Fiber-Optic Communication Systems and Techniques
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Lecture – 50
Review of communication concepts - II
(Signal and vectors, Signal energy, Orthonormal basis functions)

Hello and welcome to NPTEL MOOC, on Fiber Optic Communication Systems and Techniques. In this module, we will continue the review of some topics that we started in the previous model.

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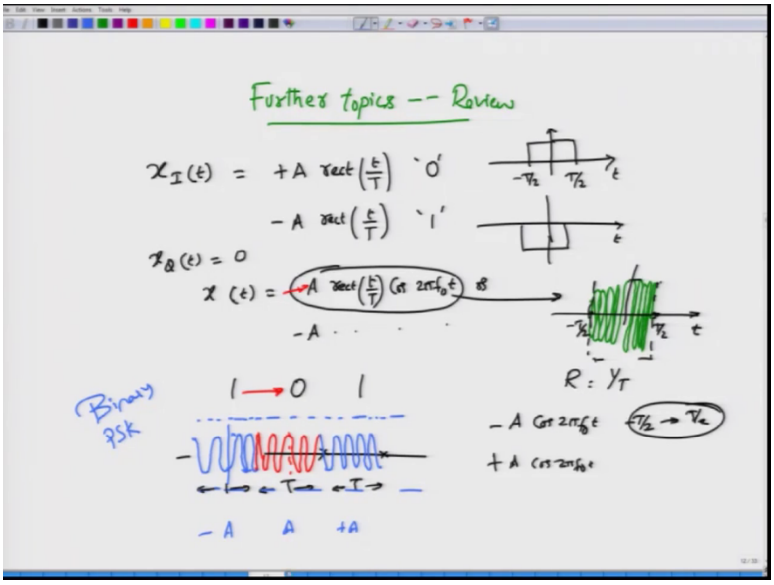

So, we saw in the previous module that given x I of t and x q of t, you can actually construct the band pass signal. Suppose, x I of t could be plus A rectangular function of duration T. This is our notation for defining a rectangular pulse of duration T which in the way that we have defined we will extend from minus T by 2 to plus T by 2. And then, x I of t can also be minus A rectangular pulse of the same duration.

So, if you want to send information in the form of a digital communication system, when a 0 occurs you may choose to transmit the baseband signal x I of t as A rectangular function of t by T or the rectangular pulse of duration t. And if you want to transmit a bit 1in this baseband, you can transmit it with minus A. So, the amplitude would be reverse. So, minus a rectangular pulse of the same duration would look like this.

So, this is alternatively or depending on what is the sequence that you want to transmit, you can transmit these signals in the baseband. Of course, on a fiber this is not going to go as a baseband signal, but it will actually go as a pass band signal or a band pass signal so, which is x of t in our notation in the previous module. Because, x Q of t in this case is actually equal to 0, x of t will be either plus A rectangular pulse t by T cosine 2 pi f naught t or it would be minus A and the same thing out there. Suppose I want to transmit a sequence which is say 1 0 1.

What I have to do? I have to first split up the transmission into three slots. Each slot is of a duration T seconds. Depending on the rate at which we are transmitting this T will be determined. So, the rate at which you are transmitting will be 1 by T so and so symbols per second is what we would call this rate as and what is the idea here.

So, when you want to transmit x of t with a bit 1 0 or 1or 1 or 0, you have to choose appropriately between A rectangular pulse t by T times cosine2 pi of 0 t or minus A. So, in this case it is minus A. So, what you actually have will be minus A cos 2 pi f 0 t over the duration minus T by 2 to plus T by 2 or essentially to say that, over one time duration, you are either transmitting minus A cos 2 pi of 0 t, which is a time limited function or you are transmitting plus A cos 2 pi f 0 t over the same time limit, that you are looking at.

So, if you were to look at how this would be seen, if you were to plot that one. So, this is over time and then this is minus T by 2 to plus T by 2, over which the rectangular pulse itself is defined to be equal to amplitude 1 and here you now have a cosine wave also being transmitted, right. So, and since this is a cosine wave; obviously, this will start at max here at t equal to 0 and then go out like that.

So, this is your cosine omega 0. You have to imagine that there are a lot of periods out there and then it will end up here. So, this will be lot of those cycles up there because, f 0 is usually considered to be very very large compared to 1 by T here. Why 1 by T because, that is a bandwidth approximately, that is the bandwidth of the rectangular pulse one half of the bandwidth of the rectangular pulse.

So, if you want to transmit in sequence here, what you have to do is to essentially first draw the envelope here and then start putting in either plus A cosine 2 pi f naught t or minus A cosine 2 pi f naught t, right. So, with this as my center here, this is the cosine wave that I am writing. So, that is how it would be. So, you should also ensure that there are integral number of cycles in this one, for various results this will be integral number of cycles in one duration.

But, now, when you actually have a 0, you now have to transmit minus or rather for 1, we should have transmitted minus A cosine t, right. So, for that one, we should have actually transmitted something like this. So, we should have transmitted a minus cosine wave and then this is one cycle on this side and then this is similarly the other cycle on this side. So, I have just reduced the value of f naught, so that you can clearly see the waveform.

But, now, I have to transmit a 0 and transmitting a 0 would mean, I have to now switch to plus A. So, at plus A, it would go like this and it would be in this manner so, 1 2 and 3. This should actually have been because, I am starting with minus A, this sequence here should actually be like this. So, it should have been, this is your cycles or if you want one more cycle, this will be the cycle.

Whereas for the bit 0, since you are transmitting this one with a plus A cosine t, this would come here. So, what is actually happening is, there is a sudden change in the phase, when the symbol makes a transition from 1 to 0. So, perhaps it was not very clear the way I have written, but you know hopefully when you look at and draw it correctly or use MATLAB to get this one correctly, you will see that this is all right.

And when you have a transition again at 1, you will see that there is a further 180 degree transition from here to this one. So, this type of transitions are the ones which actually send you the information. So, if you were to move your window appropriately and sample the waveform, in this case sampling does not really help. But the idea here is essentially the same thing you have to correlate this one with the appropriate cosine waveform and then extract this one.

But, when you extract them, the samples you will essentially obtain amplitudes which are like plus A and then minus A in the absence of any noise. And by looking at the sign of your output, we can determine whether you transmitted a 1 or a 0. Incidentally, this type of modulation is called as binary phase shift keying. This is in the class of those modulation signals called as modulation formats called as phase shift keying and this is binary phase shift keying. So, we have written x of t, given that I had x I of t I did not start from x of t and then come back to x I of t.

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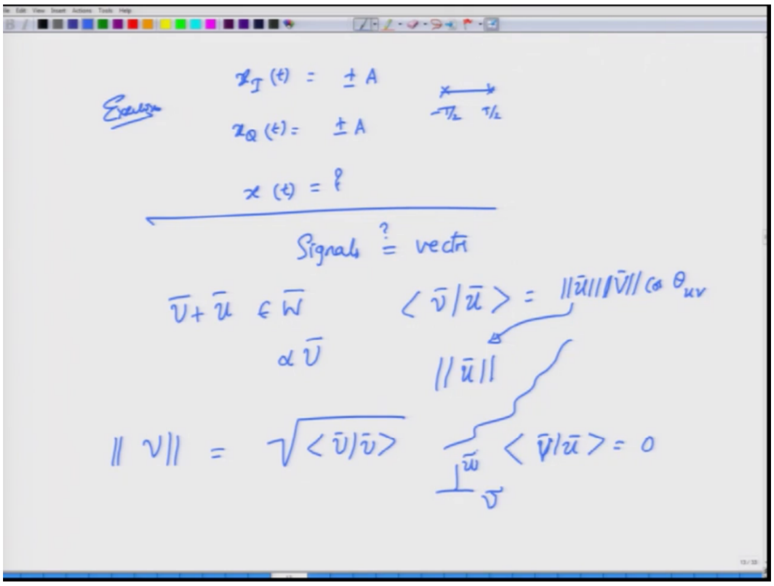

Suppose you take this as an exercise here. Suppose, x I of t can be plus or minus A. And x Q of t can also be plus or minus A over the same time duration say minus T by 2 to plus T by 2. Then, what is the possible values of this signal, the carrier signal or the band pass signal x of t could be? And you can write down, plot it and then see for yourself that once I have specified the in phase and quadrature components it is rather easy for me to obtain the bandpass signal.

So, this is bandpass signal x of t right. So, this is an exercise for you, you can take a look at this, but do not be conditioned into thinking that this is the only way we can actually describe signals. There is in fact, an important way of describing signals which is called as the vector or treating the signals as a vector and then utilizing many of the geometrical properties of a vector space to understand more about the signals. So, you have to understand something that is very interesting out there. So, signals are usually continuous quantities or may be discrete quantities, but if they are continuous, they would have an infinite number of values over any finite duration, right. Because, it is a cosine 2 pi f 0 t.

For example, if you take a duration of 10 seconds, in that 10 seconds the number of times is cosine 2 pi f 0 t or the values that it would take, but actually be you know infinite, the numbers are so, there are so many numbers because, it is changing continuously. But, then when you think of a vector, you do not normally think of a vector having this infinite number of values. Although you can, but this kind of a mapping or understanding or equivalence between a signal and a vector does not really seem to be that correct, that is what you would think at first, right.

But that is not correct. There is in fact, a one to one correspondence between a signal and a vector and whatever operations that you can do to a vector, you can actually perform the same operations onto the signal and in fact, treat a signal as a vector. The defining characteristic of a vector is not it is direction or magnitude. The defining characteristics of the vector are slightly different and we are going to talk about those things.

So, vectors are those objects, which usually follow certain rules. For our purpose, if v is a vector and u is another vector then, their sum v plus u should also be another vector w and they should also fall in the same space as we would call it. It is possible to take a vector multiplied by a scalar real or complex and then get a new vector, but the most important thing about a vector is it is geometrical properties. We define the inner product of two vectors v and u or the dot product of these two vectors u and v as the length of u, the length of v and then the angle between the two vectors. Here, we have used the length in a very specific manner up there.

So, this is a symbol called as norm. And in the norm essentially gives you the length of that particular vector, indeed the norm is actually defined as the inner product itself of the vector by itself and the square root of that one. So, the norm of v itself is defined by first defining the inner product and then talking about the or taking the square root of the length and you can show that when you are looking at vector of the same direction, I mean, the inner product of a vector with itself; obviously, that is going to give you the length square of the vector. Correct this is something that we already know from our earlier vector space theory.

So, the norm is actually square root of the length and it is usually taken to be a positive quantity. So, these are all the geometrical properties; why because, if two vectors are orthogonal to each other then, the inner product of the two vectors will be equal to 0.So, the dot product being equal to 0 indicates that, these are vectors which are orthogonal or perpendicular to each other. So, one of them is v; the other one is u.

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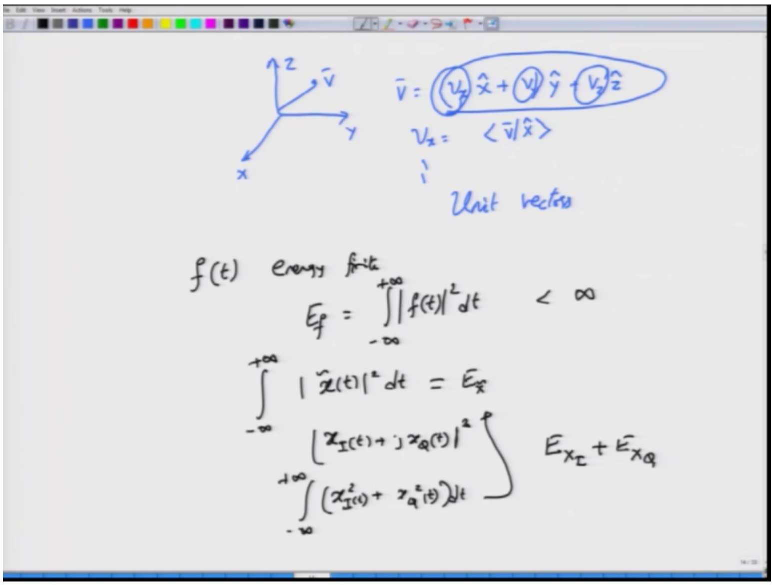

Then there is another way of thinking about a vector, is to first think of a coordinate system. So, in the 3 dimensional cartesian coordinate system we are all familiar with it. Any vector here can be described by giving it is values or components as we would call it, v x x hat plus v y y hat plus v z z hat. What are these values v x, v y and v z? v x is, in fact, given by the inner product of the vector v with the unit vector x and similarly v y and v z. And this vector x hat y hat and z hat are called as unit vectors that completely specifies all the other vectors in this 3 dimensional space.

So, if you know this x, y and z any other vector can be constructed as a linear combination of these vectors x, y and z. And you can obtain the value of that combination or the weight of that combination v x, v y or vz by going to this expression. That is taking v and then taking the inner product of that v with respect to the appropriate unit vector.

Now, in a similar manner, suppose I am given a signal f of t whose energy is finite. So, this condition I am imposing to avoid some mathematical problems, but there are various ways to overcome this constraint, let us not go into too much of a detail there. But if I have f of t whose energy is finite, recall that the energy itself is given by whatever the value of f of t that exists over or the value of f of t square or the magnitude of f of t square over the time interval over which f of t is defined in case it is not specified, we will let this be minus infinity to plus infinity.

And we want this quantity to be less than infinity so that we can talk about energy of these signals. And I have used magnitude square to denote the fact that f of t could be complex. For example, a complex envelope was actually a complex signal. So, if you go back to the complex envelope, take the magnitude square of this complex envelope and then integrate it over whatever the range the complex envelope is present or it is defined, then this will give you the energy of the complex envelope. And you know what is x tilde of t that is actually x I of t plus j x Q of t, x I and x Q are real.

But, if you take the magnitude square, you are going to get x I square plus x q square of course, this is a function of time here, no doubt and then integrate this one individually, I mean integrate this one over time to obtain the total energy E x tilde and clearly you can see that this energy will be split into the in phase component energy plus the energy in the quadrature component.

So, you have E x I plus E x Q, where x I and x Q are the energies or Ex I and Ex Q are the energies of inphase and quadrature components. So, this is something that is now intuitively appealing to us because, energy of a complex signal has been defined.

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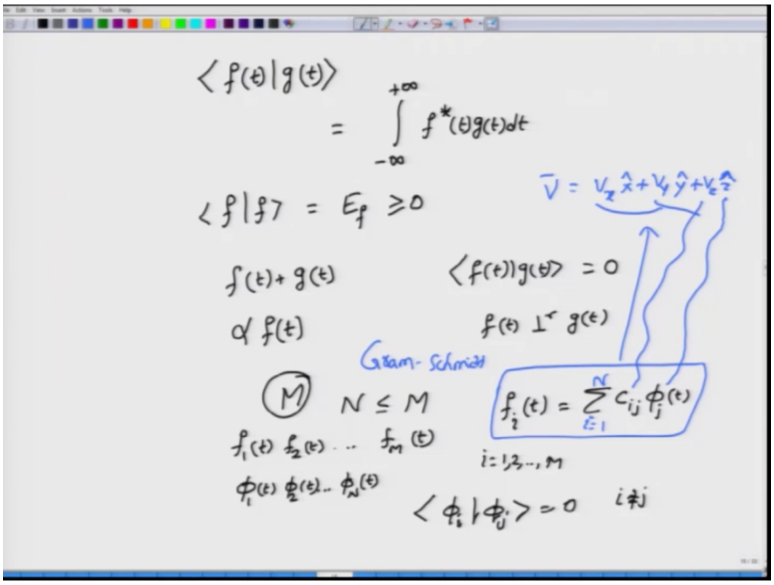

What if we define the inner product of two complex signals which we will call this as f of t and g of t in this manner, assume that both are energy signals and both are complex.

So, you have over the integration range whatever that might be, if I define this as the inner product will it satisfy all the properties of an inner product? Yes, because, if you take the inner product of f with itself, what are you are going to get? You are going to get energy E f. Why do you get this one? Because, f conjugate times f will be magnitude of f square dt integration, that is precisely what we have as E f. And E f being energy quantity will always be greater than or equal to 0.When will it be equal to 0? When f itself is equal to 0, right. And to any signal f of t you can add another signal g of t and the sum will be denoted as the point wise addition of the two signals f and g. And you can also take a scalar multiply that scalar to f of t.

So, it seems possible that those energy class that is all signals whose energy is finite actually define a vector space. The dimension of the vector space is something that we are going to talk about it, but the idea is that they can be thought of as vectors. And again in the analogy of the perpendicular nature of the vectors, you can talk about it, right. So, if f of t, if the integration of these 2 waveforms turn out to be 0 then, we will call f of t to be orthogonal to g of t. And given that you have some M energy signals, then it is possible for you to find N energy signals whose energies are actually equal to 1 and N will be either less than or equal to M depends on the signals that have been given to you. Suppose M signals are labeled f 1 of t, f 2 of t and soon all the way up to f M of t, then it is possible for us to find the signals phi 1of t, phi 2 of t all the way up to phi N of t, the number N will being smaller than M.

So, you have phi 1 of t, phi 2 of t and phi N of t, such that any ith signal, which where i can be 1,2 all the way up to M can be expressed as a linear combination of this phi j of t signals. The meaning of phi j of t being orthogonal simply means that, integral of phi r or you knows now that we know the inner product itself we do not need to go to the integration thing.

So, what it simply means is that, phi inner product with phi j will be equal to 0, unless i is equal to j. So, if i is not equal to j then, these two will be equal to 0 and of course, we know that these signals are then called as orthogonal to each other, but because their energy is actually equal to 1, they are also called as orthonormal and because in terms of these orthonormal functions, you can express any other signal f of t in the same class of functions of finite energy, then these are called as orthonormal basis functions.

And given M signals, it is always possible for you to find these N signals by utilizing many procedures, one of them is called as the famous Gram Schmidt orthogonalization procedure. So, the Gram orthogonalization procedure will tell you this. So, as a combination of f i of t or the combination of phi j of t with appropriate weighting factors, you can express any signal f i of t of the original M signal that you had.

Now, here is an interesting thing. So, now, look at the way we write a vector v which is written as v x x hat plus v y y hat plus v z z hat in 3 dimensions of course, otherwise you will have to write this one in multiple dimension. Now, it is very clear that each of these phi j's are like the unit vectors, each of these c ij's are like the three components or many components of the vector v along x, y and z.

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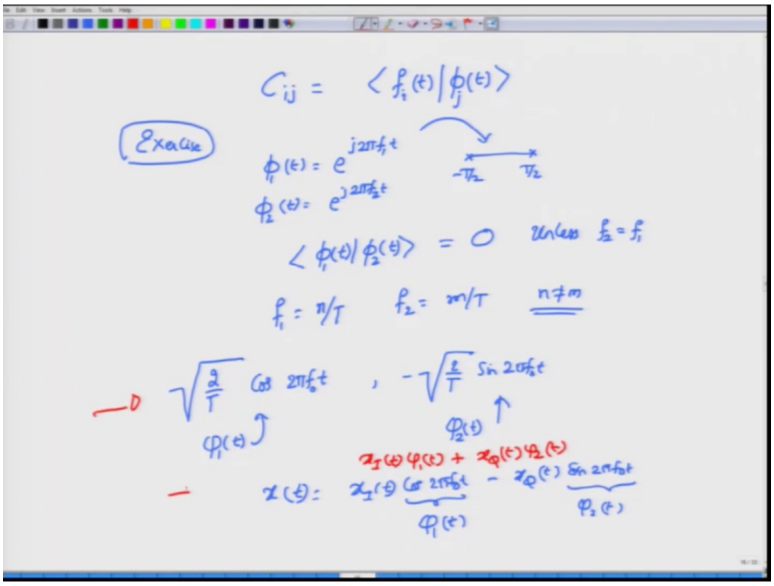

So, clearly I can obtain c ij that is, I can find out the component by taking the inner product of the signal fi of t with the signal phi j of t. If they are all real then, all the complex conjugates will disappear. So, it your life becomes much more simple. So, this is something that we have. I will give this as an exercise to you, given that you have phi 1of t equals e to the power j 2 pi f 1t and then you have phi 2 of t equals e power j 2 pi f 2 of t defined over an interval of minus T by 2 to plus T by2.

That is, all these signals or all these two signals phi 1 of t and phi 2 of t are defined to be in this manner over this duration can you show that phi 1 of t is orthogonal to phi2 of t that is, their value or the inner product will be equal to 0 unless f 2 is equal to f 1. You can also further assume, f 1 to be an integer multiple of the duration or inverse duration 1 by T and f 2 is some m by T, n is usually not equal to m.

So, I believe this exercise this is very very important exercise please do workout this one so that you can better appreciate the concepts of orthogonality. Let me also give you two additional functions, one is square root of 2 by T cosine 2 pi f naught t and the other is minus square root of 2 by T sin 2 pi f naught t. What do you think about these two functions? This function is phi 1 of t and this function is phi2 of t. Again, I would like you to see whether these two are orthogonal to each other, f 0 is the same for both.

So, you would actually expect that these two signals to be orthogonal because, they are actually in quadrature and if you do that you will be actually right. And you can use these phi 1 of t and phi 2 of t to represent any modulated signal, because, see, any carrier signal x of t would have been written as xi of t cos 2 pi f naught t minus x Q of t sin 2 pi f naught t.

Now, in terms of, you know basic functions, phi 1 of t and phi 2 of t, you can write this one as phi 1 of t and this one this minus sign including as phi 2 of t and in fact, you can write x of t as x I of t phi 1 of t plus x Q of t phi 2 of t. So, in this case x I of t and x Q of t correspond to the time varying coefficients that would get multiplied to phi 1of t and phi2 of t and of course the small difference between the way we have written that that when write the square root of 2 by T factor is also present. You can include that square root of 2 by T factor either in definition of phi 1 and phi 2 or you can push them in the definitions of x I and x Q. If you have done this exercise, after showing that this asorthogonal and understood this one.

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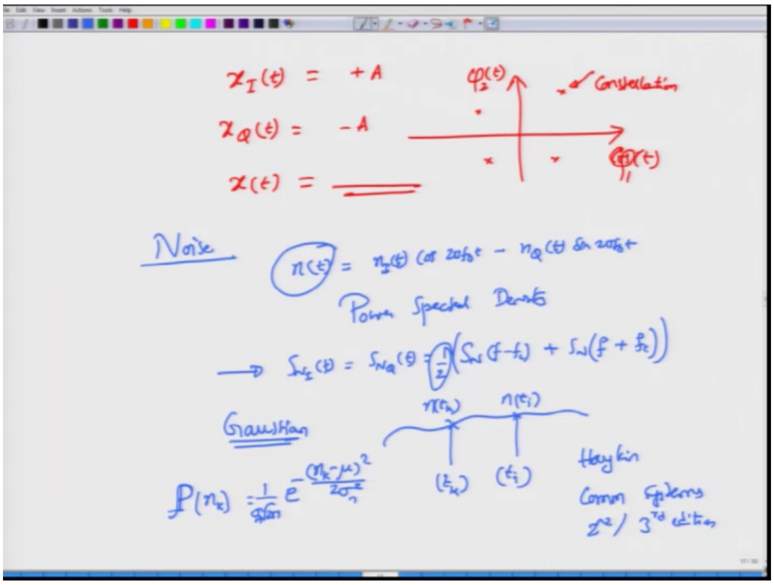

Just try your you know skills of writing a signal as a vector provided x I of t for this problem, where we have given x I of t as plus A and x Q of t, we have given this as minus A, right.

So, then you can find out what would be your x of t in terms of I and Q and then you can construct 2 dimensional space out there. One corresponds to the cosine or rather phi 1of t term and the other one would correspond to phi 2 of t. You can think of cos 2 pi f naught t as the x axis sin minus sin 2 pi of t as the y axis and then locate what would be the corresponding x of t signal here. And whatever that you would locate will actually correspond to what is called as the symbol point and a collection of such symbols would then be a constellation.

And the spacing that you have kept between these constellations the way you have put them up will all determine what kind of modulation format you are actually going to use and are we going to transmit. So, in understanding the modulation formats, digital modulation formats, it is important for you to know that these constellations are essentially 2 dimensional diagrams of the inphase and quadrature components, which in this case we have written as phi 1 of t and phi 2 of t.

So, this is the thing that I wanted you to understand. Now, we talked a lot about signal, but many similar ideas also apply to noise, meaning that noise also can be expressed in the canonical band pass way that you have n I of t cos 2 pi f 0 of t minus n Q of t sin 2 pi f 0 t; however, noise is not characterized by it is Fourier transform, but it is characterized by it is power spectral density and you can actually show that the power spectral density of the inphase and quadrature components, given that this power spectral density of the noise itself is given by this expression.

So, that would be S N of f minus f c times S N of f plus f c. The exact proof or the better ideas about this one, you will not be you know discussed here, rather if you want to know this particular thing you will have to maybe this is not just one this is about half. So, these factors of half, one they are all just scaling factors, if you have to be consistent with that, but for you know understanding the ideas this scaling factors are not really important.

But, this is exactly what you are going to get for the in phase and quadrature components. So, that is something that you should know and you should also know that, we are mostly going to deal with noise which is Gaussian in nature. What it means is that, if this is a noise process, when you sample this noise at a time t k and then sample the noise at another time t i, what you are going to generate will be a noise, a random variables actually random variables. And these random variables are assumed to be uncorrelated with each other and the probability that or the probability density function of this random variable will actually be equal to or given by e to the power minus n k minus mu whole square divided by 2 sigma square one by sigma root 2 pi. Where sigma or sigma n, let us say is the noise variance as we would call it, mu is the average most of the times we are going to deal with noise sources which are actually having an average of 0.

So, in that case mu will be equal to 0 and in case the amplitude and what this equation this is called as the Gaussian probability density function. What it would tell you is that the amplitude n k of the noise would follow certain equation and it would look be most of the times average it will be near average, but occasionally it will be deviating from the average. And again this is a statistical description and I cannot you know go to a lot of other interesting things about noise here, I will leave all that thing to you know review for you from the undergraduate textbook.

One of the books that I recommend to know more about these properties of signal as well as noise as well as representation is Haykin Communication Systems, third edition or second edition. So, this plentiful detail about all these properties and please review these properties because, we are going to use lot of these properties in understanding the performance of optical fiber communications in the next module, starting in the next module.

Thank you very much.