Fiber – Optic Communication Systems and Techniques
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Lecture – 52
BER discussion for OOK systems

Hello and welcome to NPTEL MOOC on Fiber - Optic Communication Systems and Techniques.

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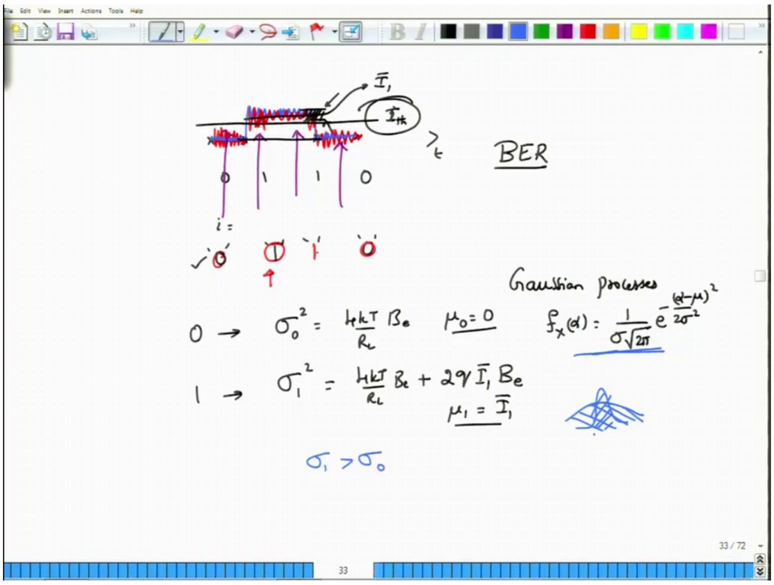

In this module, we will continue the discussion of on off keying systems and we specifically are interested in finding out what is the expression for bit error rate so that, we may use this expression to gain further insight into the system. And we are also concerned with finding the optimum value of this threshold value right that is used to distinguish whether you have received a bit 0 or a 1.

Now we have said that when you receive a bit 0, the total noise variance we will call that as sigma 0 square will be given by 4 k T by R l B e. Clearly I am neglecting the short noise contribution from the power P 0 because I am assuming P 0 itself is quite small ok. And then the noise variance when you transmit a bit 1 will be denoted by sigma 1 that would be in addition to this R l B e. They will also be 2 q I 1 average. I 1 average stands for the average photocurrent that we have received times B e, B e being the bandwidth that we are considering ok.

Now, we have written down the variances sigma 0 and sigma 1 and we have also said that these are the variances of the noise sources, but we have not fully characterized what kind of a distribution does this noise possess ok. We will assume that these are Gaussian processes which means that the random variables that you obtain after sampling the photo current, here will be corrupted by noise which is random variable itself and that random variable is Gaussian distributed. The probability density function of a Gaussian process or a Gaussian random variable x, which can take a value of say alpha is given by 1 by sigma square root 2 pi exponential minus alpha square by 2 sigma square ok.

So this is for the case when the mean is 0, but in sometimes you will find that mean is not 0 so, you will have to then put alpha minus mu square ok. When you receive a bit 0 the current is 0; so, you can assume that the mean mu 0 in this case will also be equal to 0. Whereas, the mean when you receive a bit 1 will not be 0 of course, the mean will be the current that you receive right. The current will actually so, you can see that one here. These red lines when you have launched bit 0 are centered at the output current which is also 0. Whereas, the current or rather the noise that you see here is actually centered at the received photocurrent; the average photocurrent and then this is the noise variance that you are actually seeing because of this; and this value is your average photo current I 1 when you launch a bit 1 ok. So, you can see that difference right here and this width around what I have drawn these 2 lines is actually the width or the 1 sigma variance, sigma 1 when you have launched up bit 1 ok. So, when you transmit a bit 1. So, this is what you need to remember. So, mu 0 is 0. Mu 1 is 1.

Now let us draw on the horizontal axis, the current that we receive the photo current that we receive and if we locate 0 and we locate I 1 average which is the average photocurrent that you have received and then draw the Gaussian statistics ok. So we have assumed that when you have bit 0 or bit 1 received the noise will still be Gaussian, but its variances will be different ok. So, which variance is actually larger? The variance sigma 1 is larger than the variance sigma 0 and one of the properties of these distribution functions is that the larger the variance, the broader will be the curve because the total area under this should be equal to 1. This should remind you of the dispersion case that we talked about because we had an input pulse and the output pulse was actually broadened version of it. So, the amplitude drops, but that is alright because the pulses broadened. So that, the total area under the pulse will still remain the same and that is exactly what we have in this scenario.

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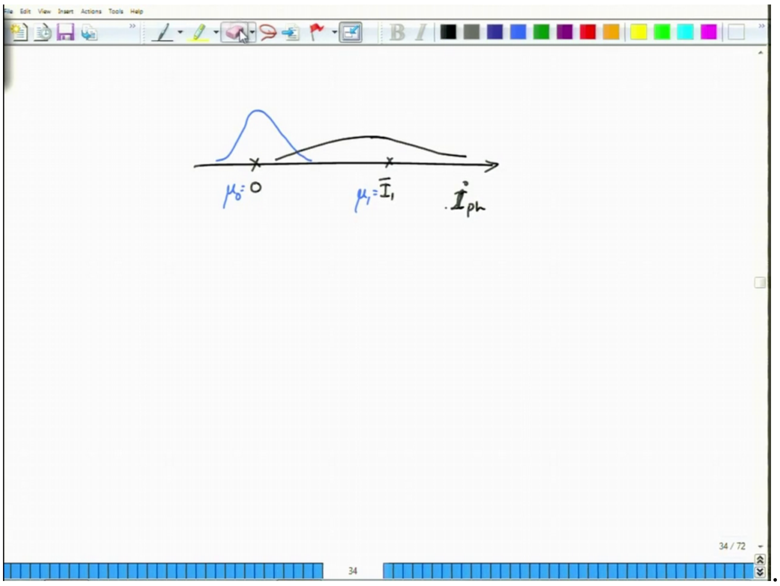

This is mu 0; this is mu 1 which is equal to 0 and I 1 respectively. The variance with 0 will have a Gaussian distribution that goes something like this whereas, the variance with 1 will be much broader. So I should have put them closer together. So, that you could have had a better appreciation of this and I put the peak not exactly at this point

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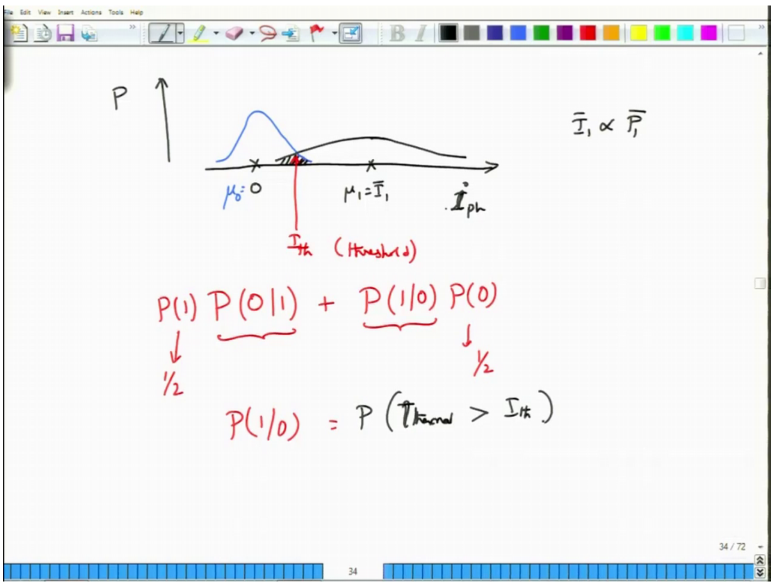

So, the peak is here. So, this is the peak. This is how the noise around 1 would look like or no not exactly is noise itself, but this is the photo current that you would obtain the amplitude of the photo current and on the y axis is of course, the probability itself ok. So, you can clearly see that if this I 1 is very very large compared to 0, then these 2 tales of the distribution curve will hardly overlap. But when I 1 is brought closer to 0 because you are not putting out enough optical power. Remember, I 1 is directly proportional to the received optical power P 1 ok. So, if you have a transmitter which is very powerful putting out lot of optical power, then I 1 will be further away on this axis. But in most scenarios this maximum value of P 1 is constrained by several other factors. One of them being the constraint imposed by the laser source itself, that it cannot produce an output power which is larger than a certain maximum.

So there will always be a chance that these two axis or these two tails will actually cross each other ok. And it is at this crossing point that you should actually locate this value of the threshold I th. Please note that this is a threshold value; and not the thermal value. This is the threshold that we are using and not the thermal noise that I have written out there ok. So, this is hopefully you understood these two pictures. This is for the distribution of amplitudes when you launch 1 and this is a distribution when you launch 0 ok. So, this is how the 2 noise variances are. Now when can you make an error? You can make errors of two kinds. You can erroneously decide that a bit is 0, but you have actually transmitted a bit 1 and then you can decide that the bit you received was 1, but you transmitted actually a 0.

So you have 2 kinds of an error and these errors will have to be waited with their appropriate transmission probabilities. That is, if you launch 1 with a probability of P 1 and then you make an error you get, I mean error of thinking 1 as a 0. This would be the probability of error when you launch 1, but you erroneously decide as 0. So, this is sometimes called as bit error probability for 1 ok. So, this is called as bit error probability for 1 and similarly this would be the bit error probability for 0 ok.

Again these are you know whether you use these terms or not is something that you do not have to worry about, but you know that you are going to make these two kinds of errors. Now, how do I evaluate these errors? Now before I can evaluate these errors, let me assume that both P of 1 is equal to P of 0 and which means that both 1's and 0's are equally likely to occur. So, you have P of 1 equals P of 0 equals half each.

So you are now going to decide P of 0 1or derive the expression for P of 0 1 or P of 1 0 ok. Let us consider P of 1 0 ok. I will leave P of 0 1 as an exercise to you ok. I chose this because there are positive numbers involved here, there is nothing specific here. The derivation is actually quite simple for both cases. When will you make an error in thinking that the received bit is 1, but you have actually transmitted 0. You will make this error when the thermal noise is so large that the total current that you receive when bit 0 is transmitted actually exceeds the threshold right and you can actually put a probability for this case.

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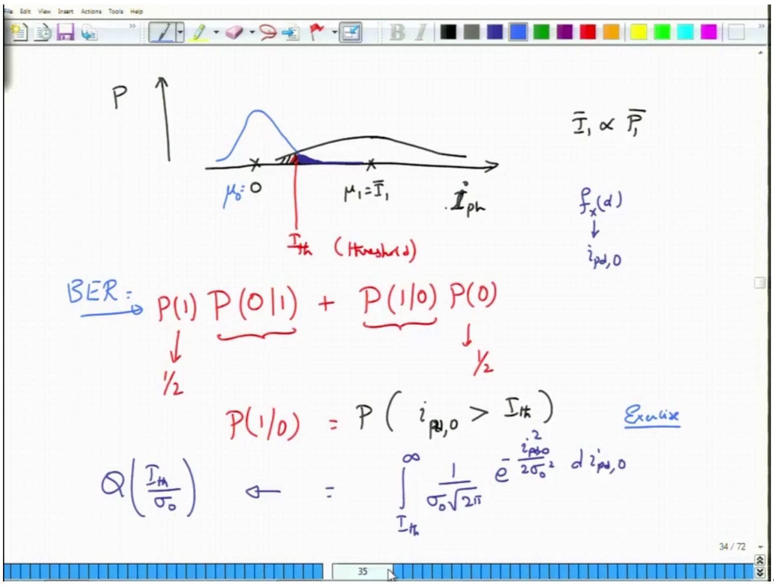

So instead of of course, focusing only on thermal noise you can put whatever the total noise that you would have received or the total current that you received when you have sent in a bit 0 which will call as I0 or rather I P d comma 0. If this current exceeds the threshold value, then you would be making an error and this current will exceed the threshold meaning you are actually looking at this dotted area.

So, this area that you are looking at is the probability that your current i pd of 0 is actually exceeding the threshold value ok. And how do I find this dotted or the shaded area? It is actually quite simple. I know what is the probability density, I have already written down the probability density. In this case, simply take x as the current that you would obtain when you transmit a bit 0 and alpha is of course, the value of the current and the probability density function needs to be integrated from I threshold all the way up to infinity. Although you don't see infinity here, there is actually value considerably starts to decay decay and eventually only at infinity it would reach 0.

So, you can actually integrate the probability density function from threshold value to infinity ok. And what is the expression that your integrating? This would be 1 by sigma 0 square root 2 pi exponential minus say we will call this as x square the value mu 0 is 0 anyway 2 times sigma square or rather not x we will write it as current, right. That is what our variable is. So, i pd comma 0 square divided by 2 sigma 0 square d i pd comma 0. So, this is the expression that you need to evaluate and you can actually evaluate this expression and show that this will be equal to Q of I threshold divided by sigma 0 ok.

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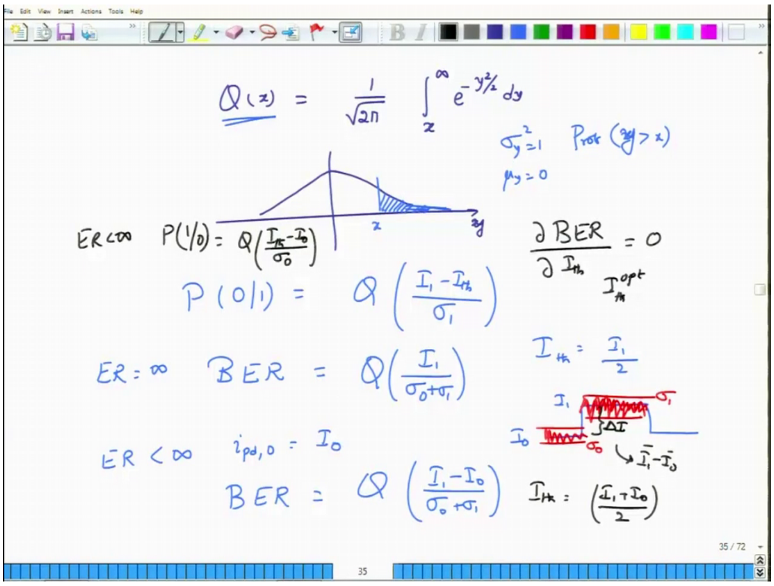

What is this Q function? The Q function is a commonly used function and it is actually the integration of the Gaussian variable. It is given by Q of x equals integral of 1 by sigma root 2 pi x to infinity e power minus y square by 2 dy.

So this is actually Gaussian variable. A Gaussian variable whose mean is 0 and Q of x would represent starting from x to infinity all the way, that would actually represent this particular area ok. So, this is the probability that the received amplitude is actually greater than x of a random variable y.

So if you have a Gaussian random variable y whose variance is equal to 1. So, sigma y equals 1, sigma y square equals 1 and mu y equal to 0 then the probability that the amplitude x exceeds or the probability that the, the received or the Gaussian variable amplitude y exceeds x is actually given by this integration and that is precisely what this Q of x function is. And you can show that that is you can actually transform this equation by making change of variables. I will leave this as an exercise to you ok.

So, you can use this exercise and then show that this will be given by Q of I th by sigma 0 ok. Similarly, you can show that P of 0 1 is given by Q of I 1 minus I th ok. Because I one is the mean value that you are anyway going to get divided by sigma 1 ok. I 1 being the photo current that you had received or the average photocurrent that you have received when you transmitted a bit 1. So, these are the 2 bit error probabilities that I wanted you to find out. And using these 2 values, you can show that under infinite extinction ratio.

So, when ER is equal to infinity not in dv, but in linear scale if it is equal to infinity then bit error rate, which is actually this expression here. So, this expression is the bit error rate because it will tell you what is the probability of making bit errors when you transmit a 0 and when you transmit a 1. So, that expression is BER and you can actually find out the expression by substituting onto these values. And in fact, you can combine this to show that this is given by Q of I 1 divided by sigma 0 plus sigma 1 ok. So, this is what you can actually show. And in this case, the threshold optimum value is actually given by I 1 by 2, but if your extinction ratio is not infinite then when you launch bit 0 or when you received bit 0, there will also be an I 0 current that you would receive. Because there will be non-zero value of P 0 which you translate to non-zero value of the current I 0. And they should also enter the equation and it does so, because you can show that BER in this case will be given by the Q function of the difference I 1 minus I 0 divided by sigma 0 plus sigma 1. And please note that I 1 and I 0 are the average value. So, in the absence of any noise, if this is I 0 and then this is I 1 that you receive any launch a bit 0 and a 1 and top of it you actually have the noises. So, this is how the noise is present, so this is larger noise.

So, this is the noise variance sigma 1, this is a noise variance sigma 0, but what you are looking at is this average photocurrent difference. That is, delta I which is defined as I 1 minus I 0 ok. I have omitted writing this bars to simplify the expressions. And in this case the threshold value, that is, when the extinction ratio is not this one, the threshold value will be roughly given by I 1 plus I 0 by 2 ok. Again, these expressions are slightly what are called as approximate expressions because the optimum value of the threshold can actually be found by minimizing the BER expression with respect to the threshold setting ok. And in this case, you don't you know you have all the P of 0 1 and P of 1 0.

So, in that case for an extinction ratio being less than infinity P of 1 given 0 which we derived in the previous slide will actually we modified. It will be a Q function of I th minus I 0 divided by sigma 0 ok. And you can go back to the full expression for BER, substitute for P of 0 1 for the case extinction ratio is less than infinity. And for the case for P of 1 0, again for the case of ER less than infinity. And then differentiate that expressions with respect to I th set this to 0 To obtain the optimum value of the threshold current ok.

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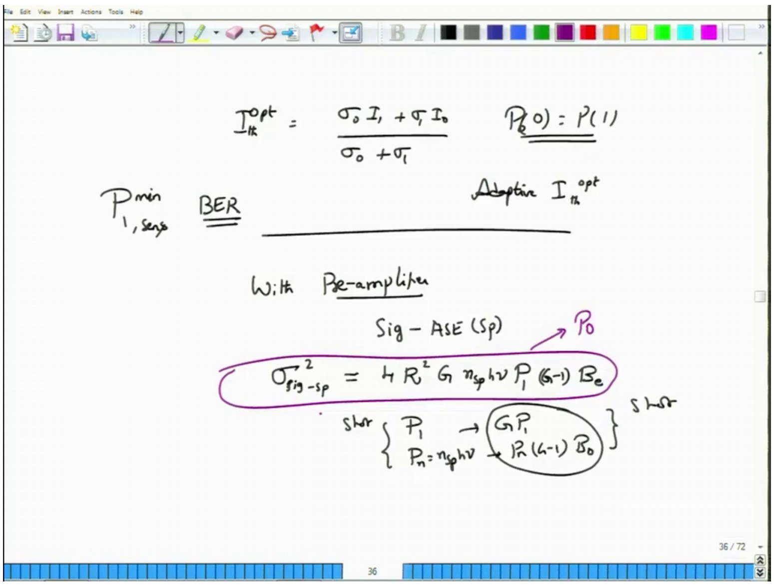

And that is something that we are not going to do here, but that is you can show with some approximations that that threshold optimum value is given by sigma 0 I 1 plus sigma 1 I 0 divided by sigma0 plus sigma 1 ok.

So, this is the optimum value. Again you have to assume here that probability of sending a 0 will be equal to probability of sending a 1. Because, if these probabilities are different, then again the optimum value will be different. So in practice, most receivers actually have a way of changing this optimum threshold. They do it by continuously monitoring the error rate that they are receiving on a particular channel and they adaptively change the optimum threshold value by either increasing or decreasing by looking at the matrix of the signal that have been received. So, this is the case that we have seen for the simple direct detection for the on off keying wehen we had no preamplifier. And the most important lesson that you need to learn, when there was no preamplifier is that, when you launch a bit 0 with the extinction ratio is not 0, P 0 will be non 0 which means I 0 will be non-zero.

So, there will be small amount of short noise, but usually that short noise can be neglected in comparison with the thermal noise that is present. So, I 0 can be approximately taken to 0. If not, you have to modify the equations only slightly. The noise variance will be weekly dependent on the signal power P 0. Weekly dependent because short noise when the extension ratio is very high is actually quite small for bit 0; However, the situation for bit 1 is completely different. Sigma 1 will be very large compared to sigma 0 and sigma 1 is a strong dependent function on the receive power P 1 because I 1 will be equal to RP 1 and for large value of P 1, this will be actually quite large. But usually you don't. I mean, so essentially to sum up for this case, sigma 1 is large, sigma 0 is usually smaller, sigma 0 is only usually thermal noise, but most importantly, the total noise statistics. Although it is Gaussian, the variance is dependent on the launch power. And if you start changing the power that you transmit then these variances will also be changing.

So, they are not independent of the bit that is transmitted. It is also common instead of talking about BER; to ask what is the minimum signal that one needs to transmit ok when you launch a bit 1. Because usually again bit 0 doesn't really give you much of an error or the extinction ratio is reasonably high sigma 0 is reasonably small size in comparison to sigma 1. So, we don't normally worry about that, but as you start reducing the optical power then thermal noise will also matter. But in any case, you normally ask for what is the average power for bit 1 such that a given value of BER is maintained ok. And this minimum receive value is what is called as sensitivity of the receiver, and sensitivity is a important parameter of optical receivers but the way I have defined sensitivity is I have masked lot of complications in this definition. Because the sensitivity depends on the optical preamplifier that is commonly used, depends on the exact receive filter designed, it depends on the probability that or the capability of us to adaptively change the threshold. Because if you do if you are not including an (Refer Time: 18:53) change in threshold then optimum value cannot be adjusted and minimum sensitivity may not be achieved ok.

So, all these factors means that sensitivity calculation is an important topic and in fact, in the 70's, late 70's and 80's and even in the 90's this sensitivity calculations were done; lot of calculations were done to find out the optimum sensitivity and there are lot of or there is the huge literature available for you to go and study. We won't touch that particular topic, but to tell you that coherent receivers which we are going to discuss in the next modules offer for the same operating conditions about 3 db more sensitivity. That is, you need 3 db less power to detect or to maintain the same bit error, bit error rate compared to the direct detection receiver. So, this also you should keep in mind.

Anyway, so all that we discussed previously was without the preamplifier. Now, when you include preamplifier, things change dramatically because preamplifier is usually a high gain amplifier that we incorporate to rise the signal levels that we have received. The dominant noise source that you are going to get will be the signal ASE beating or the signal spontaneous noise beating. Spontaneous emission gets amplified because of the gain that we have put in and this spontaneous noise beats with the signal to result in what is called the signal spontaneous noise and this variance of the signal spontaneous noise can be approximately given as, for the case where we are using the optical preamplifier is given by 4 R square G n s p h nu optical power received.

So, if you are looking at power P 1, then this is P 1G minus 1 times Be ok. So, this is the signal spontaneous noise. The short noise will also increase it will also increase because, the input power would have we increased. So, previously it was P 1. Now the new power is GP1 therefore, the short noise also increases here the noise power is say Pn which is say n sp h nu. This will also increase because now you have a certain amount of gain, that is, increased gain, but this increase gain will be within an optical bandwidth of B 0. So, this is the noise component. This is for the direct detection case, this is for the optical preamplifier case, of course, direct detection case P n will be equal to 0, but this will affect your short noise. So, overall short noise will actually incorporate the increased optical power. Because you have put in a gain here and then the noise that comes in will also have some average non-zero value. That will also add up to the noise.

So, if you multiply this P n G minus 1 B 0 with 2 Q and then within the bandwidth of Be you will get the extra shot noise that comes from the amplifier. Whereas, when you been you multiply this GP 1 with 2 Q Be, you will get the increased short noise because you have put in the optical amplifier, but again these short noises can be kind of neglected in addition to this thermal noise can also be neglected because most of the dominant noise comes from this signal spontaneous beating noise.

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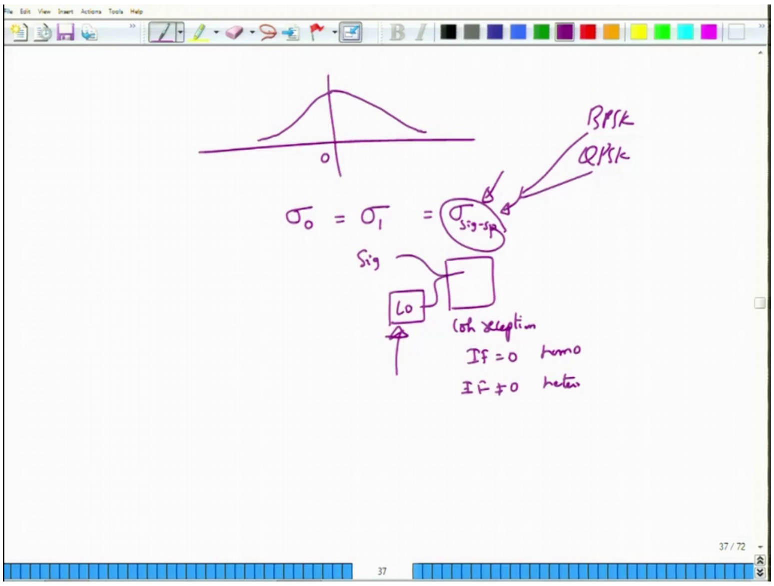

And one of the interesting things for us with this signal spontaneous beating noise is that

it can be modeled to an extremely good approximation that it's a nice Gaussian function having a 0 mean. What it means for us is that sigma 0 will actually be equal to sigma 1. Because both of them will then be equal to signal spontaneous variance, ok. So, this is an important part and because signal spontaneous noise is equal to both sigma 0 and sigma 1 you don't really find the problem of having to deal with two different values of sigma 0 and sigma 1.

So, this is an important part of how the preamplifier will actually modify. There is still some difference because you know, this is P 1 is what we have received. So, when P 0 is present again this signal spontaneous noise will be very small so you still go back to the thermal noise case itself, but when you will see that when you go to advanced modulation system such as BPSK or QPSK and so on, the received optical power will be usually constant and the signal spontaneous noise will not change for the particular bit that you translate. For the on off keying, it will still be different but for the higher order modulation noise this will usually not be signal dependent.

So, you can actually change from being signal dependent to signal independent when you have the signal spontaneous noise beating. In fact, it is not even that this signal spontaneous noise which will be dominating in most receivers ok. If the receiver incorporate what is called as a local oscillator at the input and mixes the incoming signal with what is called as a coherent reception; coherent reception can be either an IF 0 reception or IF non-zero reception. IF stands for intermediate frequency. IF equal to 0 corresponds to 0. IF case or the homodyne case, whereas, 0 I mean non-zero IF, IF corresponds to hydro dyne case and again in this cases the noise that is dominant comes not even from the amplifier, but rather from the local oscillator that is used ok.

So, we have this noise statistics being different for different modulation system and one has to take this into account when you are dealing with this BER calculation. Having said that, when you go back to on off keying, signal spontaneous noise will actually increase the sensitivity of the system because, when you do the expressions completely correctly you will find out that the BER would have been significantly reduced for the same launch power P 1 and P 0 because you have actually put in an optical amplifier in before you gave that optical power to the photo detector. And because of this preamplifier the signal experiences tremendous gain. See the signal the current grows as G times P 1 and the power goes as electrical power that you detect will go as G square P 1 square. Whereas, the short noise power or the short noise variance grows only as GP 1 or the signal spontaneous noise also grows roughly as G time P 1 ok because the variances or the noise power is growing linearly with the gain and the power that you received. Whereas, the signal power actually grows quadratically, the ratio will be much more dramatically increased. Because there will be an extra factor of G in this preamplifier case which will bring down the sensitivity by roughly roughly a value of the gain that you have put in the receiver.

So, you can actually of course, eliminate this preamplifier but use an APD which is an avalanche photodiode which has a larger sensitivity, but you can obtain the same sensitivity using the simple pin photodiodes, but using an optical preamplifier. So, optical preamplifier by raising the gain will decrease the received power or the requirement for the received power and therefore, give you more sensitivity and when you actually do look I mean coherent receiver you will gain much more sensitivity improvement than what is possible from this direct detection systems say. In fact, that is what motivates us to consider this higher order modulation formats in the next module.

Thank you very much. [noise]