Lecture –02

Distributed Circuit Model of Uniform Transmission Line-I

Hello and welcome to NPTEL course on Electromagnetic waves, in guided and wireless media. This is module 2, of our course and in this module we will consider the circuit.

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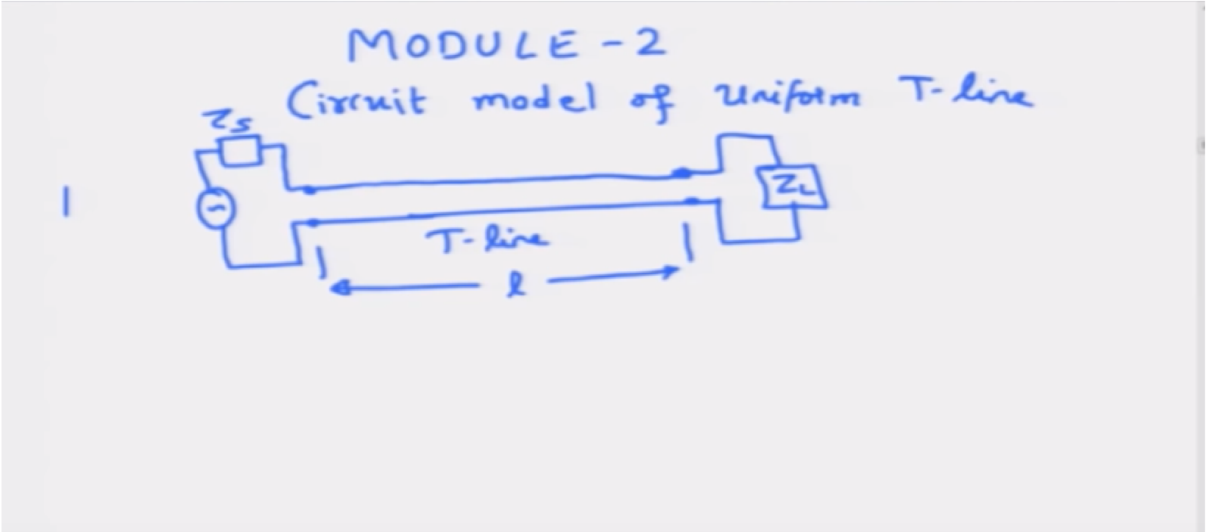

module or the circuit equivalent, of a uniform transmission line. Recall from our first lecture or first module, that the transmission line is physically, you know, system, which usually consists of, two conductors or two pair of conductors, which connects different parts of the circuit. So, you may have for example a source, which we shown her with a source resistance or source impedance set as. Then there is a load ZL and these two are connected by, in this case, what we have shown is a pair of wires, one conductor and another conductor. One of the conductors is usually grounded. So for example; so for the bottom conductor can be considered to be the ground connector, or ground connected. While potential difference between the two, at least at the input end can be established by the pair of wires. However we've also seen that, the representation we have chosen to draw two lines, actually comp, encompasses many different type of transmission lines. For example this could be just two ordinary pairs of wire which are drawn, to connect the source and the load or it could be on a printed circuit board. It could be ground plain and then you are drawing the traces, on one of the layers'. Right? So that is called as a, 'Micro Stripline', and that is also generically represented by the same picture. Although you should understand that, physically they are different, but in our model, we will represent them by, drawing pairs of lines in this particular manner. And the characteristic of a transmission line is that, its length is comparable to the wavelength of the signal that is propagating along the line. Okay? If the length is very, very small, compared to wavelength of the signal, then we can go back to the circuit approximation and then treat these pair of wires, as simply wires with, no delay between them, Meaning that, if the voltage at that particular source point changes, then the changed voltage, without any loss, is immediately available to the load as well. This is the case when, the lengths of this wires are very, very small, compared to the wavelength of the signal, that is being launched. Okay? On the other extreme, we have also seen that, when the wavelength is very small, while the circuit dimensions are very large, then it is possible to treat the signals, in the form of rays and then, you know, talk about, what is called as, 'Ray Optic Approach' something that we will not be concerned with, at least at this point in this course.

And we will of course we will not be concerned with the, case where we can treat wires as, simple wires without, in the ordinary circuit language as well. So what we are actually going to deal which is the case, where it is intermediate. That is our signal wavelength, is comparable to the length, the physical length of the transmission line and that is, that is what we call as the, 'Transmission Line

Model.’ Okay? To study Transmission Line Models or to study transmission lines, inorder to answer questions, such as; what would happen if the voltage is applied at one end of the line? What would happen to the voltage at the, far end line, at the load side? We have two approaches. Okay? The first approach is to; relate voltages and currents to electric and magnetic fields. Because that is how they are actually inter-related, as we will see later on. And then study the behaviour of electric field and magnetic field down the transmission line. And at the load side, you convert back, that information from electric and magnetic fields, into voltages and currents. How do we study the pro, you know, the behaviour of electric and magnetic fields, along the line. We have to use, Maxwell’s Equations. because, this particular study is purely the branch of, Electro Magnetics. I mean, it is, under the purview of Electro Magnetics that we understand how electric fields behave, how magnetic fields behave. And then we convert those, using the relationships, we convert those information, in the electric and magnetic fields into, voltages and currents. Because ideally, that is all, what we can actually measure. Okay? This approach, while fundamental and is infact used in many cases, suffers from the problem of having, to deal with too much of complexity, right at the beginning of the course itself. We would not like to do that one, we would like to postpone this difficulty. Instead, we will consider the extension of the usual circuit theory loss. That is, ‘Kirchhoff’s Current and Voltage loss. And then, see how we can actually come up with a model, which consists only of the, Circuit Element, such as, Inductors, Resistors, Capacitors and Conductors. Okay? Or Conductants, I should say, to mimic the behaviour of the real world, transmission line. Please remember, the transmission lines, in real world are going to be transmission lines in real world. Okay?
How do we understand that, is the, subject of Physics and Mathematics. Like, we make a model, and then we test out model against the actual behaviour. And if the model accurately predicts or reasonably predicts the behaviour, then we have a faith in that particular model. Yes the model may not encompass everything that is act, you know, that is, to b, studied in a real world transmission lines, however, what the problem under consideration, mainly to know how the voltages and currents behave at the or along the transmission line. If our model, which includes only the circuit elements and extension of circuit theory, to this scenario is sufficient, then we have faith in that model and then we can use that model to make newer predictions. Okay? That is what we are going to do. We are going to come up with a circuit model for a, Uniform Transmission Line.

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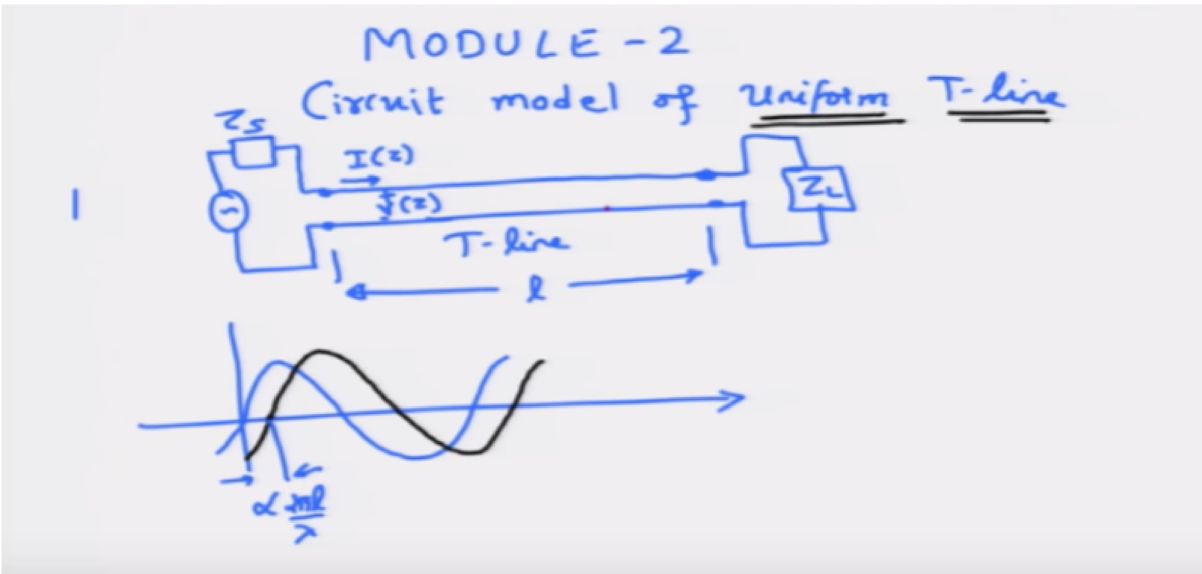

What is a Uniform Transmission Line? although the diagram here, is not really drawn nicely, because I did not use a scale to draw it. What we mean by Uniform Transmission Line is that, the cross section of the Transmission Line, whether it is a coaxial line or whether it is a two wire line or is a twisted pair wire or it could be a micro strip line or strip line or any kind of a transmission line that you can think of, which has these two conductors. If you look at the cross section, at different points along the transmission line, the cross section would essentially remain the same. So this would be the cross section, two pairs of conductors and the same pair of conductors. Okay? If you are looking at, a coaxial line, the coaxial line cross section would remain in this manner, there is an inner conductor and an outer conductor and in-between you have the di-electric field. But please note that, this cross section remains the same, as you extend the line in the longitudinal direction. So the cross section here, would also be the same, at the cross section at the, at, at one end of the line. So such transmission lines, where the cross section remains the same, along the line, are called as, 'Uniform Transmission Lines.' Okay? I sometimes also will use an abbreviation of, T-line. T-line stands for, 'Transmission Line' Okay, so this is the Uniform Transmission Line and please remember, we are interested in knowing, what the voltages and currents along the transmission line are going to look like, when exide the transmission lines. Right?

So I want to know, how the Transmission Line, voltages and currents or rather the voltages and currents behave on the Transmission Line. I would like to know that one, by postulating this Uniform Model. Or the Circuit model for this Uniform Transmission Line. Now how do we go about, building this circuit model? Well for one, we know there is a current, propagating along the Transmission Line and there is a Voltage, across this one, the potential difference between the two points, so we have the Voltage and Current. And unlike in the circuit, these two are actually the functions of the, distance along the transmission line. So clearly, the voltages and currents, at one end, will not be the same, as the voltages and currents at the other end and that is infact the behaviour of a Transmission Line. If you recall our first lecture, we had considered, a Sinusoidal Source, being launched or a Sinusoidal Signal being launched by a Sinusoidal Signal source, on to the transmission line and then we found that, at the output, you know the voltage of the current, whatever that you can, that you want to think of, was actually face shifted, by a certain value. So this amount of face shift was infact or I told you that it was infact directly proportional to the, length of the transmission line. Infact, it could be appropriately given, by 2pi, by lambda, times L, where lambda is the wave length of the, particular source that we have considered or the signal that is propagating along the line. Okay? It is not too difficult to imagine that, this face shift should be present, for every finite length of the line. Right? So, the amount of face shift keeps on increasing. Of course, in a Sinusoidal case, once the total face shift exceeds by 2 Phi, you really cannot distinguish whether, there has been a face shift more than 2 Phi, because the, of the periodic nature. But if you consider the pulse behaviour, we will see that, the length of the transmission line and the fact, that, these voltages and currents are not going to be the same, along the transmission line, is an important point, to understand how the pulse would be presented to the load, after it has been launched at the, source or the input end of the transmission line.

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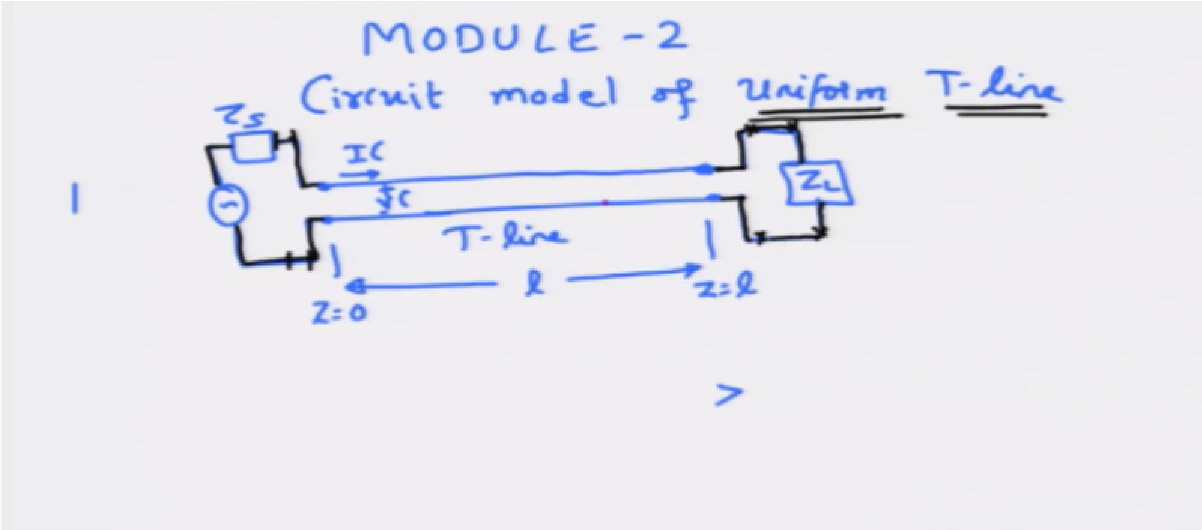

Now before going further, let me also tell you that, I have used additional wires here. Right? So I have connected this one.

So this for example, if I had, the patience to write this one everytime, I would use a different colour, to indicate that although this length resembles that of a transmission line itself and infact this will also be a wire. This length is very, very small, compared to the length of a transmission line itself. Okay? And that is also the reason why you see that I've twisted this wire, because the voltage at this point, okay? Is definitely not face shifted from the voltage at, this point? So if you mark off two points here, the potential difference between these two pairs, is the same as the potential difference between these two pairs, indicating that this is simply a, no delay wire that we have inserted. Okay? The same thing can be said for, the wires that actually connect the load to the circuit as well. So these actually have, you know, in the circuit language, they don't introduce any delay and whatever the voltage, that is impressed at this particular point, on the circuit, shown by the cross points, will be immediately available, across the load as well, which is definitely not the case on the transmission line. Okay? So please bear this particular difference in mind. And now we come back to the problem of, obtaining the circuit model for the, 'Uniform Transmission Line'. Okay? **[11:33]** that current and voltages are going to be dependant at, at what point you try to measure this current and voltages. So we have considered that as the axis, along which we measure the length of the transmission line. So you may of this as Z equal to 0, at the input end of the transmission line and then you have, Z equal to L, corresponding to the, load side of the transmission line. Okay? Or the, so that, L represents the, total length of the transmission line. So we have said that, voltages and currents are going to be dependant, at what point you are going to make these measurements. And this dependency is shown, by writing, current and voltages, along the line, as functions of, Z. Of course you may, object that, they cannot be only functions of Z, there will also have to be functions of, time as well. Because, you do expect the voltages to actually change, if you apply a, you know, time varying, signal out there. If you apply constant, then, it is a different matter.

But if you apply a, voltage which is actually changes, from the source, if the voltage is changing with respect to time, you also expect the current and voltages along the length, also to change with respect to time. How they will change, is the question that we are trying to answer. But you do, you know, appreciate that they have to change, as a function of time as well. So in simple sense, or in a slightly extended case, the current on the transmission line and voltage across the transmission line depends, not only on the position, but also on the time. We of course knew that currents and voltages can depend on time, from out earlier circuit theory. But what we never considered was that it would also depend on the position, where I'm going to measure these voltages and currents, on a simple interconnection. Right? So that is what we are actually trying to, that is what, distinguishes transmission line from a circuit theory.

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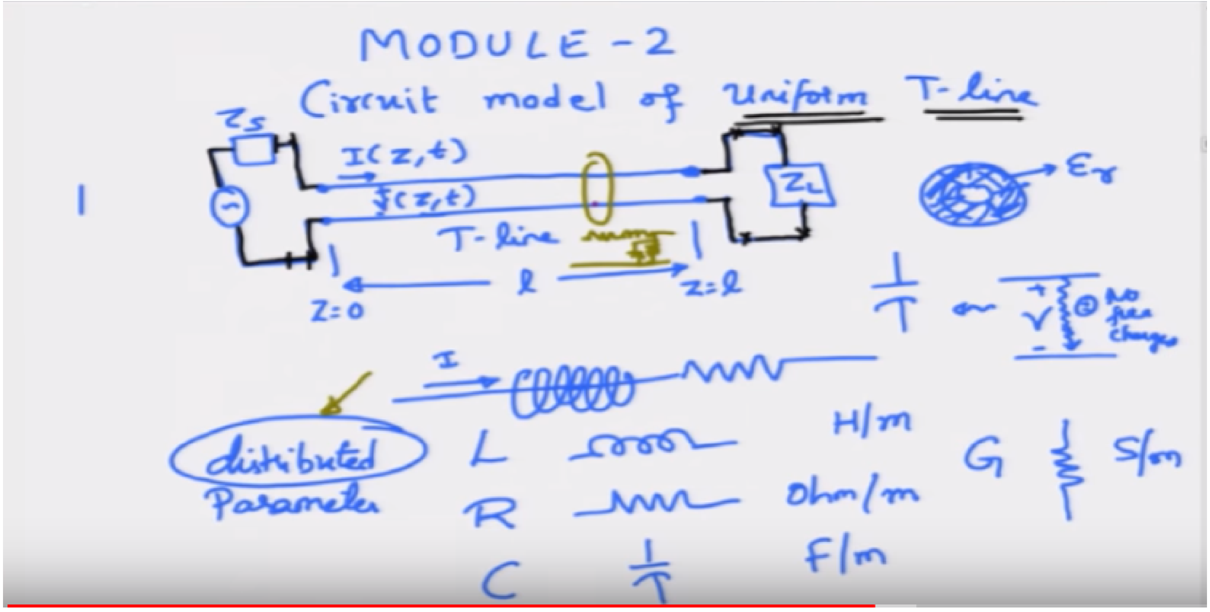

Share, circuit theoretical models and we are trying to understand, how to, describe this behaviour; Z and T, dependence of voltages and currents, along the transmission line. Okay, now, from your Physics courses, you may have, studied or recalled the fact that, if a conductor carries a certain current, then there will be magnetic flux or magnetic fields, surrounding this conductor. Right? And, one of the characteristic of magnetic fields is that, it will be captured or the effect of this magnetic field will be captured, by, you know, denoting a particular element, called, L, denoting a particular element called, 'Inductor', which is denoted by this letter, 'L', and this is the circuit symbol for the, inductor. Right? So current carrying wire, has magnetic field and this magnetic field, links the conductor back, giving rise to an inductance. Sometimes to distinguish Self-inductance, mutual inductance, but for this circuit model, we simply call this as, 'Inductance'. And how to determine this inductance is separate question. That is a question that can be answered only by, going to, electrical magnetic fields and from there, finding out how the inductance can be calculated.
But in this circuit model, we will assume that, someone has already done that tough calculation for us and they have given us the, you know, value of this induction of this line, which is usually measured, not as the overall inductance, but inductance as a, per metre quantity or per unit length quantity and which is of course given by, Henry per meter. So, this inductance, because it is given, as a per unit quantity, is called as, 'Distributed Inductance'. Okay? Or 'A Distributed Parameter, of the Transmission Line'. Okay? So you have current, which is time varying; you have inductance, which we will, consider to be not time varying. Okay? For a reason that is completely, out of scope of this course. So but you have an inductance L, which models, the interaction between current and the conductor. Okay? Of course, we may think that this wire is ideal, having more resistance whatsoever. But in practice, there will always be some amount of resistance, because the wires are made out of finite conducting metals. So there will be some amount of resistance or a finite conductance, to these wires, which can effectively modelled, by going to the resistors. Right? Again, I cannot specify this resistor as a whole, for the transmission line, I can specify this resistor, only as a per meter quantity and as you know, I can represent this resistor as, 'Omega per', sorry, 'Ohms per meter'. Right? So this

resistor is, Ohms per meter, we'll use the word, Ohm itself. Okay? So I have Ohms per meter. So this, of course is symmetrical, for both, you know, upper wire, as well as the, the wire at the bottom. But we will assume that, the bottom wire is conducted or you know, connected to the ground and attribute all of the effects, only to the top wire. Okay? Infact you don't have to do this. You can split the inductance and resistance, equally between the top wire and the bottom wire. However, as I told you, for this circuit model, we will simply assume that, these two are, referring only to the top wire, because the bottom wire is considered to be a ground. Okay? But of course, the bottom wire actually carries the return current, because the current has to go through the lower and then return back to the source, inorder to form the complete loop. And that we will ignore for the moment. Okay?

Now consider the voltage here. The voltage is being defined, between two pairs of wires and if these pair of wires are standing in air or suspended in air, then the dielectric medium or the insulating medium, is the air itself. But in the case of a coaxial cable, we have seen that, the space between two conductors, the inner and outer conductors, is actually filled, by a dielectric material, who's, you know, or an insulator, whose permittivity, will be different, than that of, air. So and this can be represented by, Epsilon R, where Epsilon R stands for, relative permittivity of the, insulating medium or the dielectric medium. Dielectric and insulator mean both same, in this particular, in our course. Okay? So the voltage between these two pairs of wires and you know that a capacitor is a device, where there will be a potential dismiss, between two conductors and the space between two conductors is filled, by a certain material, which is called as an, 'Insulator' And because in this arrangement, you have the voltage difference, between the top wire and the bottom wire. This should actually look like, a capacitor, to model. Right? So for a modeller, this looks like there is a capacitor out there. So you have a capacitor here, and capacitors measured in, Farads per meter, again this is a distributed parameter. Finally, the insulator here, an ideal insulator is one, which will not have any free charges. Right? However in practice, most insulators have a very small amount of recharges, meaning that, there will be a leakage current, flowing from the top wire, to the bottom wire. And this leakage current is because of the free charge, that are available in the, imperfect dielectric and this two, can be modelled as a, 'Resistance', and because in this case, resistance appearing between the pairs of conductors, we will model this as a, 'Conductance', and the circuit symbol of course for the conductance is the same. You know the, units of conductance, is given by, Siemens and of the distributed quantity, this is Siemens per meter. Okay?

So we have seen that, because the current is flowing along the transmission line and there is voltage difference, between the pairs of the transmission line. For a modeller, one who is familiar with circuits, it would seem natural, to associate an inductor, resistor, capacitor and a conductor or a conductance across the pair of wires, to model, the behaviour of voltages and currents. But if you ask that modeller, 'Hey, you know, drawing all this L, R, C and G, but where exactly are these L, R, C and G are located? Well the answer is, they are not at a particular point. They are located throughout the wire that is reason, why we call them as, 'Distributed Parameter'. It is not like, I can zoom into only one particular portion of this wire and then an L, an R and a, C and a, G here, and then say, 'Oh now this is where I have all these four models. No, that is not going to happen. What will actually happen is or what will actually be there, is that, this inductance and capacitances are distributed, and throughout this pairs of wires and that is ofcourse the reason why we call them as, 'Distributed Parameter'. Okay? So now what we do is, we put together these parameters. So imagine, that I have this uniform transmission line. Okay? And we consider a small region of this transmission line. We zooming that small region. Ofcourse, the accuracy of the model improves, as you make the length of this region, as small as possible. Okay? And infact in the, usual way of deriving these equations, you will see that, you will actually stop with these small cells or small regions, wherein you are going to zoom and then, you know, put your model up there, called as, 'Unit Cells', and then when you start

having these unit cells distributed throughout the wire, you will be let, from a discrete number of unit cells, to a continuous domain. Okay? and that what we are going to, look at.
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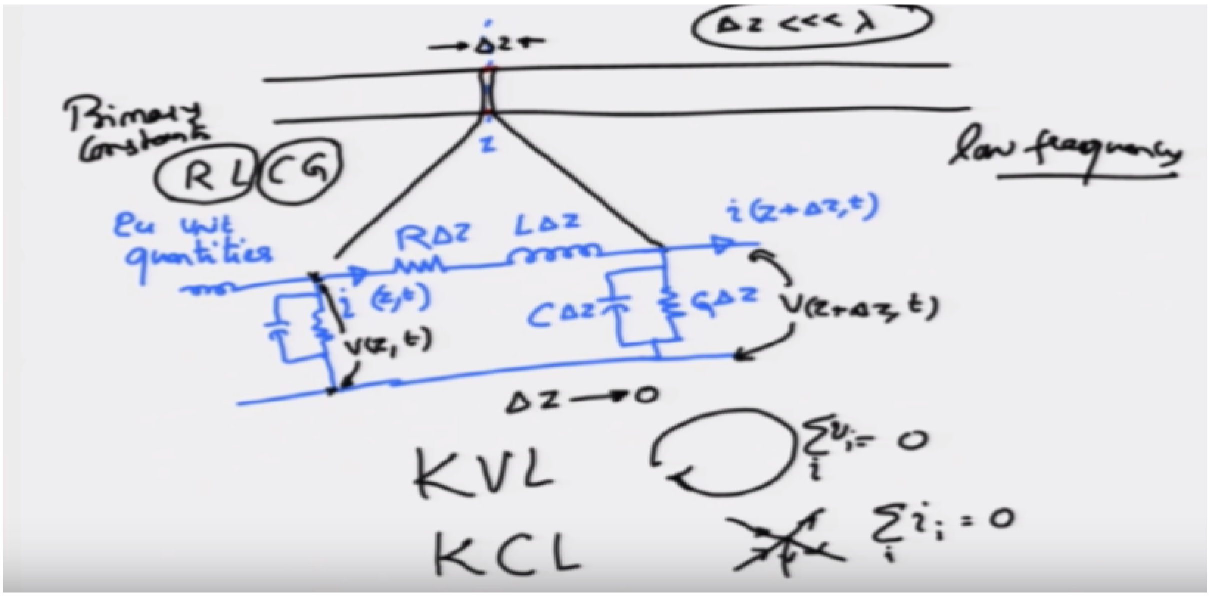

So imagine, that someone who's trying to model this, picks up a small area and then zooms it up and then see's that, if you consider the length of this small region to be delta Z and then say that, delta Z is very, very small, compared to lambda, then it is alright to consider only this particular piece of wire. Okay? which I have shown in this red colour, as a circuit. And for that circuit, we have already seen, the parameters will be defined in terms of, R, L, C and G. R and L are in series. C and G are in parallel because R and L are associated with a particular wire or individually with the 2 wires, but as C and G are quantities that are defined between the pairs of wires. So the model would look something like this. You have a resistor R. Which is again, please remember, these are all per unit quantities. Because they are, distributed parameters, they are per unit quantities. So you have R and then you an L and then you have a G and then you have one C. So this is the model. Okay?
So this is what one would consider equivalent of this short section, whose length is, Delta Z. So if R is, unit, no, unit length resistance, the total resistance, in this particular piece of wire will be, R times, delta Z, similarly the inductance will be L, delta Z, capacitance will be C, Delta Z and conductance is G, Delta Z and because you are assuming that is at a particular point Z. Okay? On the transmission line or around that, value of Z, around the transmission line. You may also see that this current will be i of Z, of course it would be, i of Zt; I'm going to use a smaller case letter to denote this current. Okay? And this is the current, i of Zt that is entering this Z. Minute self, which is located at a distance Z, from the transmission line, along the transmission line. And then, the voltage between these two points is defined by V of ZT. So the voltage between these two points is given by V of Zt and the current in the next unit cell will be, i of, maybe say, Z plus, Delta Z, times t and the voltage on that side will also be, V of Z plus, delta Zt. Sorry, this is actually the voltage. I forgot or other I did a small mistake here. This is the Z unit cell and this is the other unit cell, the previous unit cell. So the voltage here is actually V of Zt, other, this voltage is, sorry. This voltage is V of Zt. So this voltage is, V of Z and t and this voltage here, V of Z plus delta Z, t. Okay? So to recall this, I mean to recap, what we have been doing, is that, we said that transmission line can be modelled, as a distributed circuit. It's distributed because, the effect of resistance, inductance, capacitance and conductance, has to be considered throughout the transmission line. And when I consider a small section of a transmission line, at a distance said, having a width, Delta Z, which is much smaller than lambda. Then I can

consider the voltages and currents, in this particular unit cell, as we have called, to be related to the, model of this circuit in this particular manner.
Please remember, R comes in the finite conductivity of the wires. L comes from the fact that, current and you know, carrying wire will be, having a magnetic field. And that magnetic field links back to the conductor, giving rise to an inductance. Potential difference or the voltage difference, between two, of the pair of wires, in the unit cell, gives rise to capacitance. And the fact that, the insulator is not ideal, will give rise to a, you know, will give rise to the conductance of the finite resistance, connected in parallel, denoted by, G. Okay? So this R, L, C, G, are called as, 'Primary Constants', of the transmission line. These are called constants, because they are usually, supplied to us by, a you know, by someone, who is actually doing Electro Magnetic models of this one. So this value of R, L, C, G will be different, for different, transmission lines. So, an RLCG, for a two pair wire, will not be the same, as the RLCG of a printed circuit board micro strip line. Okay? So that you have to, keep in mind. And these are constants, because usually they are denote, I mean, they are derived under, what is called as, 'Low Frequency Approach'. So strictly speaking, they cannot be constants. But, to calculate the, you know, if, if you want to calculate them at very high frequencies, then, the complexity of those calculations will be so much, that we don't really bother, with that calculation. And infact we don't, many times, calculate the primary constants, because the calculation is possible only for simple geometries. In many cases, we have to actually do measurements and then in further equivalent RLCG. But no matter, what RLCG value you get.

The model that you are going to consider will remain the same. The model that I have shown, here is the same. Okay? Now what we need to do is to relate the voltages and currents and then, go the continuous domain, by making this Delta Z, actually go to Zero. Meaning that, we are now going to shrink this unit cell width, to as small value as possible, ideally you are going to shrink to Zero and then see what happens to the, expressions for voltages and currents. And we can do this, because, we have assumed that, this delta Z is, much, much smaller than the wave length lambda. And therefore, the usual KVL, meaning, the Voltage around the loop will be, equal to Zero, will hold. Okay? So the voltage, around the loop, summed, over all the paths, will be equal to, or all the elements through that particular loop will be equal to Zero. And then you have KCL, wherein the sum of currents, leaving and entering, will be Zero. Right? So this two lost, will be used now, to determine the voltages and currents, the relationship between voltages and currents. And since we are running out of time, in this module, we will do this derivation of, Voltages and Currents and then solve the resulting equation, in the next module.
Thank you very much.