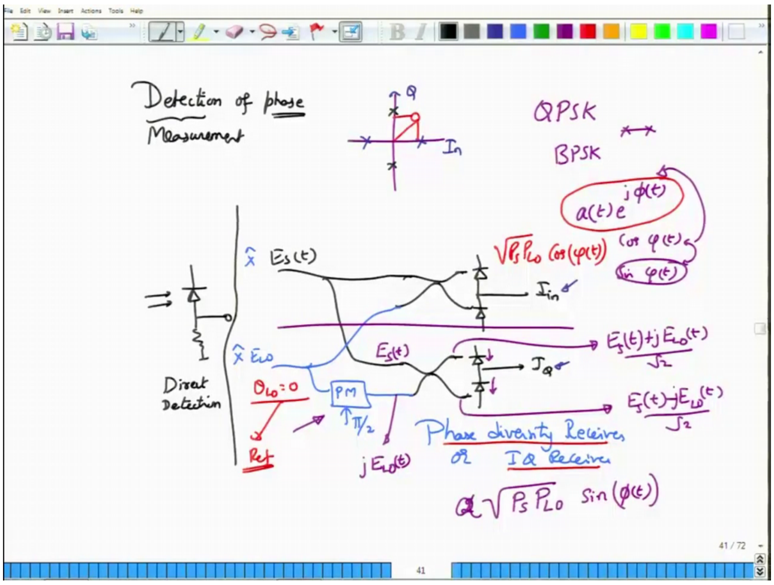
Fiber - Optic Communication Systems and Techniques
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 54
Coherent receiver for BPSK systems and BER calculation

Hello and welcome to NPTEL MOOC on Fiber - Optic Communication Systems and Techniques. In this module, we will continue the discussion of Coherent Receivers.

(Refer Slide Time: 00:25)



Just to recap, this is a direct detection receiver or simple intensity detector ok. You can detect intensity by putting up a photo detector and then, have a load resistor and look at the current that is produced by the photo detector. No phase information will be detected using the simple system.

On the other hand, this upper part is the, what we have already seen right. So, this is the coherent homodyne receiver because I am assuming the local oscillator frequency is exactly tune to the signal frequency that is incoming signal frequency and this arrangement gives me the in phase component. But I know that I usually do not have only the in phase component. My received signals will actually be a complex signal in this fashion.

So, I need to have both cos phi of t and sin phi of t to be available so that I can add these two essentially obtain the complex phase that I have transmitted ok. Of course, in the cases of BPSK that we have seen the sin phi of t did not enter into our considerations because we had consolations which was just lying on the in phase access. However, in other consolations or other modulations such as QPSK or MREPSK or quadrature amplitude modulation, we will have consolation points containing both the in phase and quadrature component.

So, in that case we want to be able to obtain the quadrature component as well. How do you do that? This arrangement is quite simple; you take the incoming signal E s. You know you take one part of that, let that part come to this other (Refer Time: 02:08) which is essentially the same as a top one. The difference between the top and the bottom ones with respect to this line here is that there is a extra phase modulator to which the local oscillator signal is used so that the signal that comes out at this point will be j E LO of t.

So, this j stands for a pi by 2 phase shift and using this phase modulator and biasing the phase modulator at pi by 2 gives me a phase or know phase shift of 90 degree which in the complex form, I capture it by writing this as j E LO of t. Please note here that this would still be E s of t and the 2 electric fields that you are going to get in the 2 branches would be E s of t plus j E LO of t and the electric field that you are going to get here will be the difference between you know the incoming signal E s and this j E LO of t. So, there will be j E LO of t.

So, you can see that you have on one. So, there will also be a 1 by root 2 here and 1 by root 2 and I will now leave it as a exercise for you to get the photo current here generated on this top photodiode and the photo current that will generated here and then, take the difference between the two; when you take the difference between the two, we will actually see that the output will be something like square root P S. So, there will be a 2 or may be there won’t be a 2 because you have a two mind whatever this half part is there. So, I will just simply write this as proportional.

So, proportional to P S P LO whose amplitudes I am again assuming to be constant and now I will have sin of phi of t right. For the case where I am going to perform BPSK, but I am going to perform BPSK on the phi 2 axis, this phi of t could be zero-phase or pi phase on this axis. So, I am actually going to get or rather pi by 2 and 3 pi by 2. So, I am going to get either sin pi by 2 which is positive or sin minus pi by 2 which is actually negative.

So, I can actually obtain these two again by looking at the sin of what I am actually receiving. Now when I combine that with this constellation right because on this branch the in phase branch, this is the quadrature branch, I can distinguish the in phase signal phase of the phase of the incoming signal on the in phase axis by looking at the sin of I in and I can distinguish the phase of the signals on the quadrature axis by looking at the sin of the current I Q and in this way, I can actually distinguish any phase that I would actually receive.

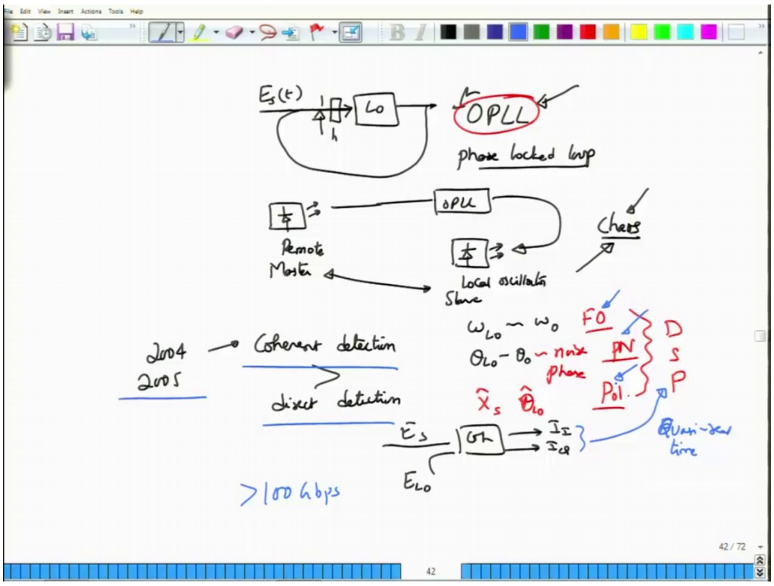
So, for example, if my received signal happens to be this particular constellation point ok, in this case the in phase component would be positive, the quadrature component would also be positive and I would actually look at the sins of I in and I Q to make that decision ok. So, what I want to tell you is that this arrangement can give us full information about the complex envelope of the transmitted signal ok, depending on what value of phi of t you have.

So, this is for the quadrature component, but for the in phase component this output would be photocurrent would be square root P S P LO cosine phi of t; where, phi of t is the phase of the input transmitted signal I am assuming that theta LO is equal to 0 or I can use these theta LO simply as a reference and this detector scheme which I have sketched, which involves two different couplers or in fact, more couplers, but there is a extra phase modulated to induce a 90 degree phase shifts is called as a Phase diversity Receiver.

This is diversity because there are two branches out there and these two branches are receiving two different signals in the form of phase two different phases. And therefore, this is called as Phase diversity Receiver or sometimes called as IQ Receiver ok. This is how you detect phase in optical domain.

So, if this entire slide can be written as detection of phase or I should actually use the proper word, instead of detection I should call this as measuring of the phase or measurement of the phase of the optical signal. Now, why is this considered to be quite an achievement because this is considered to be a quite an achievement over the last 15 years is simply because earlier if you had to recover the phase what you should have done is to let the incoming signal ok.

(Refer Slide Time: 06:41)



And then, have a VCO or local oscillator here. So, you of course, had to first putting the photo detector. You have the photo detector output driving the local oscillator or rather the local oscillator would be generating something, the phase detector photocurrent after filtering would have driven the local oscillator and the output of the local oscillator would again become no combined again with the incoming signal E s of t and then, detected and then did all this things ok. This arrangement which I have not really sketched it out here correctly is called as Optical Phase Locked Loop. So, this is optical because this is being done using laser. So, this is a phase locked loop which is a very very common devise in RF system.

So, you can actually get very nice PLL's, very high performance PLL's with very narrow bandwidths and impact you can use this PLL's to synthesis whatever the frequency is that you want. Unfortunately in optical phase locked loop, the major problem is that you are actually dealing with two different lasers ok; you are dealing with two different lasers, one of them happens to be at the remote site. This is a transmit laser and then, you have this one as the local oscillator here right. So, this at receiver side, but then you are trying to synchronize you are trying to make this is a master laser and then this one as a slave laser by somehow putting this OPLL in the picture.

So, OPLL what is it is suppose to do is to take the signal from the master local or the master laser and then, tune the local oscillator such that the local oscillator would them track with the same frequency as the transmitter frequency having a phase fixed phase difference from the transmit phase theta 0 and theta LO. LO being the local oscillator phase.

But it turns out that this problem is actually very very hard to solve for various reasons. For example, two lasers when you try to match them or when you try to make them into a master and slave can start producing what is called as chaos and that is actually a big problem because; that means, any small change in one of the setting that you are going to use will result in an extremely large and unpredictable change called as chaos ok.

So, in fact, this arrangement of using two lasers is used to produce chaos because 2 lasers getting them to lock each other is an extremely difficult problem and this is precisely the reason, why even the coherent detection was shown to perform much better than direct detection system that we discussed in the previous modules ok. It was not really taken up as a viable option until about 2004 or 2005 when people decided instead of working with an analog OPLL system, we will let the signal come in as it is you have your coherent receiver. You generate the in phase and the quadrature components; I am showing a single coherent receiver.

But this is a phase diversity receiver of course. So, this coherent receiver of course, also takes in E LO. I am going to stop tuning E LO with E s, but what are the problems I could have? Well, omega LO may not be exactly equal to omega 0, this problem is known as the frequency of set problem. Theta LO minus theta 0 might not exactly be constant, but it would actually be kind of a noise and this noise actually has a physical reason why it comes out because what is called as phase noise. We will study phase noise in the next module or in the end of this module. This phase noise is going to cause the phase of the signals to drift in a random manner ok.

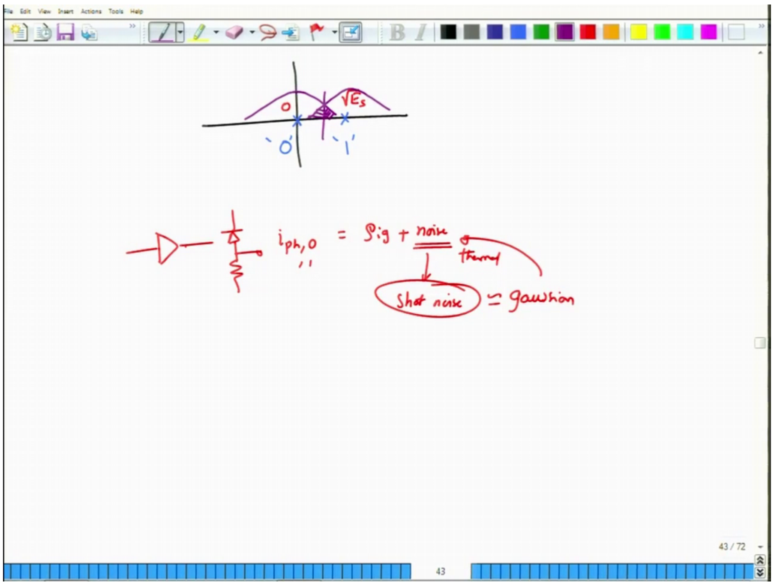
So, this is the phase noise problem which we will call as PL. It may also happen that the transmitter maybe x polarized while the local oscillator exactly not be x polarized, but it may not be exactly y polarized also. It may lie at some polarization and you want to tune the polarization right. So, this is the polarization problem which can also be.

So, all these problems that exist because of not being or not implementing the optical phase locked loop will be handled by a block known as digital signal processing. There have been various algorithms that people have used in the wireless domain or in the traditional digital communication domain to address the problems of frequency offset, phase noise and not exactly polarization, but similar to polarization people have worked on it and what more you can actually have an adaptive procedure using this DSP.

So, you can actually look at the samples at your generating for the samples are the input to the DSP and then, you can tune the parameters in the digital domain such that you can make FO equal to 0. You can estimate precisely what is PL and composite for phase noise and then, you correct for whatever the polarization crosstalk that these two would have entered. So, you can overcome this problems using DSPs. Granted that most of this DSPs are not real time there what is called as quasi real time, but in principle using this DSP algorithms you do not have to implement optical phase lock loops and this is actually a big achievement in fiber optic communications over the last 10 or 15 years.

So, this is a very important development and this important development has made it possible for us to reach data rates which are about 100 gigabits per second and beyond ok. We will look at some algorithms for frequency offset phase noise and polarization tracking in the later modules.

(Refer Slide Time: 12:27)



But at this or in this module, what we want to do is to basically look at our signal space analysis and to bring out a simple comparison between on off keying and coherent receiver for a BPSK system. Please remember that this comparison actually is a very simplest comparison in the sense that lot of choices that you need to make between direct detection receiver and coherent receiver are not going to be covered.

For example, coherent receiver requires more optical components and this optical component required their own power and their own stabilization. So, it will consume more power compared to a simple direction receiver. So, in cases where absolutely power considerations are maximum or power consideration are important, we would tend to prefer a direct detection system with the loss of spectral efficiency over a more complicated coherent receiver. On the other hand, if performance is the main goal and you donot care about the cost, then you donot normally use the direct detection receiver, but you use this coherent receiver.

So, the choices actually depend on the application that is there, but purely from the performance point of view, performance meaning if you send him the same transmit power which of the 2 constellation will perform better can be answered by looking at little more details onto this constellation itself.

So, for the BPSK constellation or for the on-off keying constellation that we will consider first, you have 2 symbols which are say, 0 and 1. So, these are the symbols that are transmitted for 0, you transmit nothing and for 1, you transmit some power in terms of constellation this is sitting at 0 and this 1 is at square root of E s. Please note that as I told you many times I could have used P S also, but I am sticking with energy considerations here because that is more common in traditional digital communication literature ok.

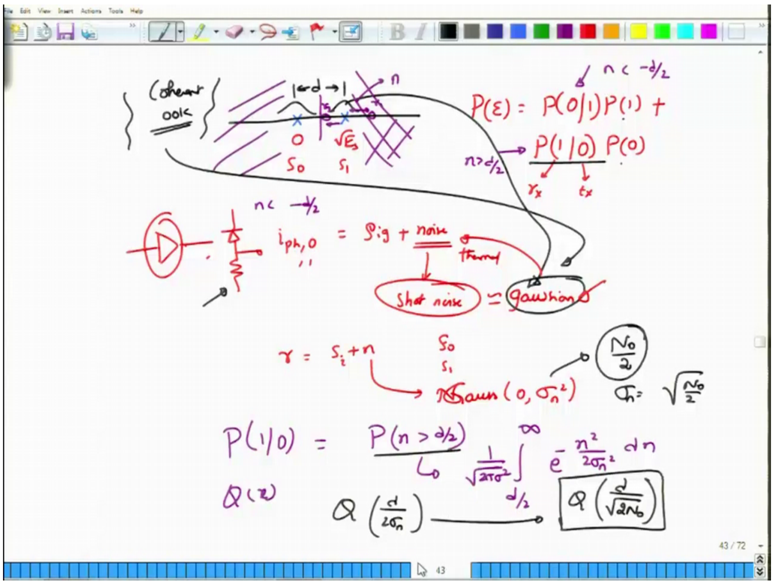
Now, what? Well, we have in a photo detector if I am going to look at the photo current for a transmit 0 or transmit 1 or whether a receive 0 or whether I receive 1, this photo current would actually be proportional to or rather this would also include in addition to the signal component. It would also include the noise component and we have seen what different types of noises are going to affect our systems.

We will assume that it is the thermal noise that is prominent in this case or if I am going actually put in a pre amplifier to overcome whatever the losses that the signal has actually experienced, then the noise that would be mainly for us would be the short noise ok. Turns out of the short noise actually is not exactly having Gaussian characteristics.

So, it will be a little difficult for us to assume that or compare the performance with the short noise because short noise is because short noise is dependent on the signal power. But what we will do is to approximate this short noise as a gaussian noise or we will work with the thermal noise itself ok. So, this thermal noise will be more or less gaussian, all though it is not exactly correct, but if you take this thermal noise as an approximation to what you are actually receiving the photo currents that you would receive will now we distributed around these points right.

And we have seen how to evaluate the bit error rate. This is a threshold that I have taken. This would be the error that you are going to make when you transmit bit 0, but incorrectly assume that this is bit 1 and this would be the error that you would make when you take that the bit has 0, but actually have transmitted bit 1 ok. Now, this is simple one dimensional thing that I am actually looking at and if I go back to this idea of the signal space, I concept that we talked about; in that signal space concept what we actually had, so this are the noises I am just for movement removing the noises. But this is my constellation point right.

(Refer Slide Time: 16:26)



This is my constellation point here; the two constellation point at 0 and square root E.

So, I will reliable this one. So, this is square root E. So, this is I am going to make this as 0 and square root E s. So, this what I have right and I am assuming that the noise is gaussian. So, please keep this in mind and take it for now that this is actually operated in the thermal noise limit, although we know that when you have a preamplifier noise I mean when you have a preamplifier or when the power itself is considered; then, the major noise sources are short noise and the signal spontaneous beating noise. But for now, we will assume that this is all gaussian noise right. So, I have a bit 0 and bit 1, I will make an error.

So, what is the probability of error? The probability of error will be the error in thinking that I have actually received is 0, but when have transmitted a 1 times probability of sending a 1 itself plus probability of thinking that have actually received a 0 or received a 1 when I have transmitted a bit 0 multiplied by the probability of sending a bit 0 itself right. So, this is what I have. Now, if I assume that my received signal r will be signal si plus n; where, si could be either s 0 or s 1. And n is a thermal noise or gaussian thermal noise with a gaussian distribution. So, assume that this has is a gaussian distribution. So, I will right this as gauss of 0 mean, but a variance of say sigma n square ok.

Then, what I have here and please note that I am also going to call this distance between that 2 as D right. When will I make an error? Suppose, you are received waveform happens to be this one right. Clearly you would have transmitted one let us say and then the received signal has fallen at this point ok. So, this is my received signal. This extra amplitude must have come from noise right.

And if this noise is going to add only positive contribution to this when I transmit a bit s 1 or symbol s 1 and then, there is no problem I can keep adding the positive noise and if I am now looking at whatever is the closest point to r 1, the closest point would always be equal to s 1 over s 0 right. So, when I have noise which is positive, it would always be positive and it would be in this particular manner.

But when n starts to become negative, suppose I receive a signal r 1 or r 2 here when clearly if even if I transmitted 1, the noise became negative and its actually starting to pull the received points closer towards s 0. If the noise reduces below minus d by 2 if noise reduces below minus D by 2, then it will move the received points on to this region. In this region all the points are actually closer to s 0 and therefore, we will actually end up making on error of the kind this one right.

So, you might have transmitted 1, but you would have received a 0 when n becomes less than minus d by 2. Alternately for the this particular case, noise can be negative does not matter, but when noise becomes greater than D by 2, then you are in trouble because s 0 might be transmitted, but it would actually put it into this region and in this region all points are actually closer to s 1 the rather than to s 0. So, the condition for making the error say P 1 given 0 seems to be that the probability of noise being greater than d by 2.

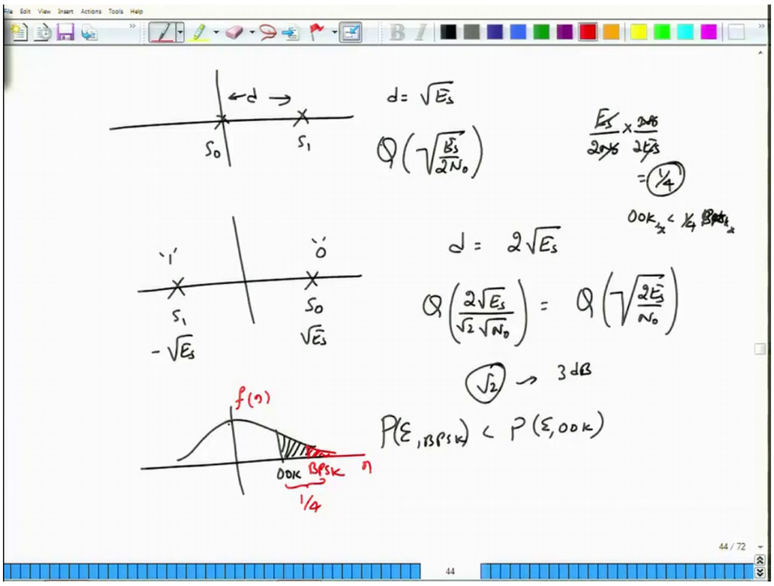
And what is the probability of noise being greater than 2? n itself is given by or rather this probability can actually be evaluated assuming that we already had a gaussian distribution for the noise. So, this would be integral d by 2 to infinity. E to the power minus n square divided by 2 sigma n square. There is a 1 by square root 2 pi sigma square d n.

So, this corresponds to the probability that noise is greater than d by 2 and in terms of your Q of x function that we already have seen in the previous modules, you can actually rewrite this probability of error or probability of n being greater than d by 2 as Q of d by 2 sigma n ok. So, you can write this as Q of d by 2 sigma n. Now if you take this noise variance to be some n 0 by 2 right that is sigma n square equals n 0 by 2; where, n 0 by 2 is the power spectral density of the noise than sigma n will be equal to square root of n 0 by 2.

So, I can actually rewrite this one as Q of d divided by square root of 2 n naught ok. I am assuming that the noise variance is the same whether you transmitted bit 0 or bit 1; although again go back to our assumed definition of gaussianity ok. So, this is what I am actually looking at. So, this would be the probability of n greater than d by 2 which is given by this. Of course, I only evaluated this error. If you assume P of 0 equals P of 1, then the errors are symmetric and this would actually be equal to the total error that I am actually going to make.

So, this is the probability and multiplied by half plus the same quantity multiplied by a you know, or the same quantity again repeat. So, and there for this is the probability of error for this system in which the two symbols are separated by a distance d. Please note that the only assumption that we have made is that of the Gaussian noise distribution and having same variance whether you transmit bit 0 or bit 1 or symbol s 0 or s 1.

(Refer Slide Time: 22:07)



Now, it becomes easier for us to evaluate you know the on-off keying as well as the BPSK system. So, in the on-off keying what I have here will be a symbol s 0 and symbol s 1; the distance between these two points is actually equal to square root of E s. Therefore, the error will be Q of square root of E s divided by 2 n naught ok. I forgot to mention here that this kind of a condition can also come in when you have a coherent on off keying systems.

But usually coherent on off keying system is not used because this is more simplified structure rather than to use a coherent receiver for on off keying. It is possible to use coherent receiver for on off keying works exactly in the same way that we have talked about and in fact, the noises that you get with coherent on off keying actually approximates the Gaussian noise very much.

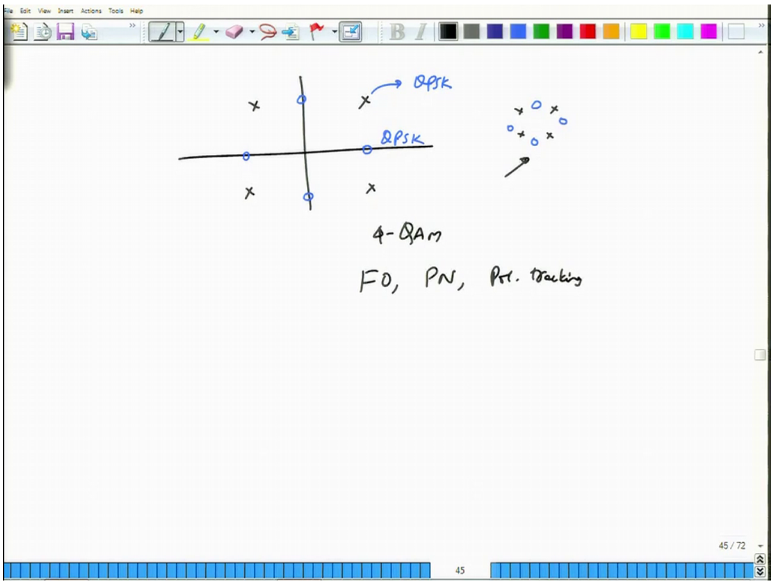
So, that is the small information out there for you. But now we have looked at the on off keying system even if it is coherent on off keying that is all right. This is the approximate expression that we are actually going to get ok. Now let us go to coherent BPSK systems. So, I have a s 0 and s 1 and please note the way we have defined the systems in this case. You have square root E s as the constellation point minus square root E s as another constellation point.

This should be familiar to you from the previous module. Now, the catch here is that the distance is now given by 2 times square root E s and when you plug in to this expression of Q of d by this one. What you will actually see is that the distance is now 2 square root E s and here you have square root of 2 square root of n naught right which of course, can be simplified by writing this as square root of 2 s by n naught and you can actually show that the square root of 2 factor translates to about 3 dB improvement in the performance system ok.

So, this is the error that we actually make and you can also clearly see that probability of error for the BPSK system will be lesser than the probability of error for the coherent on off keying or the simple on off keying system. Why it is so? Because this Q function will actually give you the area under this particular curve right and if you compare the two arguments the argument for the on off keying system is E s by 2 n naught if you take the ratio of these two you are going to get n naught divided by 2 E s.

So, this E s E s cancels n naught and n naught cancels and the argument is actually 1 by 4 that is to say on off keying corresponding x value is only 1 by 4th of on off keying are BPSK x value here. So, if this corresponds to on off keying, then BPSK would be 4 times larger and therefore, that would correspond to this point and clearly the area under this red curve will be much smaller than the area under the on off keying system. The ratio of these 2 points is about 1 by 4. So, BPSK is higher than the on off keying system. So, and this is the Gaussian distribution that we have drawn as a function of noise here ok. So, clearly BPSK systems are you know performing better than the on off keying system.

(Refer Slide Time: 25:23)



You can extend this BPSK system into additional modulation scheme such as this one which is called as 4 QAM or you can also consider 4 different phases here right this is called as QPSK system sometimes this is also called as QPSK. So, we have to be little careful in exactly what is the terminology being used, but the probability of error calculation for this is slightly more complicated, but it can be shown that the probability of error for this constellation is the same as a probability of error for this constellation because rotations do not change the probability of errors.

So, we are not going to prove all these and we are not going to consider the probability of error calculations for these higher order modulation formats. I will leave this as exercises and in the assignments for you to work on it. What we want to do now is to actually look little more carefully about the noise that we talked about namely the frequency off set, the phase noise, the polarization tracking. So, what kind of an algorithm can be used to remove polarization problems and all those things. We will do that starting in the next module.

Thank you very much.