$. Okay.

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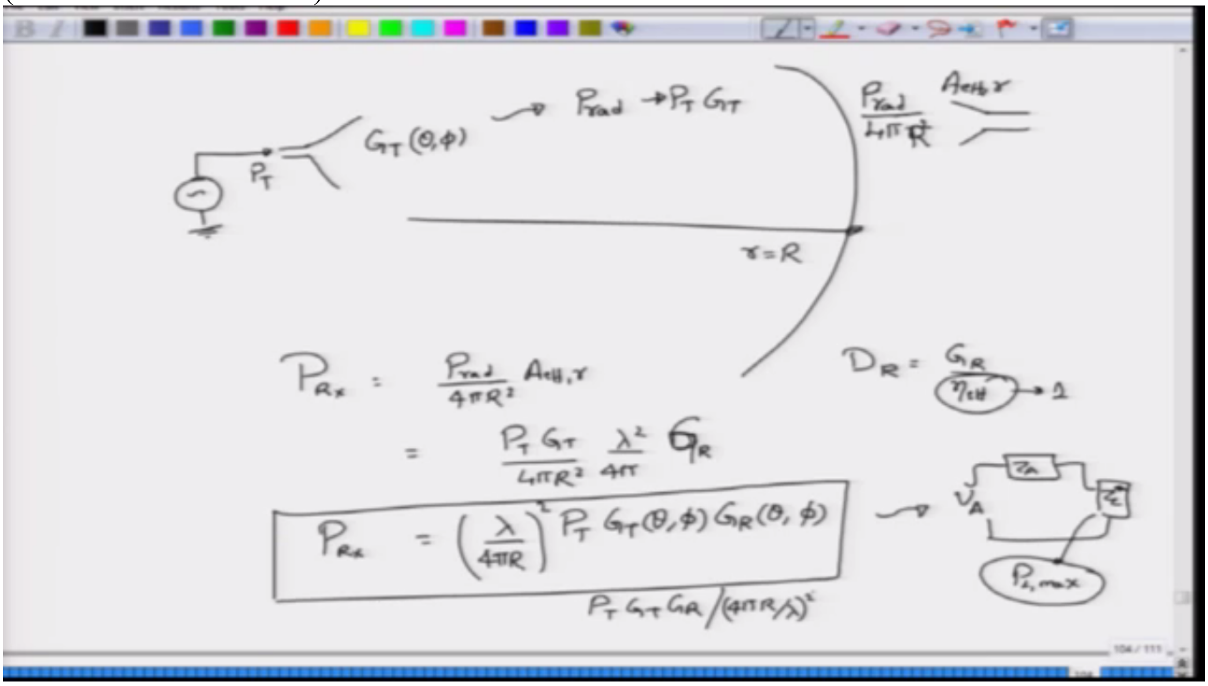

So you can write it in this particular manner and call this factor $(1/4\pi R)\lambda^2$ as the path, sorry, L_P as $(4\pi/\lambda)^2$ times R^2 as the path loss. Okay. And you can see that the path loss is kind of going as 20 dB per decade if you actually convert all of this L_P in terms of DB by taking 10 log L_P and I am taking 10 log because this is power, not voltage, right?

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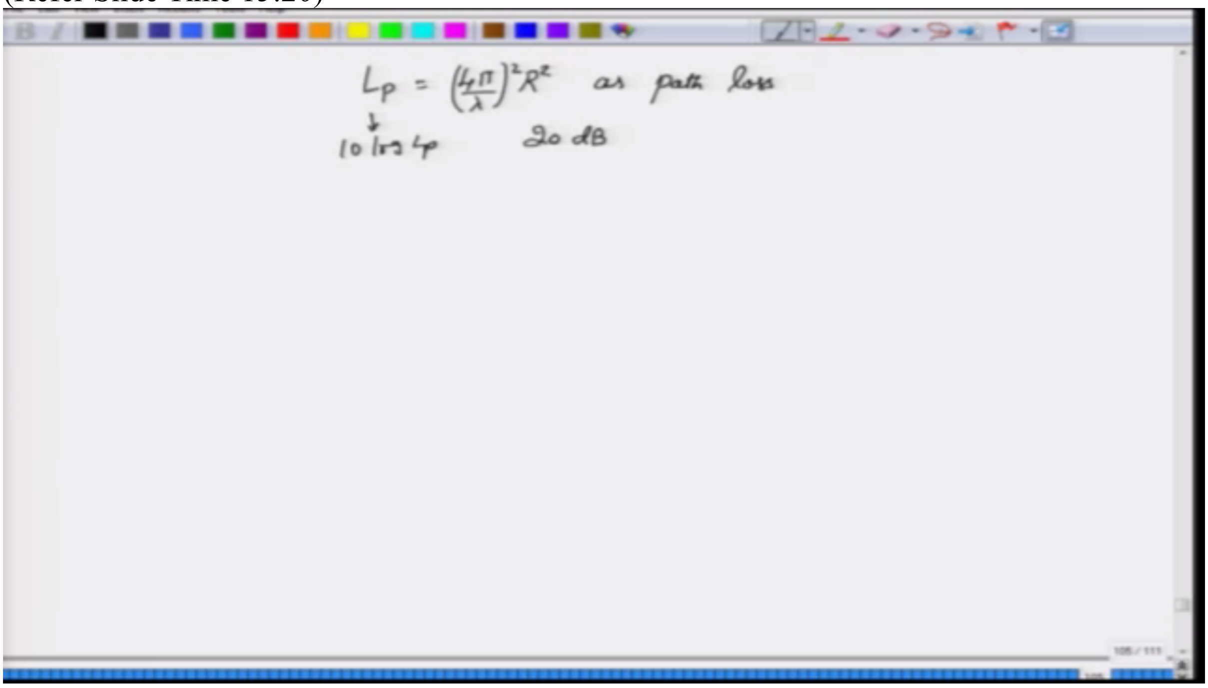

So this expression of of Friis Transmission Formula is relating the power, not the voltage. Therefore, I take 10 lo of this and express the path loss then and remember L_P comes in the denominator. So you have $P_TG_TG_R/L_P$ and of course L_P is a function of R. So may be better we will write it as $L_P(R)$, and for the ideal free space transmission, this turns out to be 20 dB per decade. Okay.

So if you plot as a function of the distance, right, R, what is the loss that you are going to get or rather the total power that is received, then this power steadily decreases with a slope of about 20 dB per decade. So decade doesn't really mean that you are changing in frequency. Here it means that it is a distance that changes. So you take R = 1 and let's say the value of the received power is 110 dBm. Okay. This is extremely large value. I am just giving you some numbers to tell you what actually happens, and if you now go to R = 10, right, then you have, you know, 10 m.

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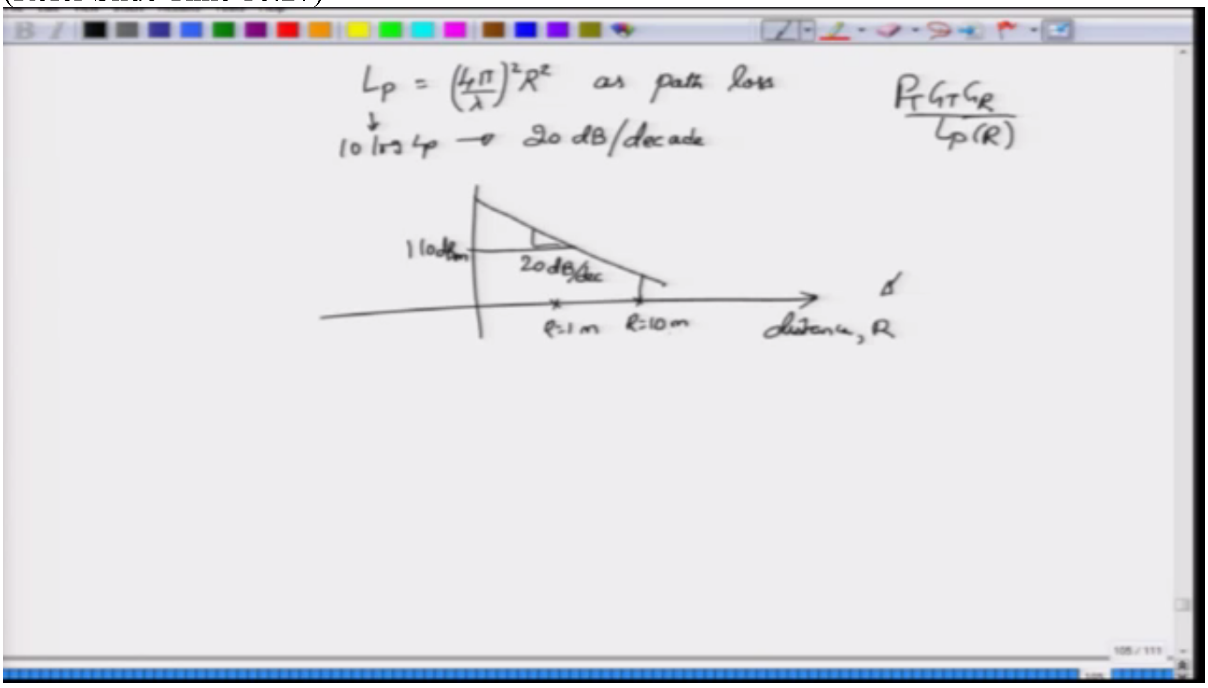

So just say R = 1 m and R = 10 m, then the power that you are going to measure here would have dropped by 20 meaning from 110 or rather let's keep it realistic. So we will keep it some -70 dBm. So what you are going to get will be -90 dBm of power. Okay. That is what it meant by 20 dB.

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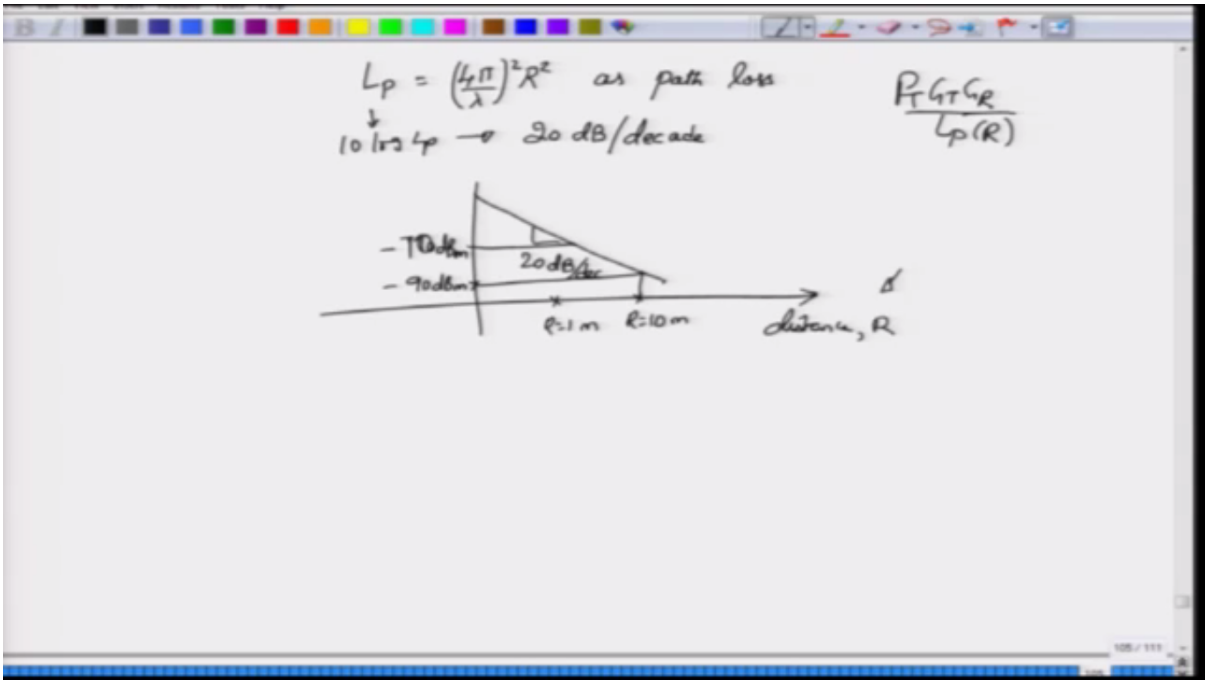

Let me also tell you that this is the best possible scenario that you can have. Okay. You have in the entire universe nothing else except the transmit antenna and a receive antenna, okay, so that the power loss that happens between these two is directly proportional to, I mean, is actually the best case scenario of 20 dB per decade. Okay.

Now this happens because the amplitude scales as 1/R meaning that as you move away from the transmit antenna, the amplitude goes as 1/R and because power is related to amplitude square, the power goes as $1/R^2$. Okay. This is very important. You are looking at voltage, which is going as 1/R and power is going as $1/R^2$. Okay.

What happens if R instead of the input power decaying as 1/R, it starts to decay as $1/R^{1.5}$? Then the power would go as $1/R^3$ because you are going to take this and square it up, right? So you are going to get $1/R^3$ and now you can immediately see that path loss in dB actually goes as 30 dB/decade. Okay.

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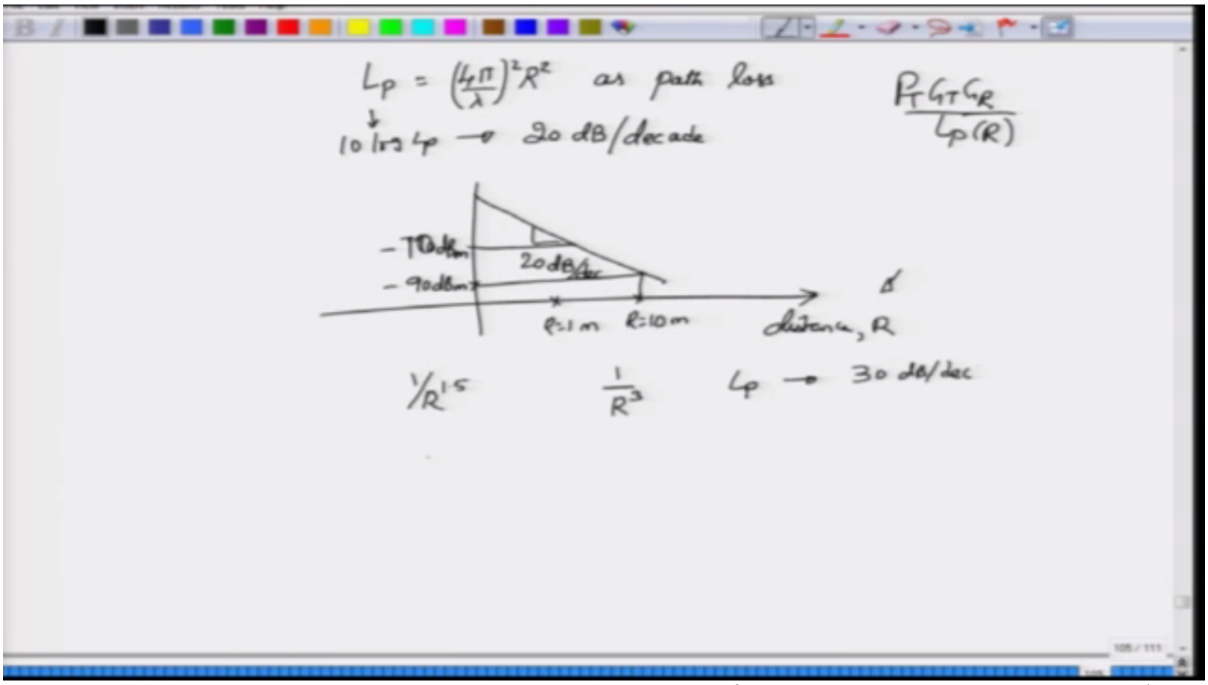

In the other case, if R, if the amplitude itself falls as $1/R^2$, then the power will go as $1/R^4$ and L_P will go as 40 dB/decade. Okay.

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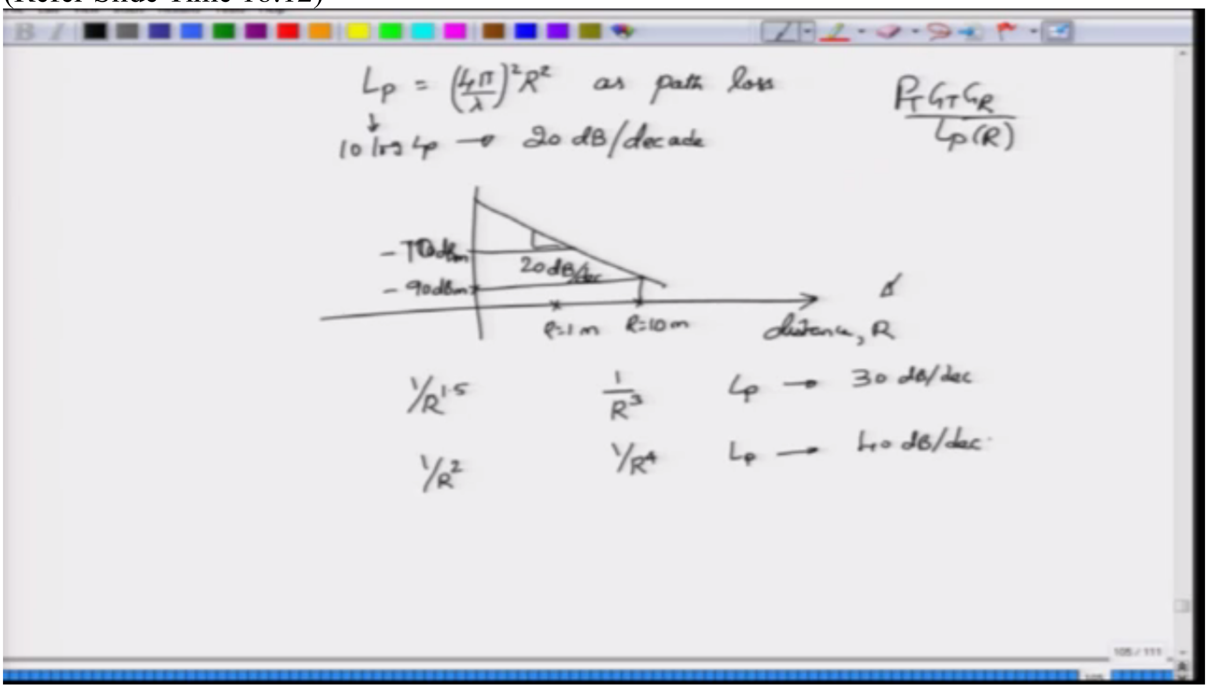

You may ask where are these 1.5 factor and a factor for the amplitude coming from? They did not exist in the, you know, equations so far. Even the spherical wave that we considered was actually e^{-jkr}/R. So you don't understand where this amplitude going, I mean, growing faster than 1/R coming from.

In fact, this path loss exponent that you will see with an ideal value of 1 or you know if you are looking at the power in the path loss exponent, then that would be the power going as R^2. So 1/R amplitude, power goes as $1/R^2$. So this is the path loss exponent. Okay.

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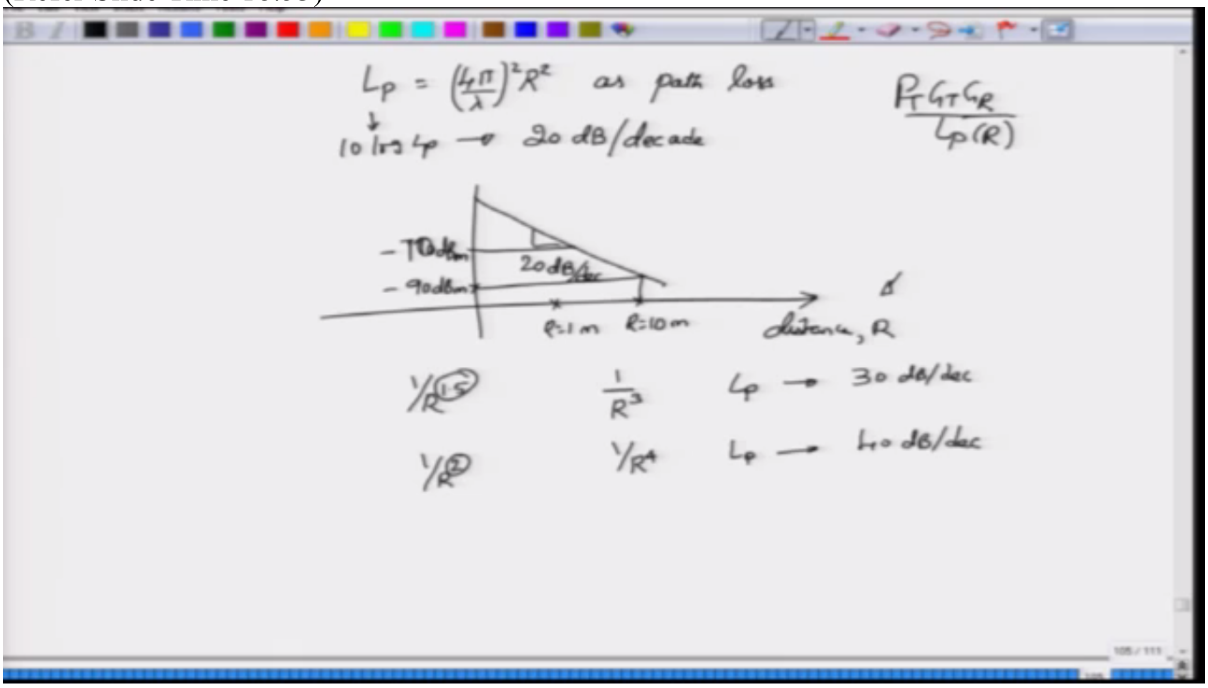

If the path loss exponent, the minimum value is 2, it cannot be less than that at least in the far fields that we are considering. Then R^3 means that the path loss exponent is 3. Here R^4 means the path loss exponent is 4. Steadily as your path loss exponent changes, the losses increase rapidly. Okay.

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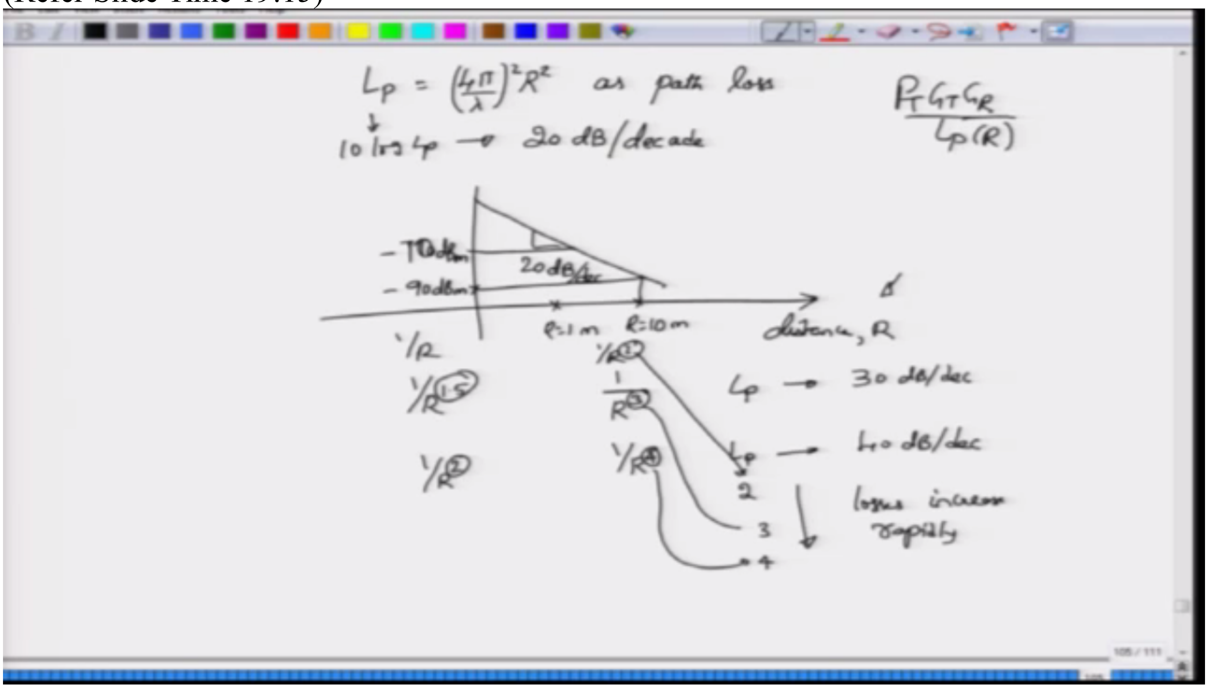

And this is a situation that you will find yourself when you are working with real-world scenarios. In real world, you may put up your transmit antenna. Okay. On some building you may put it up and or maybe close to the building let's say, and then you have a receiver antenna somewhere else.

So if you are imagining the wireless scenario, you have a base station and the base station antenna is, you know, put some on a tower you locate the base station antenna, and you are a user who is, you know, walking around nicely talking to someone and we don't, of course, expect that between the base station antenna and yourself there is absolutely nothing else in the world.

Unfortunately, there will be buildings. There will be trees. The atmosphere may not be nicely, you know, weathered condition. It may be raining. It may be foggy. It may be snow. All these things essentially contribute to what is called as a clutter, and they will increase the losses and this path loss exponent is one way of representing those increased losses. Okay.

So if the expected power is not falling as 20 dB/decade, but it is falling as 40 dB/decade, then you can, I mean, then you can actually go back and change the, you know, value of the path loss exponent from 2 to 4 or rather 2, yeah, 2 to 4 and that is you are attributing the extra losses into an effective path loss exponent. Okay. So that is what you are doing.

Of course, we will have lot more to say about these buildings, trees, and other things that come around and reduce your signal levels and you know when we start talking about wireless channel model in the upcoming modules, but for now please keep in mind that path loss exponent of 2 and hence a loss of 20 dB/decade is kind of the ideal that you can push.

In practice, you don't get such an ideal scenario. The losses are either 30 dB/decade or in worst case, in urban, semiurban, there are also again variations with respect to what environment you are looking at. In a forest, that will be even drastically low, right, because there is lot of trees in the forest I hope, right?

So all these attributes or all these losses can be attributed to an increased path loss exponent, and you can tweak the equation of Friis Transmission Formula to reflect these additional losses. Okay.

Life would have been very easy if it was just the addition of this or rather increase of these losses, but what actually happens as people have figured out by measurements is that the path loss variation whether it is 20 dB or 30 dB/decade depending on the loss thing is actually not the final loss profile. Okay. It won't be like this single line. The actual path that you are going to see would involve some local variations. Okay. These local variations are called as shadowing or shadow losses.

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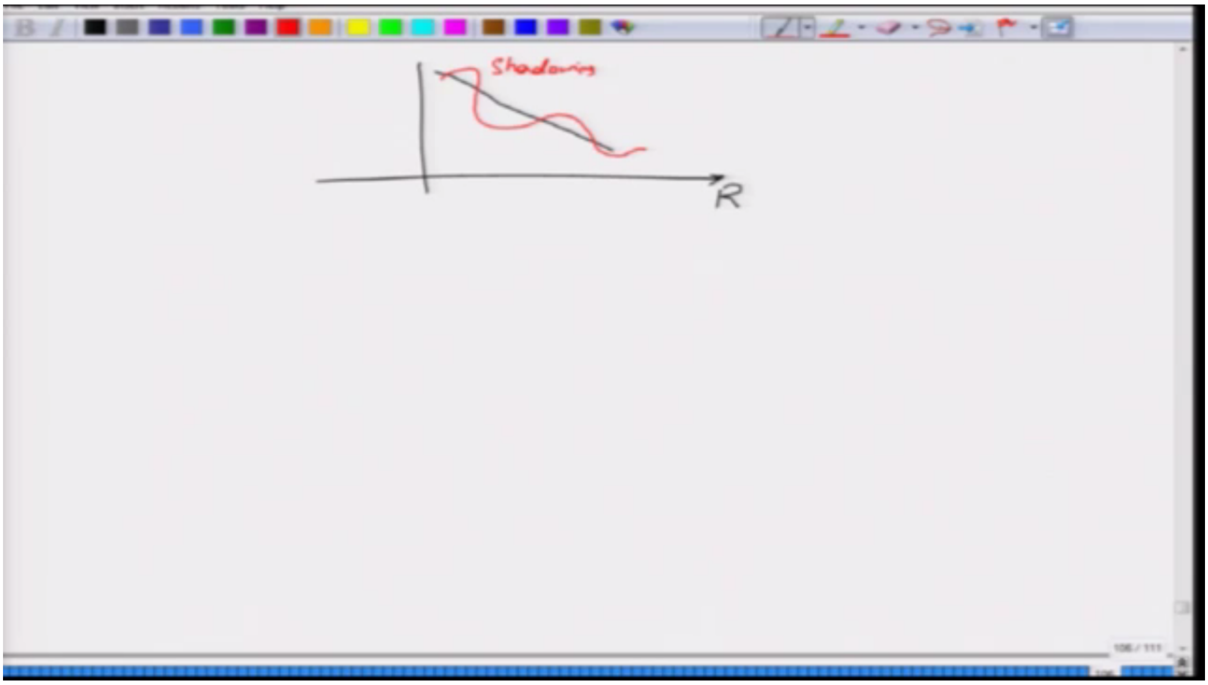

Shadowing means that if the electromagnetic wave is coming in, there is an opaque screen somewhere in between, right? So a tree suddenly comes up. The tree is already there, but because you moved in a direction that you are now seeing a tree, the tree will act like an obstacle or let's say you are receiving a call in your home, but for some reason you come out of the home, right? Then there is a building behind you. That is an obstacle, right? Yes, electromagnetic waves go around these obstacles.

Suppose you are travelling in near mountainous region, the electromagnetic waves actually go around the mountains also, but those actually happen at very low frequencies. Okay. The reason why that happens at low frequencies and what this phenomenon is called, we will discuss it later on in the other modules, but what it actually does is that sometimes these obstacles are not so bad. They can actually give you slightly higher loss. Sometimes they can give you lower loss, but those things actually depend on how many obstacles are there, what is their position, what is the base station position, what is the receiving antenna position and so on.

So it's kind of rather difficult to predict. So people rely on many empirical formulas and if you know the, and especially in the indoor wireless channels, you know where the obstacles are. For example, if you are sitting in your room, you know where your computer screen is. You know where you have hanged the clothes. You know where the television is, you know, kept or you have a almira. So you know those locations.

So in that case because you know all the locations perfectly, you can do what is called as ray tracing and then estimate what is the total loss. Okay. It will also show a large-scale reduction. So if your source happens to be at the top of your head and you are in one corner of the room, that length will be different. Okay. And that would be the direct part that you are getting, but because of the obstructions and obstacles, you may also get additional paths.

Okay. And sometimes those signals can add up. Sometimes those signals don't add up and you get an increased or a decreased loss. Okay.

In addition to this shadowing effect, technically, the shadowing effect occurs because of the obstacles blocking EM waves and to understand that correctly, you need to know what is called as diffraction, and we are going to look at diffraction in the upcoming modules.

Diffraction is also closely related to interference. Interference means two waves can interfere with each other and can either add up or subtract, okay, depending on how they add up in terms of their phase. So if they are in phase, then they can add up. If they are out of phase, then they don't add up. Okay.

In fact, the point of, you know, interference will also be taken up in the next module when we discuss antenna arrays or at least two element antenna array. There you will see that sometimes the signal from two antennas can add up. Sometimes the signal don't add up. In fact, it can be out of phase and you will get it 0.

But coming back to the wireless channel model or the channel model that we are considering, the black line would represent the, you know, free space path loss that you are going to get or the large-scale path loss and L_P may not be, you know, 20 dB/decade. It may be more than that. Okay.

In addition to shadowing, which can increase or decrease the losses, but this is also depending on the environment that you are working in, there are very, very small scale channel variations. Okay. So there are these small-scale channel variations and the scale of these channel variations is roughly $\lambda/2$. Okay. It is less than $\lambda/2$, but this is approximately λ/decade and what it actually means is that suppose you are at this position, you know, suppose you are at this R, now if you move to R, which is about $\lambda/2$, you will see that the power that you have, sorry, I will use this green one, you see that the power that you get is lesser than what the power that you got previously. Okay.

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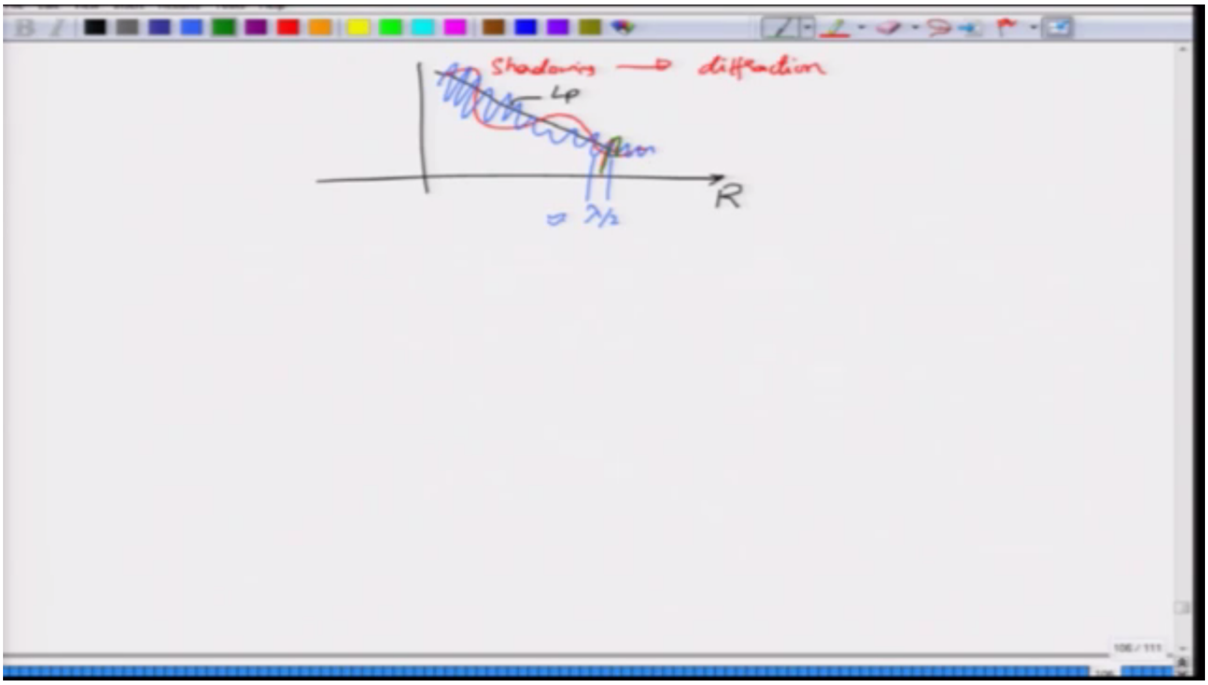

If the timescales that you are looking at are so sensitive to the received power, then this, this kind of variations that you are going to get around the large-scale variations is called as small-scale variations or small-scale fading. Okay. You may have heard of what is called as relay fading and relay fading is one such type of fading that occurs because of the small-scale variation.

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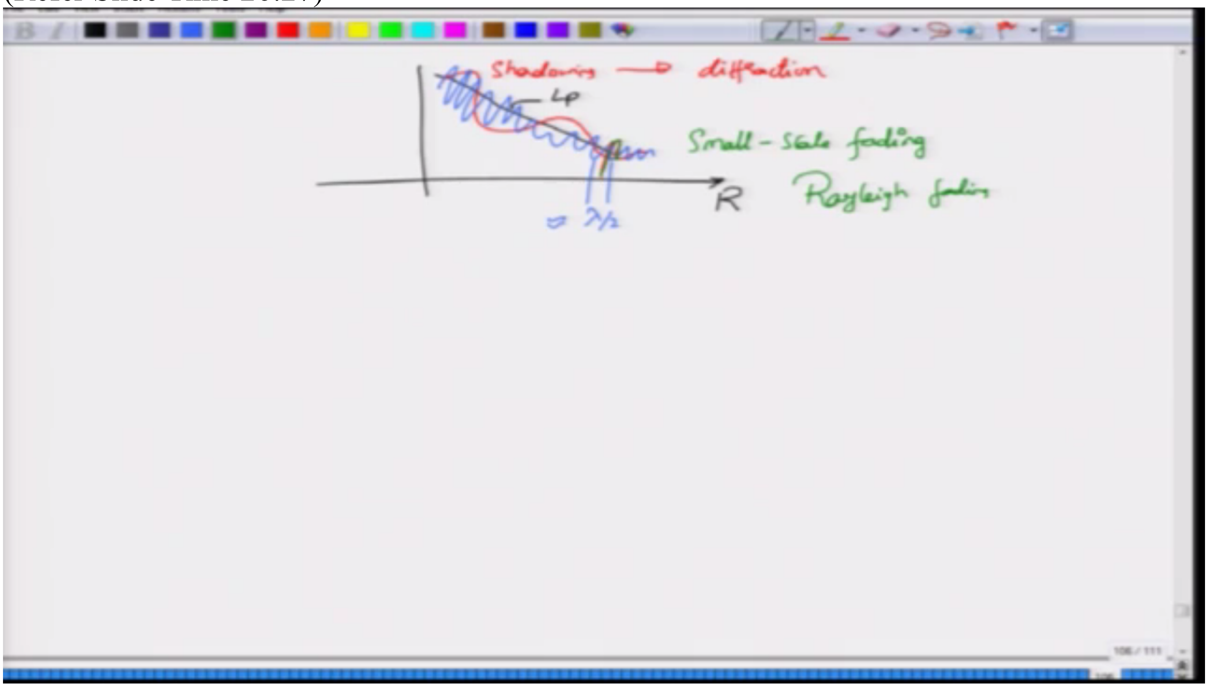

So you have actually three types of path losses that you can expect in a wireless channel. One, there is a large-scale path loss that depends on, you know, like that is the basic path loss that you are going to get, and that is simply because transmit and receive antennas are kept separately. Okay. They have to falloff as 1/R amplitude or $1/R^2$ as power.

Typically, they fall off slightly higher than that and those effects can be taken into the large-scale path loss. Then there is shadowing effect because there are additional obstacles and EM waves can be reflected from those additional obstacles or diffracted from the additional obstacles increasing or decreasing loss around this basic variation.

And finally, at the very micro level, you know, as you move in terms of λ, then there are variations in the received power and those small-scale variations are called as small-scale fading or relay fading. Okay. In general, wireless channel is what is called as a Multipath channel meaning that there could be more than one path, and in fact there is a transition from one path to multipath channels and the corresponding losses that you are going to get.

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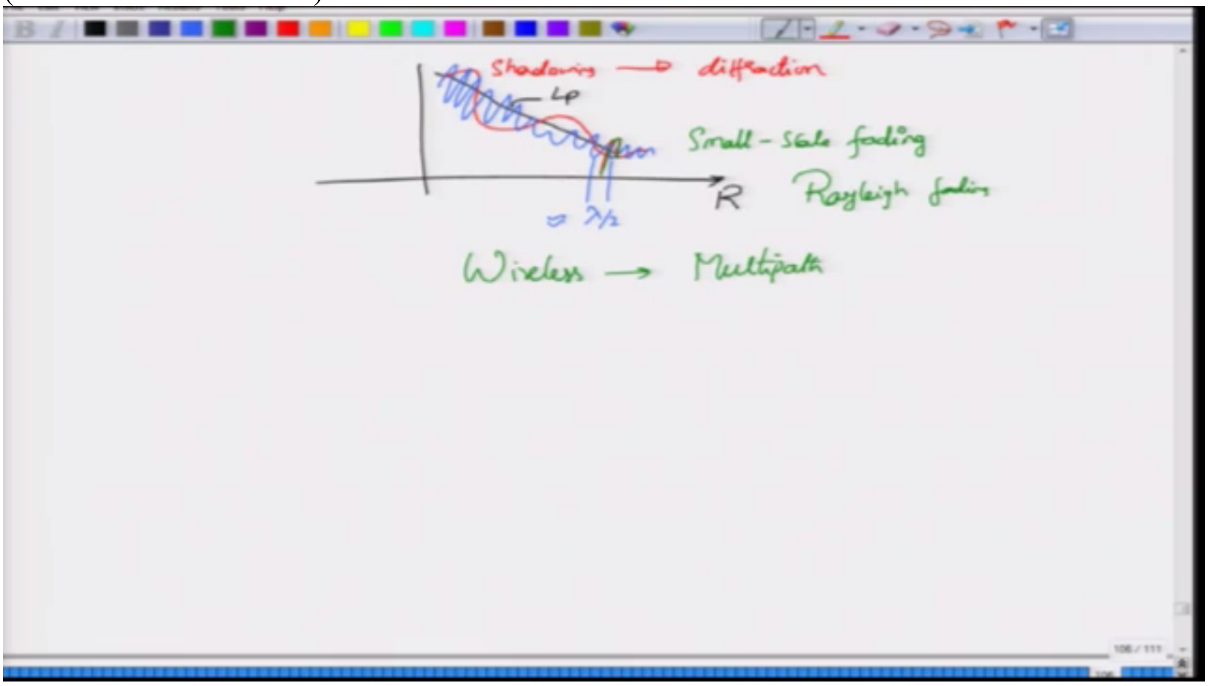

Multiple channels are characterised by small-scale fading and large-scale path loss. Okay. Large-scale path loss is usually represented by √G and small-scale path loss is represented by small h normalised to unity. This is in amplitude. So power wise it would be G.

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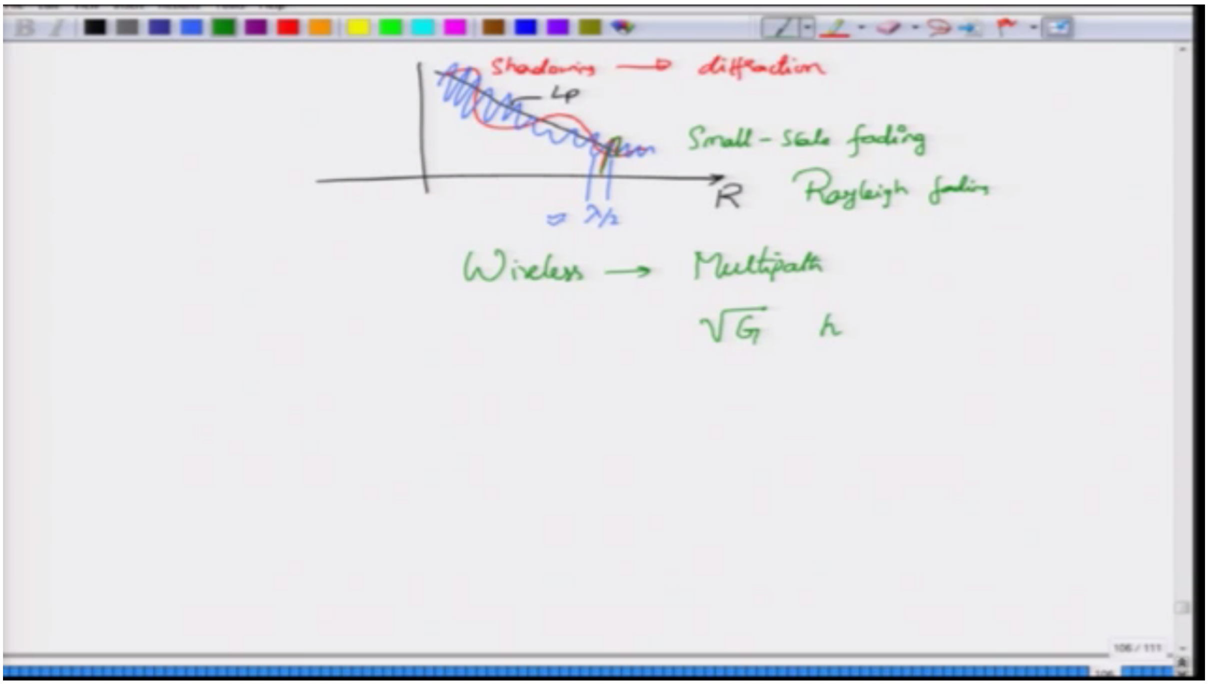

The only surprising thing is that losses are being represented by G where G is usually used for gain, but in the literature for channel modelling, unfortunately, someone back in time chose these losses to be called as path gains. Okay. They are not really gains. They are only losses. You don't have an amplifier in between. They are all losses, but someone labelled it as gain. So now we are stuck with that terminology. So we call the large-scale path gain or rather large-scale path loss as large-scale path gain although we know that it is loss and we denote that one by $\sqrt{G}$. Okay. The small-scale path loss for the amplitude is denoted by $\sqrt{h}$. And in general the wireless channel will have a path loss of $\sqrt{G.h}$ and if you have transmitted an information, which is represented by x(t), the received signal y(t) can be written in this particular manner. Okay.

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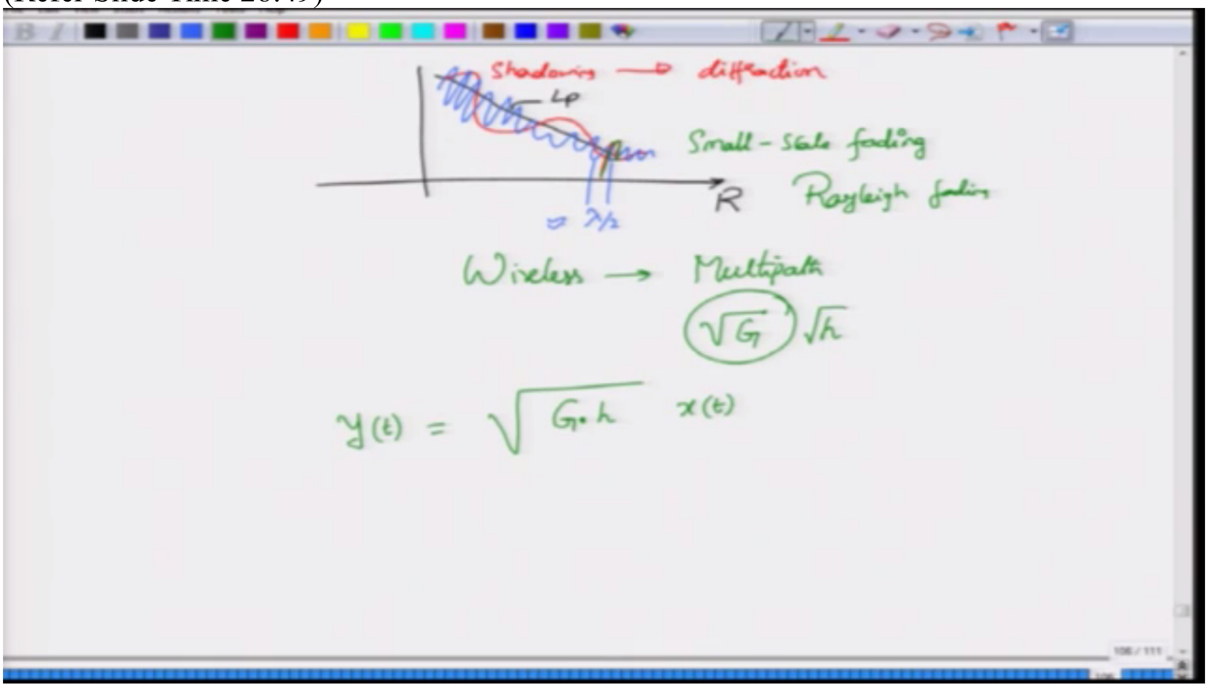

So we will come back to these, you know, losses and other things after we have covered one additional topic in antennas and that is of arrays. Okay. Thank you very much.