Fiber - Optic Communication Systems and Techniques
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Lecture - 07
Obliquely incident waves-II
(Reflection and transmission coefficients, Brewster angle)

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques course. In the previous module we discussed what happens when a ray of light or an electromagnetic wave which is essentially light is incident from a medium of certain refractive index or certain permittivity, it is a perfect dielectric medium that we considered and it is incident is while it is traveling it is incident on another medium ok.

So, it comes across another medium whose properties are different so it is directly constant is different and then we figured out that we can decompose the incident light wave into 2 polarizations, one we called as transverse magnetic polarization. In the transverse magnetic polarization the wave whatever the direction it is propagating it could actually make a certain angle with respect to the normal to the interface and the electric field would be lying in the same plane as that of the incident wave.

So, in other words if we consider z equal to 0 as the plane of interface, so above which we have a dielectric medium which is the transmitted medium or the second medium and below this interface is the first medium in which the wave is traveling, so this is what the situation that we considered. So, we had a wave approach the second medium or the interface at an angle theta and which how did we measure this angle theta, this angle theta was with respect to the normal to the interface.

So, this is a normal to the interface right and to with respect to that normal the incident light was actually at a at an angle of some theta i right and the electric field which was perpendicular to the direction of propagation it remained in the x and z plane that is how we define the coordinate. So, we had this as the x axis and the normal direction is what we considered as the z axis.

So, we had this kind of situation and the magnetic field of the incident wave vector was perpendicular to both these vectors. So, this is the propagation vector this is the electric field corresponding to this propagation vector and the magnetic field of course would be perpendicular to both these vectors, in fact it would be perpendicular to the plane that is formed by this thumb and this first finger right.

So, it would be something like this, so this would be the magnetic field perpendicular to both. So, when such a wave which we called as a transverse magnetic wave hit the dielectric medium the second dielectric medium and after it was propagating in the first medium, then what happened was there was a partial transmission. So the wave that was incident partially got reflected and in the reflection case the wave was actually propagating in the other direction.

So, it was propagating away from the interface and back into the first medium right and this first finger now or the 4 finger whatever the finger that is there, this indicates the direction of propagation and correspondingly this was the electric field that we drew and magnetic field continues to be in the same direction of the incident wave even for the reflected one, meaning that it was perpendicular to both electric field vector which is the thumb as well as the direction of propagation which is now away from the interface and back in the first medium.

So, this was the magnetic field this was the electric field and this was the direction in which the wave was traveling or reflected back into the first medium right and then if you go to the second medium in the second medium the wave would continue to propagate of course partially only, because part of the wave has actually reflected. So, part of the wave that has not been reflected is propagating into the second medium and notice that the direction of electric field vector, the direction of the propagation vector and the magnetic field vector are oriented almost as it would be oriented for the incident wave, except that the propagation vector makes an angle of theta 2.

So, while the initial or theta t as we called it while the incident wave makes an angle of theta i, the reflected wave makes an angle of theta r the transmitted wave would make an angle of theta t and these are not arbitrary angle values.

So, these angle values of theta i once you fixed theta i and once the medium properties are fixed, then theta r is fixed because Snell's first law of reflection tells us that or Snell's law of reflection tells us that whatever may be the incident angle that would be exactly equal to the reflected angle. So, the angle which the reflected wave makes would be exactly equal to the incident wave again with respect to the normal ok. In some literature and in some textbooks you would also find these relationships measured with respect to the interface that is instead of considering the normal to the interface they consider the interface plane itself as the reference and then refer all angles to that ok.

In this course whenever we talk about incident angle or reflected angle or the transmitted angle we are always talking with respect to the normal to the interface. So, theta r is theta i as per Snell’s law, but we also figured out that Snell’s law is really not a law in the sense that it is just the necessary condition for the incident wave, the reflected wave and the transmitted wave to satisfy the boundary condition. So, it came out as a phase relationship or a phase matching condition and it simply was the result of that phase matching condition right and remember what boundary conditions did we apply we applied the continuity of the tangential electric field and the tangential magnetic field across the 2 perfect dielectrics that we considered.

So, Snell’s law of reflection which tells you that the law in the wave which is reflected would make an angle theta r and that angle theta r is equal to theta i, is simply the result of boundary condition and matching of the phase values right. So, because I mean this is what we saw yesterday or in the previous module. Now what about the transmitted angle theta t, theta t the transmitted angle in our high school kind of a you know exposure to light we would have used Snell’s second law or the Snell’s law of refraction to calculate what would be the angle theta t and it turns out again that this Snell’s law of refraction which is usually written as n 1 sin theta 1 equals n 2 sin theta 2 is simply a consequence of boundary condition again.

So, whatever the information that Snell’s law is telling us is already contained in the boundary condition and the phase matching and that we went one step ahead where we actually calculated the reflection coefficient meaning the ratio of the reflected electric field amplitude. Please note this, this is the ratio of electric field or reflected electric field amplitude to the incident electric field amplitude and this ratio that we calculated we called as the reflection coefficient and because this is for the transverse magnetic case right.

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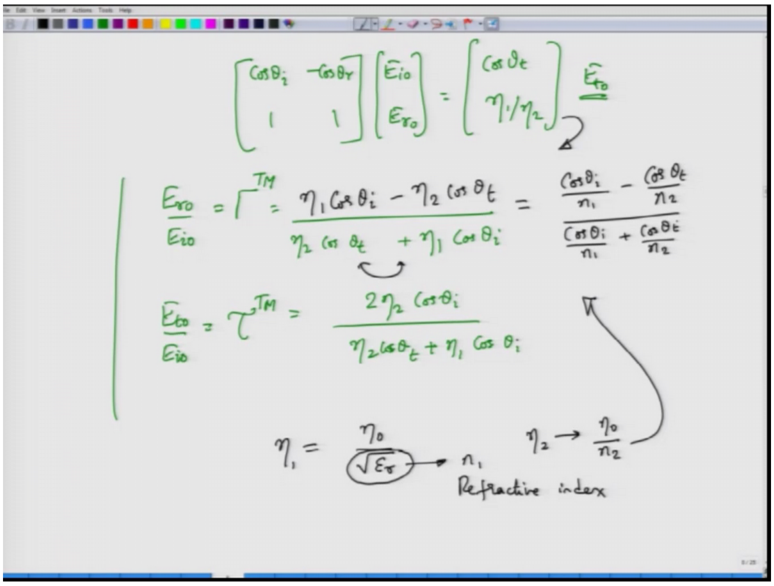

We put a superscript called TM onto the reflection coefficient which we denoted by gamma and we actually did it by a nice matrix kind of a method. But what I did not do correctly in the previous module is that the equation that I wrote was slightly wrong, the equation for the transmission coefficient was correct. Which is of course for the TM case is given by 2 eta 2 cos theta i divided by eta 2 cos theta t plus eta 1 cos theta i right. Where this eta is the wave impedance I will come to that wave impedance in a minute.

But the numerator for the reflection coefficient was carrying a wrong sign. So, the actual sign for this one would be that this is positive and this fellow will be negative. So, I should have put a minus sign to the overall expression that we wrote yesterday to make this correct. So, since I did not I mean I missed out that this one I am going to write that now.

So, what we actually have please note that this is the correction, of course this was left as an exercise from the matrix to obtain this equation so and if you have done it correctly then you should actually obtain this equation so eta 1 cos theta i minus eta 2 cos theta t is the expression for transverse magnetic polarization.

One could of course, write down a similar equation for the transverse electric case one could follow the same steps ok, except in this transverse electric case it will be the electric fields which would be along the y axis and the magnetic field will be lying in the x and z plane with again z equal to 0 plane being considered as the plane of interface.

Now we will not do that here but the final expression for gamma TE and tau TE we will write it, but I want to write all these equations not in the form of wave impedances.

Wave impedance is something that is quite popular when you studied transmission lines for example, where you talk about the characteristic impedance of a transmission line of given permittivity and for another medium of a different permittivity there it is right. But in optics we do not normally talk about wave impedances but we talk about refractive index because, that is a quantity that is more closely associated with optics and it is also something that can be reasonably measured ok, so easily measured.

So, I want to convert these formulae for the reflection and transmission coefficient from wave impedances to refractive index and how do I do that. Well we go back to the expression for eta I will not tell you how this expression has come out, but if you go to any electromagnetic theory textbook and especially look at the chapter on plane waves or on the transmission line coefficient you would realize that, this wave impedance eta is given by free space wave impedance eta 0 divided by square root of epsilon r. Where epsilon r is the relative permittivity of the medium of course, I am assuming dielectric medium these are not magnetic medium at all.

So, the wave impedance eta is written as eta naught divided by square root epsilon r and in the parlance of optics square root epsilon r is what we call as the refractive index and we choose the simple small n to denote this. Now in printing it is quite easy for me to distinguish eta and n, but in writing it is sometimes difficult for me to distinguish eta and n or at least even if I do it may be confusing to you.

But rest assured from now onwards once we convert these formulas in terms of the refractive indices, we will not go back to impedance. So, you do not hopefully get confusion between eta and n and whenever you get confusion between eta and then check your formulas for eta is always inversely proportional to refractive index. So, the larger the medium refractive index the smaller will be the value of the wave impedance eta.

So, now since we have 2 media right, so we have one media which was the incident media for which eta must be equal to eta 1 and the second media for which eta must be equal to eta 2. Clearly the first medium has a refractive index of n 1 the second medium has a refractive index of n 2 and the first medium is traditionally what we take as the incident medium and the second medium is usually the transmitted medium. So, going back to this eta 1 of course will become n 1 and eta 2 will of course be equal to eta naught which is the free space wave impedance divided by n 2 ok. Now let us substitute those into these expressions, so I will do that a simplification right here after hopefully erasing this yeah I erase this box here, now I am going to replace this one.

So, this expression will be cos theta i divided by n 1 because, eta 1 is proportional to n 1 and because there is proportionality constant eta 0 which would appear both in the numerator and in the denominator that can be taken as a common factor and canceled out ok. So, I am not writing eta 0 everywhere if you wish to be little more mathematically correct and we want to show the intermediate step, then you can include this eta 0 for every term. But you know of course that when you include this one for every term that term can be dropped because it simply cancels out.

So, I have the numerator becoming in terms of the refractive index ok, so this is the refractive index right. I am stressing this refractive index again and again because, there is possibility that you know if you are not paying quite close attention to what these formulas are, you would mistake only take eta as n and n as eta and then those formulas would actually give you wrong results.

So, anyway hopefully that confusion should not be there, now we write this as cos theta i by n 1 minus cos theta t n 2 ok, I do the same thing in the denominator. So, I have cos theta t or maybe we will interchange the terms in the denominator as well. So, first we will write down this eta 1 cos theta i then add eta 2 cos theta t, well interchanging these 2 here won’t really make a difference to the numerator I mean denominator right. So, I have cos theta i divided by n 1 plus cos theta t divided by n 2. Now our formulas are all expressed in terms of refractive index which we normally simplify further ok.

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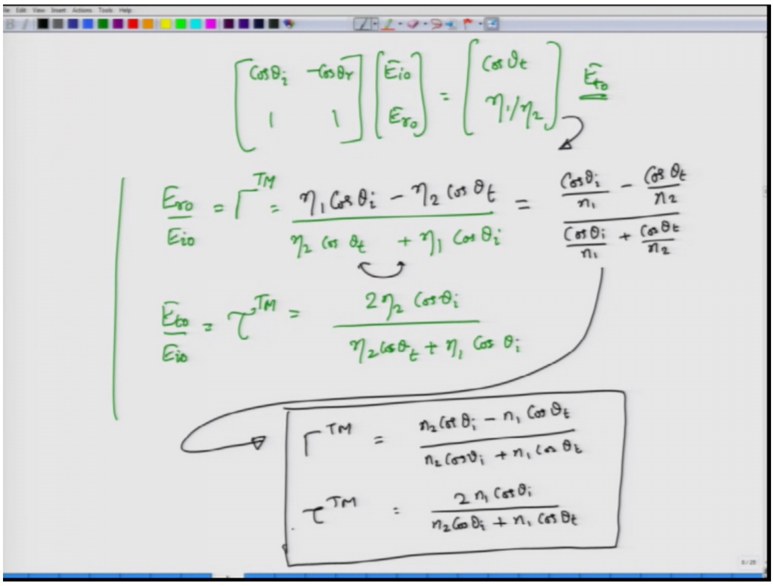

So, we simplify this expression further by actually cross multiplying with all these n 1 and n 2 and the modified expression which is quite commonly used in optics is given by n 2 cos theta i minus n 1 cos theta t where theta i is the incidence angle theta t is the transmitted angle divided by n 2 cos theta i plus n 1 cos theta t ok.

Similarly, you can show that the transmission coefficient can be written as 2 n 1 cos theta i divided by right. So, you can write this one as n 2 cos theta i plus n 1 cos theta t. So, we now have 2 new equations which essentially capture the same information as the earlier equations, but they all have now been written in terms of n 1 and n 2.

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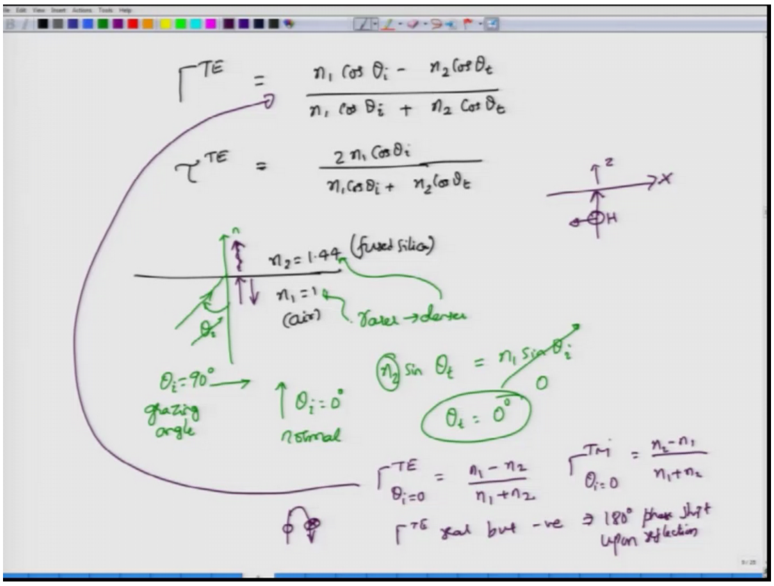

So, if you are curious about what the formulas for transverse electric would look like again in terms of n 1 and n 2, this would of course be the ratio of the reflected electric field to the incident electric field except that the electric field is now along the y axis.

So, the answer for this one will be n 1 or the formula for this one will be n1 cos theta i minus n 2 cos theta t divided by n1 cos theta i plus n 2 cos theta t ok. So, where again n1 n 2 n are the first and second media, the transmission coefficient and again please note that these are all ratios of amplitudes there are not powers ok. So, the transmission coefficient is given by 2 n1 cos theta i divided by n1 cos theta i plus n 2 cos theta t. So, the denominator would essentially be the same for both ok.

Now, let us look at certain cases ok, first let us assume that we are in a situation where n1 is 1 you know this could for example, be air and n 2 is equal to say 1.4 or 1.5 ok. I am taking 1.44 because this is very close to the fused silica refractive index. So, this could for example be fused silica which is essentially glass or used as a substrate in most of the integrated optical devices. So, I have a ray of light or I am sorry the wave propagating and making a certain angle of incidence, what I am going to do so this is the normal what I am going to do is that I am going to vary this angle theta i ok.

So, we vary this angle theta i from what values can be vary this angle theta I, well one incident angle that would be at one extreme would be when the light or the propagation vector or the k vector of the incident light is lined up along the normal direction itself, which means theta i will be equal to 0. The other one is normally called as the grazing angle where theta i will be equal to 90 degrees ok.

So, one case you have theta i equals 0 degrees in which case the wave is incident what we call as normally on to the second medium and the other case theta equal to 90 degree is when we call as grazing angle incidents ok. So, this is normal incidence and then grazing angle incidence, of course theta i being any greater than 90 degrees does not really make any sense because making theta i greater than 90 degrees mathematically is possible, but physically it means that you have interchanged what is incident and what is reflected medium or refracted medium.

So, the range of theta i is 0 to 90 degrees. So, now this is a case which we normally call as optically rarer to denser medium reflection or refraction ok. What is rarer to denser mean here rarer means the refractive index is smaller compared to the refractive index of the second medium. So, amongst these 2 media n1 which is equal to 1 is lesser than 1.44 clearly air is a rarer medium while this is a denser medium ok. So, now with theta i equal to 0 degrees let me ask you what would be the reflection coefficient values right when theta i is equal to 0 degrees right. So, before we do that we first need to find out what is theta t and to find out theta t we go back to Snell's equation which of course we have derived it now.

So, n 2 sin theta t must be equal to n1 sin theta i since theta i is 0 sin of 0 is 0, so the right hand side of this entire thing will be equal to 0. So, n 2 is nonzero clearly the only solution possible for us seems to be possible is theta t equals 0 ok. So, this is the perfectly valid condition, so theta t is 0 which means that any transmitted wave would also be oriented along the direction of the normal. So, the incident field is incident at theta equal to 0 or theta i equal to 0, the partial transmitted wave would also be at theta equal to 0 and what about the reflection well reflection is always equal to the angle of incidence. So, therefore reflected light would also be at the same angle that is it would be coming back perpendicularly from the interface.

So, all the 3 waves are now either lined up along or lined up in the opposite directions and they are all parallel to the normal to the interface. So, what happens is there a difference between gamma TE and gamma TM now, well now there is actually no difference between the 2 why it should there be no difference. First let us look at the case suppose I have my k vector incident k vector in this particular direction, remember this was my x axis and this was the z axis.

So, now electric field can be oriented in this plane right and the magnetic field can be oriented in this plane, alternatively I could orient the electric field here and the magnetic field in that direction. But in both cases what I have actually is a simple transverse electromagnetic wave which is incident right and these 2 solutions must behave similarly and this is true for the normal incidence and you can actually check that.

So, gamma TE will be equal to n1 minus n 2 divided by n1 plus n 2 from this expression. So, look at this expression theta i is 0 so cos theta i is 1 theta t is 0 cos theta t is 1. Of course, there has to be a small difference in the wave we have put electric and magnetic fields and that small difference comes because you know there is a 90 degree phase change between the 2 and there is an overall 180 degree phase change before you do this one and we will not go into the details. But the reflection coefficient for TM in the normal case, so let me write this one as theta i equals 0 theta i equals 0 case will be n 2 minus n1 divided by n1 plus n 2.

Now, given the values that we have chosen n1 is 1 and n 2 is 1.44 clearly gamma TE will be real but negative, which means that as the wave for the transverse electric goes in right. The electric field is along no y axis and the wave then when it comes back would actually undergo, so electric field would be directed in the opposite direction right.

So, it could be in this way and this electric field would experience a 180 degree phase shift, so indicating a 180 degree phase shift upon reflection whereas the case that we have considered here the transverse magnetic field would not experience any phase shift and the reflection coefficient will be real and positive ok.

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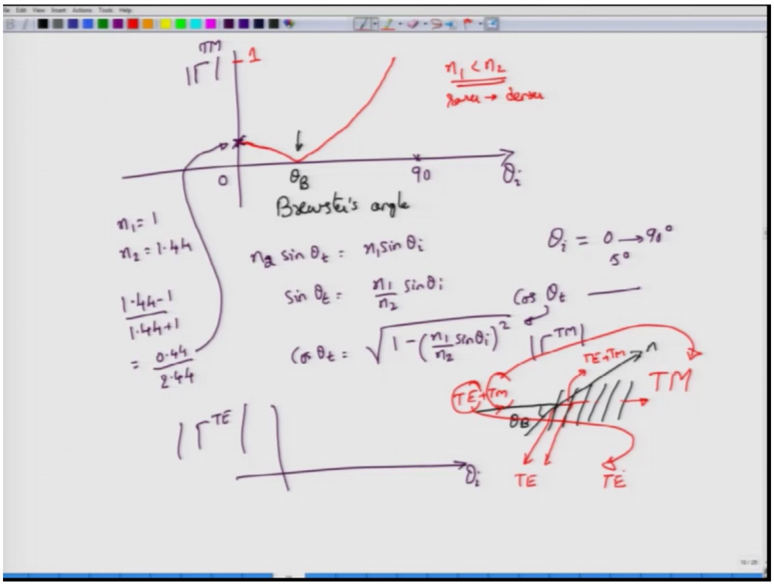

But if you were to look at only the magnitude of the reflection coefficient for TE or TM both would have a certain magnitude. So, I am only going to plot the magnitude of either the transverse electric or transverse magnetic and this is what I would actually have. So, both would have the same magnitude and then as theta i is increased you can use these equations and in these equations you can actually convert this gamma TE or gamma TM into a function only of theta i by expressing cos theta t in terms of n1 n 2 and sin theta.

So, we use Snell's law and write this as n 2 sin theta t equals n1 sin theta i, which means sin theta t is n1 by n 2 sin theta i and therefore cos theta t is equal to square root of 1 minus n1 by n 2 sin theta i whole square right. So, this is a simple trigonometric relationship between cosine and sin and this is what you would actually have ok.

Now let us actually look at the transverse magnetic case because, that is more interesting and I will leave the transverse electric case magnitude versus theta i as something of an exercise for you and you can of course either use excel sheets or you can write a very small MATLAB script ok. Where theta i varies from 0 to 90 degrees maybe in the steps of about 5 degrees you can take that 1 and then calculate theta t or essentially calculate cos theta t from this expression and then substitute these values into the equation for TM and with a magnitude ok.

You do this for the case of n1 equals 1 and for n 2 equals 1.44 that is incidence from rarer to denser medium and plot the result ok. As theta i increases from 0 of course it can go up to 90 degrees, what happens is the magnitude would start off with some value. So, in this case for example the magnitude would be 1.44 minus 1 by 1.44 plus 1 which is about 1.44 divided by 2.44 ok. So, whatever the numerical value that turns out to be you can put that numerical value here and luckily this number is positive, therefore you do not have to worry about this fellow being negative and for the case that we have shown you can even show that TM would never become negative anyway.

So, as you start increasing theta i you will observe that as theta i increases the angle actually decreases, I mean the reflection coefficient decreases. So, decreasing reflection coefficient means more and more transmission into the second medium right so this is very important. So if I have an interface and then I have my angle theta i so starting from theta i equal to 0 as I slowly start increasing the amount of light that is reflected would be less while the amount of light that is transmitted into the second medium would become larger and you would see something like this I mean of course, this is what you would see and then it would come down and then eventually touch 0 and very slowly increase off to 1 ok.

So, this maximum magnitude of gamma in this case can go up to 1 and this is not the surprising fact ok. So, this was the case where n1 was less than n 2 and you are going from what is called as rarer to the denser medium. So, this is not something that is surprising at 90 degrees that you would actually get 1 the equations would also nicely predict that. But what is interesting is this case right at some angle between 0 to 90 degrees which we call as the theta b, the reflection coefficient actually becomes equal to 0. So, there is completely no reflected light ok, when you have an incident light which is purely TM polarized the reflected light will not at all be present if you incident this transverse magnetic this TM polarized light exactly at an angle given by what is called as theta b.

So, the entire wave would be transmitted into the second medium, no wave would be reflected when it is TM incidence or TM polarized and incidence of angle is equal to theta b and this angle theta b is called as Brewster’s angle. After the person who discovered this particular phenomena and this is an extremely important device or extremely important principle that we used to make lot of optical devices. For example, I consider what we call as a piles of plate polarizer and I will transmit an incident light at an angle of say theta b with respect to the normal. Of course, these are slanted so this is the direction of the normal; if the figure is not very clear please bear with me. But the incident light was having a component both TE plus TM ok, so this was an unpolarized light having components TE and TM it is incident on these plates at an angle theta b.

What happens is because this is incident at an angle theta b, the wave that continues next right would be only TE I mean there will be some amount of TE. But there will be completely TM wave right and the there will be partial reflection for the TE wave and the next time also there will be partial reflection of the TE wave TM continues and so on at the output what you have is that the entire TM component of the incident wave has been transferred to this and very nearly all of the TE waves because it has undergone many reflection would be almost reflected. So, what you have actually managed is to create a polarized light in you know by just using a piles of plate and orienting your light which is unpolarized at an angle what we call as theta b Brewster angle.

So, having light incident at the Brewster angle has allowed you to separate transverse electric and transverse magnetic fields and this device was earlier used as a polarizing beam splitter. So, we will have more to say about this phenomena in the next module stay tuned.

Thank you very much.