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Indian Institute of Technology Kanpur

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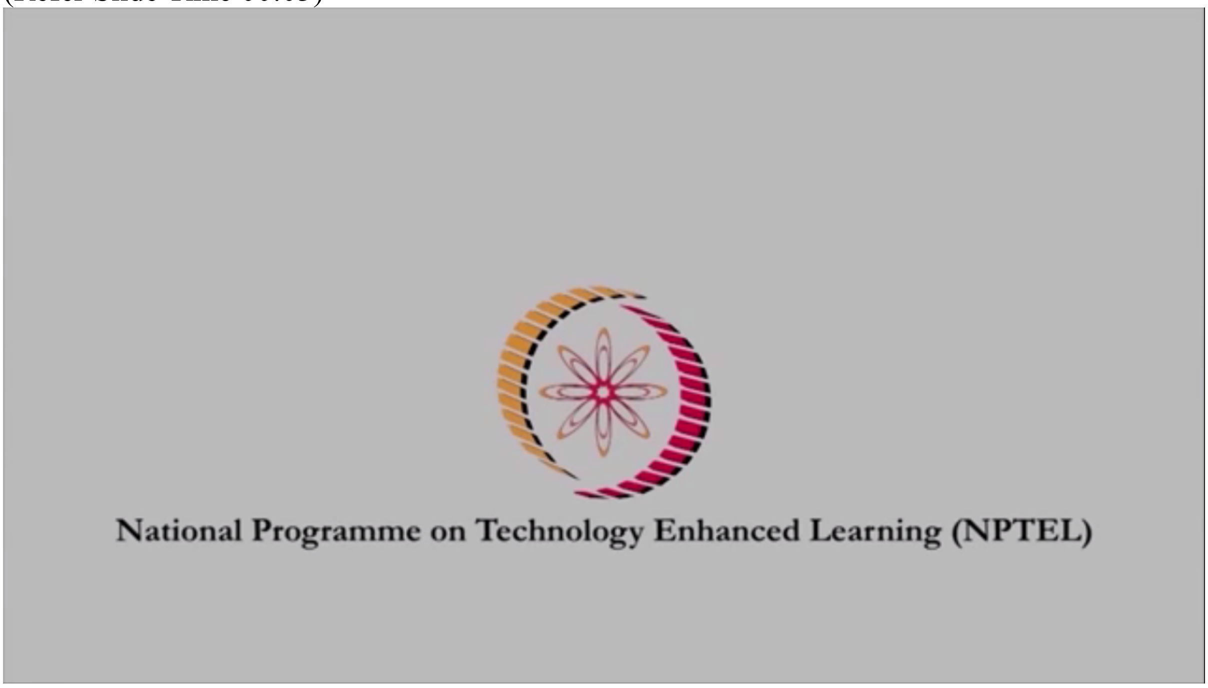

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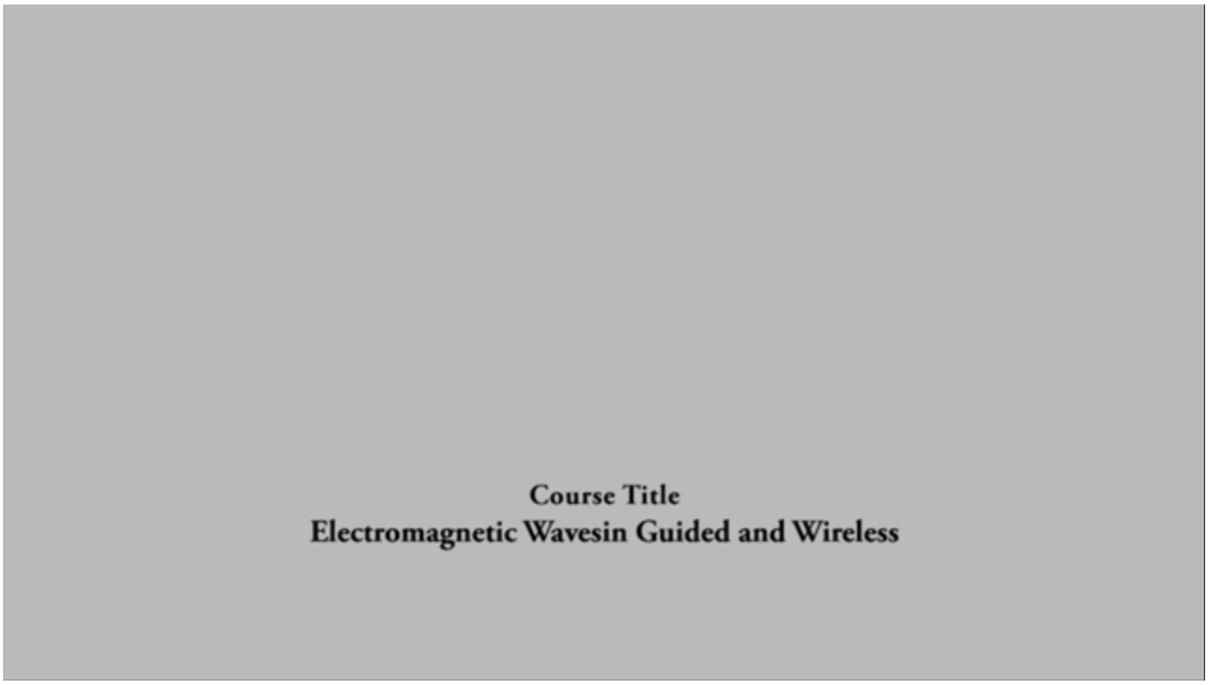

Course Title
Electromagnetic Waves in Guided and Wireless

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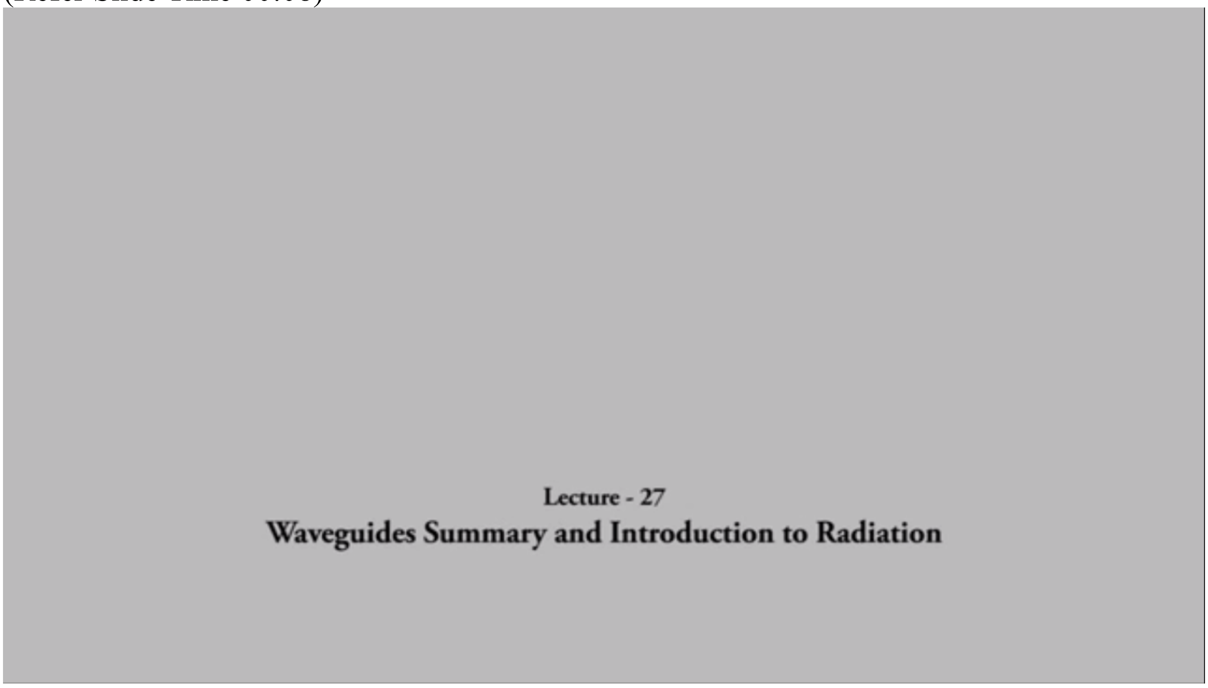

Lecture - 27
Waveguides summary and Introduction to Radiation

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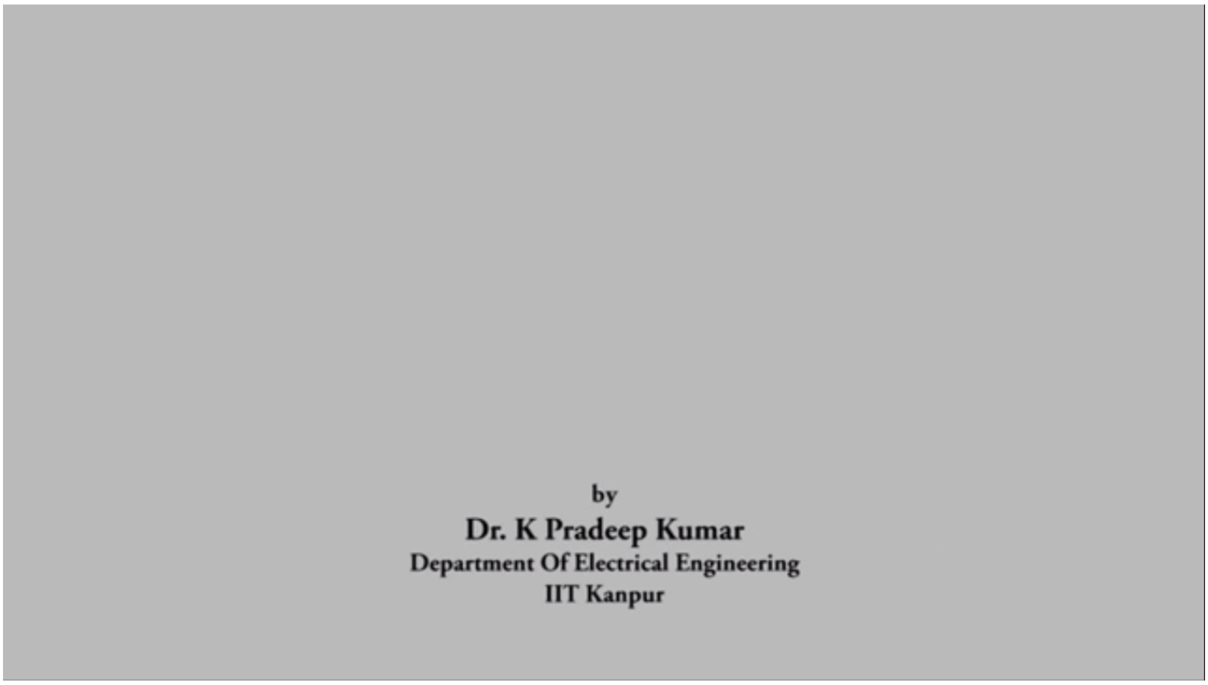

by

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Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media. In this module, the first round up a couple of facts about wave guides that we have been considering and then start discussion of a very important topic called as radiation.

Now as before, I mean, before we go to the radiation, let us recall couple of facts and then introduce additional two, three facts waveguides which I would like to tell you.

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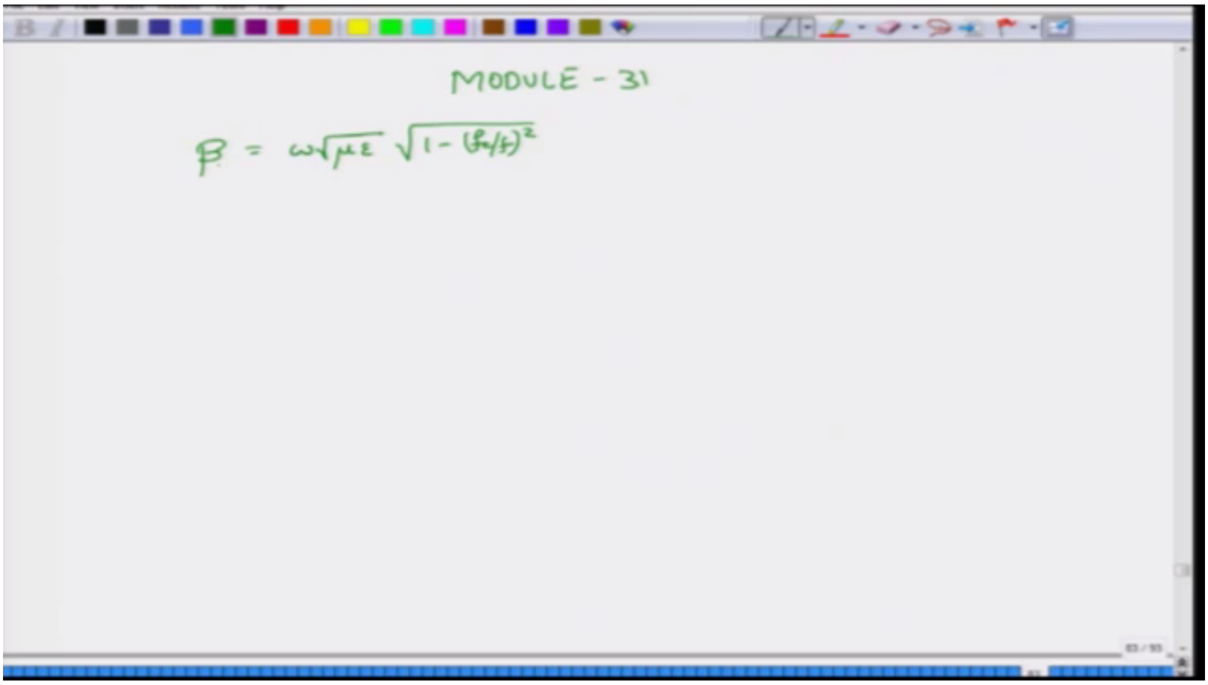

One is we already have seen the expression for the propagation constant of a given moode, right, inside a rectangular waveguide and this is the expression that we have. And if you look at this expression, of course, this expression is for a particular mode, which will have a index of m and n and that would have a refractive index of, sorry, that would have a cut-off frequency of f_{cm} and of course beta (β) will also be specific β of m and n.

Now we will keep m and n to be constant. For example, this could be a TE_{10} mode or this could be a TM_{10} mode or 11 mode or you can have a TE_{20} mode. Whatever the mode that we are considering, we will keep the mode to be fixed, but once you fix m and n, the corresponding value of β also gets fixed or at least it would be by given by this expression, and you have to also notice that this β is a function of omega (ω). That is β is actually changing with respect to frequency. Okay.

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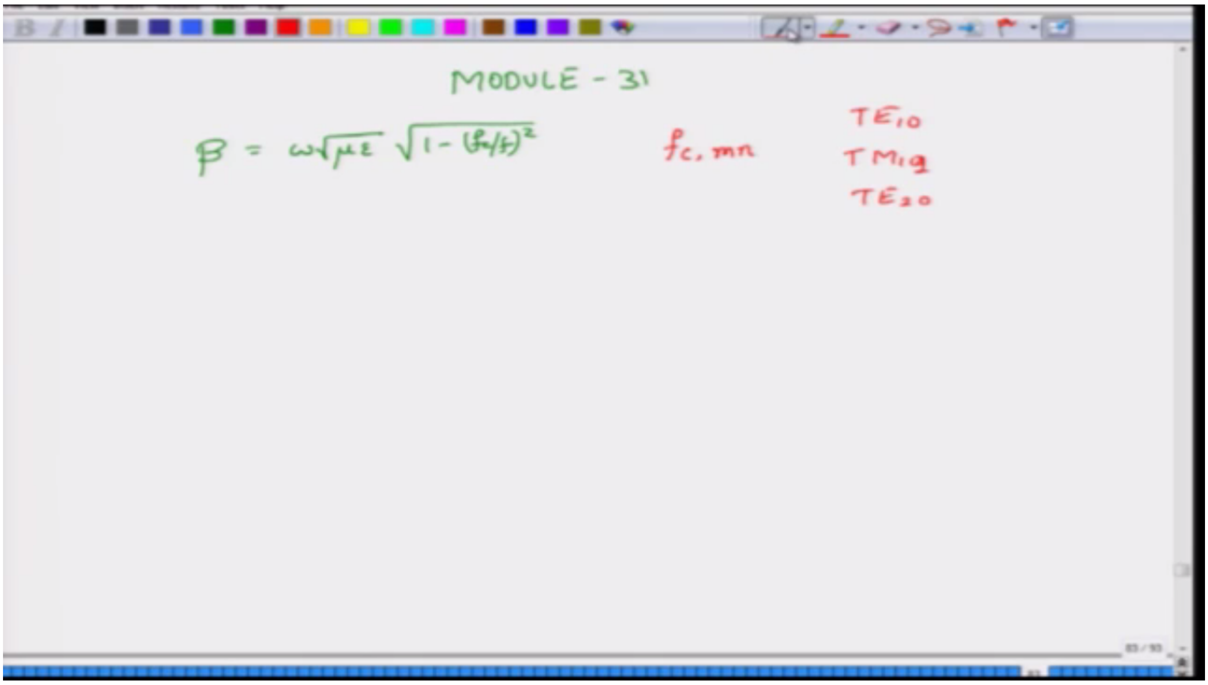

Now in the previous case where we had this waves in the free space, right, so we considered the propagation of waves in the free space called as uniform plane wave propagation. In that uniform plane wave propagation, you had the waves propagating as say $e^{-j\beta z}$, correct, as far as the z propagation is concerned, and then we said that the relationship between ω and β, right, was actually given by the ratio of ω to β was actually given by what is called as the phase velocity u_p. Okay.

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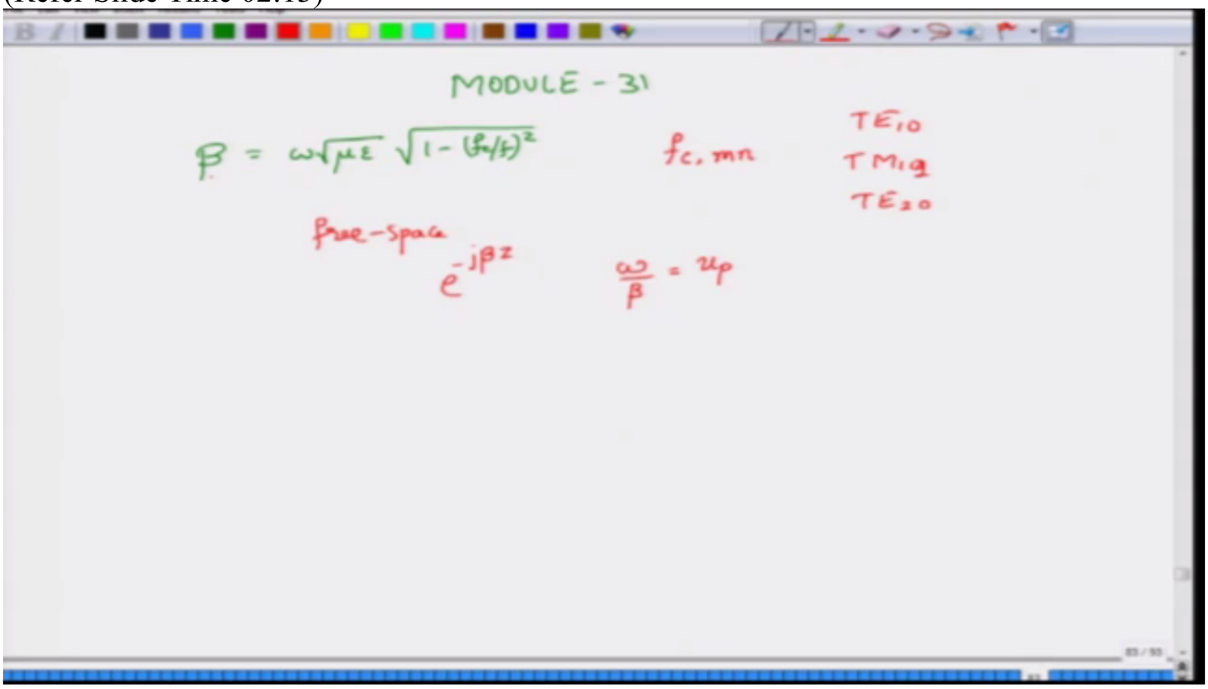

Now in this mode case, that is in the waveguides scenario, you still have the same kind of a relationship between ω and β. Okay. So we will define, without worrying too much about this, we will define what is ω/β, and we will call this as the phase velocity u_p, and using expression for β in this particular, you know, expression here in this expression, the second expression, what you get here is $\omega/\omega\sqrt{\mu\varepsilon}$ multiplied by this factor, which $1-(f_c/f)^2$. Okay.

I can cancel ω from both sides and then once I remove ω from the expression, what I get is $1/\sqrt{\mu\varepsilon}$. For a waveguide that is filled with $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ meaning that the no material is filled. $1/\sqrt{\mu\varepsilon}$ will actually be equal to c, which is the speed of velocity or the velocity of light in the free space or in the air medium, right? So, for air, ε is approximately equal to ε. So for only for the air filled case, you have $c/\sqrt{1-(f_c/f)^2}$. Okay.

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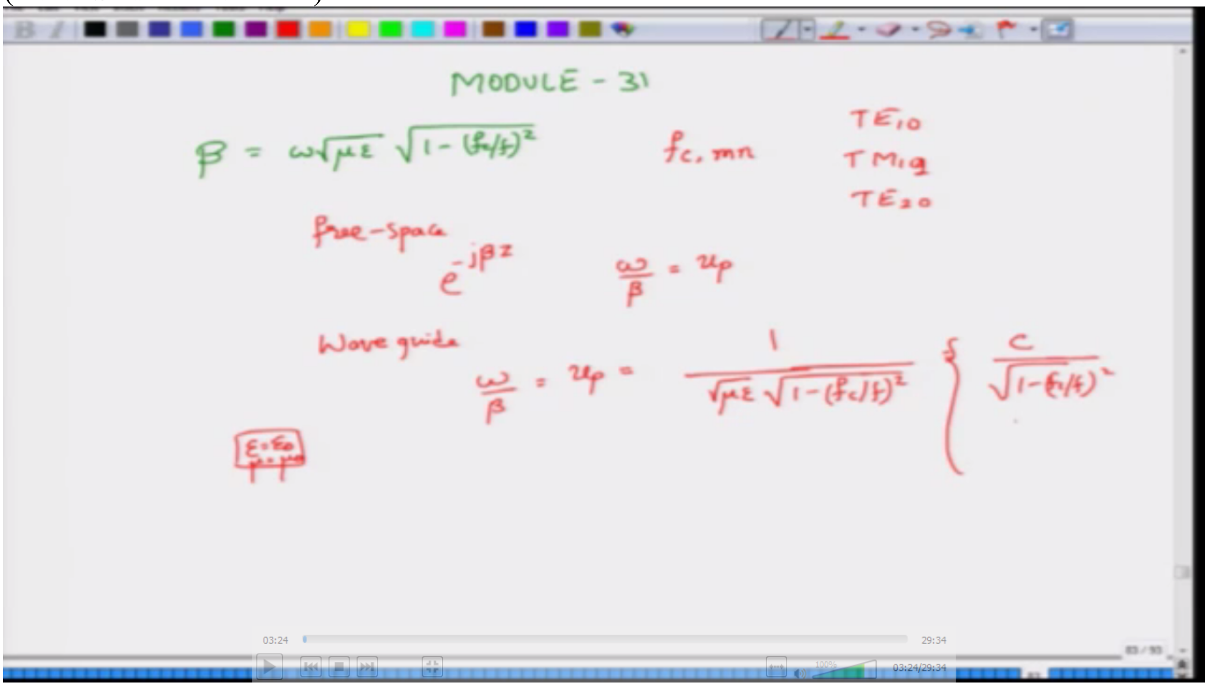

If not, you can replace this one by say some u_{E0}, which is my notation for the velocity of the wave in the medium that is given by ε of ε whatever the different values of ε Epsilon and a different values of μ. Of course, in our consideration, μ has been set to μ_0 and ε can be set to ε_r times ε_0.

So when you fill the rectangular waveguide with a material which is not air, but some other dielectric material, which is characterised by the permittivity ε_r, then the corresponding velocity we will write it as u_{p0}, which is basically the expression for $1/\sqrt{\mu\varepsilon}$ and the remaining factor that I still have is $1-(f_c/f)^{2.}$

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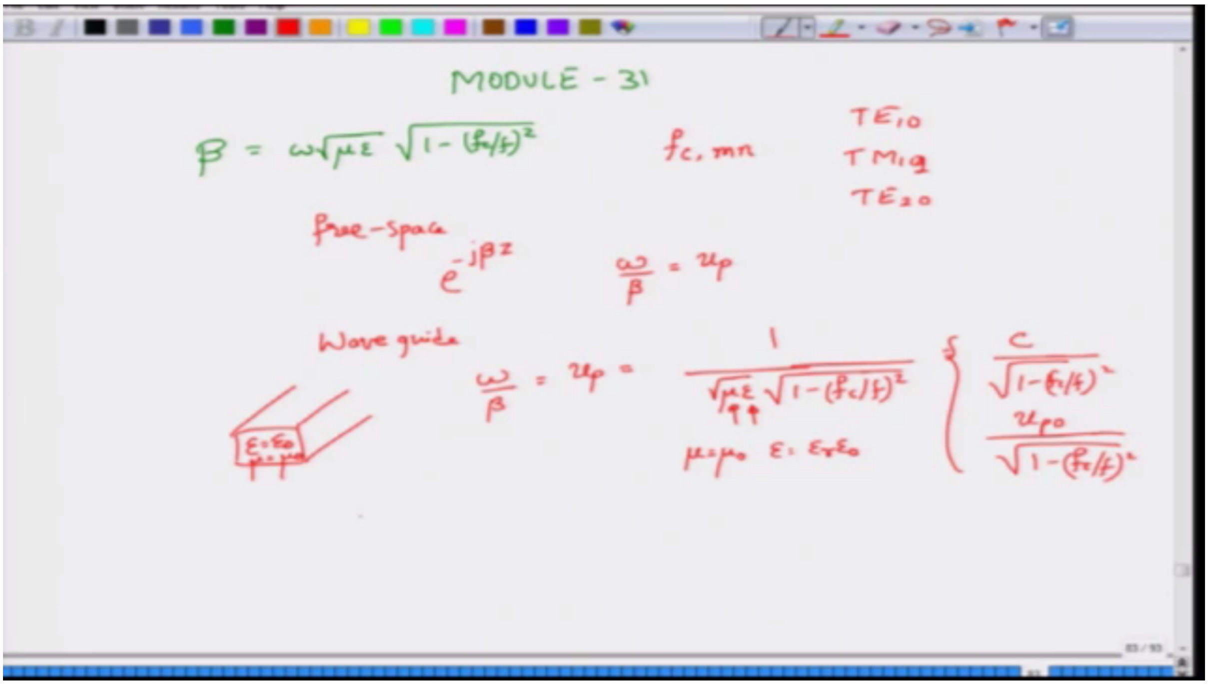

Now we have said that for a particular mode to propagate, f has to be greater than f_c. Now if f is greater than f_c, f_c/f will be less than 1. So square of a quantity that is less than 1 will also be less than 1. One minus of that quality will still be less than 1 and the square root of this will also be less than 1, but because this factor is in the denominator, what you would actually observe is that u_p will be greater than c, right? So it kind of seems to indicate that the phase velocities greater than the speed of light, right, when the material is filled with air, the speed of the phase velocity u_p happens to be greater than the speed of light, which, you know, perhaps would contradict from our Theory of Relativity, which says that no information can actually propagate at a speed which is greater than the velocity of light, which is c.

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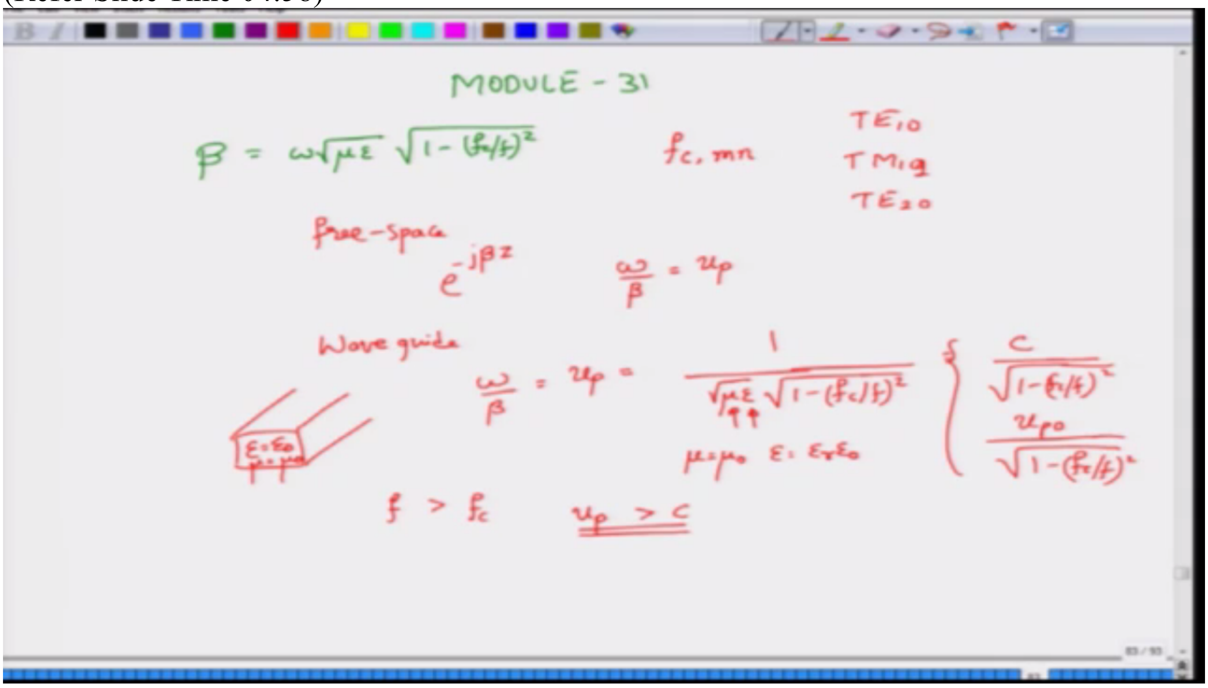

So how is it that we can reconcile the two statements that u_p, the phase velocity is greater than c, but is it also mean that can we signal or send information from one point of the waveguide to another point of waveguide at a velocity greater than c? No, it does not mean that. Please remember that no information is actually being conveyed by the phase velocity.

So no information is conveyed by this phase velocity u_p meaning that see in the parallel plate waveguide, we assume that the waves can be thought of as these bouncing back and forth from the walls, right? So this bouncing back and forth with an oblique incidence and so on.

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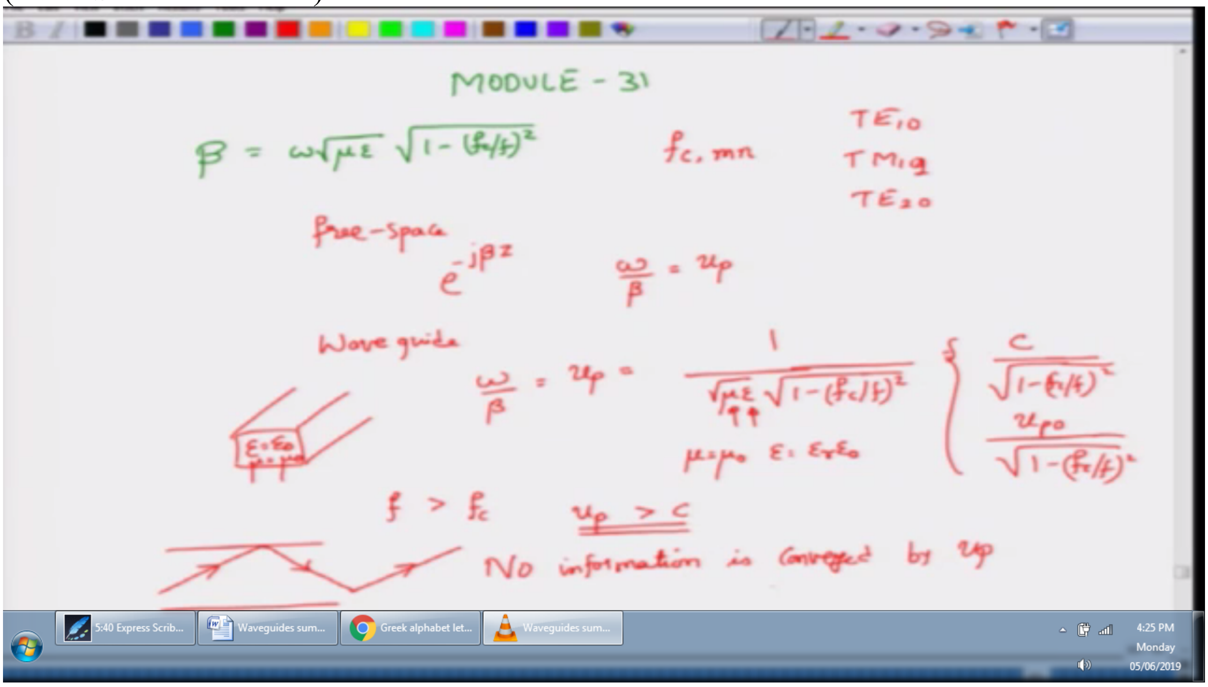

What this phase velocity would simply tell you is that what is the velocity with which this slanted ray or the obliquely incident ray should actually be moving. Because this length is larger than this length to satisfy the condition that, you know, the waves actually have to become the phase of this particular, I mean, segment and this segment should actually be in phase, it means that the wave has to actually propagate a longer distance in the same timeframe as you go to the horizontal distance, the distance that is propagated by this oblique k vector or oblique this one should be larger, but this does not mean that information is actually being conveyed because this is true only for some mathematical point of a phase, okay, and knowing that or not knowing that, you cannot know any kind of an information.

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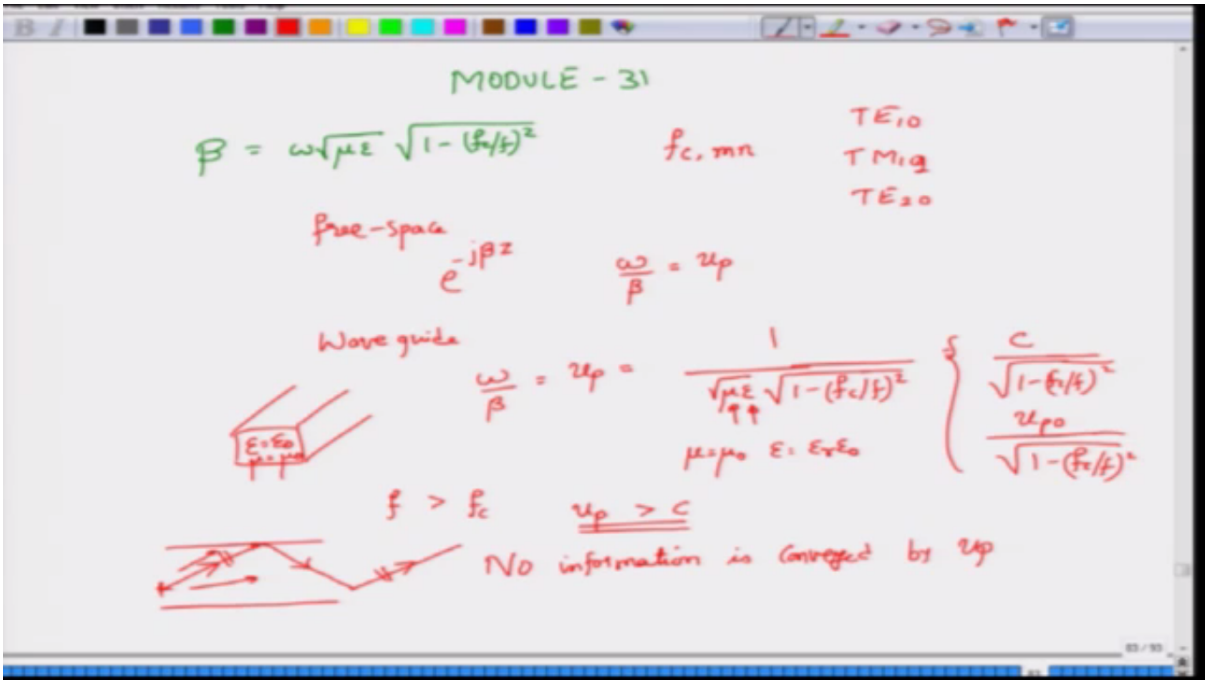

Rather information is contained only when the frequency changes or the phase changes, right, or of course amplitude changes, but we will ignore the amplitude change for now. We will, so for any change that or for any information that needs to be conveyed, the information has to be in the form of some changing frequency or changing phase because if it isn't changing, then you're not conveying any information. Okay.

So what quantity measures this rate of change of information is what the velocity at which we want to propagate. Okay. And that quantity happens to be what is called as group velocity.

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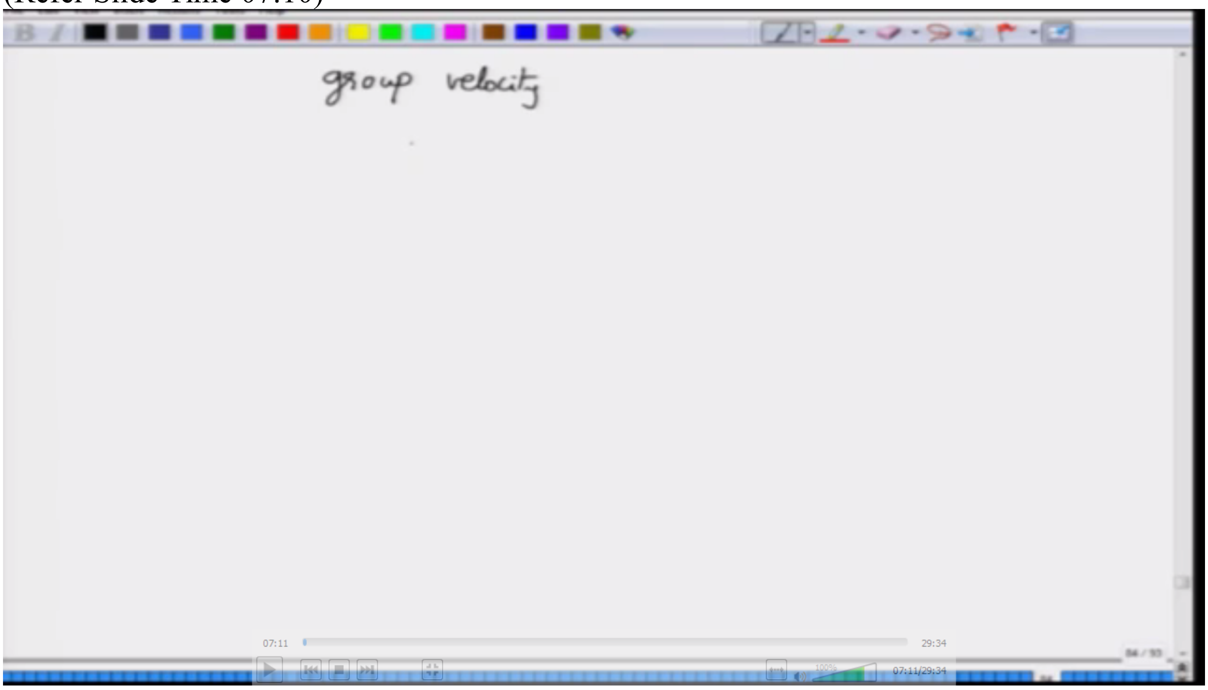

The basic idea of group velocity is that you can consider two sinusoidal signals of frequencies ω_1 and ω_2 and then see that they are basically propagating along say with β_1 and

β_z. β_1 and β_z are the propagation constants at these two frequencies. We will assume that both of these are of the same mood. For example, this could be the fundamental or the dominant TE_{10} mode and different frequency, different propagation constant, which is allowed, but both ω_1 and ω_2 are obviously greater than the minimum of the cut-off frequency for the TE_{10} mode. I am giving you an example of the TE_{10}. It could be any other mode, but we will assume it to be of the same modes, right?

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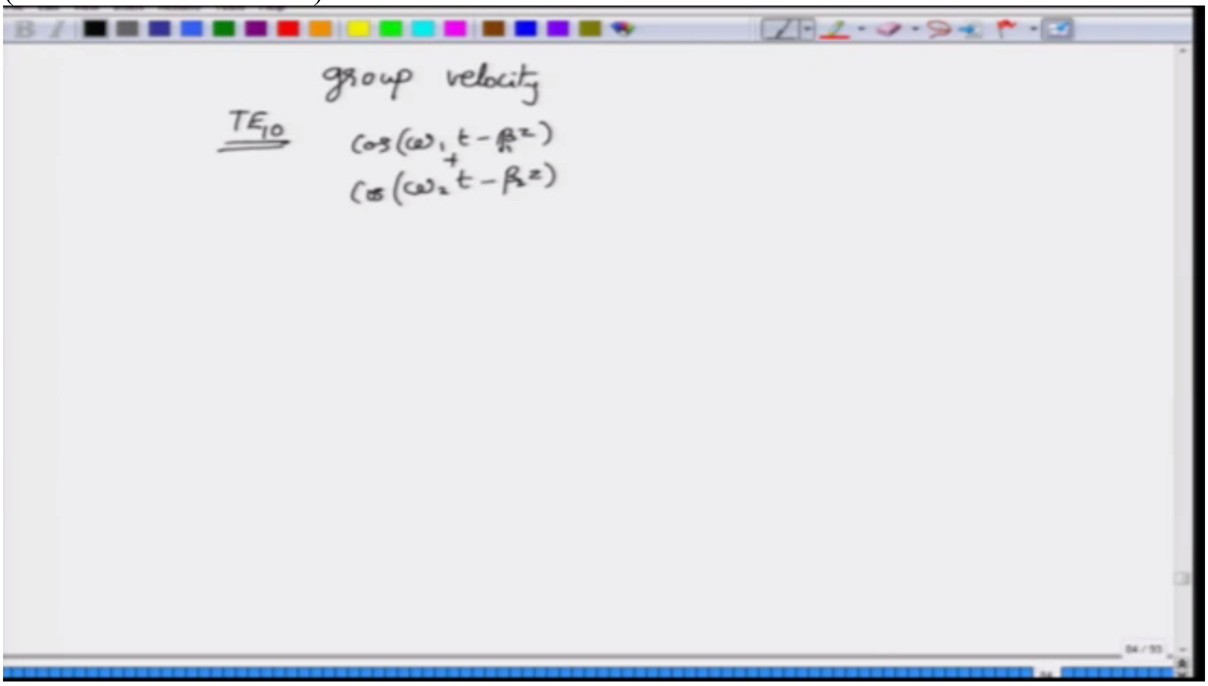

So if you now look at the overall field or overall electric field to be the combination of these two, what you would actually realise is that there will be two additional frequencies that are introduced, which are $\omega_1 + \omega_2$ perhaps by 2 and then there will be an ω_1 - ω_2 by 2, okay, because this is a straightforward cosine addition. Okay. And you can see that this is a slower, you know, a lower frequency. This one is a higher frequency.

So if you now sketch, what you would actually be able to see for a given z, the combination would actually look something that would be very familiar to you. So you have this kind of a combination, and then, you know, you have this carrier, which is what the frequency $\omega_1 + \omega_2$ by 2 would look like, and then this is the envelope, which is changing, and this envelope frequency would be proportional to ω_1 - ω_2. Okay.

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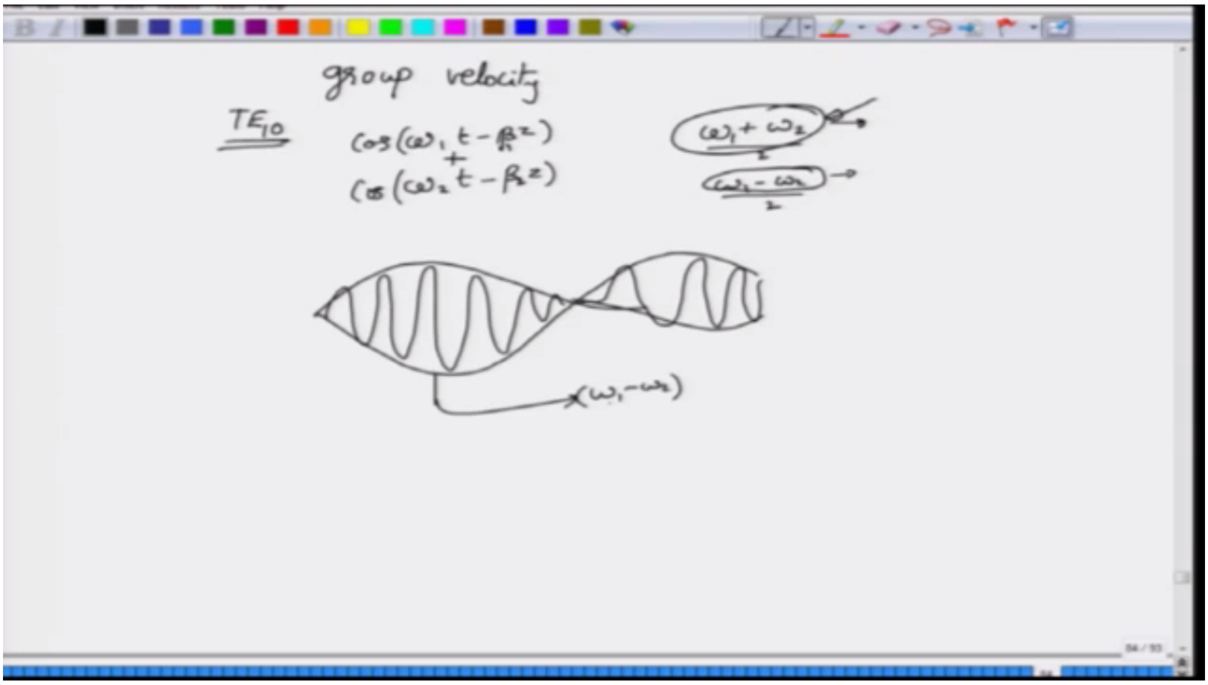

So you have a slow frequency wave riding on the high-frequency carrier and you're not interested in what speed that the carrier will move, right? So if you were interested in what speed the carrier will move, you can mark of a point and then look at what is the distance, I mean, what is the time taken for this point to go through a given phase reference of whatever and that velocity will turn out to be u_p. Okay.

However, if you're interested in knowing what is the rate at which this particular peak is moving, right, or in some sense the envelope is moving, this entire slow varying envelope is moving, that velocity will actually be u_g or the group velocity. Okay. And you can show by some detailed analysis, which we are going to skip now in the interest of time, that this d, I mean, u_g can actually be given by the derivative of β with respect to ω . Okay.

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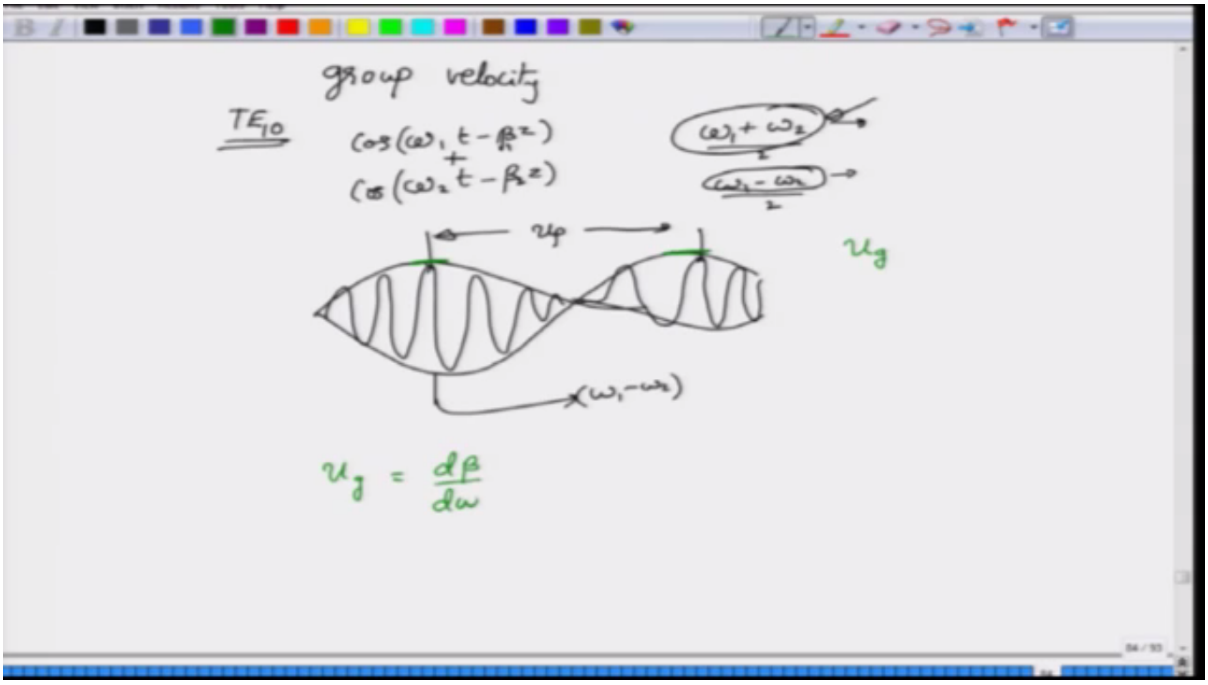

So this is what you get u_g as $d\beta/d\omega$ and it is the rate and it is this velocity with which information will actually begin to propagate or information can be conveyed. So this is the velocity of information that is being transmitted. So information velocity I can write this. Okay. And this information velocity happens to be less than c, which is the speed of light or in general it would be less than u_{p0}. Okay.

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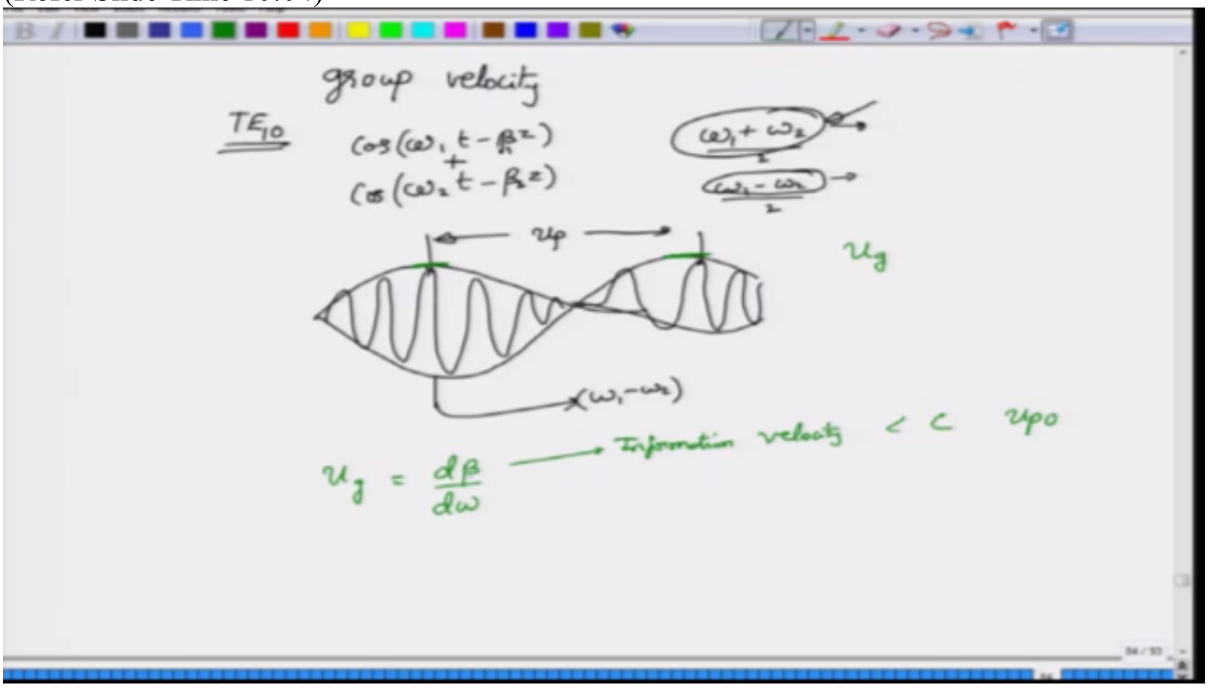

Why? Because let us go back and look at the expression for β, which we've already written there and then differentiate that expression of β. Okay. When you do that, you will see that group velocity is given by in general $u_{p0}\sqrt{1-(f_c/f)^2}$. Okay.

So if this is fc and when you operate at an f, which is much, much higher than f_c meaning that this particular mode is now well represented, the ratio of f_c to f will be almost 0. Sorry, almost, yeah, almost 0, and this further implies that u_g will be approximately u_{p0}. All the time it will actually be less than that, but this is an approximation, right?

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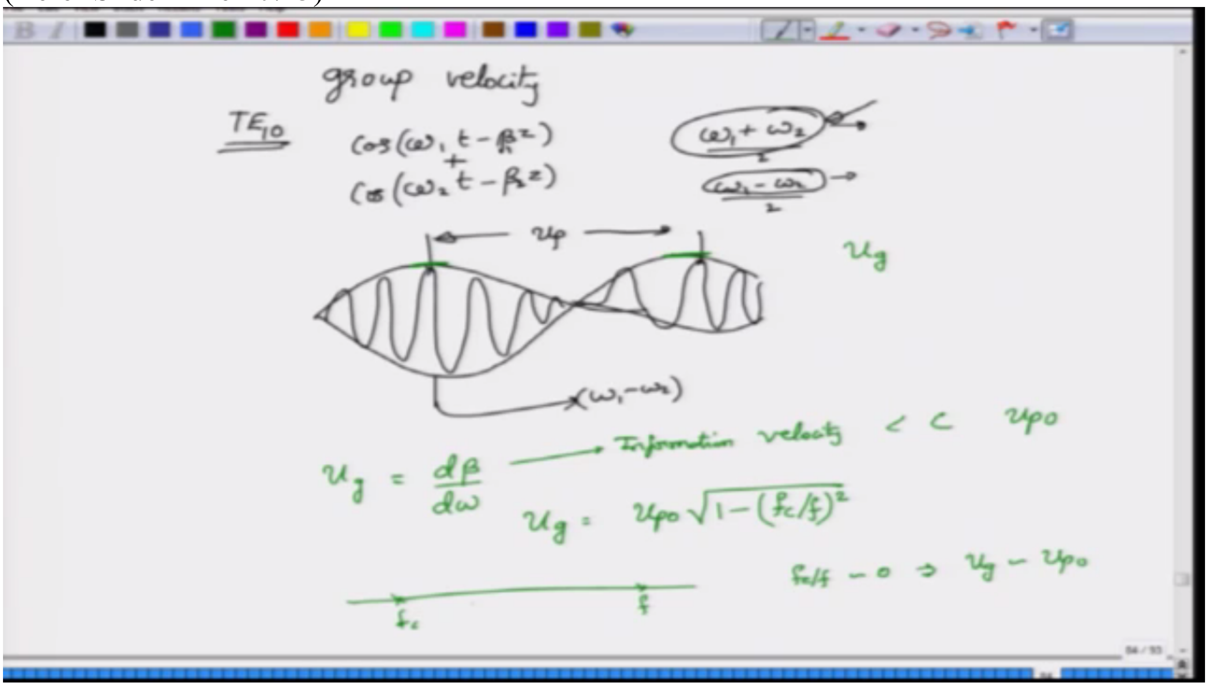

So as you move away from the cut-off frequency, the group velocity will actually approach the phase velocity and in fact it does become independent of the frequency. So if you want a good distortion less transmission, that is your pulse should not be distorted, you want to operate at a frequency which is higher than the cut-off frequency, but there is also a catch here that as I start increasing the operating frequency, there may be additional modes that would come in.

For example, this could be TE_{20}. This let's say is TE_{10}. So if you take your frequency beyond this, then you have to deal with the fact that there could be multiple such modes and information will anyway be or the pulse will anyway be distorted. Okay. So this range where you are going to operate between, you know, the first or the dominant mode and the onset of the next higher-order mode is called of the single-moded region. Okay.

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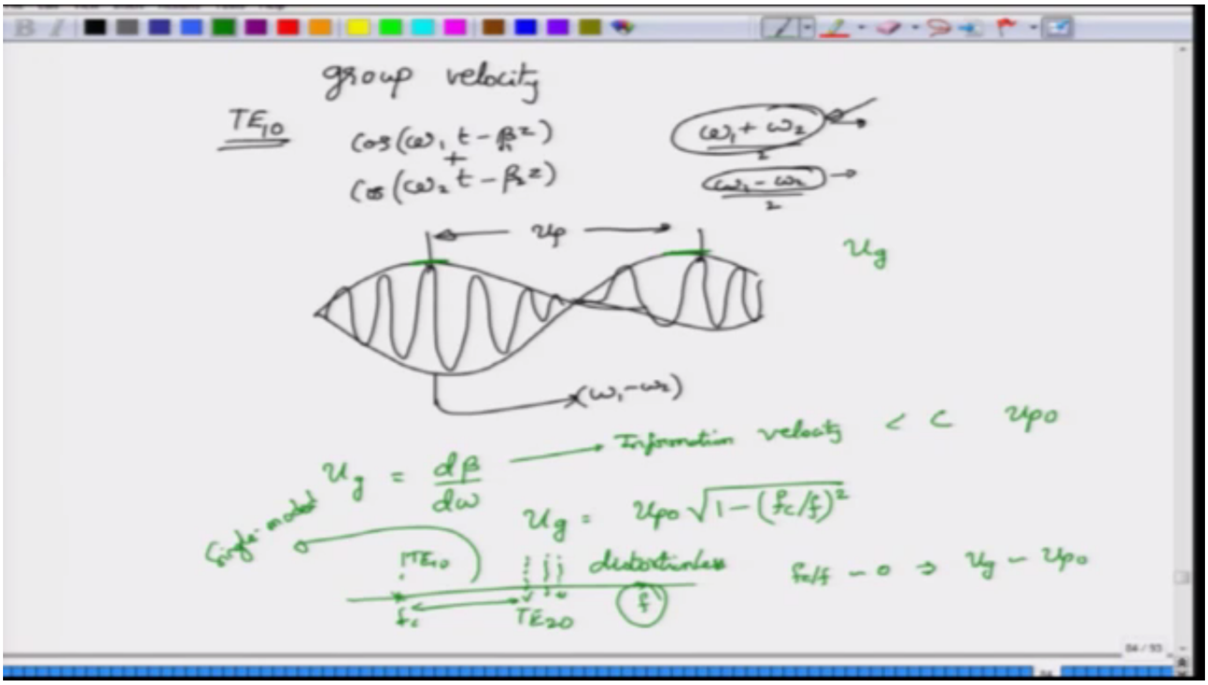

This is very similar to the fibre case that we discussed. So you had this v number and then once your operating number became greater than some 2.405 and if it went beyond that, then you would actually have these additional modes coming in, right?

So the fibre would not be single-moded beyond the cut-off frequency of 2.4 for a step index profile. In that same type you have a single-moded rectangular waveguides, which may become multi-moded as the operating frequency increases beyond a certain range. Okay. And when it does happen, distortion will be introduced. Okay.

However, if you are operating within this, you still will have some amount of distortion because this u_g will not be approximately u_{p0}, but rather u_g will be a mild function of frequency. Okay.

In fact, let us do this. This is a more convenient way of looking at what is the phase velocity and group velocity. Suppose I plot this β versus ω. Okay. I will assume that β is the independent variable and then simply plot ω. You can, of course, switch the axes; doesn't really matter.

What is interesting is there will be no real value of β until a particular cut-off frequency. This in the case of a rectangular waveguide, this would be the TE_{10} mode. So until this cut-off frequency occurs, the value of β will be imaginary and I am not representing that one in this axis. Okay.

But once β or rather once ω starts to increase from ω_c, β will actually start to increase. Okay.

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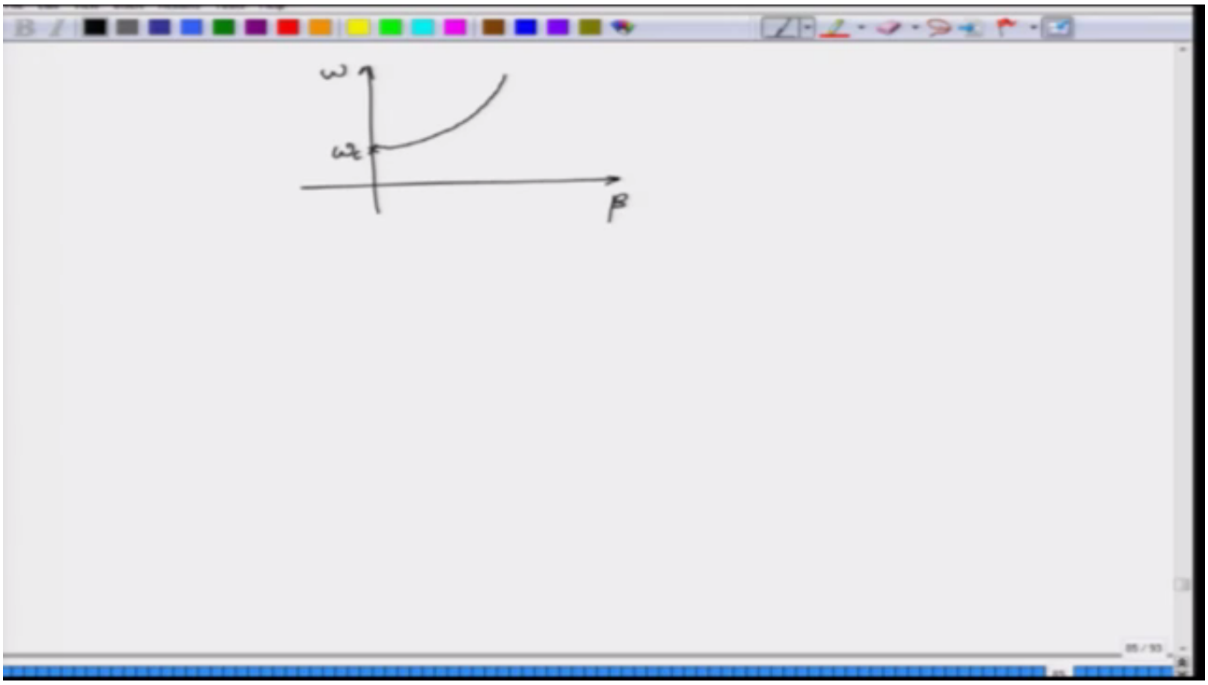

In the asymptotic case, that if this is the kind of an asymptotic case, which I should actually write it in this manner, okay, the ratio, at any point, the ratio of ω to β, so this is some ω_0; this is some β_0. The ratio of this one will give you u_p.

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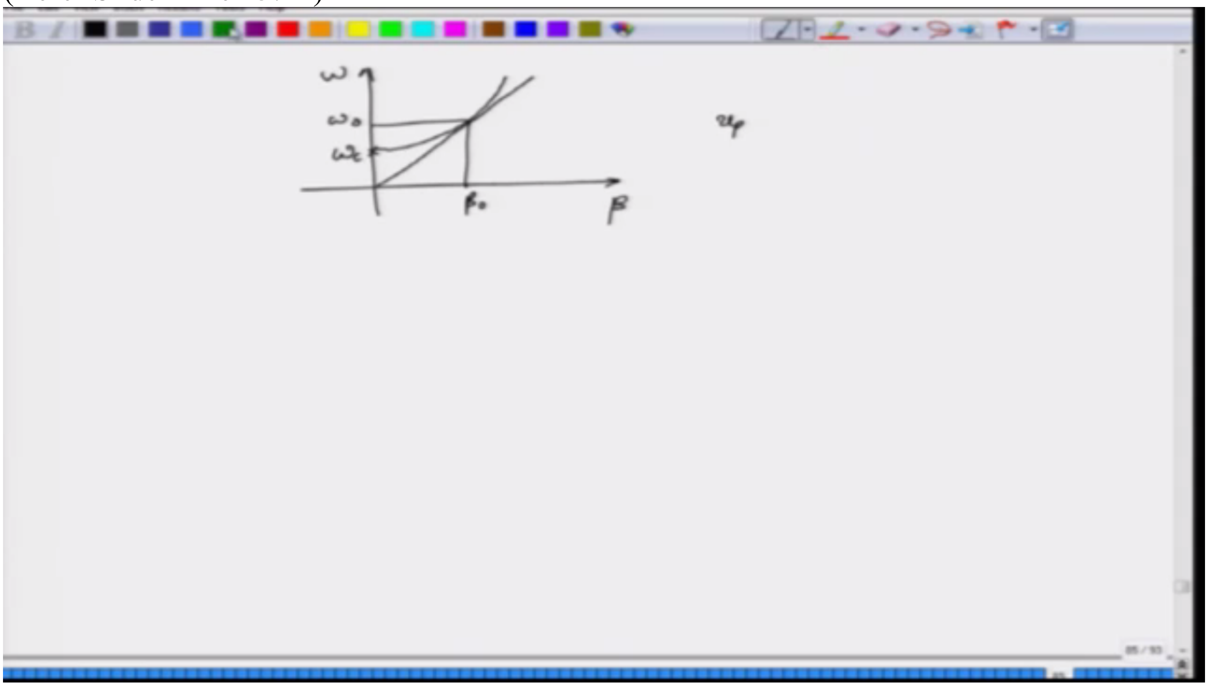

However, to obtain the u_g, that is group velocity, you have to actually draw a tangent here and then measure the slope of this tangent, which will give you u_g. In fact, take it from me that the product u_g times u_p will be u_{p0}^2. Okay. So that's another check or that's another way in which you can obtain what is the value of u_g given that you know what is u_{p0} as well as the phase velocity u_p at any given frequency. Okay.

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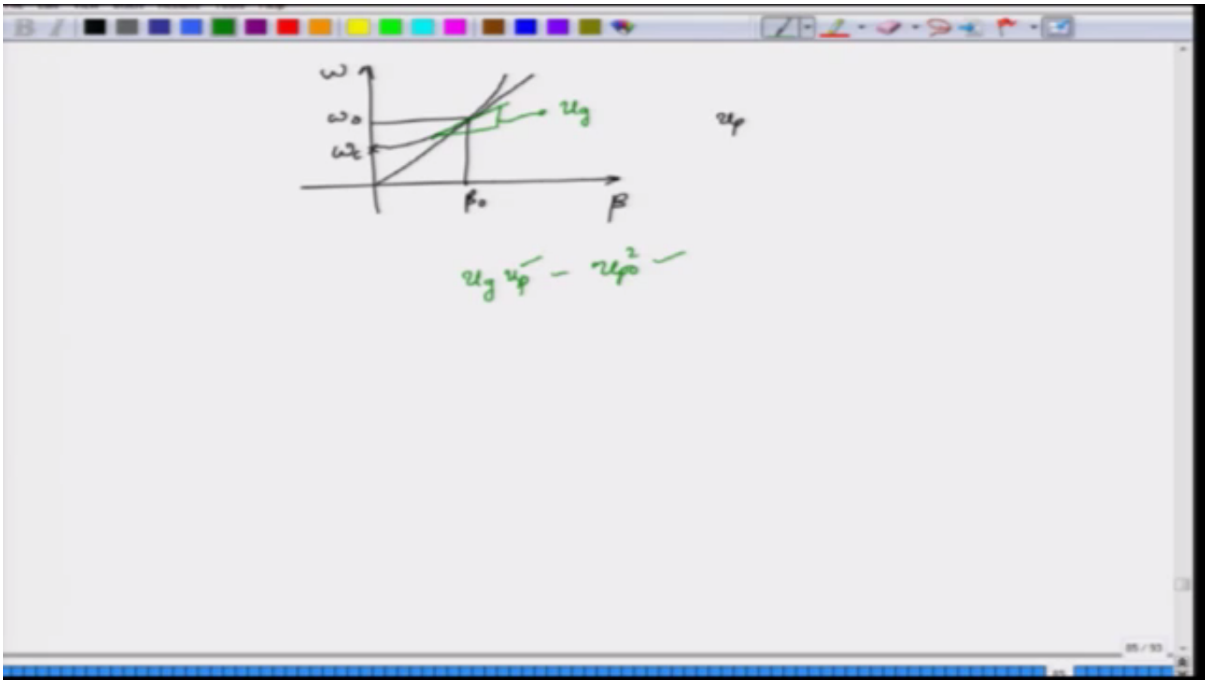

So we will stop discussion about the velocities here and then point out discussion, two discussions here about the impedance.

See when we talked about the uniform plane wave, right, so we had this electric field amplitude to the magnetic field amplitude ratio, and then we said that this ratio is what we are going to call as the wave impedance or the characteristic impedance of the material medium. But in the case of a waveguide, what could such a thing be?

Because you have the transverse mode, right, you also have the longitudinal mode. So I can't simply tell the ratio of electric field to the magnetic field. I have to specify what is the ratio that I am actually looking for.

So in the case of an obliquely incident wave on a medium interface for the TE or TM, we did introduce the idea that you can always look for the tangential component to the boundary of the electric field and then the magnetic field, corresponding magnetic field and call that as the impedance, equivalent impedance. So you remember this η Sec θ or η, η Cos θ being the two types of impedances that we found, right?

In that same lines, we can actually go back. In fact, we can go back to Maxwell's equation, find out the expression for E_X and H_y for the TE mode as well as for the TM mode.

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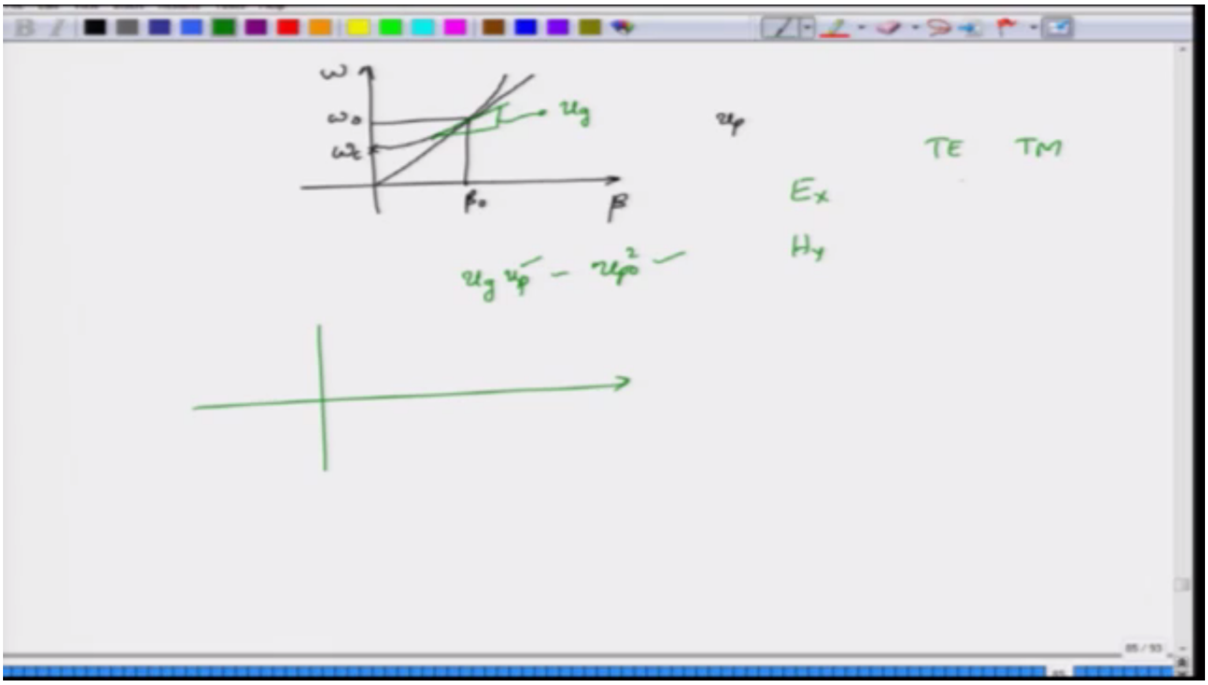

Remember for the TE mode, $E_z = 0$; for the TM mode, $H_z = 0$ and you can find out the expression for E_X and H_y and then take the ratio of E_X to H_y. Okay. If you do this for the TE case, we will call this as Z_{TE} and that would be the equivalent impedance of the mode that would be propagating in the waveguide. So that is Z_{TE}, which you can show is given by $\eta/\sqrt{1-(f_c/f)^2}$.

How do you know this one? This is I will leave it is an exercise. Remember you have actually written E_X as a function of del E_Z as well as del H_z, right? So there you had written E_X as a function of Z and H_z derivatives.

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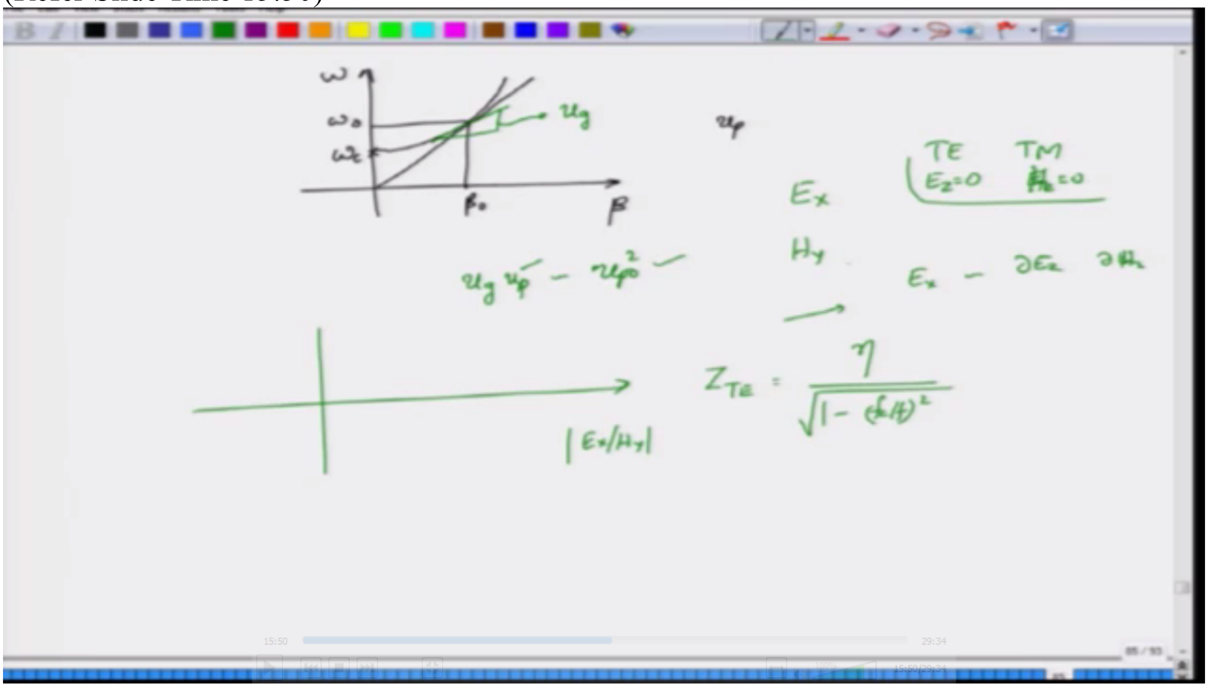

Similarly, you would have written the function for H_y, right? This was the first step that we talked about when solving the waveguide problems and from those expressions, setting appropriately $E_z = 0$ or $H_z = 0$ you are going to get this equation. Either they would be $\omega\mu/\beta$ or $\omega\varepsilon$ by $\beta/\omega\varepsilon$. Okay. So when you substitute the appropriate value of β into that expression that you're going to get under these two conditions, you can show that Z_{TE} will be η by this factor and you can show that Z_{TM} will be η times $1-(f_c/f)^2$.

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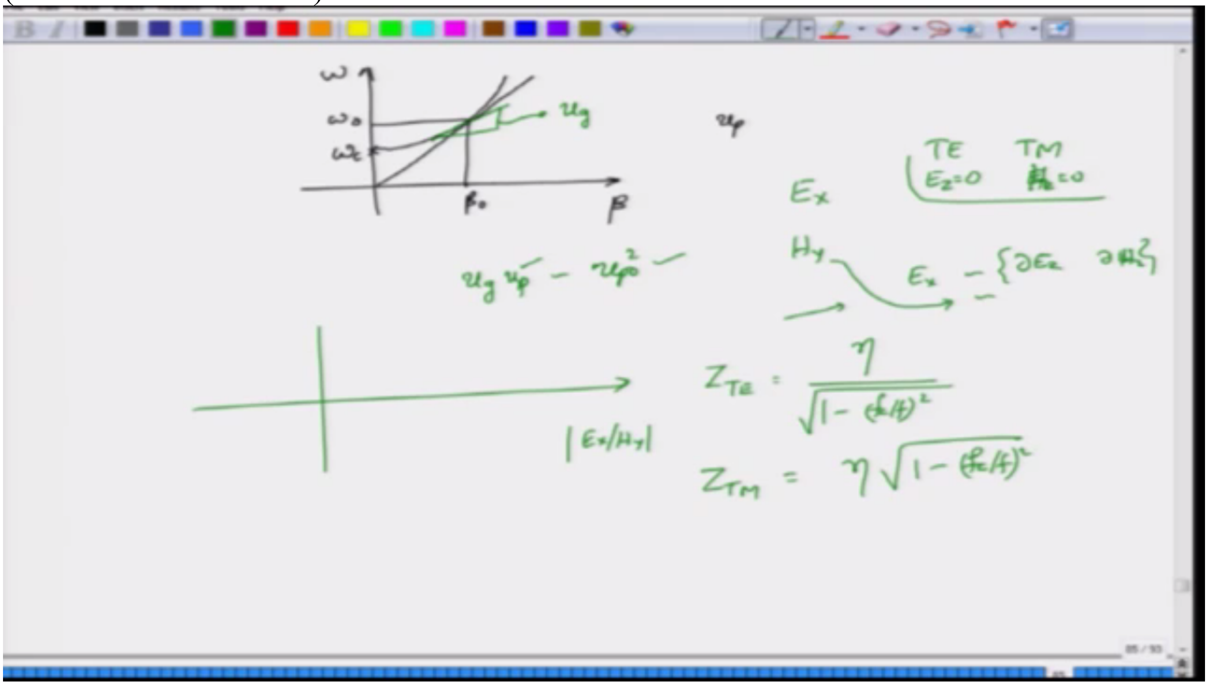

So if you're looking at a particular mode that you're interested in, then what you can see is that at $f = f_c$, of course, before $f = f_c$ there is no concept of an equivalent impedance because there is nothing really happening, but at $f = f_c$ what would happen is that this denominator will be 1 and then the impedance Z_{TE} will be infinity. So it's like an open circuit out there and then at f_c/f going to 0, Z_{TE} will eventually approach the value of characteristic impedance.

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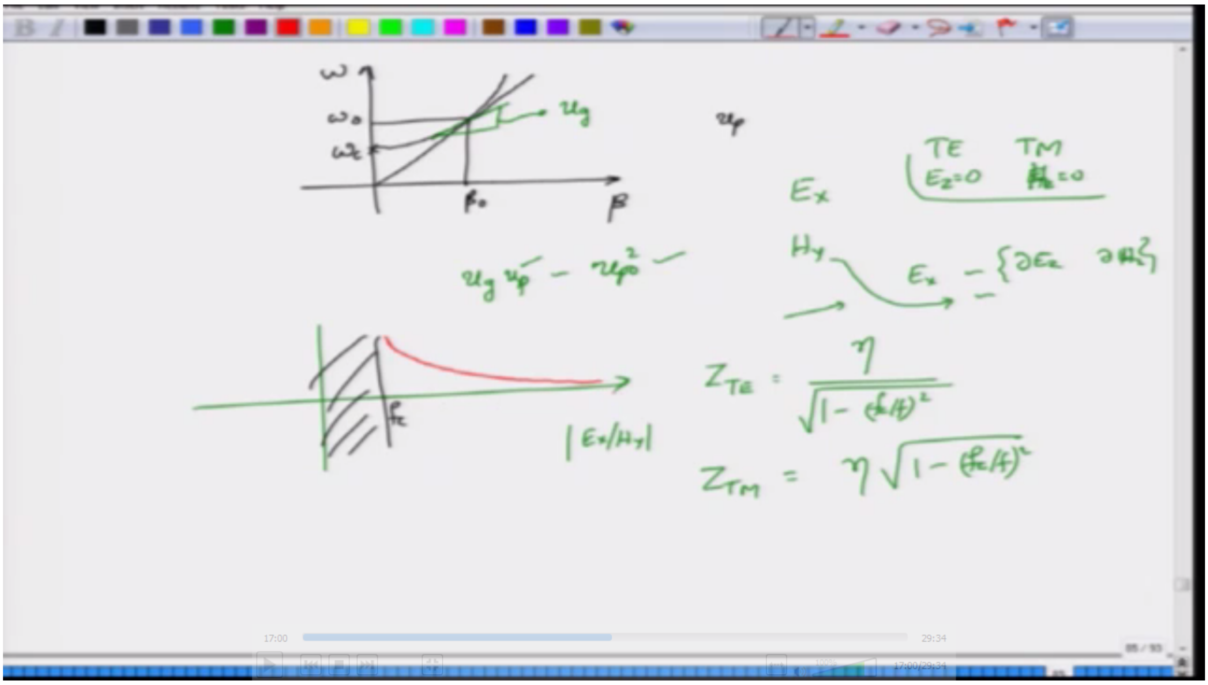

So the line that I have drawn here corresponds actually to the characteristic impedance Z_0 or the characteristic impedance η, right? So in this particular case, η will simply be equal to $\sqrt{\mu/\varepsilon}$ for the material filled waveguide. For a free space, it would essentially be free space characteristic impedance. Okay. So this is Z_{TE}.

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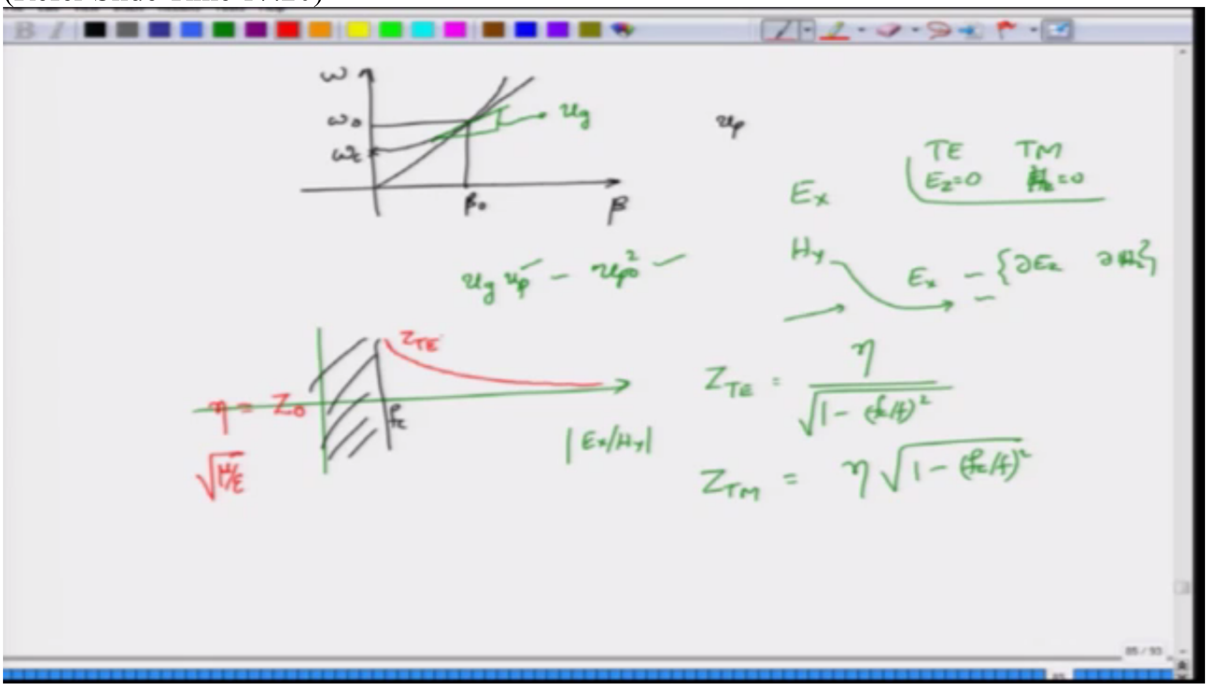

What will you see for the Z_{TM}? For Z_{TM}, you will actually see that it would be 0 when f is equal to f_c and from being a short circuit it actually again increases and eventually it becomes that of the characteristic impedance of η. So both TE as well as TM will start off with different values, but as the frequency increases beyond the cut-off of that particular mode, they both will converge to a common, I mean, common impedance of η, which is the material impedance. Okay.

So we stop our discussion of waveguide. So to kind of summarise what we have done, we have seen waves in free space, right? So this justifies our wireless media propagation kind of a, you know, subtitle for the course where we saw that the corresponding modes of the waveguides were the uniform plane waves, right? These ways had the structure that their, you know, electrical field and magnetic fields were, you know, kind of perpendicular to each other. Therefore, they were also called as transverse electromagnetic waves.

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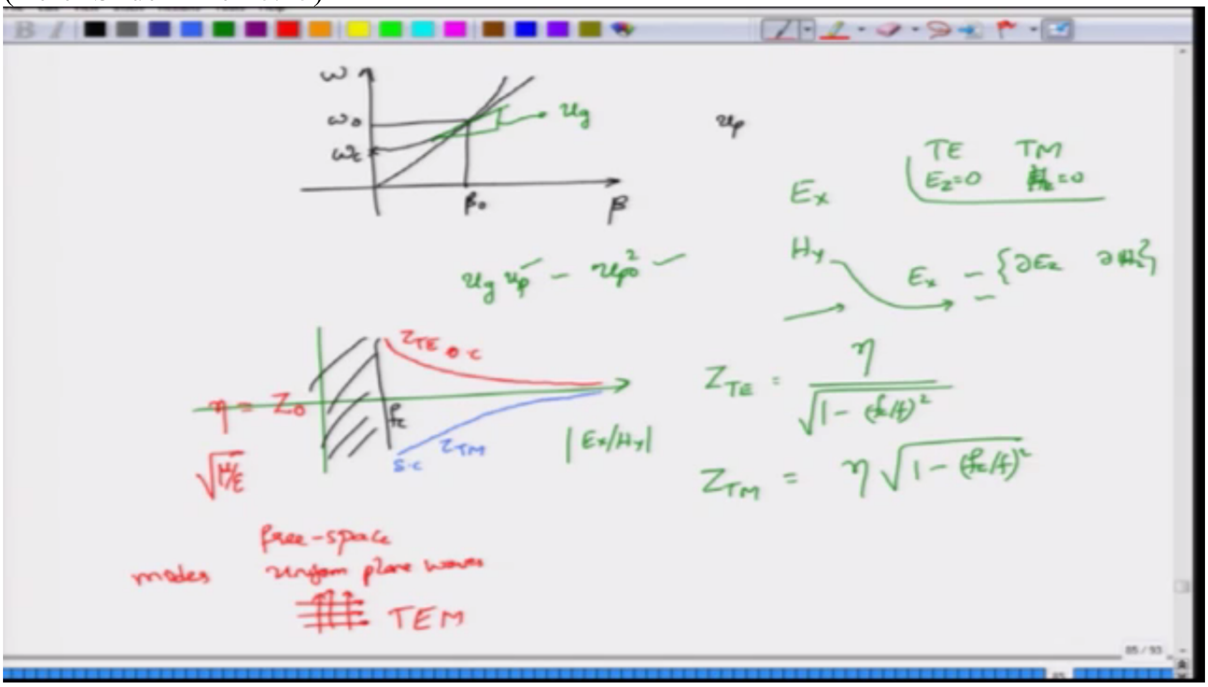

We then saw parallel plate waveguide. Okay. We saw this one either as a metallic waveguide or in terms of the dielectric waveguide. Here the modes were actually functions of x because I assume that this was going along x thing, right? So the functions in the case of a rectangular waveguide were exactly cosine and sine whereas for the dielectric waveguide they were exponentially decaying out, and therefore they were actually somewhat like this, right? So this was for the dielectric waveguide and this was for the metallic waveguide.

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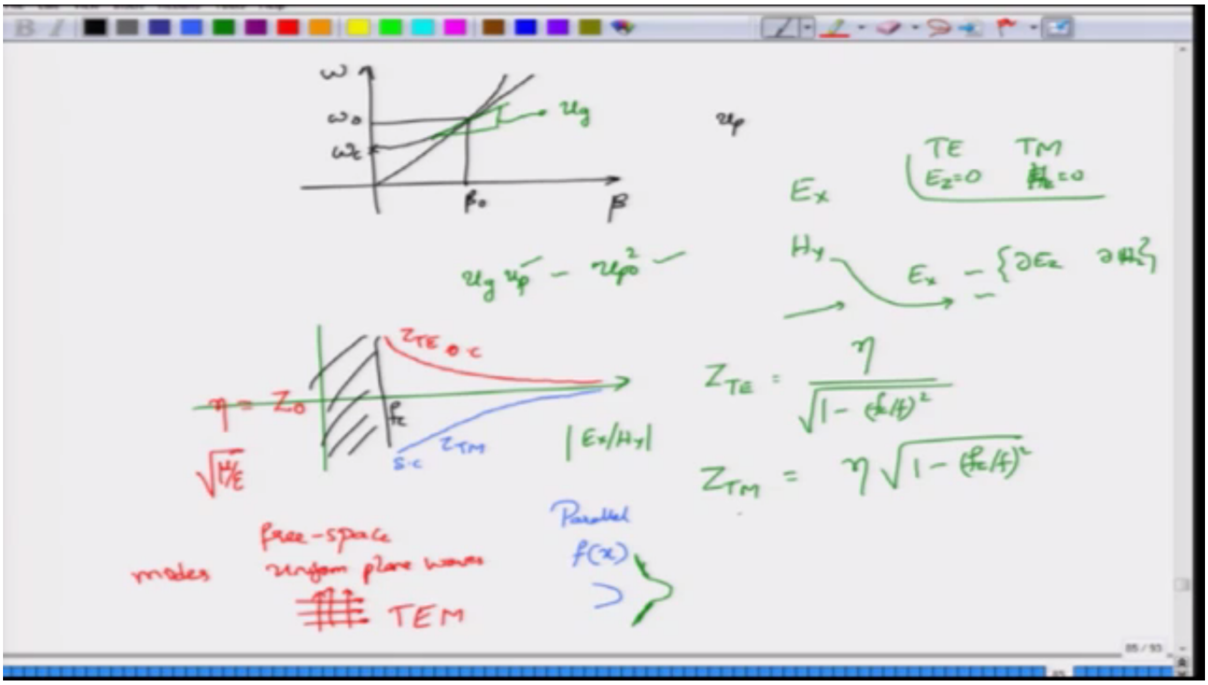

And when you go to a fibre, so that would correspond to a cylindrical rod in the sense. So, again, for the metallic waveguide, we haven't really shown what the solutions are, but you can guess the solutions to be some sort of a Bessel functions. Okay. So the Bessel functions would, of course, go to zero at the boundary because that's what the boundary condition for the metal would be, but in the case of an optical fibre, this mode would actually be these different modes HE_{11}, LP_{01} and so on. Okay.

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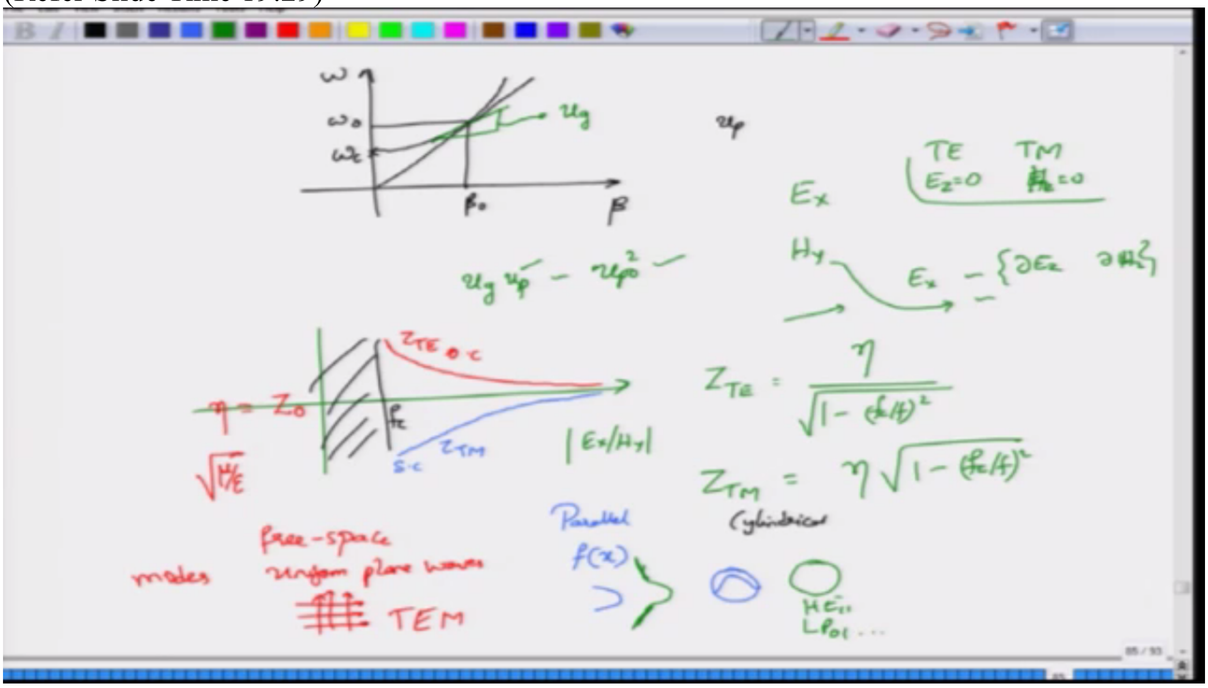

So for the case of a dielectric, I mean, for the case of a metallic waveguide, you still have the hybrid electric and other modes, but the mode structure will be something like that would be zero here whereas for the fibre, it would be some Bessel function followed by an exponential decay, right? So you can also have this other kind of a thing. So these are all valid modes and these are, so what these functions are now not just f(x), but they are rather f(x, y) or if you prefer, you can write this as f(r, φ), right?

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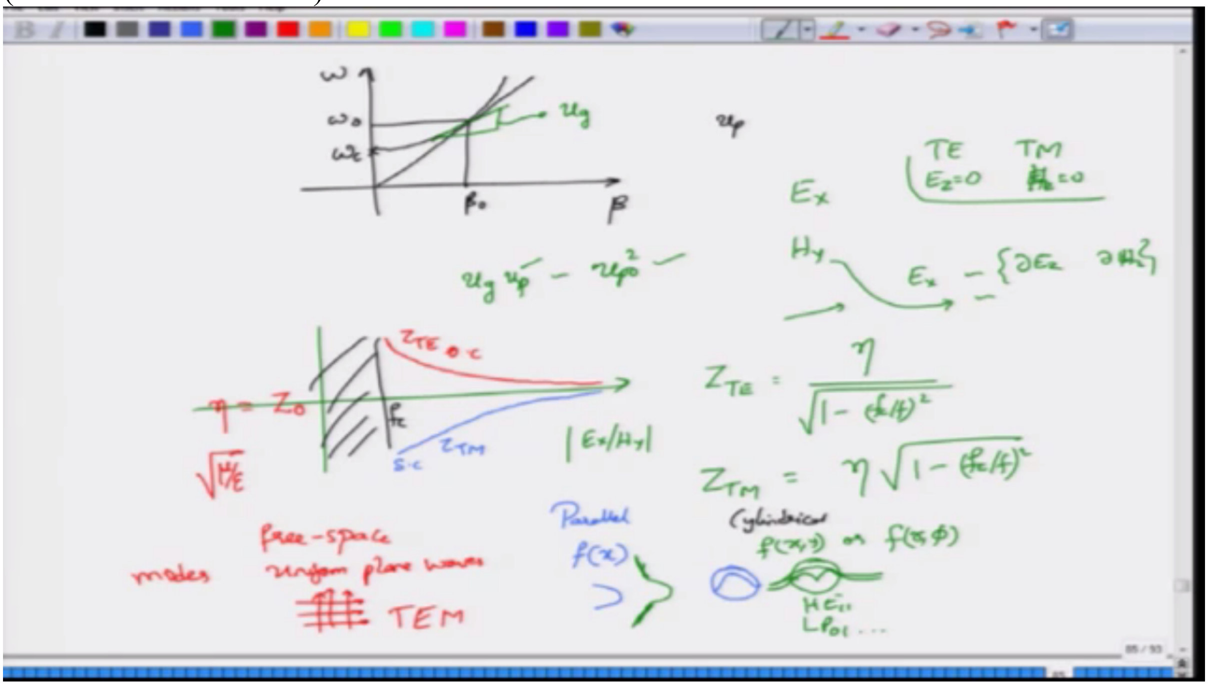

So now you see that the modes of the cylindrical rods waveguides or the fibres, they are actually becoming functions of two variables r and φ. Okay.

Then, finally, we also studied this rectangular waveguide in which case the functions were again some function of x and function of y in the form of a sine and a cosine wave, but then that also illustrates the fact that for the rectangular waveguide that we considered, right, the mode functions are again sine or a cosine waveform, but then, you know, this is the rectangular waveguide that we considered and this mode is essentially the transverse distribution, which would be propagating, right?

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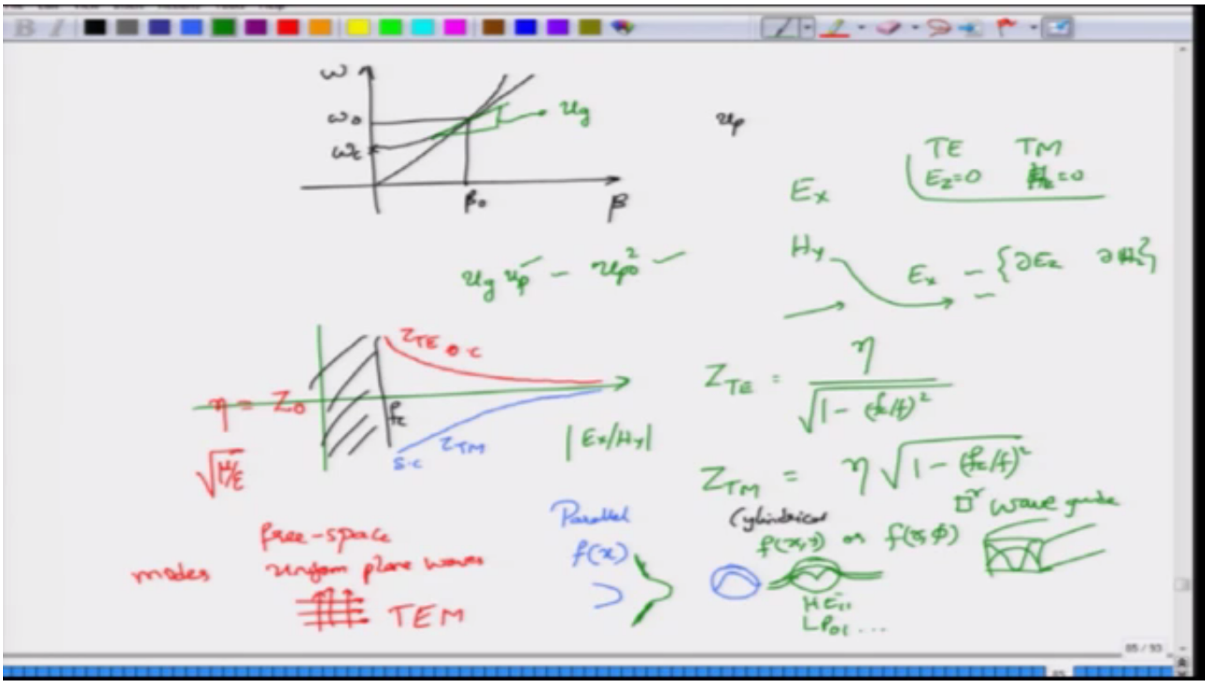

So what we've seen is that when I say something as mode, by mode what I mean is the set of electric field and magnetic field patterns that would satisfy Maxwell's equations as well as it would satisfy the boundary conditions. Okay. So that is what the propagation in different media that we have already seen.

Now we haven't really addressed one very important question here. Okay. That question is how did we generate the -- how did we generate uniform plane waves, right?

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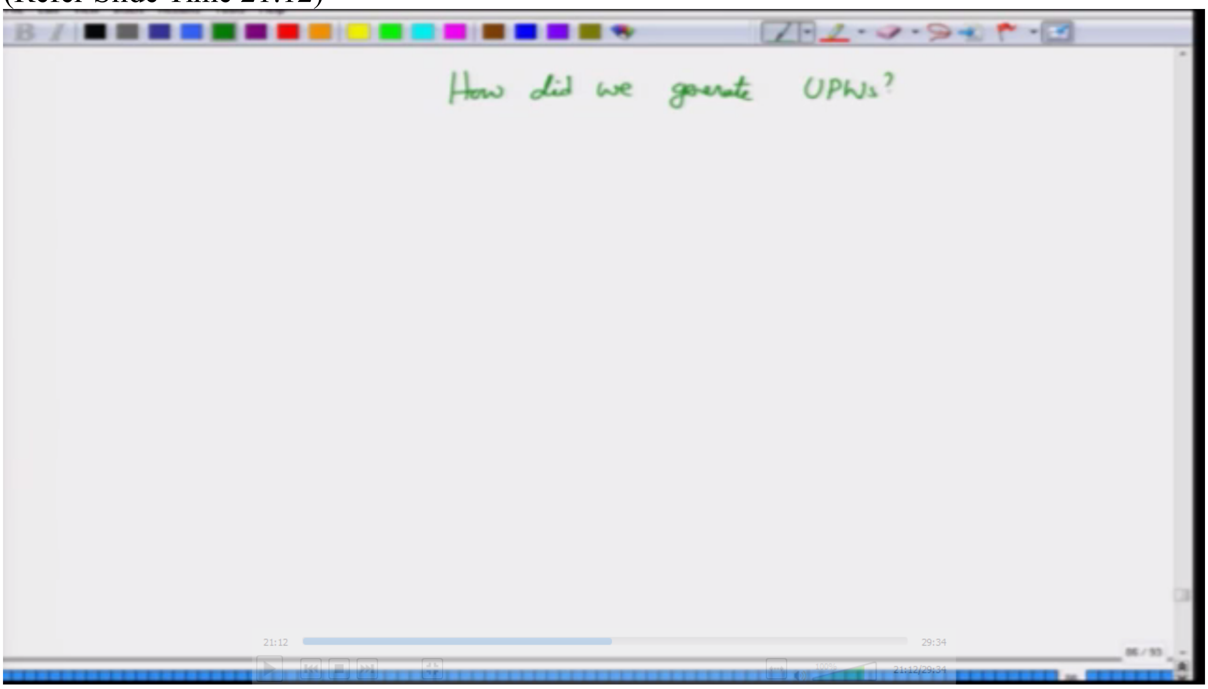

What was our methodology to generate uniform plane waves? I mean, mathematically, you could simply write a set of equation. We wrote that equation called as Helmholtz equation. We found that the solutions of that could be something and since there was no boundaries, we had the form of the solution and we called that as the uniform plane wave, and we also figured out that the structure for transverse electric, I mean, electric field and magnetic fields would be transverse to each other and we call it as a T_{EM} wave. But who was generating those waves, right?

Technically, uniform plane waves do not exist because they would imply certain other conditions as we have treated earlier. They cannot be generated. However, waves which are travelling from very far away distance, for example, light that is travelling from Sun and falling onto the Earth, the wave front or the plane, you know, the face front of that one can be approximated in a very, very good approximation as a plane wave. Okay.

Technically, they will be spherically as we will see sooner, but when the sphere radius becomes very, very large, we are talking of thousands and millions of kilometres, what is happening is that the curvature would be very small and you can almost treat the curvature to be of infinity in the sense that you're treating them as a plane wave itself, right? So there has to be some source which is kept far away from some distance and then as you keep moving away and away from the source, you would see that the waves that are generated by that source can be thought of as uniform plane waves.

Indeed what kind of sources are required and what is the mechanism that is generated or that is, that is necessary or what is the mechanism that actually gives rise to waves, which may not be just uniform plane waves? This was just an approximation. What are the actual waves that are generated, right? And if they are generated, what of the devices can generate them and what will happen for these waves as they propagate? Can they be recaptured? Can their energy be tapped by some way? The question, the answer to these questions is what we are going to occupy for the next few modules. Okay.

As a preparation to that topic, which we can combine or we can put it in the heading called Radiation and Antennas, okay, radiation is the phenomenon in which the waves which are generated from antennas actually propagate the mechanism, the waves, the way they propagate is what radiation is, and the devices that does radiation or the devices that radiate electromagnetic waves is called as the antennas. Okay. So antennas generate waves or generate radiation and this radiation propagates. Okay.

So what we are going to do for the next few modules is to look at this radiation phenomena, address some interesting, you know, effects of antennas and then we consider the radiation to be very important because radiation leads to propagation because once the fields are radiated, the radiated friends must propagate. Okay. And when they propagate, for example, in the wireless net, you know, communication scenario, they will usually be also carrying information. Okay.

So when there is an electrolytic wave, which propagates, and it suddenly sees a building, what should the electromagnetic wave do? Should it go and hit the building and come back? Well, if the building is made entirely of metal, it exactly does that, but if the metal, it's not a metal, but it is some other kind of a building and maybe the building is not completely infinity in every extent, it is just a small portion of the building or there is an obstacle somewhere, you know, someone is standing there, I can't model a person as a building, so what will happen to the electromagnetic waves?

So all these questions become very important because answering these questions will tell us many things about the way information itself can be changed by these obstacles, okay, and that comes under what is normally called as the channel modelling. So we are going to look at some aspects of the channel modelling after we have understood the fundamentals of radiation.

As a preparation for radiation, let me begin with couple of very simple equations. Okay. Let me write down this equation for you. I have this $\nabla \times E$, right, which is given by, usually, it would be given by $-\nabla b/\nabla t$ because you are normally in the range, I mean, considering in the scenario of what is called as dynamic scenario, time is varying, but for now we will simply assume that time is not of an important quantity. We will make that equal to 0 meaning that we are in what is called as the static case. Static means nothing is changing with respect to time. Time is not an important factor.

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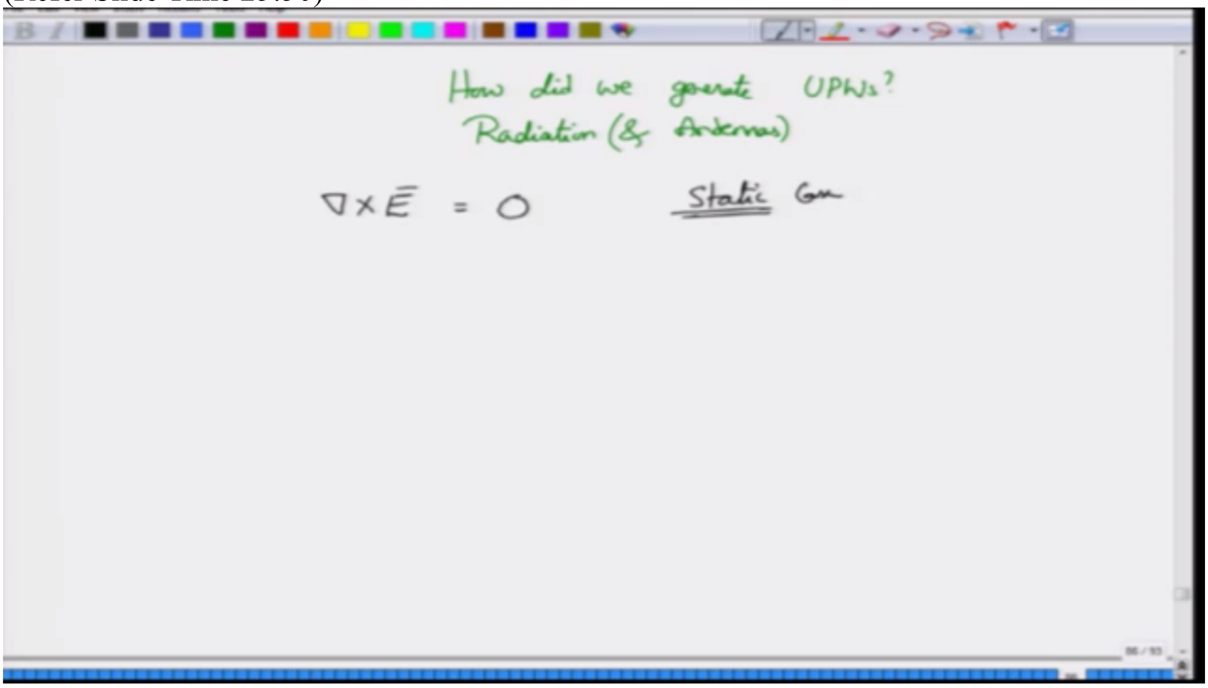

So I have this equation $\nabla \times E = 0$ and I also have an equation which says $\nabla \times H = J$. In this case I can write $\nabla . E = \rho/\varepsilon$ because you know in the free space on in the medium that we are considering, in the static case, E and t are simply related by a constant of proportionality which we will take it to be ε. Of course, in our case we can imagine that ε to be equal to ε_0. Okay. So no worries about that.

So in the medium that we are considering, we are considering the medium of a permittivity ε_0. You also have $\nabla . B = 0$. Okay.

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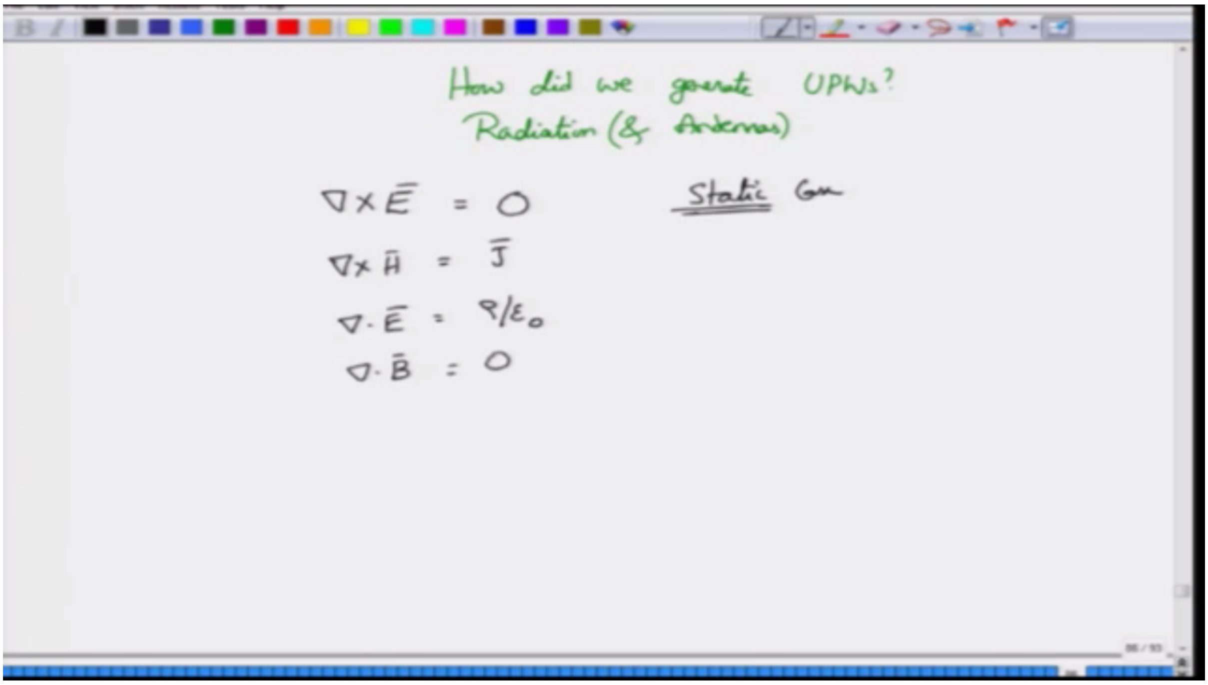

These are the equations of Maxwell's equations or rather these are the Maxwell's equations in the static case. Okay.

Now vector analysis tells us that if the curl of some field is equal to 0 and provided the field is continuous and satisfies certain conditions, then I can express that field as a gradient of a scalar function. Okay. I have put a ‘minus’ sign because of the conventional that we use in electric field. Okay. So don't worry about the ‘minus’ sign thing, but remember that if this is true, then this can be done. Okay. And this is okay because gradient of a scalar will give you a vector field, right? Okay.

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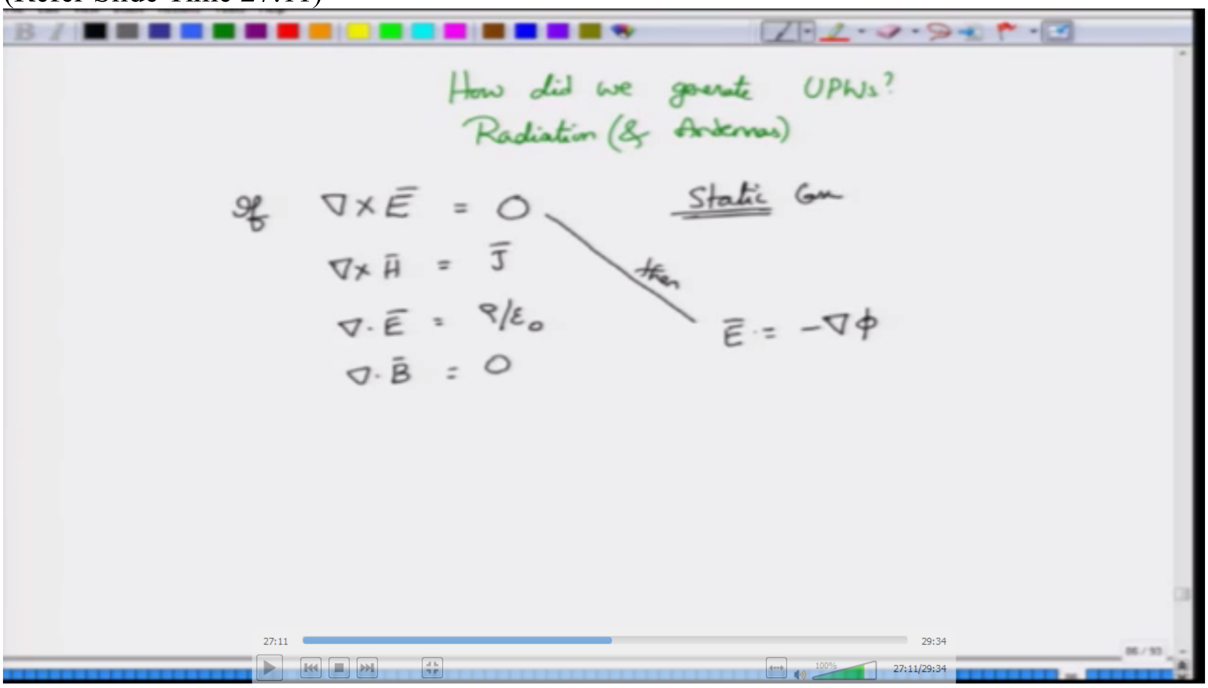

Another theorem from vector calculus tells you that if the divergence of a quantity, vector field quantity is equal to 0, then you can express that vector quantity as curl of another field. Okay.

So you may have seen these variables φ and A. This φ is called as electric scalar potential. It is obvious that it is scalar because it doesn't have a vector thing and gradient operation on a scalar will result in a vector, which is the electric field. This quantity, which you may not have seen earlier or you may have seen but forgotten, is actually called as magnetic potential or sometimes called as magnetic vector potential. Okay. And we will usually drop the word vector because we kind of understand that when we are dealing with vector potentials, we only deal with the vector potential corresponding to the magnetic field B. Okay.

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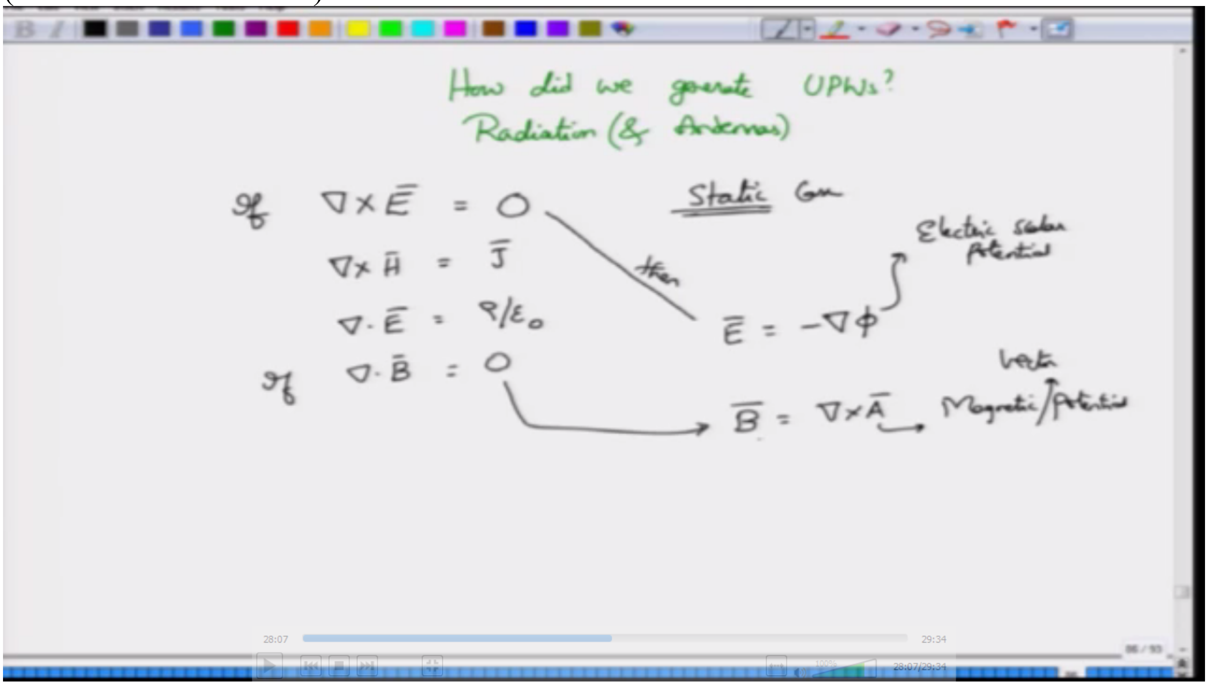

So I have these two relationships. Okay. If I use this Gauss's Law and then substitute for the electric field from this theorem, right, I will get $-\nabla\varphi$ that would be equal to ρ/ε_0.

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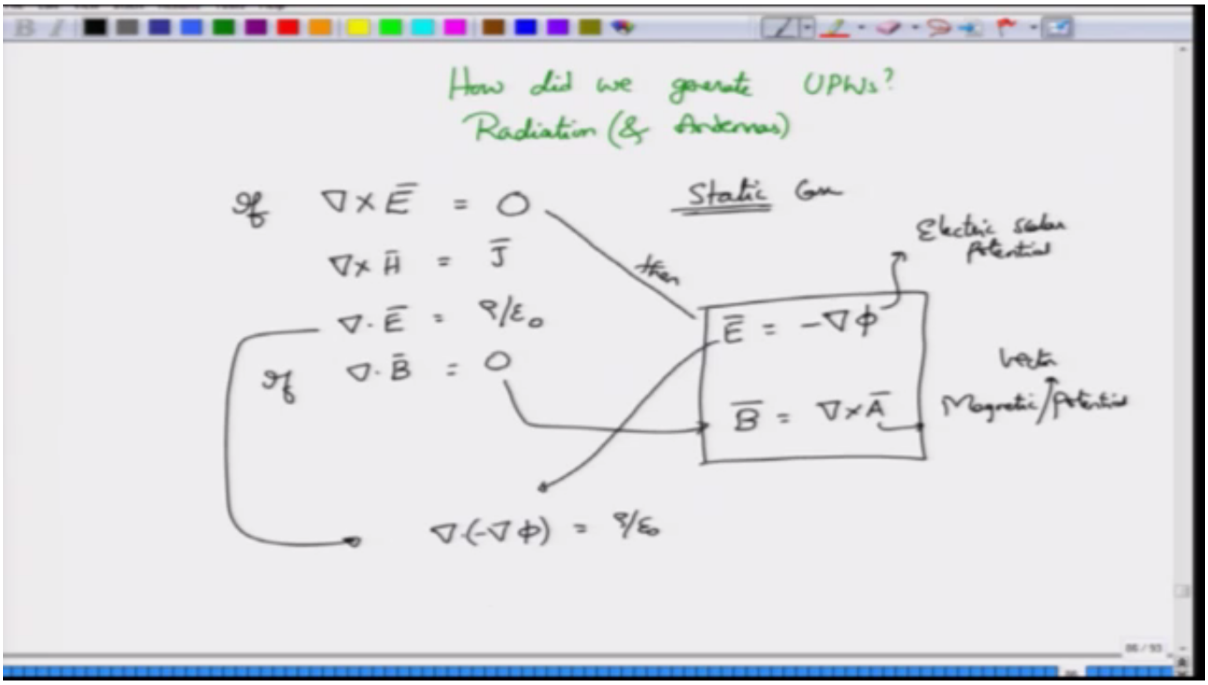

Divergence of the gradient, okay, I move minus onto the right-hand, divergence of the gradient essentially gives you what is called as Laplacean and then you have $\nabla^2\varphi = -\rho/\varepsilon_0$. This equation is called as Poisson's equation. Okay. And when $\rho = 0$, you get what is called as a Laplace's equation. Okay.

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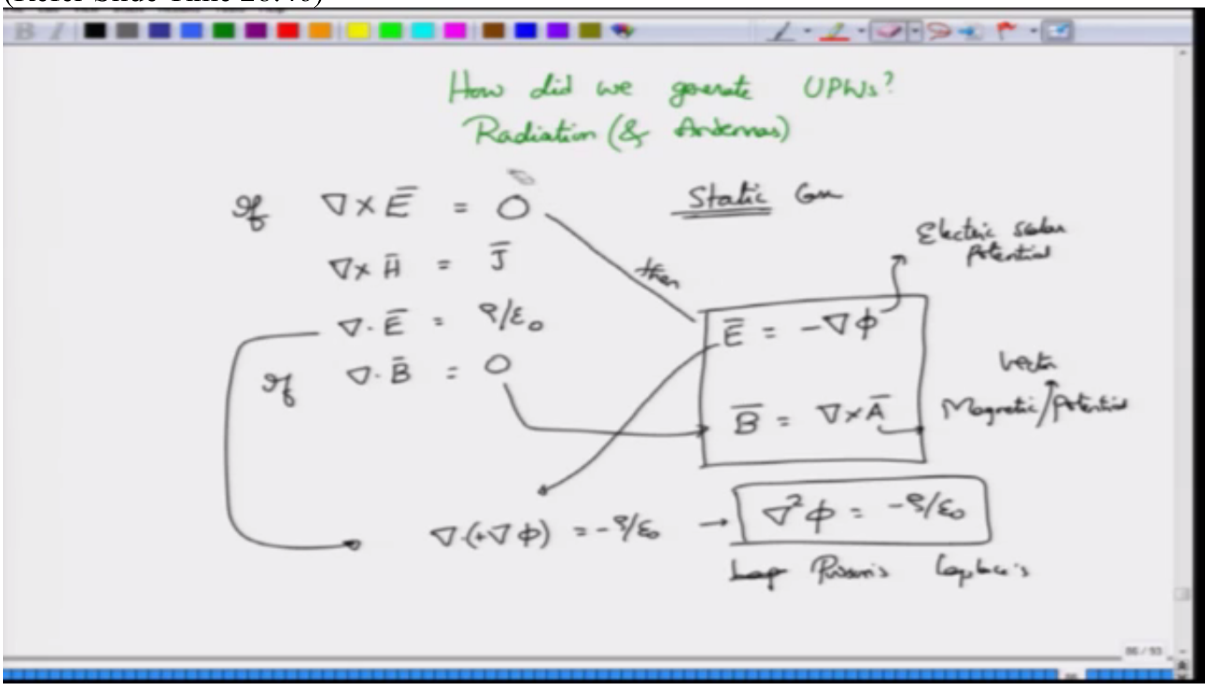

So with Laplace's equation and Poisson's equation, we find the solutions of this scalar potential and then we would also like to find the solutions of the magnetic vector potential. We will do that in the next module.

Thank you very much.

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