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Indian Institute of Technology Kanpur

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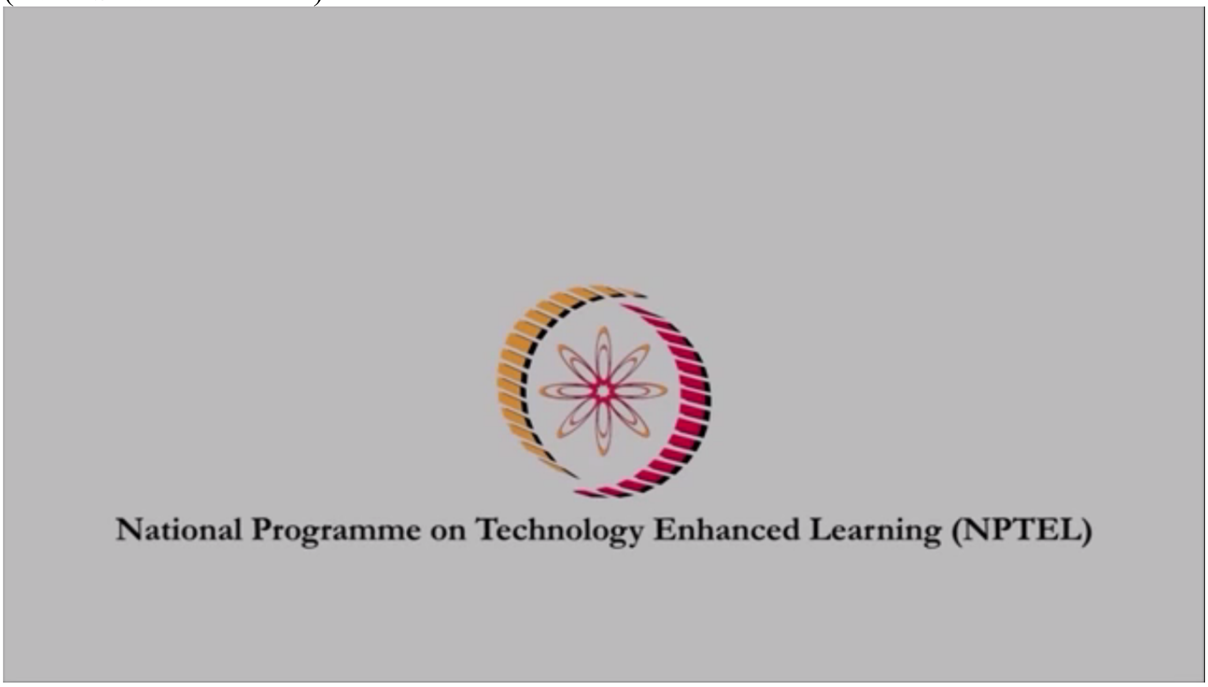

National Programme on Technology Enhanced Learning (NPTEL)

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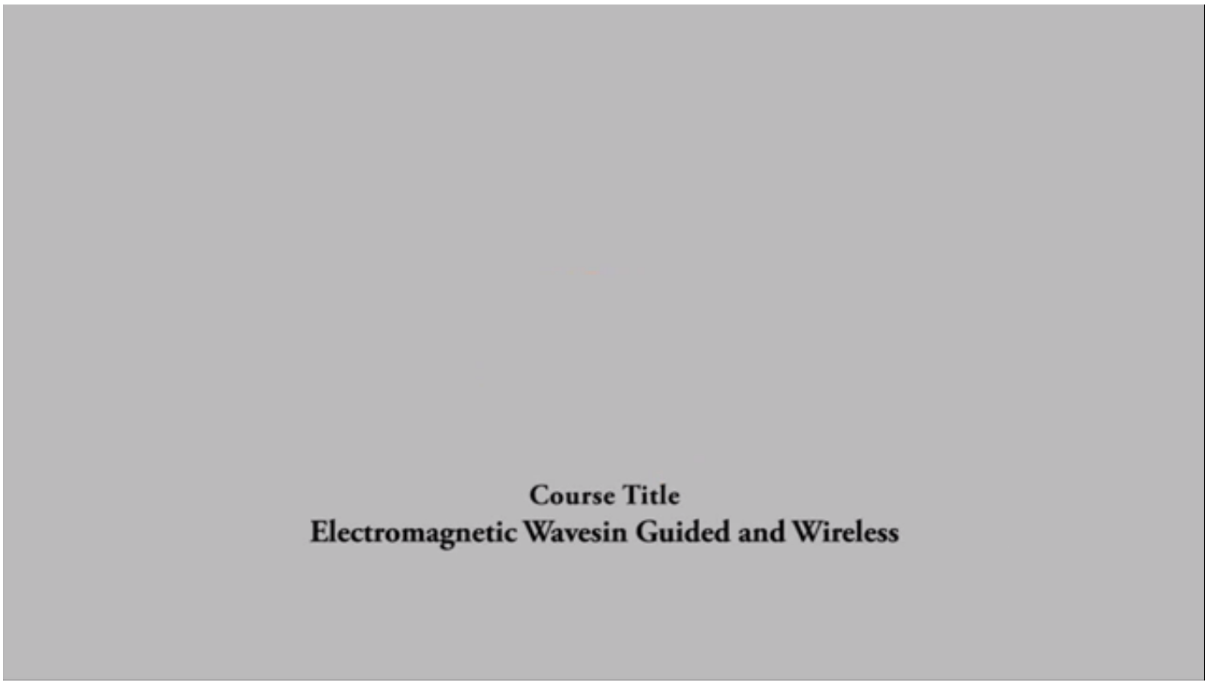

Course Title
Electromagnetic Waves in Guided and Wireless

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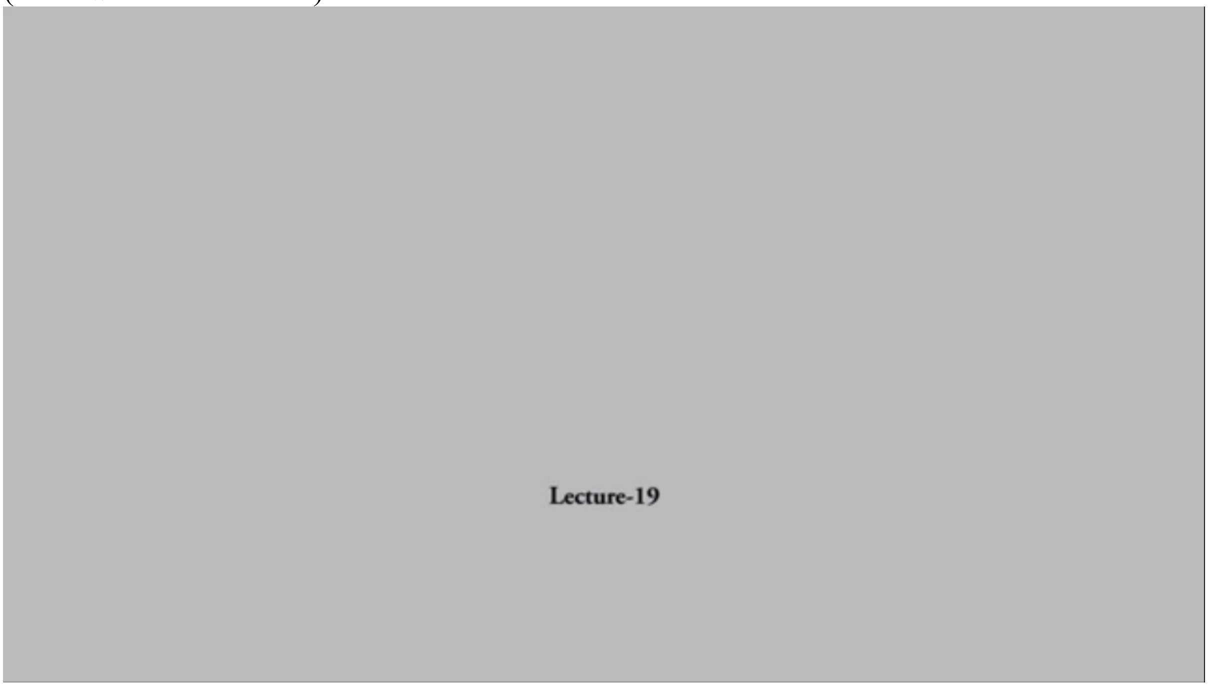

Lecture-19

by

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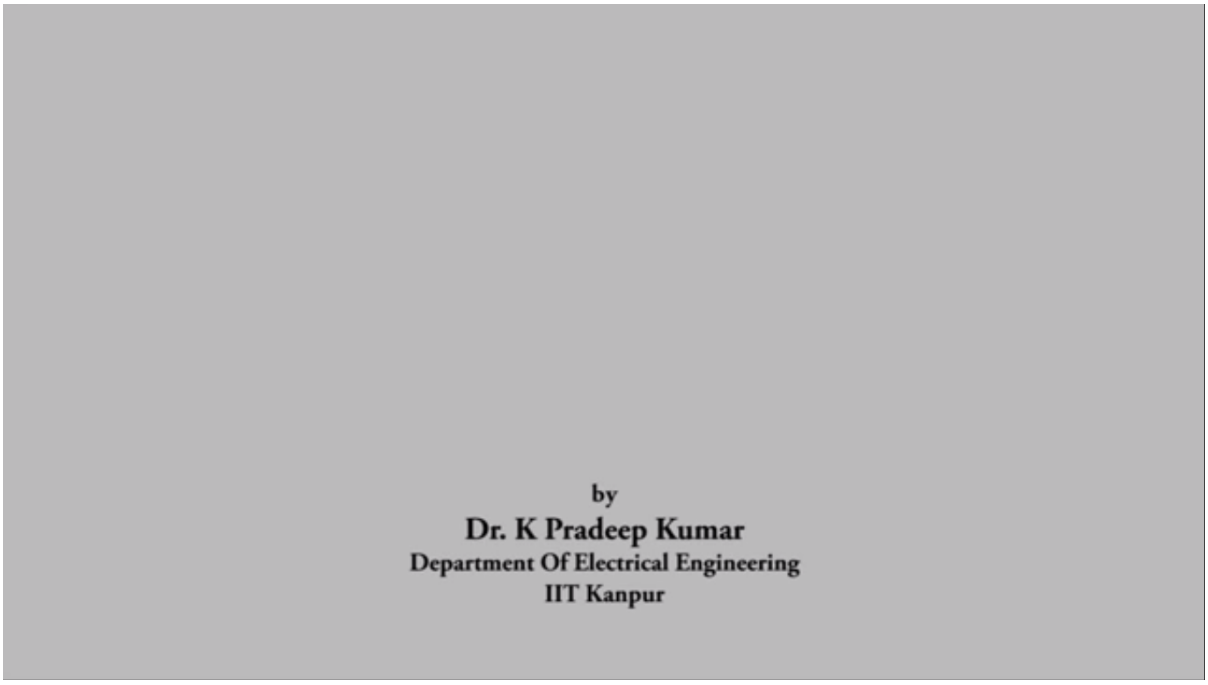

Hello and welcome to NPTEL MOOC on Electromagnetic Waves in Guided and Wireless Media.

This is Module 19 and in this module we are going to consider a different type of incidence, which is called as oblique incidence. Okay. This case is important because in many scenarios, especially, when we talk of waves in guided media, we can think of the waves or the modes propagating in that guided media as being composed of successively reflected obliquely incident waves. Okay. So whatever we are going to learn here, we can apply it to the study of waves in the guided media, which we are going to do in the next, I mean, not next, but some other modules after we finish this properties of plane waves, right?

So with that, let's actually look at what the physical situation is. The physical situation for the problem is kind of the same. So you have this plane of interface wherein you have a medium one and a medium two. Previously, we considered angle of incidence in such a way that the propagation vector was coinciding with the normal to the interface plane. So the normal to the interface plane was the Z axis, and then the angle of incidence was coinciding, sorry, the propagation vector of incident reflected and transmitted media were coinciding with exactly the same Z direction normal to the interface. Okay.

Now instead of this propagation vector coinciding, what happens when the propagation vector is at an angle theta, we will call as θ_1 because we want to distinguish two angles, what happens when the incident wave arrives at this plane of interface with an angle θ_1 as measured from the normal? So this is a normal. Move θ_1 here and this is the angle of incidence now. So what happens?

We know from Snell's law, two things are going to happen. One is that there will be a transmission into the second media whose angle of refraction θ_2 can be related to θ_1 by the following Snell's law. So you have n_1 Sin θ_1 equals n_2 Sin θ_2 where n_1 and n_2 are the refractive index of the media, indices of medium one and medium two. Okay. So this is a law which allows you to determine what is θ_2 given n_2, θ_1 and n_1.

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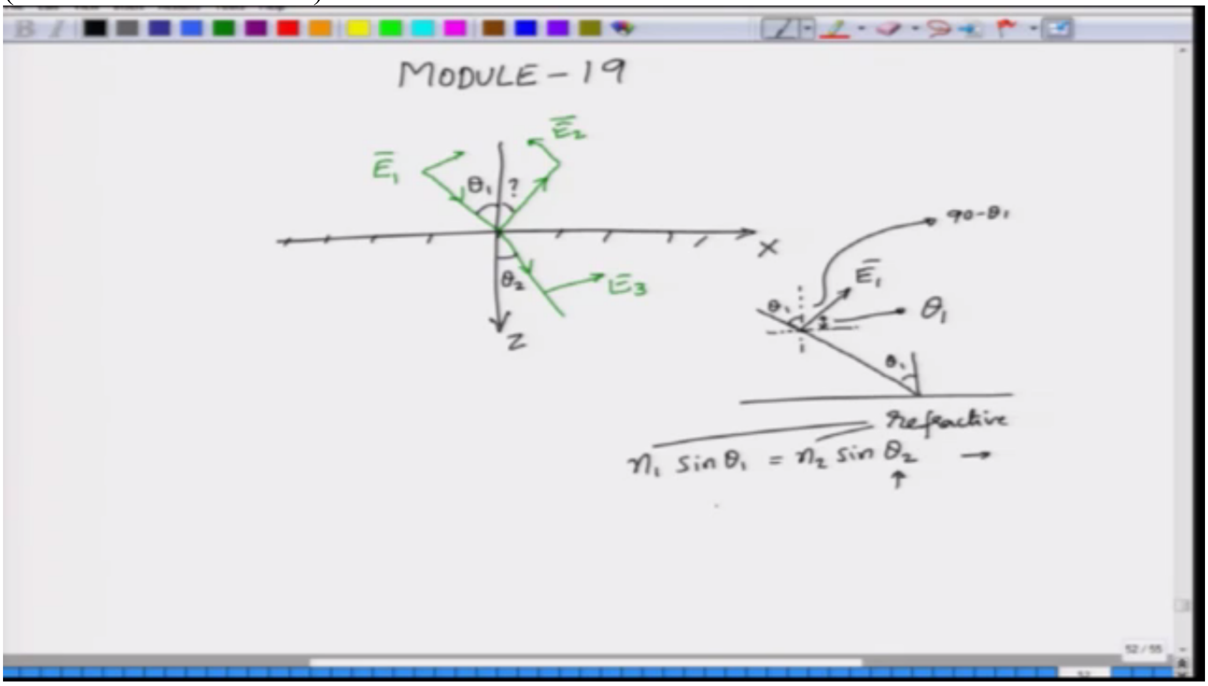

There is another law which states that angle of reflection, which we will call as θ reflection would be exactly equal to the angle of incidence, right? So what it means is that if I consider again this obliquely incident wave, this wave as it approaches, okay, part of that one will be reflected onto this side, right? So part of the wave is actually reflected onto this side and some portion of the wave is transmitted into the second medium with an angle that is different from the angle of incidence.

Now nowhere with the Snell's law you actually are specifying or you are actually able to determine how much of the power that has been incident is actually being reflected and how much of the power is being transmitted, right? So to obtain that important information, we have to go back to the electromagnetic perspective of this problem. Moreover, it is not just, you know, the amplitudes that are or rather the power relationship that are imported, but also something interesting happens with the amplitudes as well. Okay. Moreover, this Snell's law, the so-called Snell's law actually fails for certain scenarios, which will be very important when you consider what is called as optical waveguides. Okay.

So for all these reasons, we need to go back to the electromagnetic perspective, okay, starting with electromagnetic waves and then apply boundary conditions to really understand how much power is being reflected and what exactly happens if the medium of first, if the first medium has a refractive index or equivalently the permittivity greater than the medium, second medium's refractive index. So all those things can be answered by looking at Maxwell's equations or wave behaviour at the boundaries.

Now before we go further, we actually have to have two kinds of waves that we can think of. Okay. So let's return back to our picture. This is my interface plane and I have this angle of incidence. I mean, I have this incident, this one. This is the propagation vector. The black one is the propagation vector. Please imagine that this is at an angle. Okay.

Now I can have two cases. I can have electric field in this plane, okay, which would be if you look at it in this manner, so it should be perpendicular. So it should be like this let us say. So this electric field lies in the same plane of interface. Okay. Or I can have the other way round. I can actually have the magnetic field in that plane, okay, may be in that slightly different, this one.

We define the incident plane as the one that would be concerned with this particular plane, right? The one that would involve that normal and one of the tangential components. So if we take the tangential component to be along the X, then any, the electric field can lie along in the XZ plane, okay, or the magnetic field line can lie in the XZ plane. Okay.

Depending on these two choices, you have what is called as transverse electric polarised waves. Okay. In this case, you have the magnetic field in the same plane as the interface plane, which we have, I mean, as the plane of incidence, which is X and Z. Okay. Or you can have transverse magnetic wave which is written as TM when it is the electric field, which lies in the plane of incidence, which we take as X and Z.

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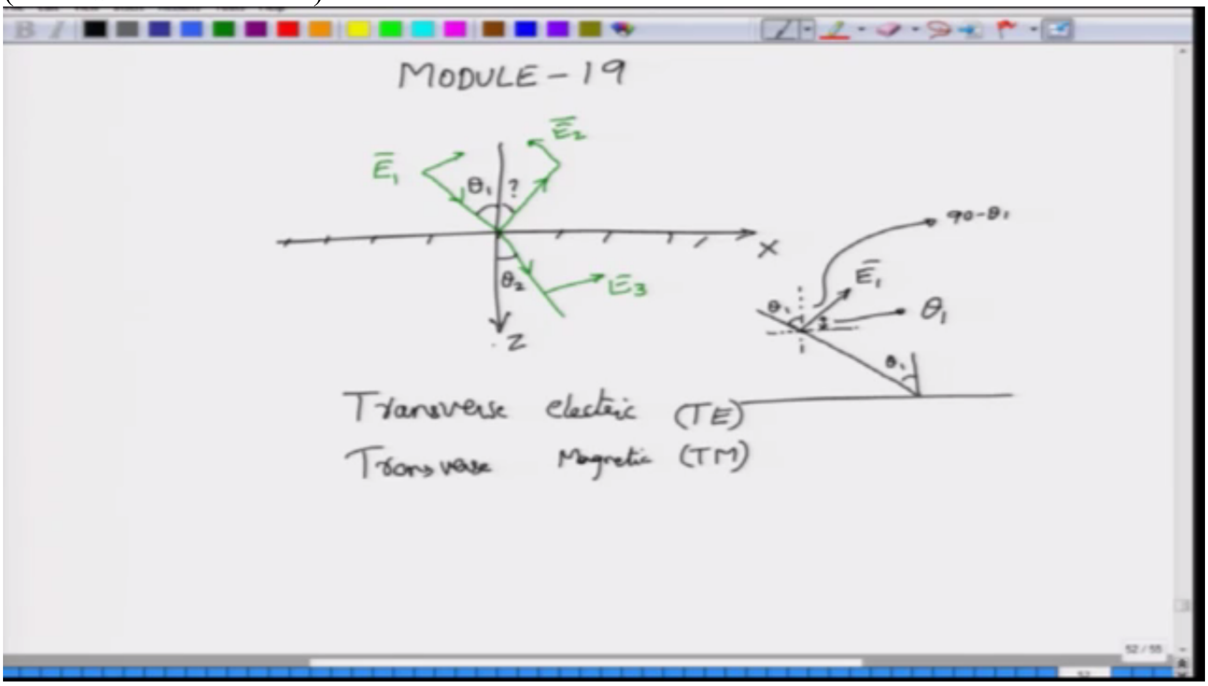

So as you have seen here, in this diagram, I am assuming that the electric field lies in this plane of incidence and therefore I am describing what is called as transverse magnetic because the magnetic field will be perpendicular to these two lines, right? So the magnetic field will be perpendicular and that is why it is the magnetic, sorry, transverse magnetic waves that we are considering. Okay.

So the rest of the ideas are quite simple. All you have to do is to find out appropriately the boundary conditions. There are four boundary conditions. E_{t1} will be equal to E_{t2}, no doubt.

H_{t1} will be equal to H_{t2}, no currents. This medium has refractive index or equivalently the permittivity ε_{r1}. This medium has a permittivity ε_{r2}. And this time I have switched x and z axes. I have taken the z-axis downwards and x-axis along the horizontal thing, and the waves are given by these green lines and the electric field components are shown.

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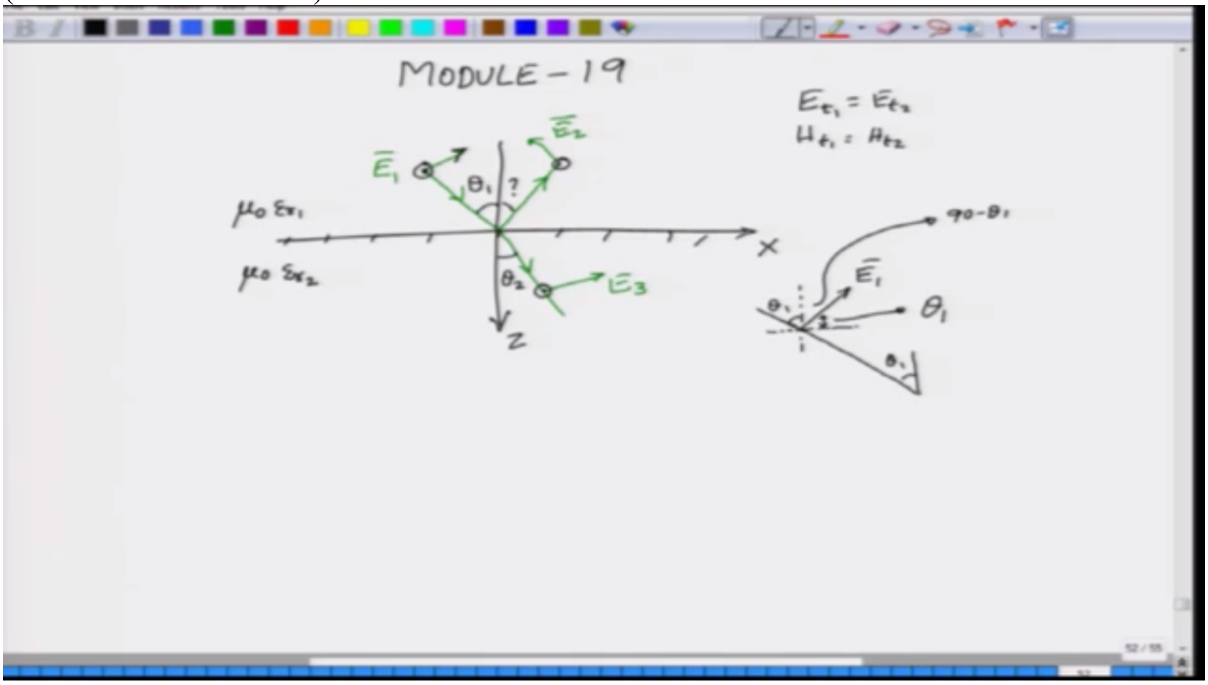

By the way, I have taken the electric field component for E_2 to be completely arbitrary. Equations will tell us whether the direction is this one or the direction of electric field E_2 should be reversed. Okay. So don't worry about that. This is the incident wave. This is the reflected wave and this is the transmitted wave. Okay. So you have these three waves and you have these two boundary conditions. You can also have, of course, the other body condition, which is B_{n1} equals B_{n2} and finally, $\varepsilon_0 E_{n1}$ or rather $\varepsilon_{r1} E_{n1}$ to be equal to $\varepsilon_{r2} E_{n2}$ coming from the D field normal relationship.

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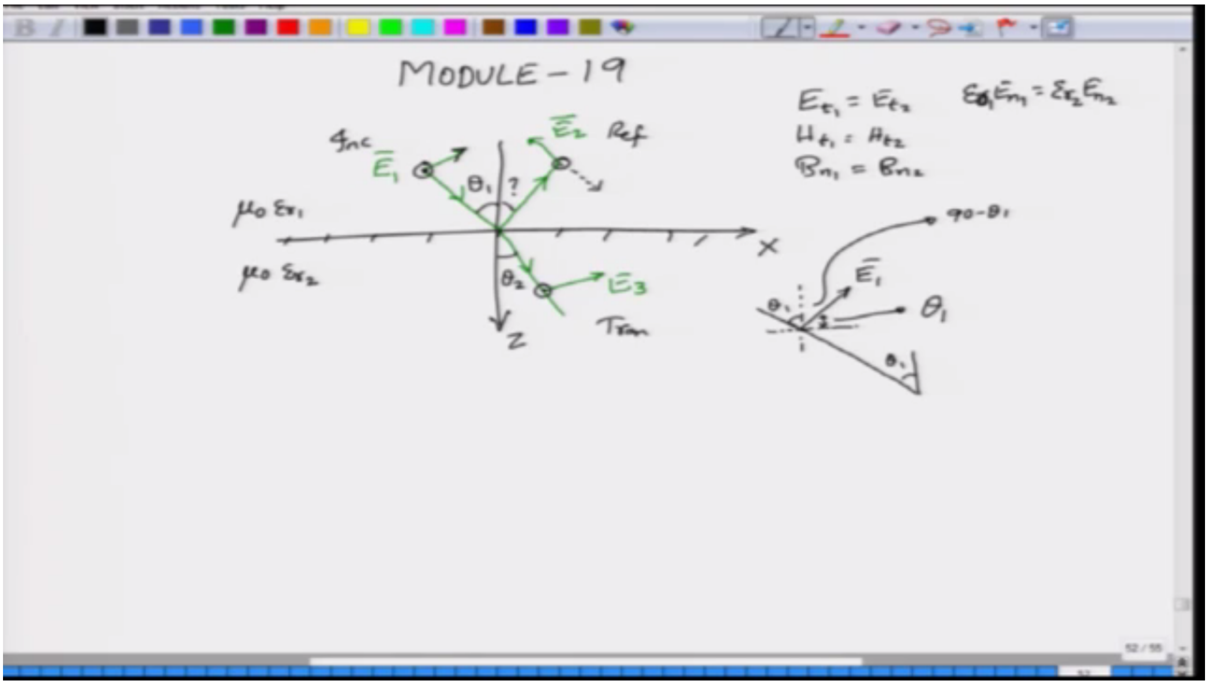

So these are the four boundary conditions that you have and you have to use these boundary conditions to tell us or to find out what would happen to the reflection, reflected power and transmitted power. Okay.

So let us go with this. So I have this k vector here, which is k_1. This k vector is also k_1 whereas this k vector is k_3. k_2 is equal to k_1 because or rather I'll write it as $k_2 = k_1$ because these two actually in magnitude they are in the same medium, right? Okay. And this is k_3. Okay.

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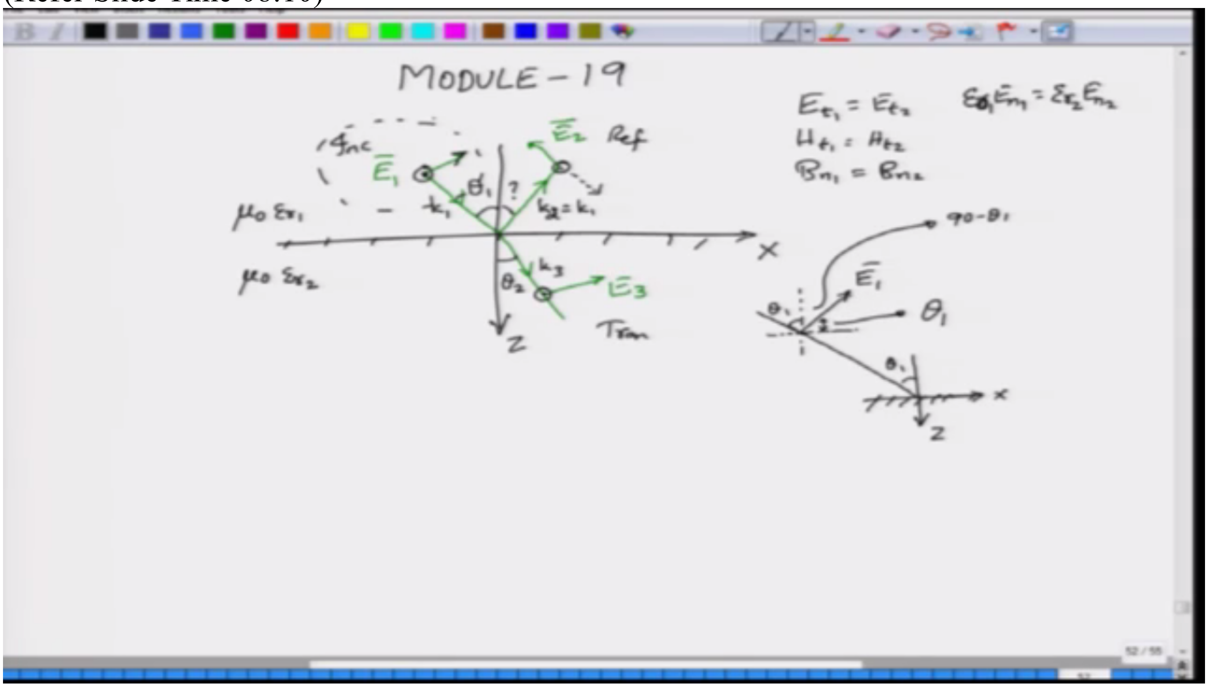

We will now look at the tangential component for the electric fields in medium one and tangential component of electric field in medium two and equate the two, right? So I have blown up this portion of the picture here, okay, because I want to talk about the electric field angles. So you can see that if this is the x-axis, this is the z-axis, this is the plane of interface, okay, I have this electric field E_1 itself having two components, which is E_{t1} and E_{n1}. Okay.

The tangential component, of course, is given by E_1 Cos θ_1 that is the amplitude, but there is a also a phase. Now what is the phase here? In so far what we have considered, our direction of k was exactly equal to, you know, it was actually equal to one of the normal or one of the unit vectors. It could be z, x or y. We have taken it to be z. So our k vector could be written as whatever the magnitude of the k vector, so medium one times z, right? So I could have written this k_1 in a vector form as k_1, which is the magnitude times the angle which is z.

In this case, that is not true. In this case, I have the k vector itself at an angle. Okay. So the k vector should actually be written k_{t1} along say x plus k_{n1} along z, right, because you can take this line and then, you know, decompose this into two lines of this particular nature. One will be along x. One will be along z. And what is the value of k_{t1}? k_{t1} will be, so this k vector can be written in terms of the, so along z it would be k_1 Cos θ_1 z plus k_1 Sin θ_1 x. So this would be the k vector, k_1 vector. Okay.

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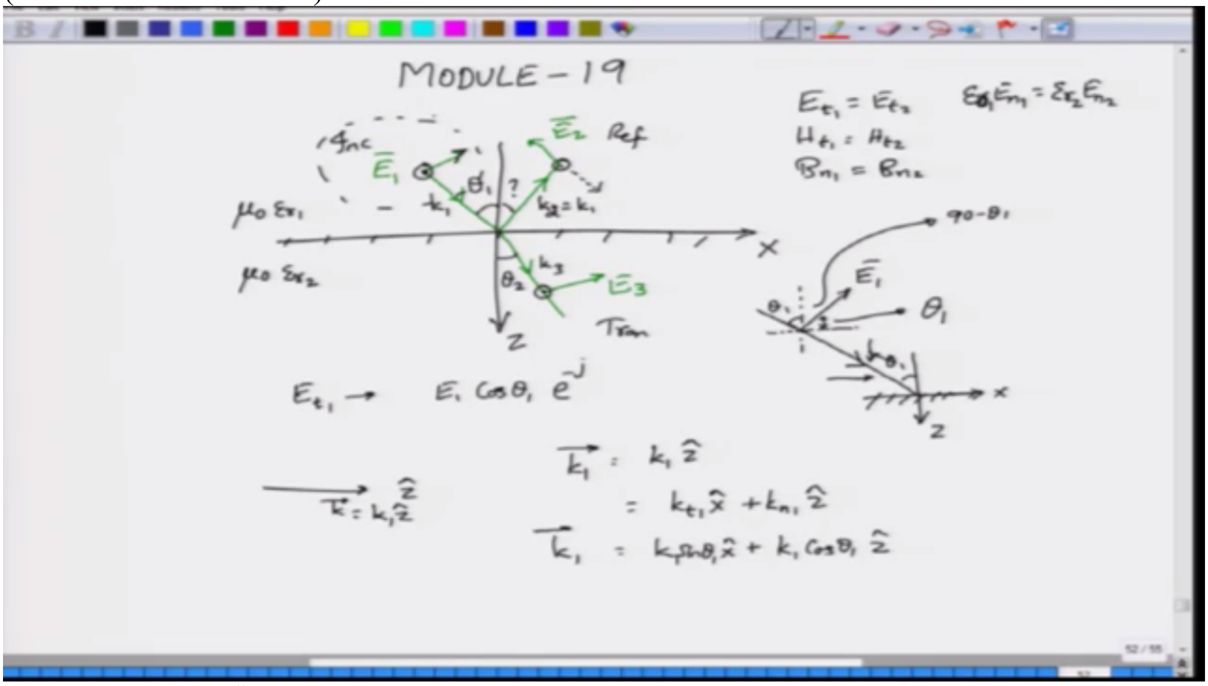

What about this e^{-jk1z} that we were writing earlier? We were writing earlier, you know, as, you know, very simple as k_1z because you could take this k_1 vector and the position vector r. In the previous case, the position vector r was simply z z-hat because z was the only direction in which the wave was propagating. So when you take the dot product of k_1 and r, this phase factor was simply equal to e^{-jk1z}, but in this case the position vector can be in the X and Z planes. So at any point that you can consider, okay, which would be described by X and Z, so that point can be described by both X and Z values, the actual, the phase factor that should be written will be $e^{-jk1.r}$ where r is given by x x-hat + z z-hat, okay, meaning that if you now combine everything, so what we have here is the position vector r given by x x-hat + z z-hat.

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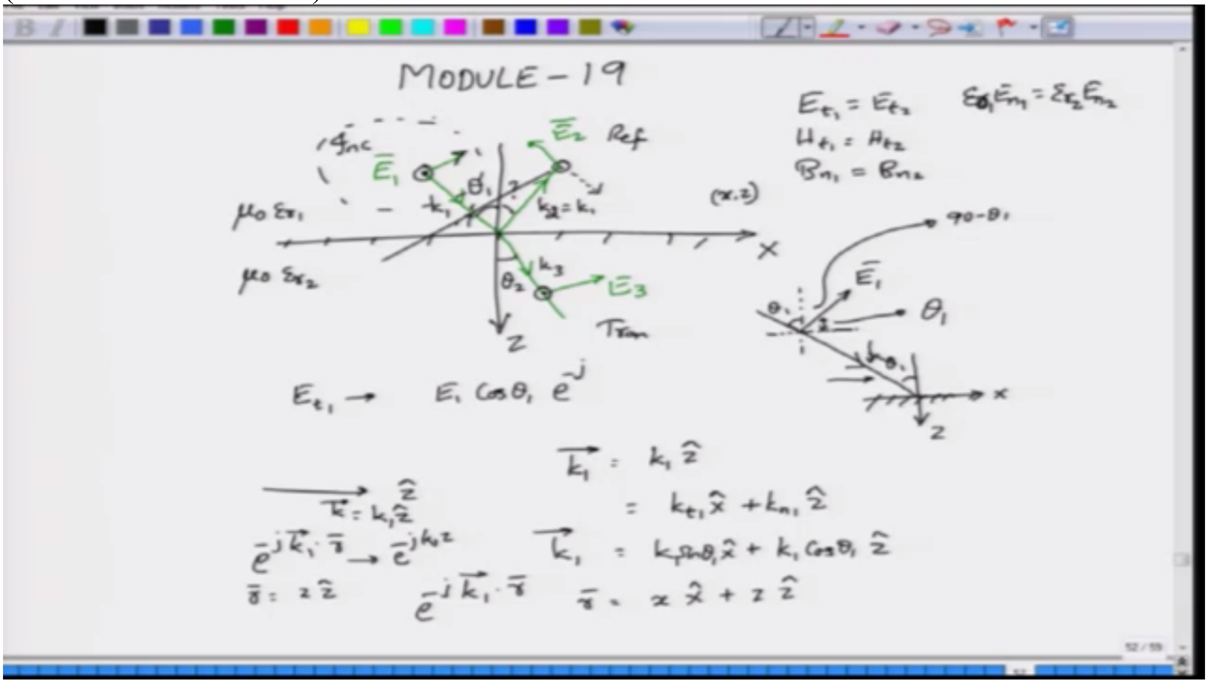

Now I will erase here itself so that these can be written correctly. The phase factor corresponding to the incident wave, okay, would actually be given by $e^{-jk1 \sin \theta 1x + k1 \cos \theta 1z}$. Okay. So this would be the phase that should be appended to the amplitude. So this $e^{-jk1 \sin \theta 1x + k1 \cos \theta 1z}$ should be accompanying the tangential component that we have for electric field E_1. Okay.

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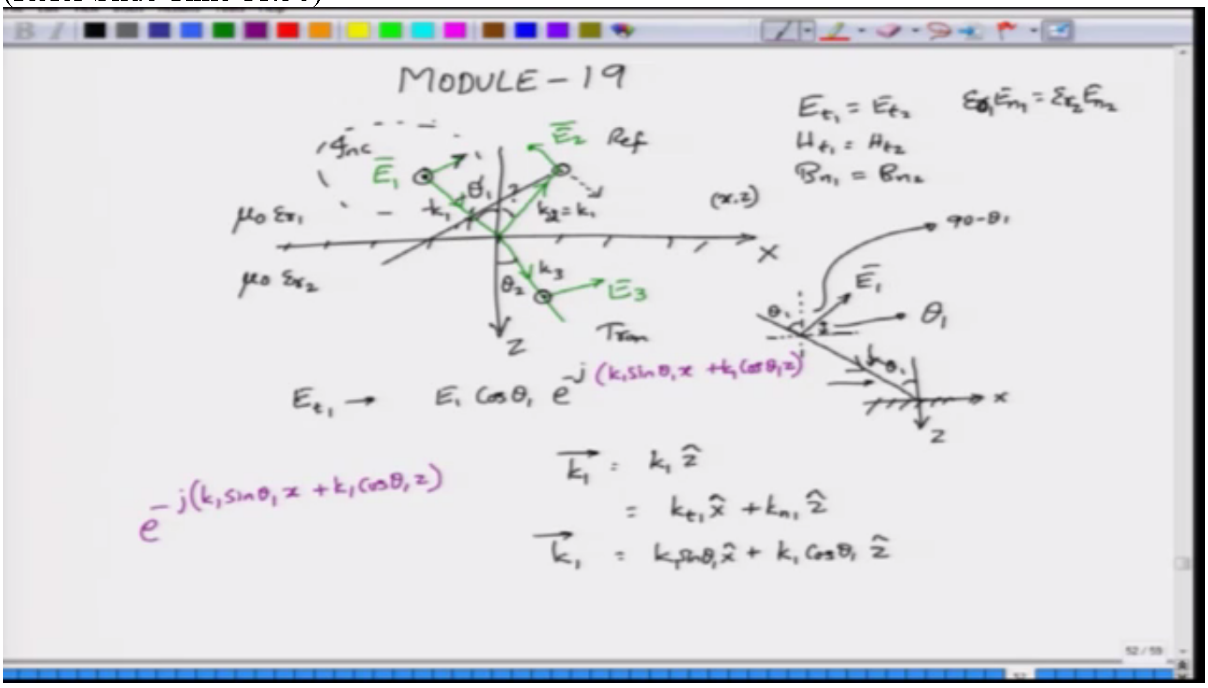

See the electric field E_1 would have the tangential component, which is given by this E_{t1}. So this is E_{t1}, which is of course making an angle of θ_1 with respect to the electric field in the medium one, sorry, with respect to this axis. So you write down this in terms of tangential as well as a normal combine.

Normally, you are not worried at this point. Tangential component has an angle of θ_1 with respect to this axis. So you simply have E_1 Cos θ_1, E_1 being the magnitude of the incident electric field; Cos θ_1 giving you this one. Okay. Now k_3 which is in the same direction as k_1 will also be given by the same expression except replacing θ_1 by θ_3. Okay.

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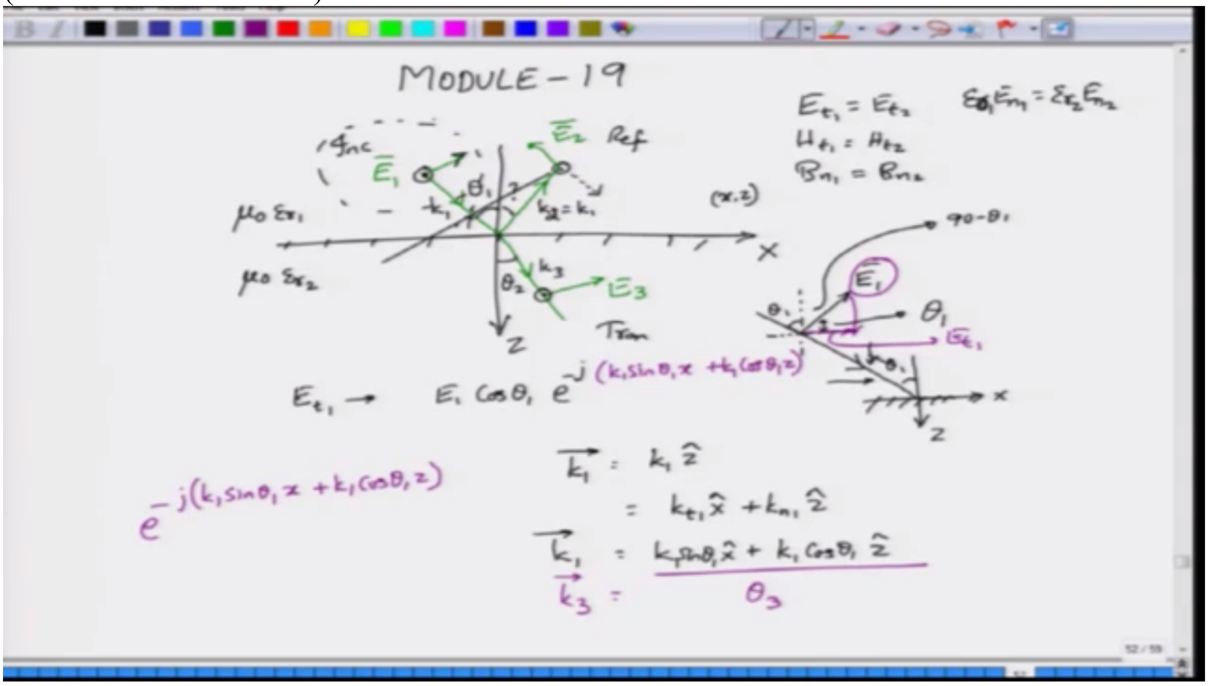

So the tangential component in second medium, okay, the tangential component of the electric field in the second medium, of course, has the same angle also. So if you look at this electric field, you see that this electric field will be in the same angle. Everything is same except θ_1 will become θ_3. So I can write down E_{t3} as E_3 Cos θ_1 $e^{-j(k3Sin\theta3x + k3Cos\theta3z)}$. Okay. So that is for the transmitted electric field.

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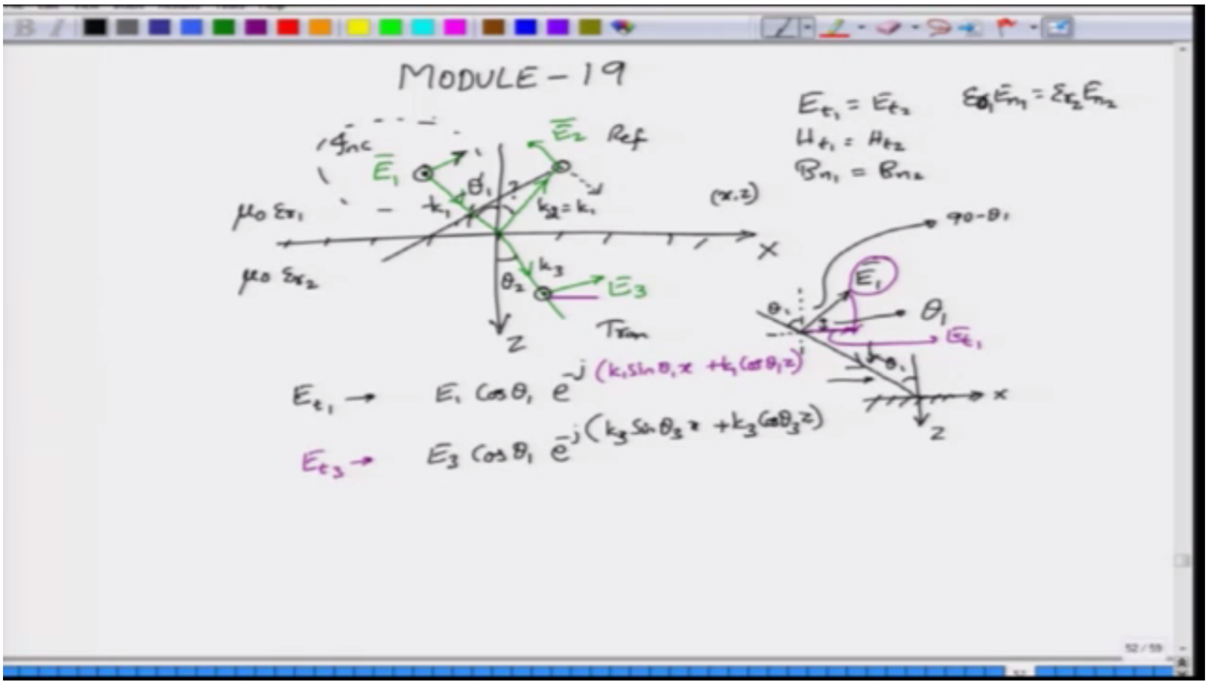

How about the electric field E_2? Now E_2 is slightly different because k_2 vector itself will be given by k_2 magnitude, which, of course, actually is equal to k_l because magnitude wise they should be the same. They are in the same medium, right? So you will have k_l itself.

Now look at this. The direction along z will be opposite to the incident wave. That is how, of course, the wave is propagating along -z direction. Therefore, you can write this as k_l Sin θ_1x. Why θ_1? Because θ or rather we will write it as k_2x at this point. Okay. So k_l Sin θ_2x, sorry, this would be x-hat, that is the vector along the x direction, plus or rather minus because now k_2 vector is -k_l Cos, so I have this k_2 Cos here, sorry, along x.

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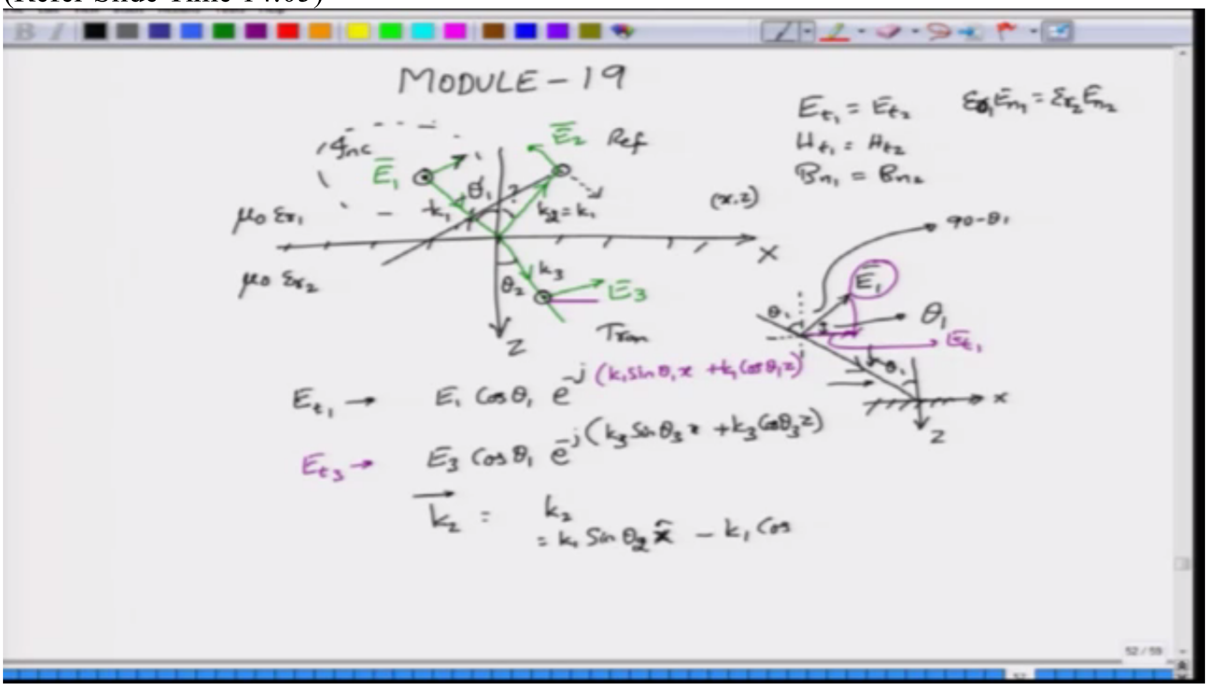

Hold on. I've made a small mistake here. It should have been k_1 Cos everywhere. We put in Sin instead of Cos. So this actually should be Cos, right?

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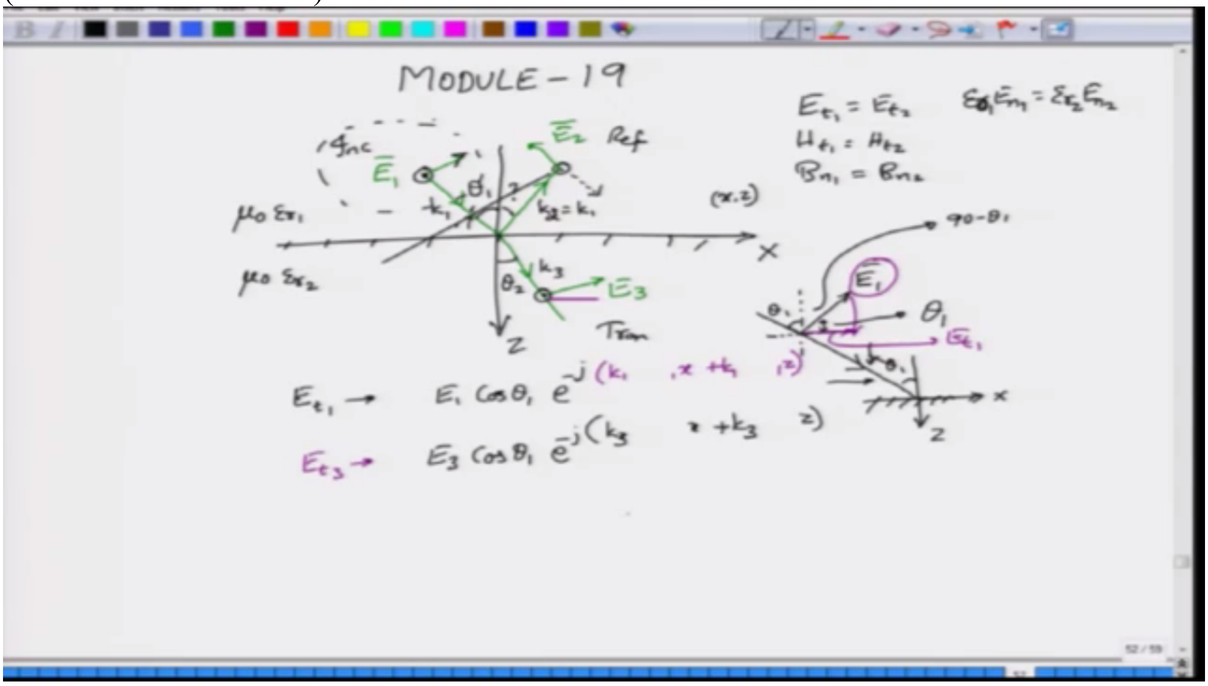

Let me write down this correctly. So this is your k_1 at an angle θ_1. So along the x-axis will be k_1 Cos θ_1, k_1 Sin θ_1. Okay. This is fine. I mean, whatever we wrote earlier was actually fine. So we'll go back and write it. k_1 Cos θ_1 along z, k_3 Sin θ_3 along x and k_3 Cos θ_3 z.

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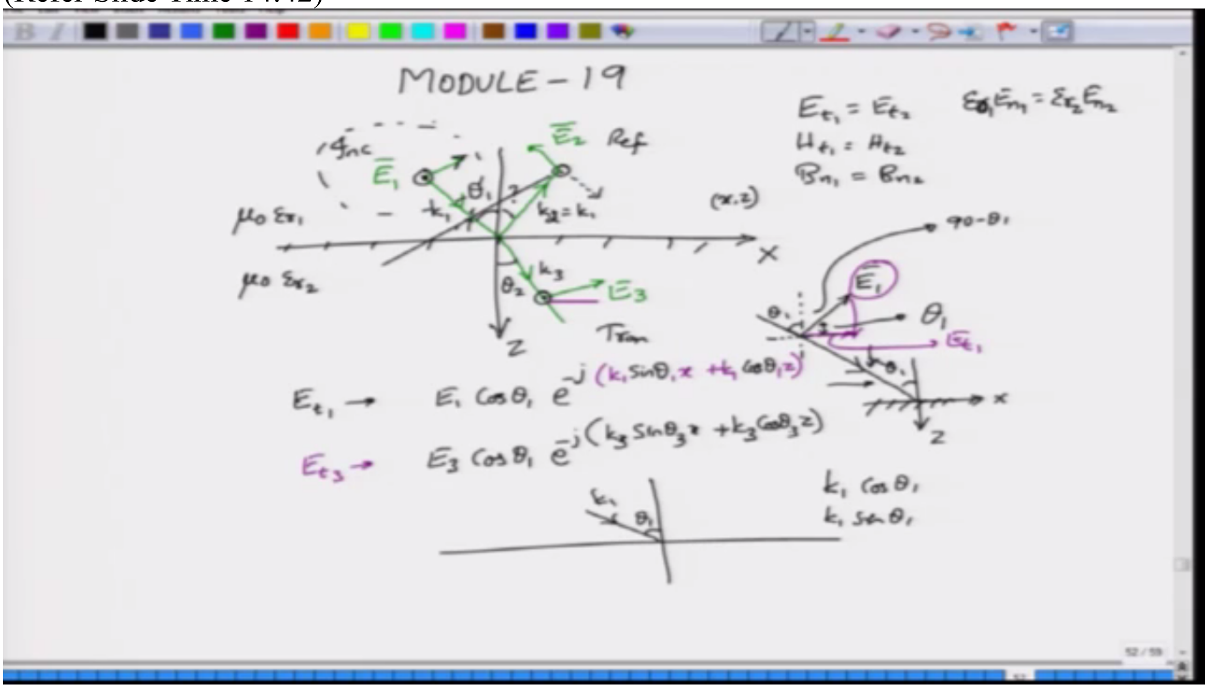

So this is all right. I mean, I thought Cos and Sin should be inverted, but no. It is actually correct. So k_1 Cos θ_1 will be along z, k_1 Sin θ_1 will be along x, but for the reflected wave which is making an angle θ_2 here, you have k_1 or -k_1 Cos θ_2 because this is along the -z direction, right? So this would be along the -z direction and along x direction it would be positive. I mean, it would be along the same +x direction as k_1. Okay. So the reflected wave vector k_2 can be written as k_1 Sin θ_2 x - k_1 Cos θ_2 z. Okay.

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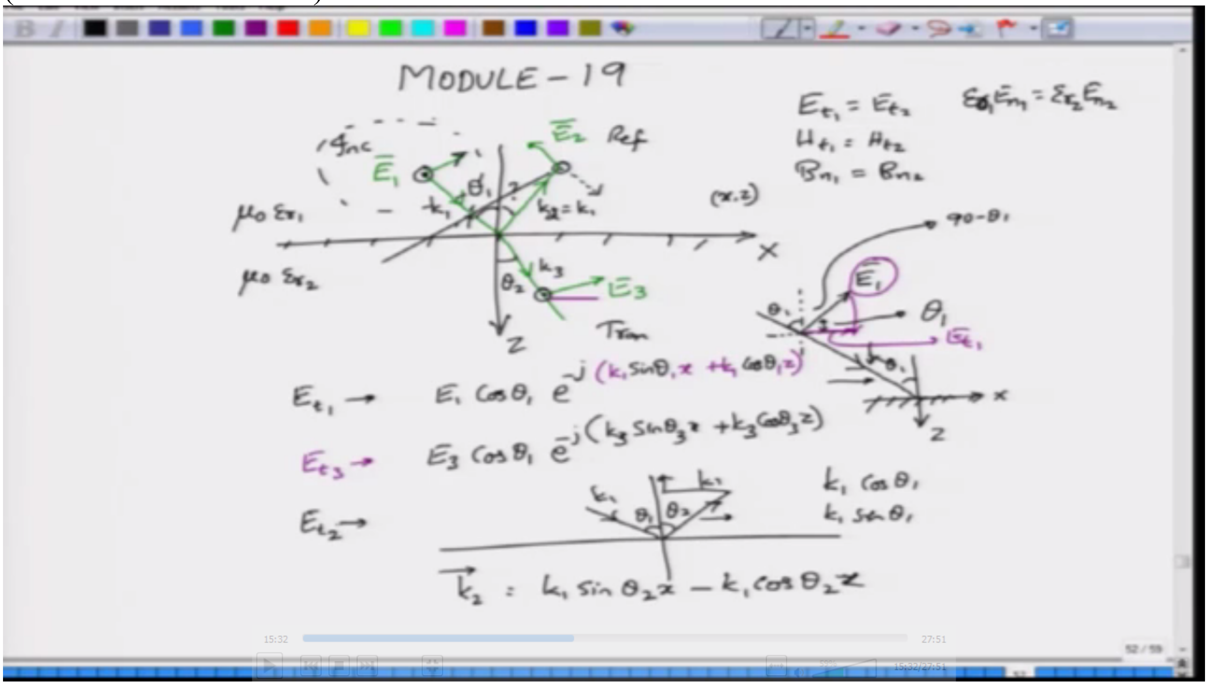

So I can write down E_{t2}, which is the tangential electric field, what is the amplitude of a tangential electric field? Well, you have to again rewrite this picture and then determine what is the responding amplitude there? So let's put down the amplitude here. The picture that I am now looking at is when electric field is making this angle E_2. This angle is θ_2. So, clearly, onto this one, this line makes an angle of θ_2. Between this line that is electric field E_2 and this normal will be 90 - θ_2. Correct? Because this E_2 is perpendicular to k_2, so this angle will be 90 - θ_2.

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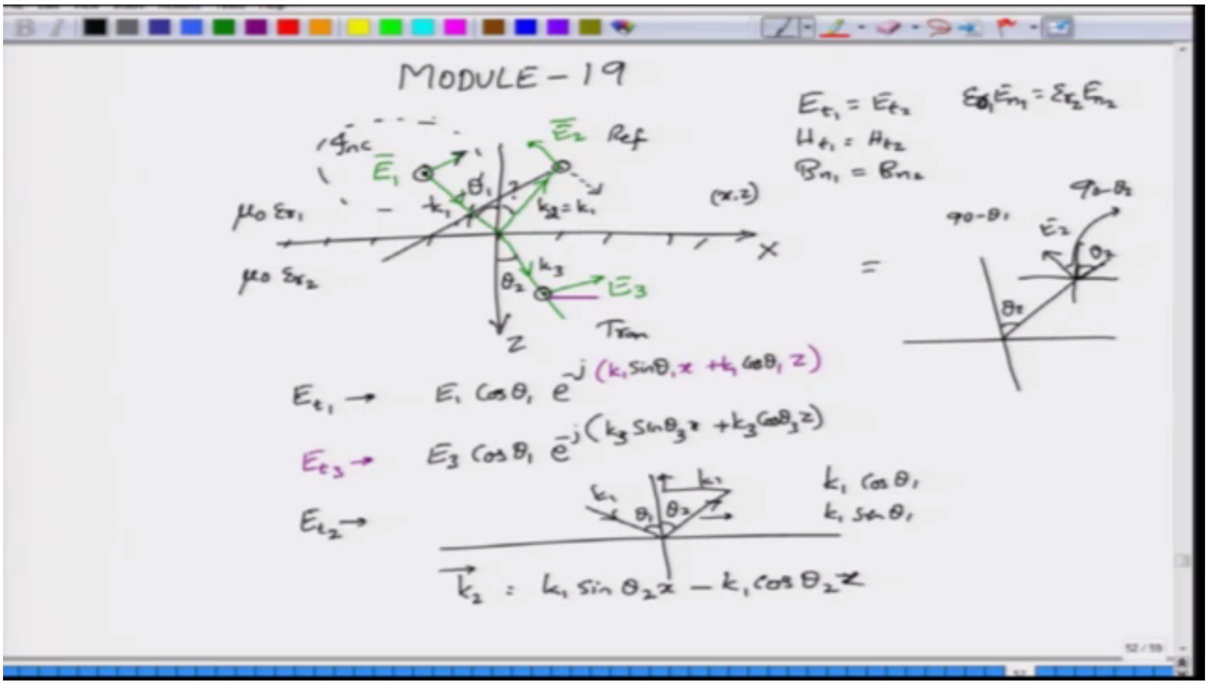

However, these two lines, which is the perpendicular line and the horizontal line, they themselves are 90° apart. Therefore, this angle is basically θ_2. Okay. So you can write this as E_2 Cos θ_2 along x direction, that would be along the -x direction, and E_2 Sin θ_2, that could be along the z direction, and it would be along -z direction.

However, we are interested in the tangential component. Therefore, I can write this as E_2 Cos θ_2, but this is along the -x direction because of the way that we have taken the electric field to be. So this would be - E_2 Cos θ_2 multiplied by this k_2 phase factor, right, multiplied by this phase factor, which is given by $-jk_2$ and k_2 Cos will be along z, k_2 Sin will be along x. So we have already written that one. So $-k_2$ Sin θ_2 x - k2 Sin or rather Cos θ_2 times z.

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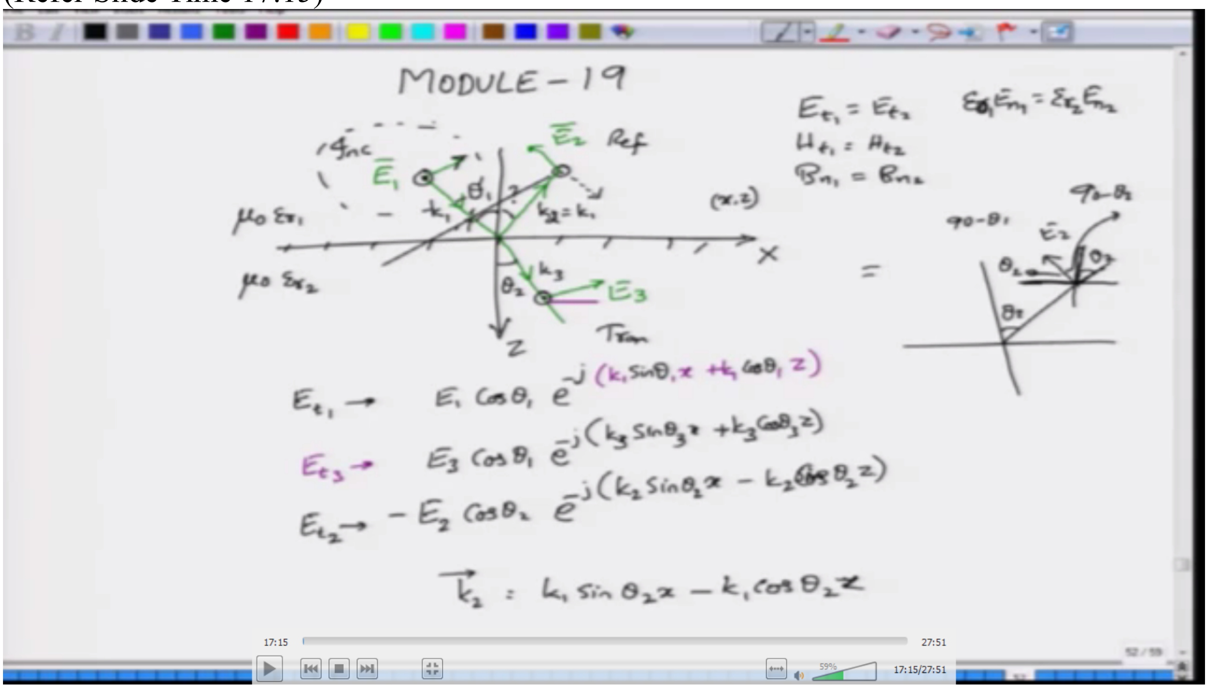

So we have the three individual components. It took us a little bit of effort to find out the correct amplitudes and other things. That is because we are dealing now with oblique incidence. Normal incidence would be very simple.

You also have seen that there is a small change in our electric field coordinates because for the normal incidence θ_1 will be equal to zero, okay. We assume that θ_3 is also zero and θ_2 is also zero. Why? We will see it shortly, but if you assume all these thetas to be zero, then you will clearly see that all the x dependent phase factors will go away as it should because there was no e^{-jk} something times x in the normal incidence case. All the waves were propagating either along plus z direction or along -z direction. All Cos θ factors will become one. E_{t1} will be along the +x direction. E_{t3} will be also along the +x direction. Okay.

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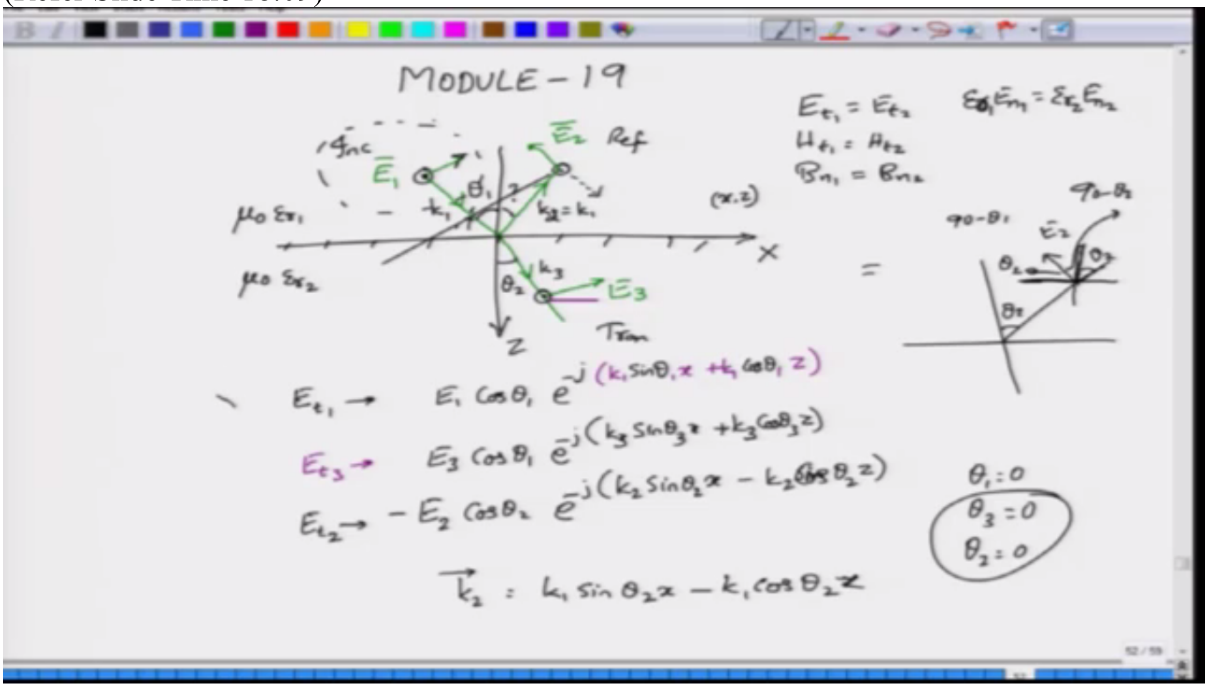

However, E_{t2} has now become along the -x direction. Okay. If you did not want this -x direction, all you could do is to simply switch this direction of the electric field, so instead of considering the electric field in this manner, you can select the electric field to be in the same direction as E_1 and then adjust this minus sign in the H case. Okay.

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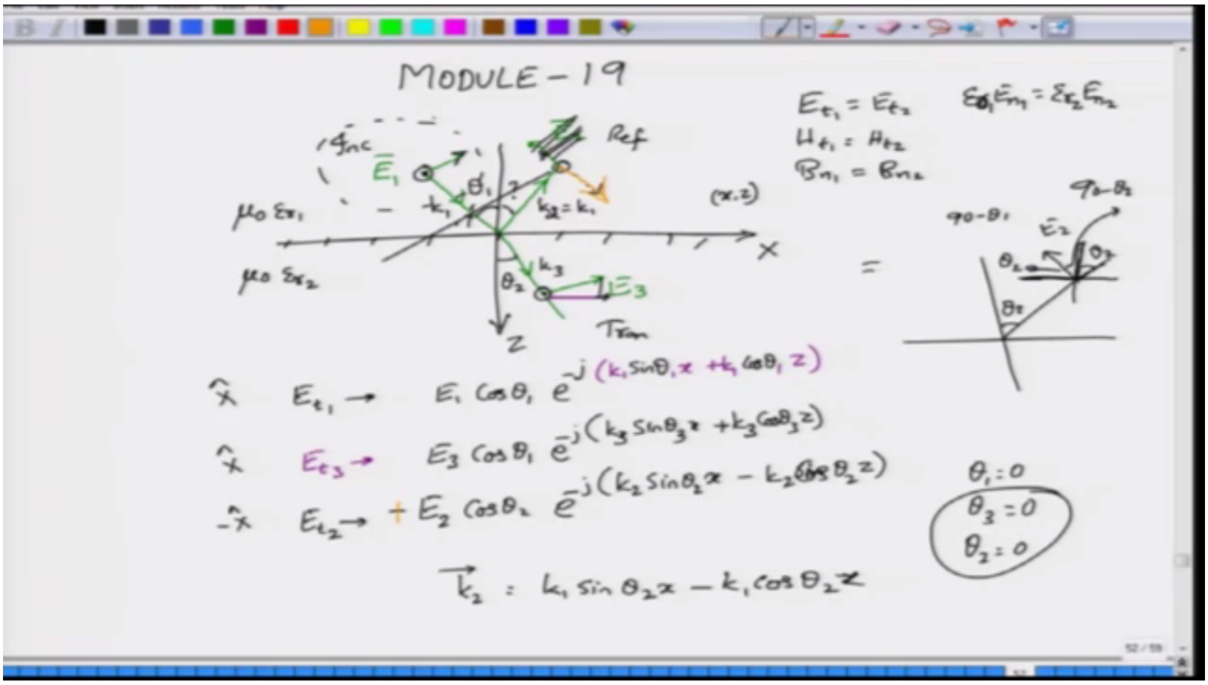

So when you do that you can remove the minus sign and make this one plus without changing any of the other arguments. Okay. This is just kind of consistency between this module and the previous module. If you wish that, you could do this. Okay. If not, you can continue with the original assigned directions and work throughout. The questions will anyway tell you that you will have a minus sign or a plus sign. Okay.

Now all I am doing here is to try and make everything to be consistent with the previous module. Therefore, even though I start off with an electric field in the direction that I showed in the green line, I have now switched it over to the orange line simply because I want to be consistent with the previous one.

However, our original expressions that we started out in this module are also valid. I would encourage you to take this as an exercise and continue that next part of the development with the original diagram as well. Okay. For now I'm simply assuming that electric field is going to be in this direction just to be consistent with the fact that when θ_1, θ_3, and θ_2 are all zero, we land up back into the normal incidence case. Okay.

Anyway, so this is k_2 we have written. Everything we have now written. So we have a set of equations which are valid for electric field. Okay. Now what should we do? Well, we know that $E_{t1} + E_{t2}$ should be equal to E_{t3}. That is total tangential electric field medium one should be equal to medium three, right? And where should this equality be present? This equality should be present at z = 0 plane, but unlike the previous case because the phase factors will be dependent on x now, this should be valid for all x. Okay.

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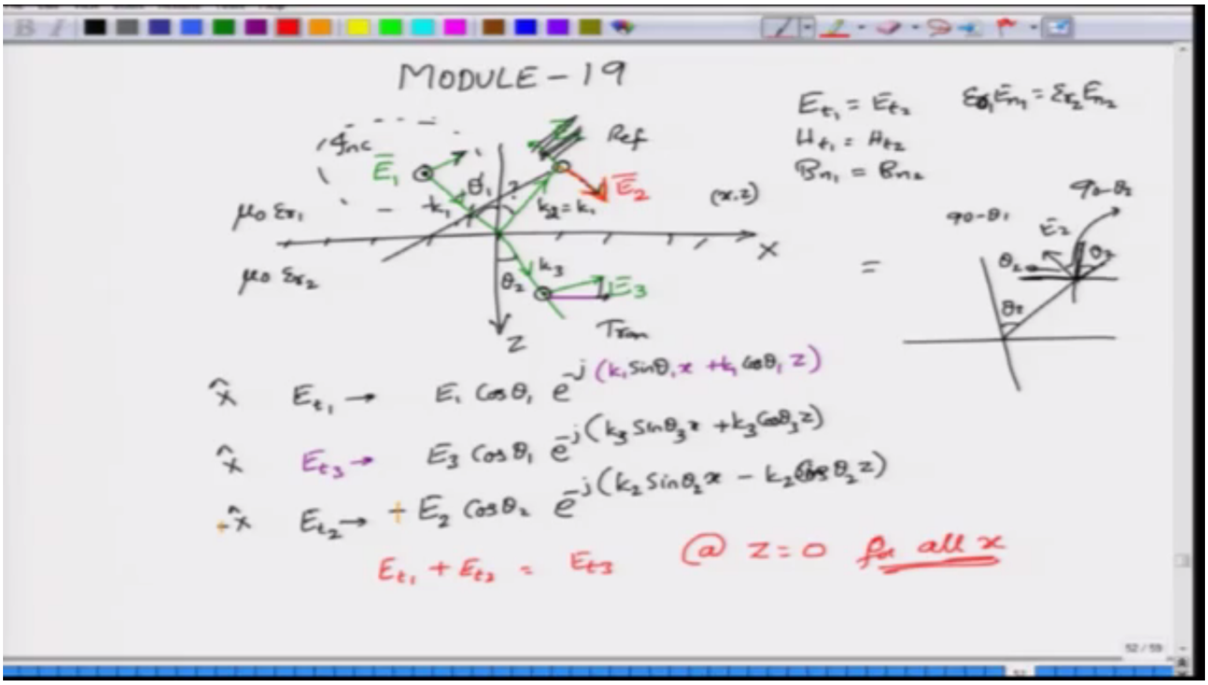

This expression that the tangential electric field should be valid for all x, what it means is that if I consider the electric field to be landing at this point and then calculate the tangential electric field, it doesn’t matter where the tangential electric field lies on the x-axis, right? At every point on the x or rather every point on the x should satisfy this equation. Okay. So that is the critical part of it.

And now when you impose the condition that at z equal to 0 and for all x, the sum $E_{t1} + E_{t2}$ should be equal to E_{t3}. You can write this as, okay, with z equal to 0, all these terms will go to 0. You don't have to worry about it. So the expression will actually become even E_1 Cos θ_1 $e^{-jk1Sin\ \theta1x}$ + E_2 Cos θ_2. Okay. Please note that I have switched the convention here. It doesn't matter, and then I have $e^{-jk2\ Sin\ \theta2\ x}$. That should be equal to E_3 Cos θ_3 $e^{-jk3Sin\ \theta3\ x}$. Okay. And because these equations have to be valid for all x, the only way this can happen is that these individual phase factors are all equal to each other. Okay.

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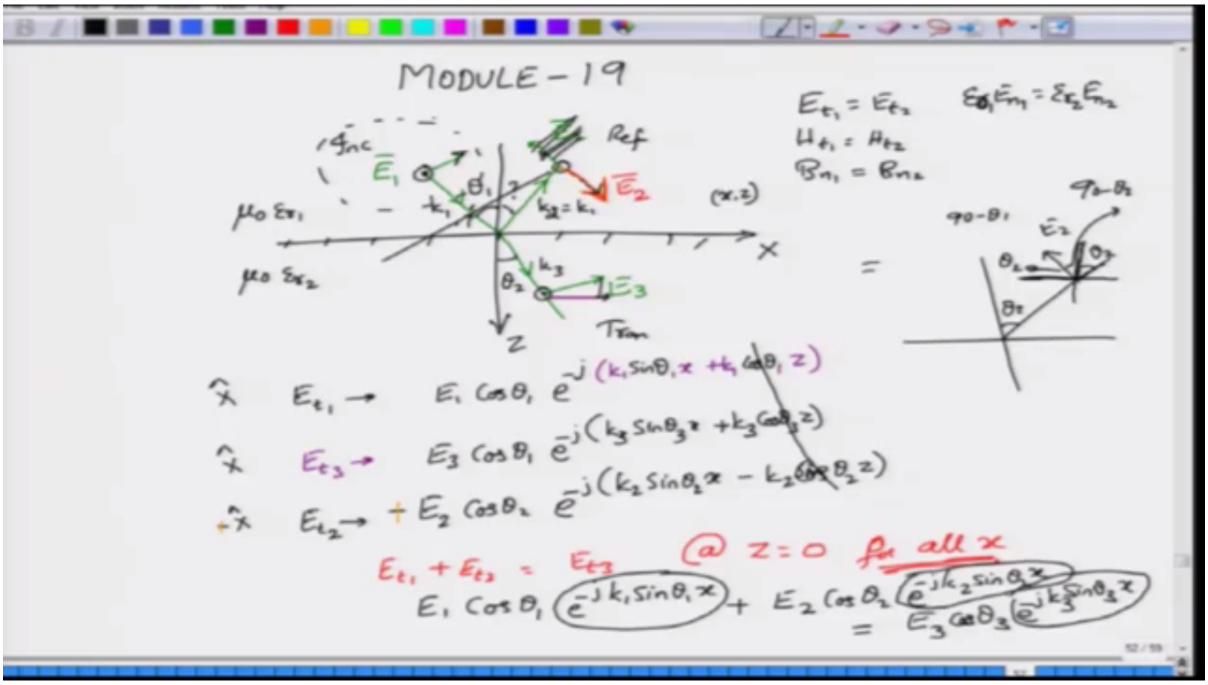

So these individual phase factors being equal to each other means that I have k_1 Sin θ_1 x should be equal to k_2 Sin θ_2 x, which should be equal to k_3 Sin θ_3 times x. Now, obviously, in this expression, k_2 is given by $\omega \sqrt{\mu_0\ \varepsilon_{r1}}$, which is actually equal to k_1 magnitude. Why? Because both incident and reflected waves are in medium one and medium one is characterised by permittivity ε_{r1}. Okay, and that immediately implies that Sin θ_1 must be equal to Sin θ_2.

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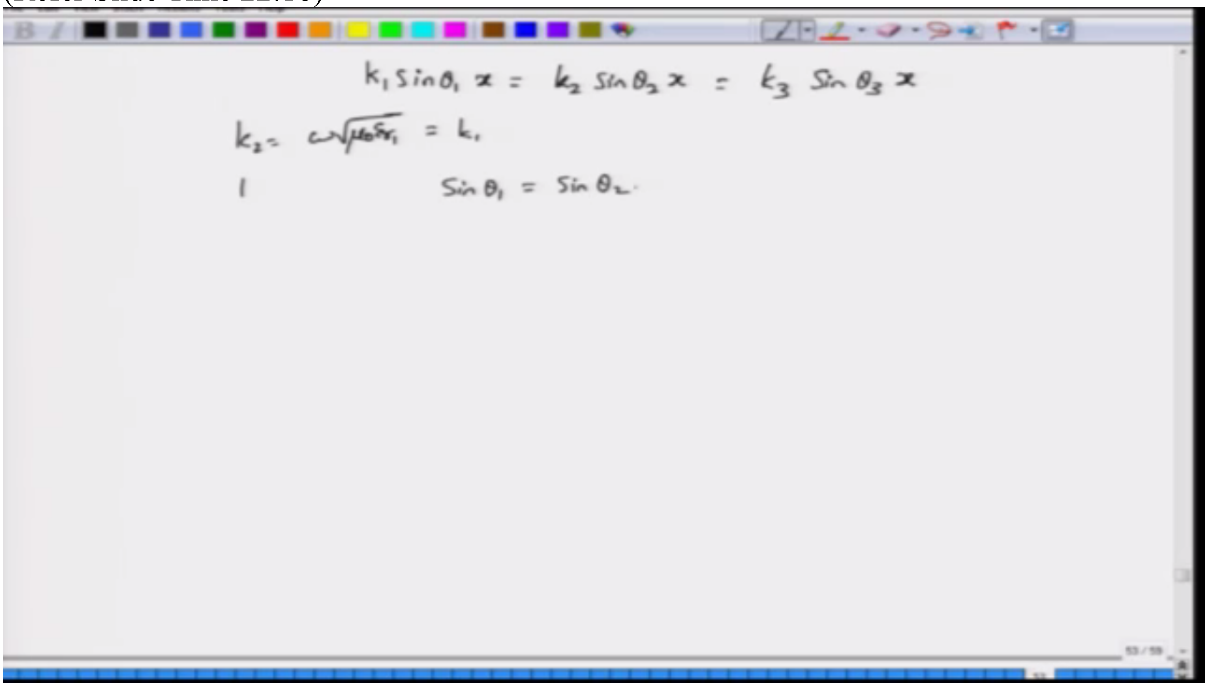

And what is the limits on θ_1 that we can have? Well, this θ_1 can be zero which corresponds to normal incidence and all the way to θ_1 equals pi by 2 in which case the wave will be riding along the plane of interface. It would be riding along the plane of interface with the electric feel appropriately directed. Okay.

So this kind of a wave, right, where the electric field is along the ground kind of a thing and this is riding along the parallel, you know, interface plane is called as grazing angle incidence, okay, whereas this wave was called as normal incidence; this is oblique incidence; this is called as grazing incidence. The wave is kind of gliding or grazing the surface and electric field line actually lies in that particular plane. Okay. Electric field is parallel to this ground. Okay.

The other way would, of course, be that electric field is perpendicular. The magnetic field would be gliding along this surface, right? So that would correspond to transverse electric or vertical polarisation. This would correspond to parallel polarisation. Okay.

So with that in mind, if you look at the equations, sorry, θ_1 can go from 0 to 90°. So if you sketch Sin θ_1, it would look something like this up to 90°. So this is 90° done. So if you have two Sin functions equal to each other over 0 to 90° interval, the only way that can happen is when θ_2 is equal to θ_1. Okay. This is, in fact, so called Snell's Law of reflection.

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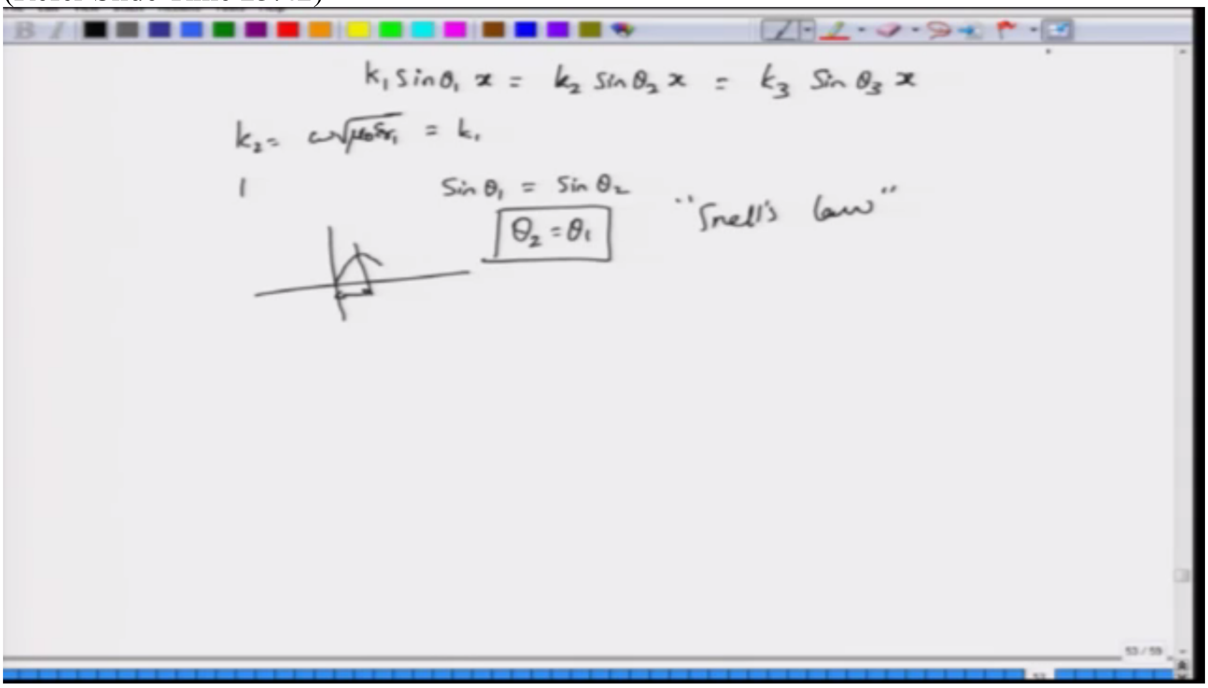

In fact, what we have seen is there is nothing like a law. It is a simple, you known, consequence of boundary conditions, right? So all these laws that we have learnt so far are nothing but consequences of boundary conditions. Okay. So that is the boundary condition.

The second equation that you have k_1 Sin $\theta_1 = k_3$ Sin θ_3, right, is also Snell's law, but this equation tells you that for the same frequency ω, this would be $\sqrt{\varepsilon_{r1}}$ Sin $\theta_1 = \sqrt{\varepsilon_{r2}}$ Sin θ_3. Okay. But because ε_{r1}, ε_{r2} square roots are nothing but refractive index, this is n_1 Sin $\theta_1 = n_2$ Sin θ_3. Okay. So this is another of Snell's Law. This is called as Snell's law of refraction.

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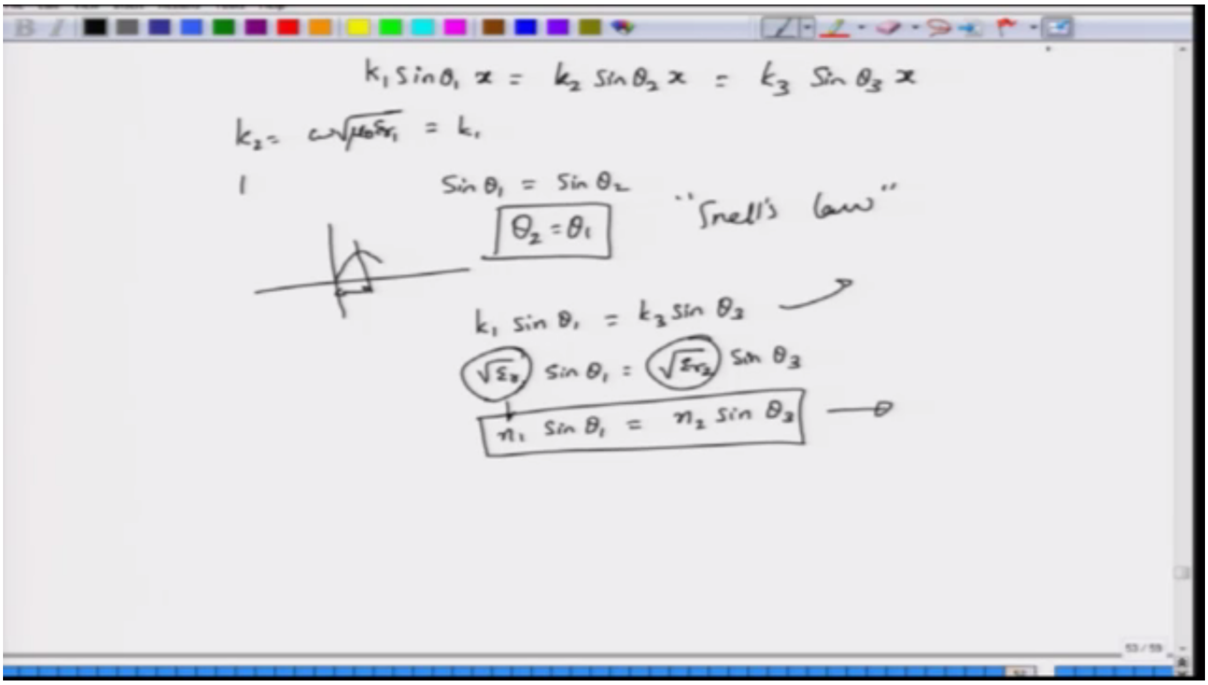

Okay. So we have recovered the Snell's law as a consequence of boundary conditions, but we are not done yet, right? What we have done so far is to simply equate the tangential electric field and get an equation in this particular manner. So we will recollect, write the equation for our use now. So we have E_1 Cos θ_1 + E_2 Cos θ_2 where θ_2 and θ_1 are actually equal to each other is actually given by E_3 Cos θ_3, right? So this is one equation that you need to keep in mind.

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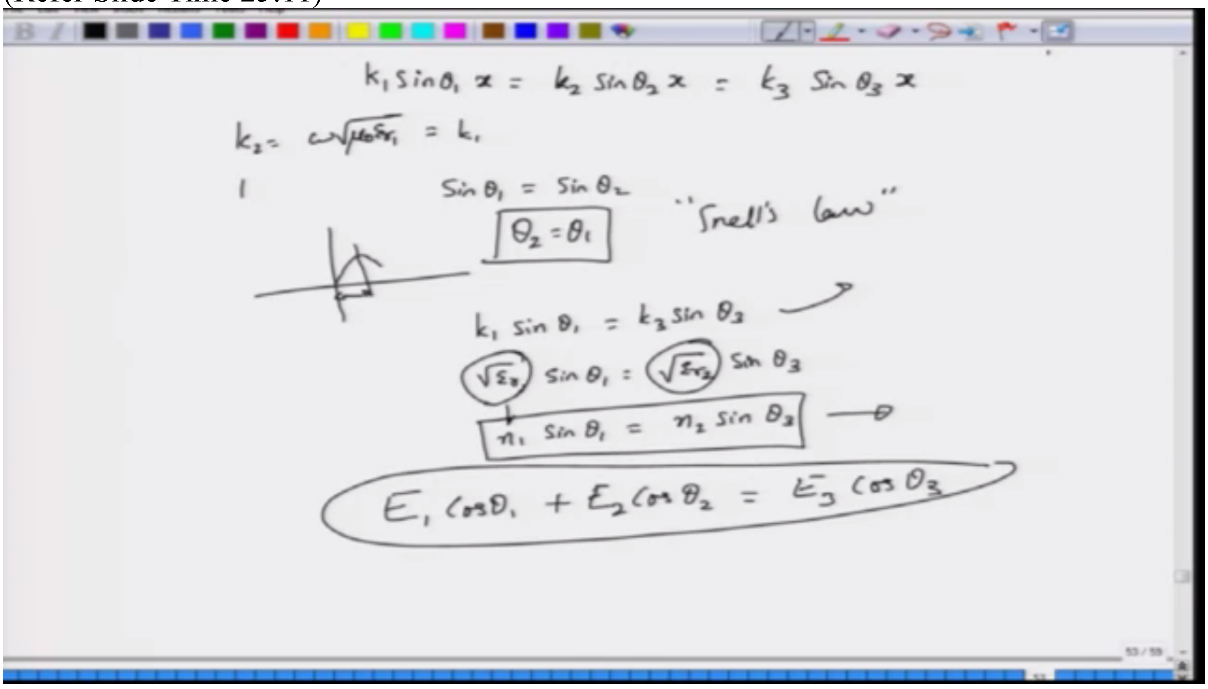

Now we need to supplement this equation with the magnetic field equation. How would the magnetic field equation be?

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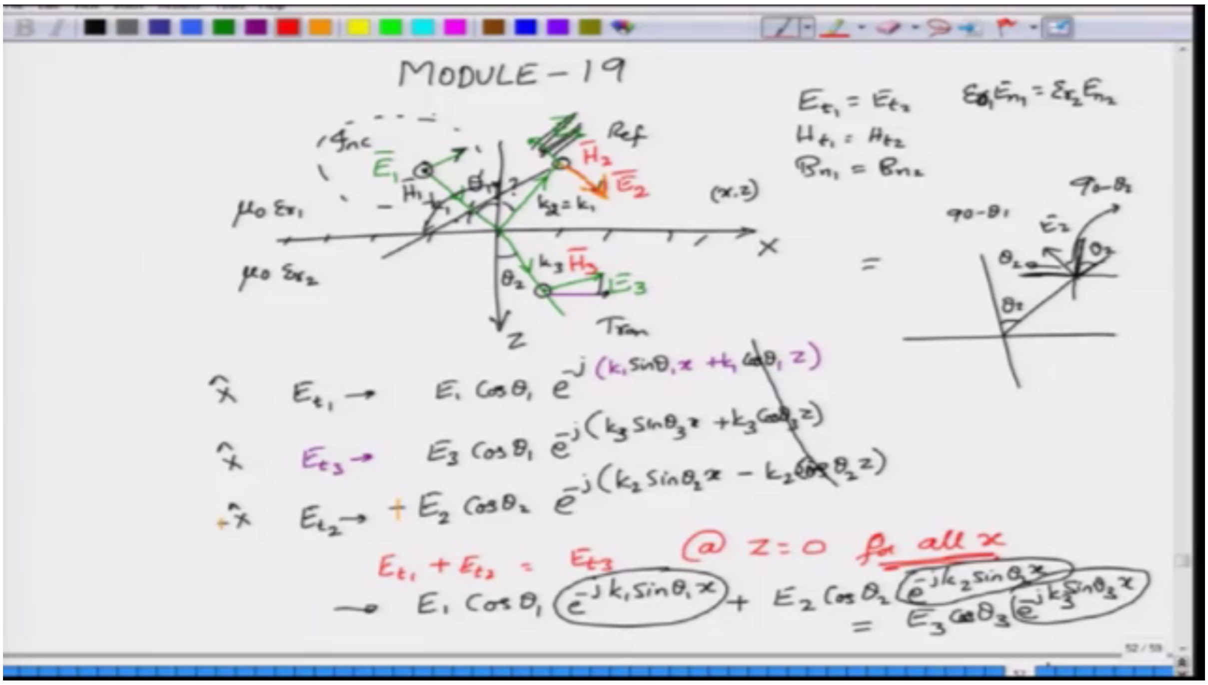

Well, you go back to this picture here. You assume that this is H_1 magnetic field, and this would be say H_2 magnetic field, and then you will have H_3 magnetic field. In all these cases, the magnetic fields are in the perpendicular direction except that for the reflected field, we will make the magnetic field go in the opposite direction so that E x H would be propagating along -z direction.

E x H, if you take H to be upwards in this manner, E x H would be propagating in this direction. E x H in the second medium would also be propagating in the given k_3 direction. If you reverse the direction of H_2 to downwards, E x H would actually propagate in the k_2 direction. Okay.

So with that, I can write down the expression for H as since H is transverse or rather tangential everywhere, it is actually along the plane of interface, I will have $H_1 - H_2 = H_3$. Okay. So I will have this particular equation, but H_1 is basically E_1 by η_1. This would be E_2 by η_2. This would be E_3 by η_3.

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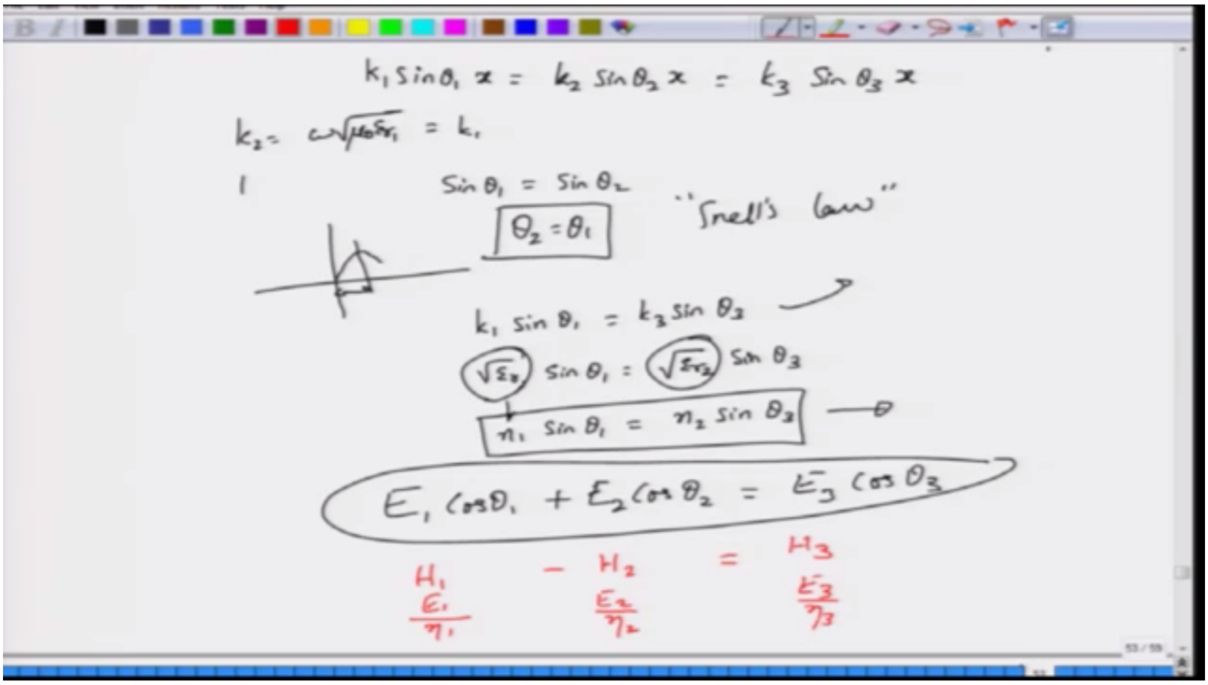

And I need to, you know, group the terms together, solve again for E_3/E_1, which we will call as the reflection coefficient. Okay, and then you will have E_3/E_1 is called as the transmission coefficient because this tells you the ratio of the electric field that has been transmitted to the electric field that has been incident. E_2/E_1 we will call as reflection coefficient, and we will get the expressions for these two. Okay.

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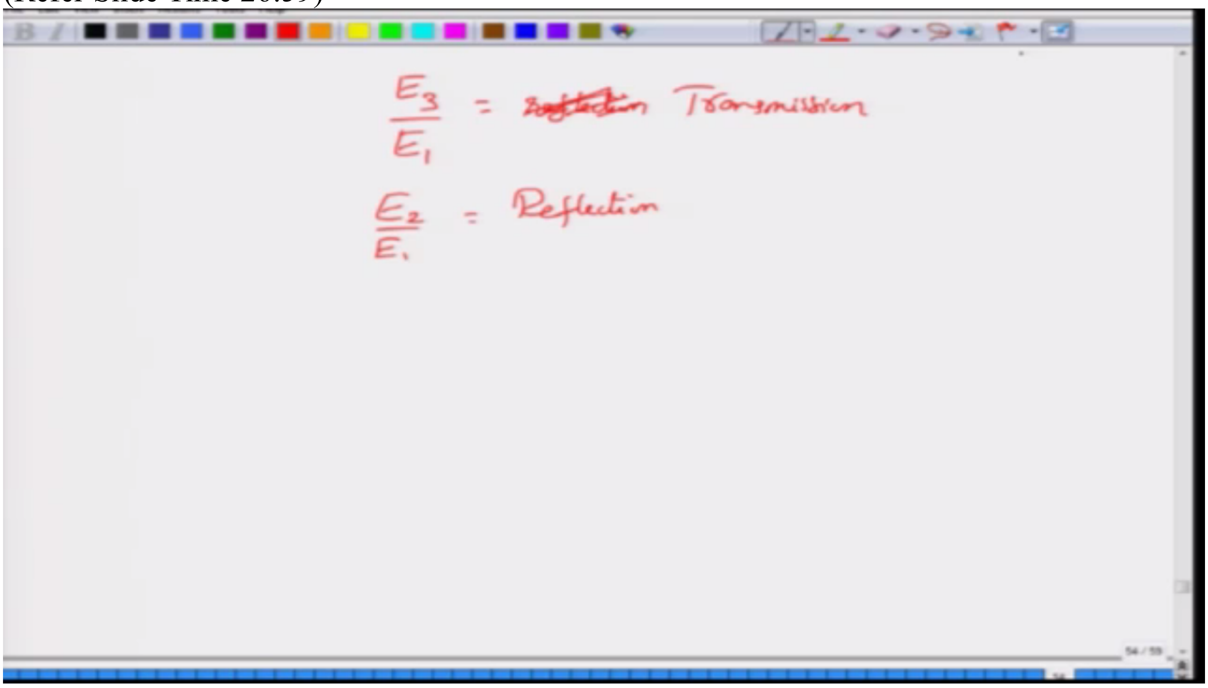

Since I'm running out of time, I will stop here, and we will, you known, start from these expressions, derive the transmission and electric, transmission coefficient and reflection coefficient and also tell you about that peculiar scenario where this Snell's law actually fails. Okay. Thank you very much.