# Fiber-Optic Communication Systems and Techniques


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**Lecture - 08**
**Total internal reflection**

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module, we will continue the discussion of varying the angle of incidence and then, looking at what happens to the reflection from transverse magnetic and transverse electric polarized wave.

So, we have already seen that transverse magnetic case actually we have at a particular angle called as Brewster angle. The reflection goes to 0 which means the wave does not reflect back anything, but transmits completely into the second medium, ok.

So, this angle Brewster angle theta B.

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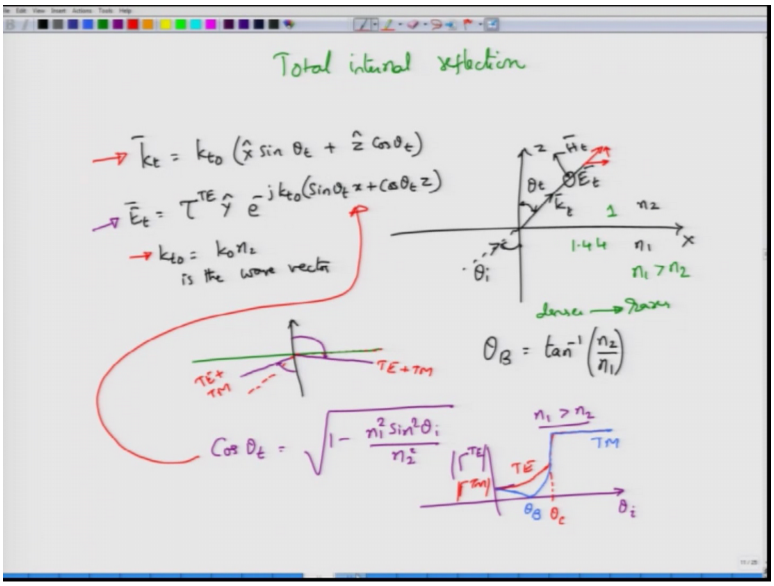


I did not give the expression for that and I did not want to derive it, but the expression for that is given by tan inverse of n 2 by n 1. So, for the previous case of n 1 equals 1, n 2 equals 1.44. This angle works roughly around 34 degrees. You can actually use a calculator and actually calculate that. Now, what we want to do is to reverse the situation, so earlier what we had considered was n 1 is 1 and n 2 is 1.44. So, what we had was say this is n 1 and this is n 2. So, this is the medium. this time I am going to consider not the transverse magnetic case that we have been considering, but transverse electric case. The reason I am considering transverse electric case is because in the transverse electric case, electric field is oriented along the y direction for incident reflected and transmitted waves. That way I do not have to decompose the vector in electric field vector.

In my expressions, of course with the little bit of more effort from my side, I could have considered the transverse magnetic polarization for the discussion of total internal reflection because there would just be one more component of the electric field that would be present, but I am not going to do that one. I want to keep my life little bit simple. So, I will choose transverse electric polarization ideas of course are easily transferable to transverse magnetic polarizations.

So, this is a situation that we are now quite familiar with since the last 2-3 modules. We have a wave incident at an angle theta i and the wave which is now transmitted at an angle theta t, I am not interested in the reflected wave at this point. I am only interested in the transmitted wave, but now with a very import consideration that n 1 happens to be greater than n 2. For example, this could be fused silica 1.44 refractive index and this could be air.

So, you now have a situation where you are moving from what is called as the denser medium to rarer medium. So, remember denser and rarer only referred to the refractive index values. The larger the refractive index, we call this as the denser medium. The smaller one we call as the rarer medium and what I am interested as I told you I am interested in the transmitted wave which of course makes an angle theta t. What is the bounds on theta t, I mean as I increase theta i on what values of theta t can or what values of theta t can it take? I have theta t equal to 0. So, in that case theta i is also 0, in that case theta t will be equal to 0 and this k vector of the transmitted wave would be lined up nicely along the z axis.

On the other hand, I would expect as theta i starts to increase that this angle theta t changes and at most it should change by 90 degrees, right. These are intuition correct in this case where we are moving from denser to rarer, of course our intuition is wrong because there exists what is called as a critical angle, ok.

So, once your light is incident at an angle greater at an angle critical angle as we have what we would call it, the ray would actually be transmitted, ray would actually start propagating along the interface and next time when you increase the angle of incidence beyond this critical angle, then we have observed from our experience that there will be a reversal of the wave. The transmitted wave actually gets reflected or you know goes back into the first medium. It would not go to the second medium at all.

So, purely from theta t value this has now increased beyond 90 degrees, right. So, up to this is 90 degrees and once this angle of incidence is increased beyond the critical angle mathematically at least theta t has increased beyond 90 degrees. When this happens what would be the expression for I mean whether this can happen with all the consideration that we have and what is the condition that needs to be satisfied for this one is what we want to see now, ok.

So, let us focus our attention on the transmitted electric field expression. There I have since this is a transverse electric case; I have used the transmission coefficient for the transverse electric case. So, the expression for the electric field vector which would be oriented along the y axis will be given by tau TE which is the transmission coefficient times y hat and I am assuming that the incident amplitude, the amplitude of the incident wave is equal to 1. Otherwise I should write down this incident amplitude as well which I do not want to write it, ok.

So, with normalized values what we have is the transmission coefficient tau TE which of course will be in magnitude less than 1 or will it be let us see that. So, this fellow will be times e power minus j k t 0 sin theta t x plus cos theta t times z, and we also know that cos theta t can be written as 1 minus n 1 square sin square theta i divided by n 2 square under root. Now, observe that we have a situation where n 1 is actually greater than n 2, correct.

Our incident medium happens to be optically denser compared to the rarer medium, and if you plot theta i as a function as this one and then, if you look at the reflection coefficient for the transverse electric case, the magnitude of a transverse of the reflection coefficient for the transverse electric polarized wave, then at 0 of course it starts off with some value, and then, what you would see is very different from what you would expect.

So, what you actually see is that while it continues to grow in magnitude at some point, it suddenly becomes 1 and stays 1 thereafter.

So, this is what is called as the critical angle and if you were to do the same thing for the transverse magnetic case, you would see that initially it would start at the same value as TE in magnitude. Of course, they are the same, but then goes through 0, right which is where you would have the Brewster angle value and then, suddenly shoots up to a magnitude of 1 and continues to remain there, and it will do so at exactly the same angle as TE case. This is interesting both TM and TE cases actually exhibit the same critical angle. The value of the critical angle is independent of what polarization of light that you have transmitted.

So, if you had initially both TE and TM right, so if you had both TE and TM the moment you have the critical angle, the output would also be TE plus TM. So, no TE or TM actually escapes and the magnitude of the reflection coefficient will be completely in this particular you know it would actually be equal to 1 ok, but that does not mean the transmission coefficient will also be equal to 0 that is an interesting part.

Normally one would expect that if the wave is actually you know completely reflected and we say that there is nothing in the second medium, this is what our traditional theory of total internal reflection would say, but it is not true. What you would actually have is a very interesting phenomena which we are going to talk about it now, ok. So, this I hope you can you know write a short matlab script or you can use excel and then, you know show that this magnitude of TE or magnitude of TM, both cases would actually exhibit this type of a behavior that is there is a critical angle beyond which the magnitude of the reflection coefficient will remain equal to 1 ok.

But what happens to the expression for the transmitted field. Well, look at the transmitted field expression, right. I have this e to the power minus j. You know k t dot r and I know that k t, the transmitted wave vector is given by k t 0 x hat sin theta t plus z hat cos theta t. That comes simply by decomposing this k t vector, right.

So, you decomposers k t vector into horizontal x component which would be sign of theta t and the vertical component along z axis, which would be cosine of theta t along z axis. So, this expression I hope you know and of course, this expression for k t 0 which is the magnitude of the wave vector in the second medium will of course be equal to k 0 times n 2, ok.

Now, go back to this expression of cos theta t and substitute that expression into the electric field expression, ok. I am going to do that one in the next slide.

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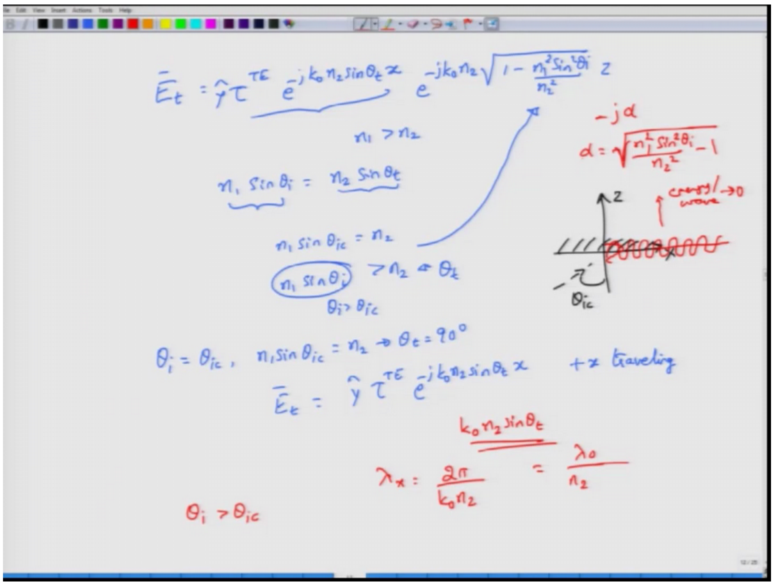


What I will be the transmitted electric field being directed along the y axis tau TE y hat e power minus j k 0 n 2 sin theta t x and I am going to just split the exponential function, ok. So, I will have e power minus j k 0 n 2 square root of 1 minus n 1 square sin square theta i divided by n 2 square times z.

So, please remember that we have a situation that n 1 is actually greater than n 2 and what is our Snells law n 1 sin theta i is equal to n 2 sin theta t. Now, look at this theta i is a component which is increasing you know going from 0 to 90 degrees. So, when theta is equal to 0, of course n this entire left hand side will be 0, right hand side will also be 0 and when theta i starts to increase from 0 to 90 degrees at some point, right the product n 1 and sin theta i can actually be greater than the product n 2 you know sin theta t, right.

Why would that happen? Because n 2 is smaller and maximum value of sin theta t could be equal to 1 and therefore, the maximum value that the right hand side can take in this expression will be equal to n 2 whereas, on the left hand side the maximum value that it can take is n 1, but you may not even need to go all the way up to theta equal to 90 degrees to reach the maximum, ok. This is in some sense you know if you go from 0 to 90 degrees and plot this n 1 sin theta i, so if you plot this n 1 sin theta i, it would look something like this reaching a maximum of n 1 as theta is increased and theta t would also be changing to keep phase with this expression, right. So, because these two expressions have to be equal for all values of theta and theta t as per our expectation, so depending on theta t theta t increases faster because it is 1 which actually has a smaller value of n 2 to compensate for the larger value of n 1 here and at some point it just cannot.

So, I am not able to write this one. So, let us say I am trying my best here at some point actually it goes slightly like this. So, this is what would happen for n 1 and this is what would happen for n 2. So, the angle theta it must change rapidly, but at some point it hits n 2 or other it actually hits n 2 at this point, but then it would not be sufficient for this one to be equal to the left hand side.

So, perhaps this figure did not convey what I wanted to try or what I wanted to write down, but you get the idea, right. See theta i is changing, and theta t is changing at a much faster rate because or much faster value compared to theta i because the product of these two should be equal to the product on the left hand side, but at some critical angle, it just so happens that n 1 sin theta i c would be equal to n 2 and if your angle of incidence becomes greater than you know critical angle, then this product would become greater than n 2 and you just cannot have you know a real value for theta t.

So, no real value of theta t can actually satisfy this particular requirement and in that case what happens is that total internal reflection takes place. Theta t mathematically becomes greater than 90 degrees, and then, you know the wave actually reflects back into the first medium. This is what the total internal reflection phenomenon is, but what happens to the electric field amplitude.

Well if you go to this one, now at theta i equals theta c n this numerator n 1 sin theta i is equal to the denominator n 2 and therefore, this one will be 1 and the exponential term with respect to z will be equal to 1. What now are left is an expression you know which is just dependent on x, right. So, let us do this at theta i equals theta i c n 1 sin theta i c will be equal to n 2 because theta t would have you know become 90 degrees and it cannot really go beyond this 90 degrees.

So, in that case this expression for the z will be the multiplier for z will be 0 and this one will be equal to 1. The exponential will be equal to 1 and the electric field would be y hat tau TE, ok. We should of course also look at what happens to tau TE at theta i or theta t equal to 90 degrees and theta i equal to this one, ok.

In any case, I will not do that one. I am interested more in this 2nd part which I will come to it. So, this will be e power minus j k 0 n 2 sin theta t times x. What kind of a expression for this electric field is this? This is an expression for electric field which is now travelling along the plus x direction. Now, what is plus x direction? Well we had this as the x axis, right. This was z axis, this was the interface.

So, what we have is that when the incident angle happens to be exactly equal to the critical angle, the wave that we now have is actually a x propagating wave. So, this is a wave which is propagating along the x axis very interesting, right. So, this wave is propagating along the x axis. The corresponding k vector along the x direction is given by k 0 n 2 sin theta t, and the lambda along the x axis for the wave which is propagating.

Now, it will be given by 2 pi by k 0 n 2 sin theta t 2 pi by k 0 is the free space wave length lambda 0. So, this is lambda 0 divided by n 2 sin theta t, ok. Sin theta t will be equal to 1 in this case and therefore, this is what you would actually lambda 0 by n 2. So, it is very interesting that the wave is actually propagating along the interface at theta i equal to theta i c.

What happens when theta i is greater than theta i c, then this expression n 1 sin square theta i becomes greater than n 2, right and therefore, this term becomes greater than 1 and this square root of a number. So, the number becomes negative, the radical inside the square root becomes negative and overall this will become complex, and if this is complex, then you can rewrite this as plus or minus j alpha where alpha is given by square root of n 1 square sin square theta i divided by n 2 square minus 1, ok.

So, should I choose plus j alpha or minus j alpha? Now physically what I know is that if I move far away into the interface that is far away from the interface, I should not expect any energy nor I should not expect any waves, right. So, these waves should actually go to 0. So, if I had e power minus j sum number k 0 n 2 and if I choose plus j alpha, right then j and minus j would actually become positive and this fellow would actually grow along with z. Therefore, the correct sign that we need to choose for this alpha is that of a minus sign. So,I have minus j alpha.

So, I can re-write I mean re-write this quantity once theta i is greater than theta i c into this form and then, alpha will be equal to this expression.

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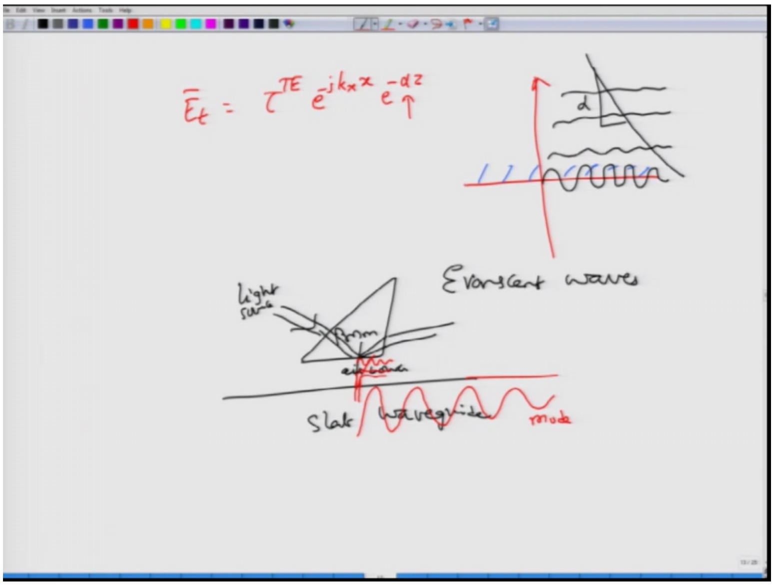


So, the electric field would become tau TE e power minus j k x, where k x is the wave vector k 0 n 2 sin theta t. Of course, sin theta t will be equal to this calculated k e power minus k times e to the power minus alpha z and it is this alpha which tells you how quickly the way would decay as you move away from the interface. So, this is the interface that we have, ok.

Then, as the wave which was propagating along the interface for the three critical angle, once you increase the angle of incidence beyond critical angle, the wave would still be a travelling wave along the direction of the interface, but it would actually start to decay along the direction perpendicular to the interface. So, maybe as you go to the interface, this would essentially decay out and it would decay out in a manner which is exponential with a decay parameter of alpha, right. So, as you move away and away, the wave is decaying further and further, but propagating along the interface. Such a wave is called as Evanescent wave, ok.

The great importance of Evanescent wave is that I could for example have a slab waveguide. This was you know earlier done this wave and then, I could have a prism or something and then, I have light incident on it. This light would be incident in such a way that this is air or some other lower refractive index material and this would be incident at an angle which is greater than the critical angle.

Therefore, the wave would be completely reflected and it would be available back to a source, ok. So, this is a light source in that we are sending in and this is a slab waveguide which is one of the important of electronic device that is used to guide light and because this is angle greater than the critical angle. There will be a decay in wave.

So, there will be a propagating, but there will be a decaying wave along the direction a perpendicular to the interface, and some of that wave if you can adjust the gap between the prism and the slab waveguide, some of that wave can actually just be sufficient to generate or excite a mode inside this slab waveguide, which would then be picked up and the corresponding mode would propagate, ok.

So, it is just used to excite a mode inside a slab wave guide and this is form of coupling is known as a prism coupling which was very popular waveguides, ok. Now, it is quite popular to excite light into the waveguides, ok. So, this waveguide that is being excited is actually in the form of this evanescent wave. So, the evanescent wave although we can show that it does not really carry any power in the direction perpendicular to the interface, it still can excite waves because it does not really go of to 0.

So, the wave will not go to 0. What does go to zero is the energy, which is perpendicular to the direction of the propagation, and as the angle of incidence is greater than critical angle beyond you know very large value, then the rate of decay would also be quite large. The value of alpha would also be quite large, ok. So, this is what goes in you know behind the scenes of new total internal reflection phenomenon. So, it is not just sufficient for us to say that.

When angle of incidence is greater than the critical angle, the entire light gets reflected and nothing is transmitted on to the other side. In fact, very important devices are based on the fact that there is an evanescent field on to the other medium. Of course, that evanescent field would not be propagating perpendicular to the interface. They would be propagating parallel to the interface along the interface and they would decay along perpendicularly to the interface, but they are useful in cases. In many cases, one of them was to excite the modes this is not the complete story.

There is one additional story that, I would like to talk about total internal reflection.

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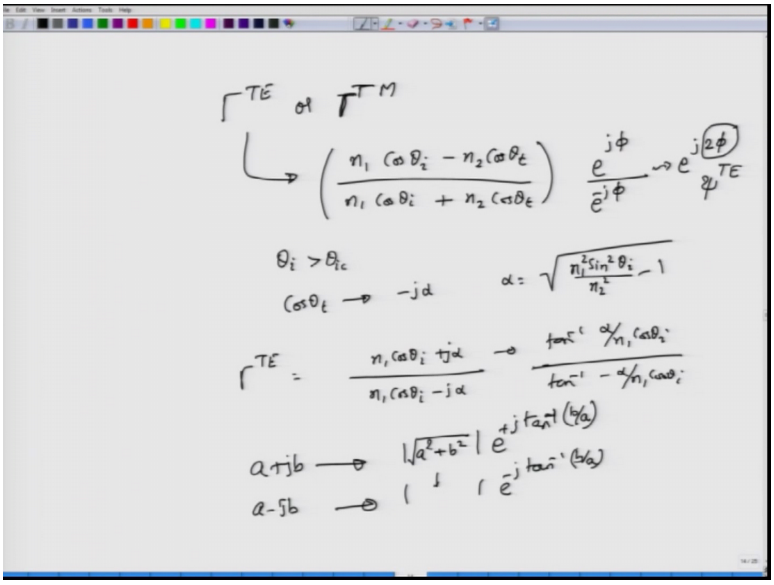

Well, if we go back to the expressions of say gamma TE or gamma t sorry gamma TM right, so let us go to the expression for gamma TE which is given by n 1 cos theta i minus n 2 cos theta t divided by n 1 cos theta i plus n 2 cos theta t, ok. We consider the situation where the angle of incidence is greater than the critical angle.

So, obviously in that case cos theta t would actually have become complex cos theta t is actually what we wrote as minus j alpha where alpha is equal to square root of n 1 square sin square theta i divided by n 2 square. So, minus 1 of course, this is what we wrote. This theta i times n 1 remains real quantity. So, what we have for gamma TE will be some real quantity plus j alpha because of minus and minus cancel becoming plus there.

Then, you have n 1 cos theta i minus j alpha. If I have a plus j b, the corresponding polar form will be the magnitude quantity which is say square root of a square plus b square times the face which is, sorry plus j tan inverse of b by a ok. I am assuming certain things about b and a that they will be positive and therefore, you do not really need to worry about which quadrant or addend certain pi or something like that ok, but anyway you can always represent a complex number in terms of its magnitude and an angle so-called polar form and because gamma TE happens to be of the form a plus j b divided by its conjugate a minus j b, the magnitude of the numerator will be equal to the magnitude of the denominator. The face of the numerator will be minus or negative or shifted by 180 degrees for the face of the denominator because one of them is conjugate of the other, right.

So, I can write a minus j b as the same magnitude, but with a face which is minus j tan inverse of b by a right. Now I go back and write the numerator and denominator in this fashion and because the magnitude is the same that would cancel out with respect to each other and what I have is e power j pi divided by e power minus j pi. Together this becomes e power j 2 pi, and you can call this e power j pi as some sigh with TE and this particular sigh with a sigh e is what is called as the phase shift upon reflection.

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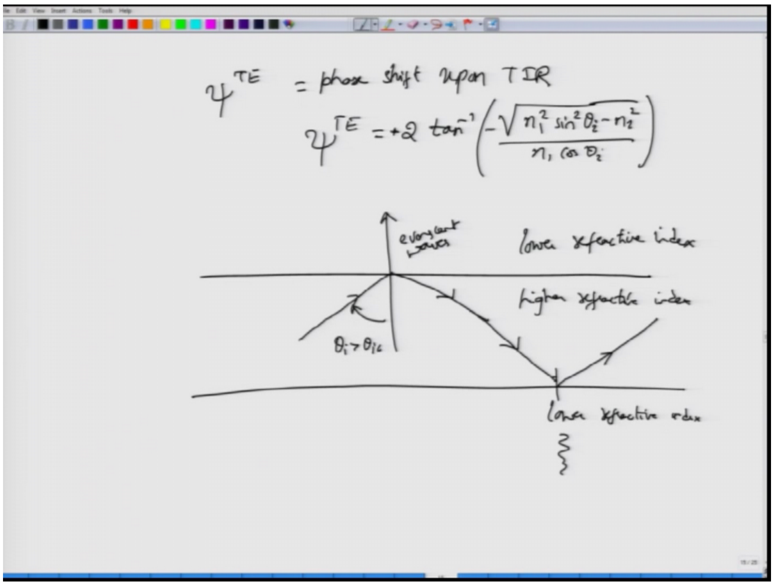


So, when total internal reflection happens it not only you know pushes a I mean now only reflects all of the light back into the first medium, ok. There is evanescent wave in the other medium of course, but the reflected light would have a certain phase shift or would undergo a certain phase shift which of course is dependent on what is this value theta i and the value of alpha. So, this expression for phase shift up on total internal reflection see this is different from phase shift upon reflection of a typical TE or TM case and for TE case, you can show that, this would be equal to 2 times tan inverse of square root of n 1 or you know what we had was n 1 square sin square theta i minus n 2 square divided by n 1 cos theta i or with a minus sign of course since minus of tan is the same. So, this can be pulled out and then, you can put a minus sign back on to this one. So, this is the expression. Of course this expression simply comes from this case, right. So, you would have n 1 cos theta i plus j alpha. So, you can express this one.

So, it would be tan inverse of alpha divided by n 1 cos theta i. There will be denominator as well which would be tan inverse of minus alpha divided by n 1 cos theta i and then, you can put both of them together and then, you would have the sum of these two. So, tan inverse of a plus tan inverse of minus a and then, you can use all those formulas with them. Sorry there is a minus sign here not on the outside and you can use those formulas and then, find out that the total phase shift for the transverse electric case will be this expression. This is a very important point which I will finally mention and leave it out here.

So, what we have done is that let us suppose that we had a lower refractive index medium here and then, we have a higher refractive index medium, and what we have done is a very interesting case. So, this is a normal to the interface as such by carefully taking the angle of incidence to be greater than the critical angle. What I have done is to essentially reflect all of my light back into the first medium right, so of course there will be evanescent wave, but it by evanescent waves are not carrying any energy, right.

So, there is no power transfer that is happening because of the evanescent waves, but what I have achieved is a very remarkable thing. When light is incident at an angle, then entire light gets reflected back, means that if I can somehow engineer another layer of interface here with a lower refractive index and cause this ray or this wave also to be at an angle greater than critical angle, then this light would be reflected further back into this wave.

There will of course be an evanescent wave, which has no meaning, but this would be reflected back in essence. What I have done is to confine this slide into this slab of higher refractive index and I have done it without any use of a mirror, ok. So, we will expand upon more on these type of devices in the next module.

Until then thank you very much.