Lecture 21
Total Internal Reflection

Hello and welcome, to NPTEL's MOOC on, on electromagnetic waves, in guided and wireless media. In this module we will continue our discussion, of total internal reflection and we have already introduced what total internal reflection is.

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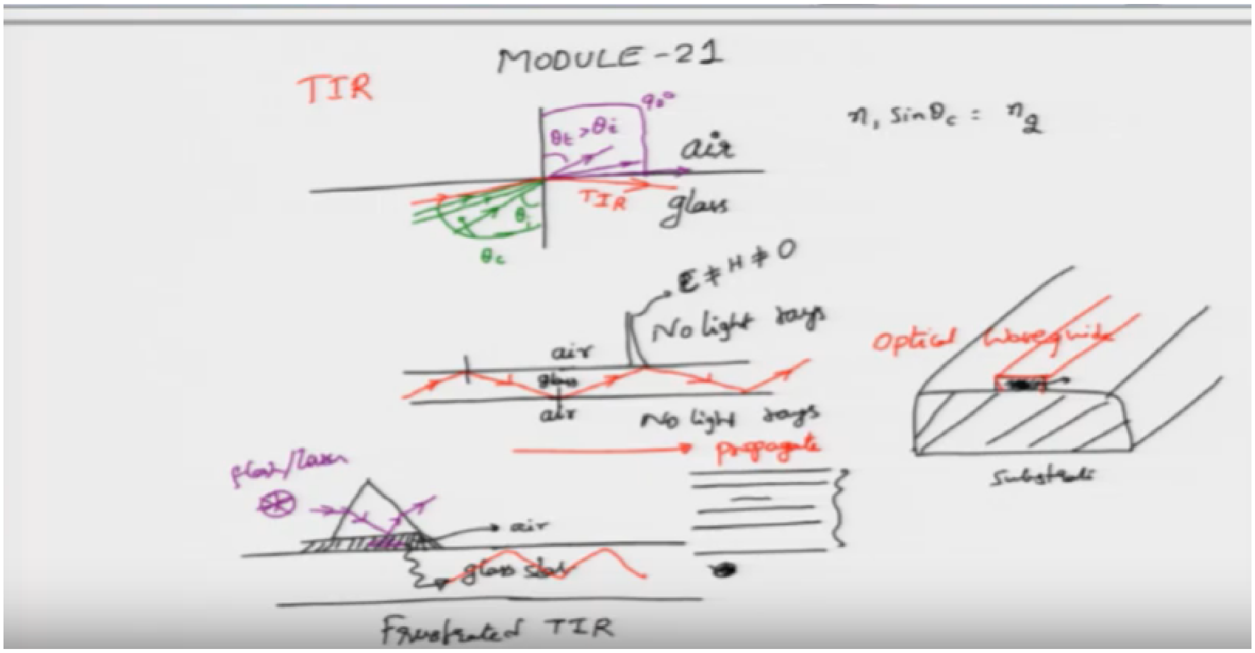

But, if you are someone, who is you know? Remembering or who can remember, physics that was taught in, high school, you would remember that, if you take two media, so, let's say air and glass for example, or air in water that, for that matter and send light, from say, glass, on to air, at an angle which we will call as, ‘Theta I’. This is the oblique incident and this is a ray that has being incident, what you would actually observe is that, the reflected ray, kind of bends away from the interface. Right? Meaning that, this angle theta T, is usually greater than angle theta I. Okay? And as you start increasing, the angle of incidence. Okay? So, for example, you take another ray, with an angle theta either is much larger, than this theta I this angle no, the transmitted ray must bend even more, towards the interface. So, let’s say it bends, in this particular manner and now, when you increase the angle of incidence further. Okay? Which we will call as an, ‘Angle Theta-C’. Okay? What will happen here is that, the light ray would remain almost parallel. Okay? In fact the angle theta T would be equal to 90 degrees and the Ray is actually, propagating along the interface. Okay? With this angle being approximately 90 degrees and Snell's law, which we would have happily applied here, to predict this one, would actually tell you that this angle of the critical angle, would actually occur, at a value of theta equal to theta C, such that, sine theta C is equal to n2 by n1, why is this so, because this sine theta T, was actually equal to one, because theta D is actually equal to ninety degrees. Okay? So, this is something that we all have, probably studied and understood quite well in, in the earlier you know, our stages of our life, when we were high school students. But, now let’s ask this, what happens if my angle of incidence is greater than this critical angle. Okay? At critical angle we know that the light wave was just, kind of moving parallel to the interface. But, now I’m actually increasing the angle of incidence further, than the critical angle, then what should happen to this light ray? Now, without much of our you know, actual motivation, we would have studied this phenomenon called as, ‘Total Internal Reflection’ which would tell you that, the Ray instead of being confined to the medium two, would actually be reflected back, into the medium one. Okay? In fact that is the reason, why we would call us, ‘Internal Reflection’. Okay? That will always be reflection, see you should cannot confuse between two things. So, one I have an interface here, so I send in light, ray at this point, there will be a partial reflection already, at the same angle as the incidence. Okay? However there will also, be a transmitted wave or transmitted ray whose angle would be in such a way that, it would be moving away from the interface. However as I start increasing the angle of incidence reach critical angle, this transmitted ray will actually, become almost parallel. Okay? But, it is still kind of in the second medium and corresponding to this, theta C there will also, be a reflected ray here. Okay? That is present all always. Okay? We are not talking about that one, the moment you increase the angle of incidence to be greater than the critical angle, then this transmitted ray which was earlier in kind of in the second medium, now that gets reflected back into the first medium. Okay? And that in fact superimposes on the actual reflected light that was already present. So, what you essentially have is that, what this geometric optics or the ray optics would actually predict is that there is absolutely no array or no power in the second medium. Okay? And all of the light actually, gets reflected back into the first medium and that is why it is called as, ‘Total Internal Reflection’. Okay? So, we are dealing with this phenomenon called as, ‘Total Internal Reflection’ and what it would tell you is that, the total internally reflected ray would actually, come back almost at the same angle as the angle of incidence, infrared the same angle as the angle of incidence, however this would be the total internal reflection ray. Okay? Now, immediately you can put this, phenomenon to a very good use, imagine that I take a slab of glass. Okay? Slab meaning I simply take a flat plate, of glass and I am somehow, you know able to send in light, into this one. Okay? At an angle that would be greater than the critical angle. Okay? So, assume that this glass slab is kept on both sides by air, well technically I can't keep a glass you know, suspended in air but, let's assume that I am tied up the glass from the top and suspended that glass slab. So, we can actually observe this well. So, I am sending in light, don't ask how, you have coupled light into this glass slab assume: that you have done that one and once you have coupled the light into glass slab or light trace into the glass slab, you would see that, if the angle of incidence at which this initial ray, would actually strike, the glass air interface, is greater than the critical angle, then we know that, this light ray would actually, be reflected and it would be traveling back, onto the glass air interface, which is down here. Okay? Now, at this point, if you again observe that, the angle of incidence is actually greater than the angle of reflection, you see that this ray would be reflected back towards the upper plate or upper interface and by consecutively, doing this total internal reflection, you can actually trap this light ray inside, in such a way that, you’re actually transmitting light effectively, along the glass slab. Okay? So, in fact, what you are doing is to propagate? This particular light ray along this particular glass slab. Okay? In fact, these plates, only to kind of hold or trap the glass you know, light rays within the glass slab and this arrangement of a simple glass slab, with air and air on both sides, is what is called as an, ‘Optical Wave Guide’. Okay? Because it can guide or trap or propagate optical race, it is called as an, ‘Optical Wave Guide’ kind of a similar trapping can be possible but, not using the total internal reflection. But, using boundary conditions, those are what are called as, ‘Metallic Wave Lets’ which we will study, later on. Okay? In practice these optical waveguides don't look anything like what we have drawn here, there would usually be, what is called as a, ‘Substrate’. Okay? And on top of the substrate there would be, another material that has been grown and then, you will actually have a slab, in the form of a thin strip in this manner. Okay? On the 3d dimension, I mean, 3d of course it would look something like this and this is what you would actually have, this is what you typically have as a optical waveguide. Okay? So, waves will be trapped, within this region, because of that, you need to make the refractive index of this part. Right? Which is basically in between not the kind of a micro strip? But, then that material itself there has a higher refractive index, than the material outside, many times you will also see multiple layers. So, you will have one layer here, you know the layer here, then propagation is happening in this layer then there will be more layers on top of it, the appropriate design, is to you know, this kind of situation comes up in many, optoelectronic

integrated circuits, where there will be multiple layers and light may be guided top, at the bottom or at the intermediate layer, depending on the nature of applications. But, all of them use the fact that, it is possible to trap or propagate waves or propagate rays, by the phenomenon of total internal reflection, by making the surrounding region to be, having a lower refractive index and the region where you want to propagate this race has a higher refractive index. Okay? So, this is the basic idea, but, the practice of that idea is slightly you know, more complicated than what I have, shown what we have observed is another thing which I have also mentioned, is that there are basically no light rays outside. Okay? There are no light rays anywhere outside, this particular slab. Okay? This is what the geometric optics of Snell's law would tell you, but in fact, there will be some light rays, in fact it goes to how we can send light into this glass lab itself. So, imagine that I actually have a glass slab. Okay? Which is acting like an optical waveguide, what I do is? I take a prism and then I take now, keep it on a on this glass plate. So, maybe I'll also have this small region where I am going to keep. Okay? Being either, made out of an actual material or I can simply take it as an air itself and what I do is? I send in light in this you know, into this prism. So, the prism light will go and because the prism usually has a higher refractive index, this light would be actually reflected back. Okay? I have not drawn these angles correctly. So, please forgive my ray picture here, but this is the basic idea, so you are going to send in light, from maybe some source that is present. So, this could be a flashlight on this, this is most of the times a laser that is coming in, so this laser would send in light it would go through the prism and after this, because this medium you know, the medium that has been shown in this crosshatch region actually, as a lower refractive index, much oscillate is actually reflected back. Okay? If you were to go with only geometric optics ideas, you would then say that there is no light that is, in there that there will be no light in the glass slab, no matter what I do here. Right? However we will be wrong, because after sometime once you make this arrangement and then observe, you will actually be able to observe light at the output of this slab. Okay? Which is not because of just you know, scattering or arbitrary light this is actually, the light that can be measured and there is significant amount of light, of course that depends on how, good the laser here is, but, geometric optics would tell you that, there should be no light in this glass lab. So, what exactly is happening? What is happening is that? The Ray picture is incomplete. But, if you go to the wave picture, you will actually see that, the wave will be having, what is called as a, 'Decaying Portion'. It won't carry power, but there will be amplitude of the electric and magnetic fields. So, E and magnetic fields are not equal to 0, in this region outside of this glass slab. Okay? And those electric and magnetic fields can actually couple, as we would call it, into this race or into the modes of this glass lab and actually begin to propagate. This phenomenon is called as, 'Frustrated Total Internal Reflection'. Okay? And this phenomenon is quite widely used or at least one point of time widely used in order to couple light into the integrated optical wave guides. Today you have different kinds of coupling mechanisms. But this basic idea, of using and evanescent wave couple, is called as an, 'Evanescent Wave Coupler' and that is, that was very prevalent in the1970s, in the integrated optics community. Okay? So, what is this Evanescent wave? And how do we describe it, well what we do is?
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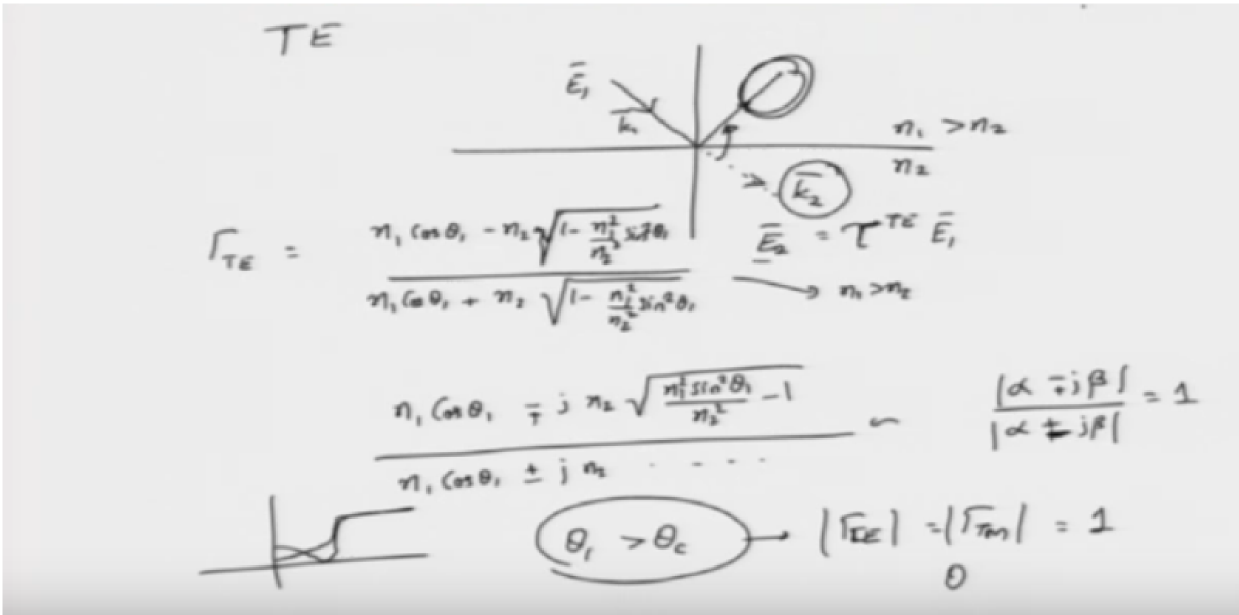

We’ll go back to the expressions for say, transverse electric polarization. Okay? Maybe that is because it's kind of simple for us to describe, the basic idea of course to medium, let's we have already called this as, ‘Medium N 1 and Medium N 2’. It's correct and then I would send in light and some light would usually come out. However if N 1 is greater than n 2, then clearly, this would not be happening in this manner. Right? When the angle of incidence is exceeding the critical angle, then light would not, come in this manner or rather they would not be in this part. There will be light that would be reflected back. However for argument’s say, let us assume that there is already K 2 here, k to being the transmitted wave, I am NOT interested in this one, I am basically interested in what happens to this, K 2 itself. Okay? In the usual case this would be e 2 and e 2would be something like gamma T E times or rather sorry, tau T E time’s E 1, E 1 will be the electric field associated with, the medium one refractive index, this of course is K 1 and you have K 2. Right? Now, recall the expression for you know gamma T E. Right? So, if you look at the expression for gamma T E you would see that, there will be N 1 cos theta 1, minus n 2 cos theta 2, now n2, now cos theta 2, using Snell's law can be written as, 1minus N 1 square by n 2 square, sine square theta one, where theta 1 is the angle of incidence that I am considering, divided by N 1 cos theta 1, plus n 2 into square root of 1 minus N 1 square by N 2 square, sine square theta 1 please remember that, N 1 is greater than, n 2 in these expressions. Now, without going to too much of a mathematics, what you can easily see is that? There's a chance: that when N 1 square by, n 2 square sine squared theta one, exceeds unity. Right? Remember it can happen, for a value of theta 1, less than 90 degrees, because N 1is greater than n 2. Right? So, when that happens, the square root of you know, the term under the square root actually, becomes negative. Now, what is the meaning of term becoming negative in the square root? That means, the number actually becomes pure imaginary. So, in fact you will actually get something like this, you will have N 1 cos theta 1, you can rewrite this as say minus and assuming that you are going to use a plus J or a minus J I don't really know, which one you want to use you can write this as minus plus, J n2 square root of, N 1square, sine square theta 1 divided by n2 square minus 1. Okay? And then in the denominator also, you will have the same thing. So, N 1 cos theta 1, again you will have a plus minus thing. Okay. So, J n 2 and this is the same, this is like alpha, minus plus

J beta, where alpha and beta 2numbers that so, these are nothing to do with attenuation and propagation consider just numbers and then in the denominator you have alpha minus or plus minus, J beta. No matter what you have, if you look at the magnitude of these two, in the numerator and denominator separately, the magnitude will actually be equal to 1. So, when theta 1 is actually exceeding theta C. Right? Then the magnitude of T E and equivalently the magnitude of T M as well, both become equal to 1, for the angle of incidences which are greater than theta C. Okay. So, this is the phenomenon that we actually saw, when we plotted this magnitude of reflection coefficient and transmission coefficient, in the previous module. Right? So, there if you remember, TM was actually going to 0 and then suddenly went up to1, for it the it actually, after a certain critical angle, it kind of moved rapidly into one, whereas for the T E K started off is a small value. But, at this point it would also, become equal to one, you can actually do this by simple Excel sheets and show that, when the angle exceeds the critical angle, your reflection coefficient magnitude will actually be equal to one, meaning that, the light will actually not be transmitted but, rather be completely reflected. Okay? However does it actually mean that, E 2 will be equal to zero, well we will show now that E 2 is not equal to zero. Okay?

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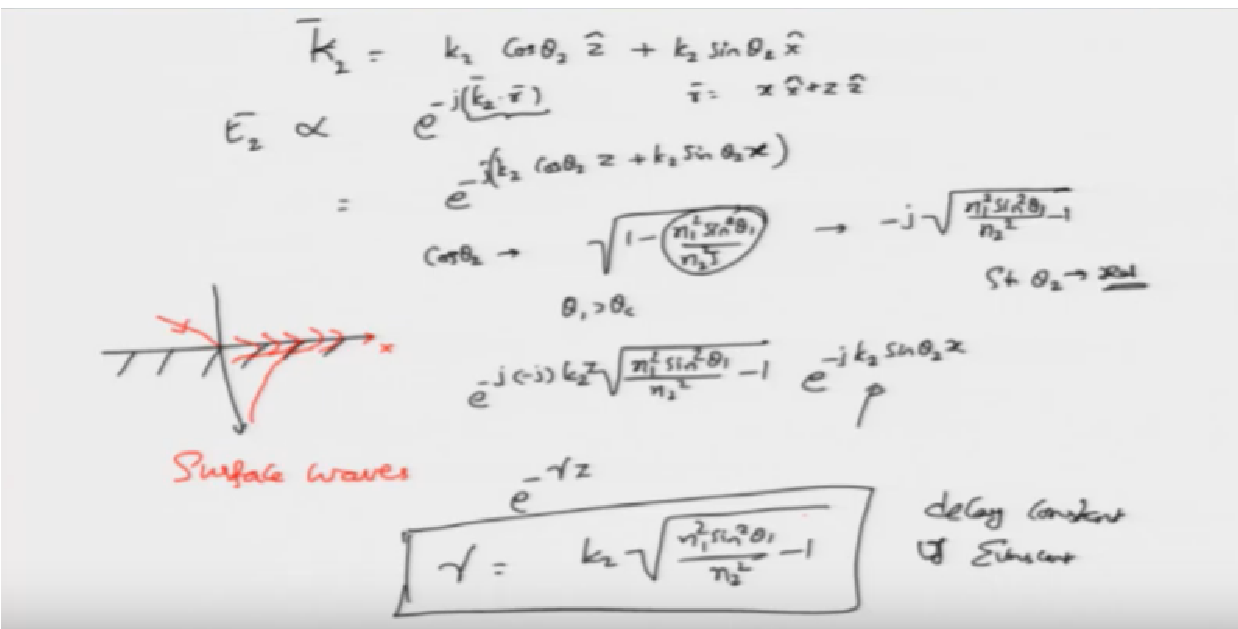

Forget about tau T E, let's just look at the K vector for E 2. Okay? So, e 2 now, the K vector K 2, referring to the previous figure, can be written as the magnitude of K 2, cos theta 2 Z, plus K 2 sine theta 2 times X. Okay. And the equation for E 2 was proportional to, E power minus J, K 2 dot R, where this was a vector of course. So, R is basically X, X hat plus, ZZ hat. So, this was the wave that was propagating. Right? So, what will happen to this phase factor? Now, look at this, a power minus J, K 2 cos theta 2 Z, plus K 2sine theta 2 X. Right? This is what you get? Now and then you observe, an interesting thing that, cos theta 2 is actually become imaginary, why because, cos theta 2 was the one that was given by 1 minus n1 square sine square theta 1by n2 square. Right? So, if you now, take this cos theta 2, in the angle of theta 1is greater than the critical angle and we know that this term actually exceeds1. So, I can take this one as say, minus J, square root of N 1 square, sine square theta 1 by n 2 square minus 1.

Okay? This time it will be purely imaginary and then substituting for this cos 2, cos theta 2 in the expression about, you will notice that this would be, E power minus J, there is also another minus J, k2 and then you have in place of cos theta 2 or rather in place of this entire thing you will have, n1 square, sine square theta 1 divided by n2 square minus 1, then you will also have E power minus J, K 2 sine theta 2 X. Okay. Will not worry about this part. Okay? Because sine theta 2 is still going to remind real, you can actually show that, this is actually going to be real. But, if you observe here, minus n minus is plus J and J is minus so, what you actually get is e bar minus, gamma times Z. So, I'm Sorry, I did not put it here I'll put this set. Okay? And what is this gamma? Gamma is given by K 2 square root of N 1 square, sine square theta 1 divided by n 2square, minus 1 and this particular term gamma, which basically is kind of 1 by meter kind of a thing, actually tells you the decay constant, of the second decay constant of the evanescent wave, meaning that, if you take this interface. Right? And then this is the normal to the interface, this is the interface that I have, a treat after critical angle. Okay? The incident wave will be in this particular manner, the wave actually decays along, the z axis however, the wave actually propagates along the x axis. So, it is kind of hugging close, to the surface it propagates in this manner, but if you actually start going away from the interface down below, the amplitude, you would still be propagating along the x direction, however the amplitude here would be very small. So, this kind of evanescent or an exponential decay is very, similar to the skin depth, or you know that we have already seen. So, even though the medium was lossless, the amplitude in the second medium rather decreases, because of this phenomenon of total internal reflection and the wave now, kind of propagates along the surface, in the direction, of the interface along X direction. Okay? And these waves are sometimes also called as, ‘Surface Waves’. Okay. You can do lot of interesting things with surface waves. So, there is an entire, you know built around utilizing this surface waste. But for us, we won't, be needing or we won't be describing this, innocent wave any more further, but it is very important for you to understand that this evanescent waves can be used in, used to couple light, in the evanescent couplers, they can be used in sensors you know, based on how, the surface way or how the decay happens, you can get low of information about region n1 as well. Okay? So, with the appropriate critical angletheta-1 and other things. So, there's lot of interesting thing that happens with, surface base, but, the take-home message is that,

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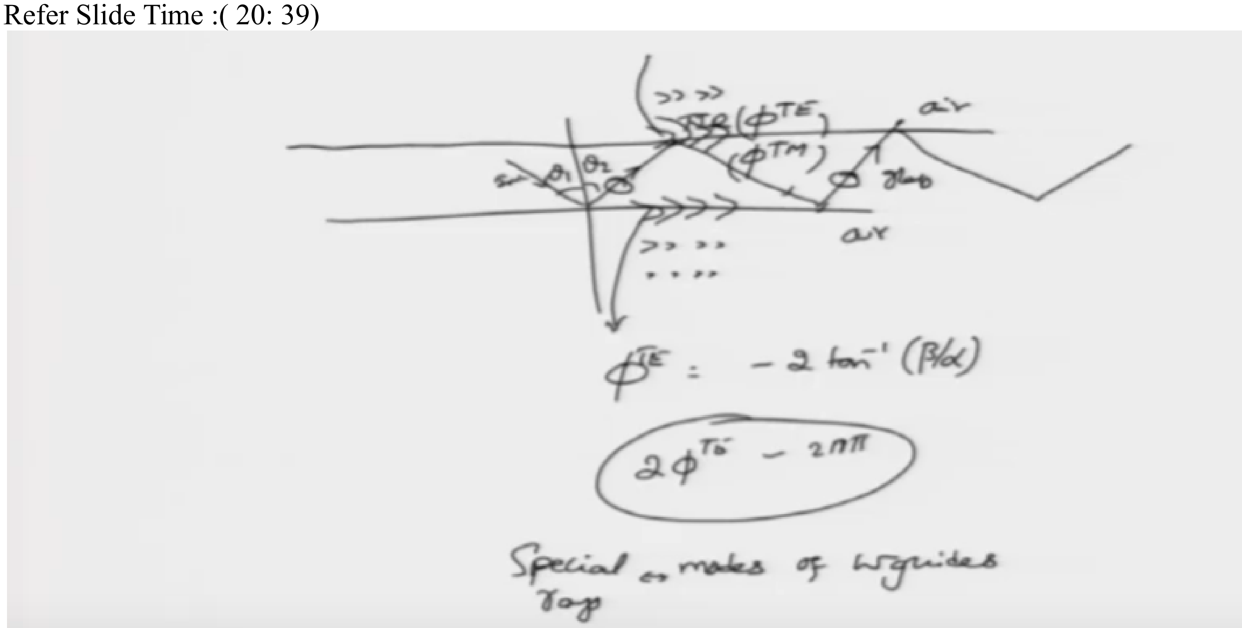

the simple geometric ideas that we are used to: that when angle of incidence exceeds, the angle of no critical angle, then light will be total internally reflected and no light will be present outside, is actually not true, there will be an electric field. But, that electric field will actually be propagating along the surface, however whose amplitude basically keeps decaying, as you go away from the interface. Okay? So, these waves actually, exist. There's one final point that I want to mention and to do that when we will go back to this, expression for gamma T E it is the same for other expressions also, I mean for gamma TM also, you will see that the same kind of equations hold, what we saw here was that gamma T E here has a magnitude which is equal to one, however if you evaluate the face, here that would be say minus, tan inverse of, beta by alpha, divided by tan inverse of beta by alpha. Right? That would be the phase: that you are going to get, by taking the converting this complex numbers into, appropriate polar form and what you get, is this phase which will be roughly, minus 2 tan inverse of, beta by alpha, please remember beta and alpha or not the propagation and attenuation coefficients, but they are basically associated with this terms. Right? So, this term and this term. So, this is what we called as, 'Alpha' this is what we called as, 'Beta'. So, if you just identify this, calculate them then you can actually calculate, what is the phase shift that you are going to get, upon reflection. So, not only that, there will be this wave which now comes back, this is the TIR wave. But, this TIR wave, will actually experience a phase shift, depending on whether you are dealing with T E or TEM polarization, the corresponding phase shift will be different and that has to be also, accounted in fact when you put one more you know, interface at the top and then make this as air and this has air and put the intermediate thing as glass and then you know, you have this multiple total internal reflection, evanescent wave should be created everywhere here. Okay? The evanescent wave, decay again on this side, also and as well as on the other side, however not only this angle, determines the propagation, there will be a phase shift that is associated. Right? So, the incident wave in this manner, will be reflected upon and then it will undergo an additional you know, phase shift called, 'Phi T or Phi TM' you can take one of the polarizations for discussion. So, assuming phi T E is the phase shift, for the TE mode that has been incident, then next time when it gets incident, I mean next time when it gets reflected at the top, interface it will undergo an additional Phi T E. Right? Phase shift so, after it comes back and then you know, just kind of begins to move away or begins to go back to the top interface, if this wave front and this wave front are to be in phase, the total phase shift. That is two times Phi T E must be approximately equal to, some two n Phi. So, there will be additional phases also that we don't really need to worry about, but basic idea is that the overall phase shift, determines which angle, of these Ray's actually, propagate along the glass slab or the optical waveguide. Okay? And this condition which needs to be satisfied in addition to simple, total internal reflection the actual thing that the phase also, has to satisfy this you know, integral multiple of 2picondition, means that only certain angles, are possible when you, you know, when these only certain angles of incidence theta 1 and theta 2 are actually possible, for the wave to be propagated or guided by the mode, of guided by the wave guide and though special rays are called as, 'Modes of the Wave Guide'. Okay? So, those special rays are basically modes, of the waveguides and that special ray or the mode, depends not only on the angle of incidence but, also on the total phase shift accumulated. So, this completes our discussion of total internal reflection, we have also introduced the concept of optical waveguides, in the next module, we will solve for the modes of a simple glass slab, as an example, of how electromagnetic analysis tells you more, rather than just a geometric optics. So, the take-home message here was again that, when total internal reflection happens, there will be evanescent waves and moreover the wave that gets internally reflected, back into the first medium, will also have an additional phase shift, depending on T E or TM. And when you, satisfy the overall phase condition, only those special rays will be propagated through the waveguide and those special rays are called as, ‘Modes of a Waveguide’. Thank you very much.