Fiber - Optic Communication Systems and Techniques
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Lecture – 43
Noise in Photodetectors

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques.

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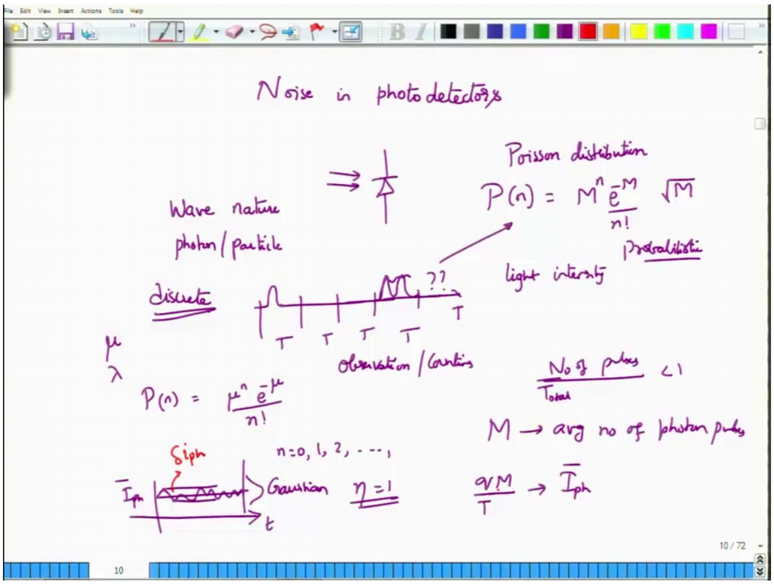

So, what kind of noise does a photo detector produce? Now, we let us look at the photo detector out there and then shine some light onto it. It turns out that this light actually possesses two complementary characteristics or one can think of light as having possessing two complementary characteristics; one is the wave nature and then the other is the photon for the particle nature,.

So, if you reduce the light intensity so much that on an average you are essentially getting less than one photon per time unit, then what you actually observe will be photons that would arrive kind of randomly ok. So, for example, if I consider the units of time to be equal and then I am observing the photon pulses arriving, then sometimes you get one photon most of the times, you do not get a photon, sometimes you actually get two photons and so on.

In fact, the number of photons that you receive in a particular time interval T, T is called as the observation time interval ok. So, in that observation time interval or sometimes called as counting interval, what you actually see is that these photons do not arrive in a steady manner and they actually arrive with some amount of randomness within them.

So, this of course, randomness manifests very nicely when the light intensity is kind of very small so that on an average that is if you were to count the number of pulses and then divide by the total observation time, right the number of pulses in the total observation time that you take then this average will be much less than 1. So, this is when the light intensity is actually quite small and then you can observe if at all you had an ideal photo detector which could record each arrival of the photon, then you would be observing that this photons arrive with a certain randomness out there.

And in fact, they are described to a very good approximation by a probability function called as Poisson distribution. You might have heard of Gaussian distribution of the arrival times, but in this case these arrival times or the number of pulses that arrive on an average at value of an observation interval T is governed by the probability distribution called as Poisson distribution given by probability of occurrence n given that there are M pulses on an average arriving.

So, M is the average number of photon pulses that are arriving when the time units have been segregated equally; so, when you know the average value, then the actual number of photons that you obtain or the probability that there are n photons in a given interval, so this is the interval T. How many photons here the probability that it will actually contain n number of photons is given by M e to the power minus M divided by n factorial, sorry M to the power n e to the power minus M by n factorial.

In most literature instead of using M, you would find it as mu or sometimes as lambda and appropriately the probability of n will be given by mu power n e to the power minus mu divided by n factorial where n of course will start from n equals 0 that is no photons arrive in a given time 1, 2 and so on. Please note that this is probabilistic we will have to say more about this probabilistic nature later on in when we start actually looking at communication systems, but the idea is that you are never predicting that given the average value M known to us that there will be exactly n photons in this interval. No, we are not predicting that there will be exactly n photons, but rather giving the probability that there could be 0 photon, there could be 1 photon, there could be 2 photon and so on and so forth.

It turns out that the average of this one is M anyway you can show that, but the variance of this distribution is also equal to M and the R M S value will be equal to square root of M ok. Each pulse that would arrive will assuming efficiency eta equal to 1 that is to say each photon will be able to convert into 1 given current pulse and because each current essentially this electron hole pair carries a charge of q right.

So, when they go in their separate ways, they give you a current or because it corresponds to a current or a charge of q coulombs crossing over the time observation time interval T each arriving pulse would give you a current of q by T and if there are M pulses on an average, the average photon current will be about q M by T ok. So, this is the average photon.

But so, if you actually look at how the total number of photons go, it would be something like this. So, there is an average value I p h and then around that there will be these fluctuations. Of course, in writing this graph or in plotting this graph, I have assumed that this interval T kind of goes to 0 that is you are considering shorter and shorter intervals and when you consider shorter and shorter intervals and raise the intensity level which is important raise the intensity level to reasonably large value say in the range of about micro and micro watts or maybe about milli Watts and so on, the intensity levels earlier that we were talking about were in the range of about pico Watt ok.

Now, if you rein raise them to about two orders of magnitude higher, you go to micro amperes or milli Amperes at that or micro watts and milliwatts at that light intensity levels, you and as the observation level shrinks the distribution Poisson distribution essentially becomes Gaussian distribution and you would not be able to, I mean you will be able to say that most of the pulses on an average contain certain average power which translates into an average current which is given by I p h.

So, this distribution around this which is the distribution, right, so if you look at what is the total number of counts, the counts can increase from the average value or decrease from the average value that distribution essentially becomes Gaussian as the average photon current increases. Now, average photon current increases means that the optical power that you have lost also increases which means that M value increases.

But please remember that although we are assuming a continuous kind of a behavior over here inherently, this is a discrete nature. Inherently, photons are not emitted in a continuous stream ok. They are emitted with this probabilistic scheme only. Only that the uncertainty kind of reduces as the average photon number increases that is to say as the intensity of the optical source increases, you start to observe more or more and more of classical probabilities compared to the earlier discreteness.

So, the discreteness kind of washes out because now the pulses start appearing closer together and overall pulse would essentially start to become or at least look like continuous, but inherently this is a discrete process and this discrete process gives rise to uncertainties right. So, this difference in the current that I am in that you would see around the average will be called as I p h with a small I or maybe we will call this as delta I p h to show that this is actually a short variation or a I mean small magnitude variation around the mean photocurrent I p h and how much would be the variance of this photo current.

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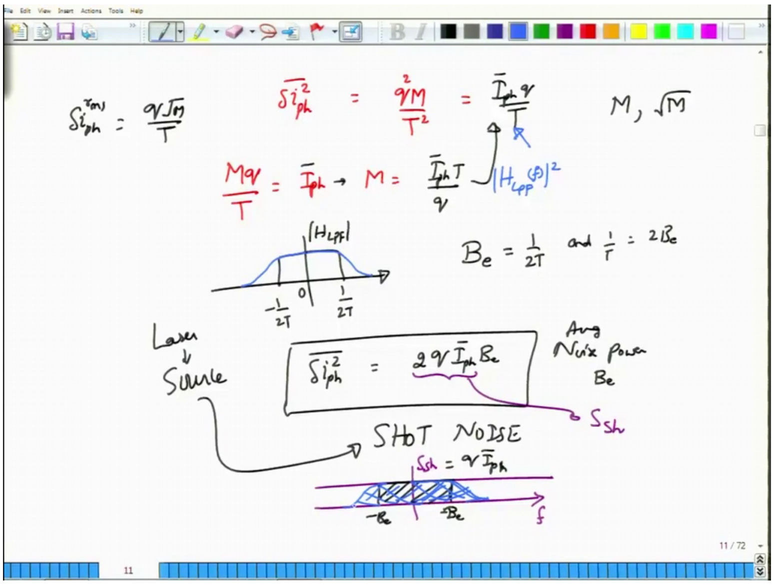

Or the you know the noise current that variance which is the squared value on an average is equal to q M by T which turns out to be just I p h. I am sorry the variance actually turns out to be M and then, you have to square this up right. So, M square M q square by T square right.

And then, you look at the value of M, I know M is given by I know M q by T corresponds to the photo current I p h and therefore, M is by interchanging this equation M is equal to I p h times T divided by q right. Of course, that is the correct expression. Now, you put this value of M into the above expression. Please note that we got this q square M by T square because the R M S value of this 1 was actually q square root of M divided by T. This was the R M S value.

Why? Because this is precisely what we said the Poisson distribution with an average value of M will have a variance of square root of M ok. So, that is where this came from and square root of M number of photon pulses would give you an R M S value current of q square root of M divided by T. So, R M S square will give you the mean square value or the variance of this one which is q square M divided by T square. I know M to be I p h T by q therefore, what I get here will be I p h q by T and if I consider this photo detector is followed by an ideal low pass filter whose bandwidth is about 1 by 2 T. This is the baseband bandwidth that we are talking about which is perfectly valid in this case because the signals once they have been converted from the optical domain to the electrical domain are now centered at the baseband level, they are centered at 0 frequency.

So, for an ideal low pass filter, this is an ideal low pass filter has a characteristic of the nature that I have given here this is the magnitude response it will be having a bandwidth of 1 by 2 T. So, if you denote the bandwidth of the electrical low pass filter as B e, then B e is equal to 1 by 2 T and therefore, 1 by T will be equal to 2 times B e what is the value of or what is the use of this expression 2 B e because I can go back and then substitute into the variance formula and then get 2 q I p h this is on an average therefore, I will put a bar over there. So, this is I photon average photon current times B e.

So, this is an important expression this is telling you the noise power. Of course, this is not any noise power; this is average noise power that you would find in a low pass filter following the photo detector with the low pass filter of an ideal bandwidth B e. This is the amount of short noise that you get or rather this is amount of noise power which is called as short noise of the device. And please note that in this derivation of the short noise expression, what we have not said I mean what we have said is all about the source uncertainty right. So, we have not actually considered the short noise of the device, but the device is assumed to be ideal. Whenever there is a photon pulse it will generate a current pulse and therefore, it is actually the source which exhibits uncertainties and this is the source short noise and this source which is a source that is our laser source. So, whatever the short noise that is resulting because of the photo detector even in an ideal photo detector is simply because of the laser itself or the light source itself. So, this is the source short noise.

And you can see that this is over the bandwidth B e, but this quantity that is here right. So, this quantity that you have 2 q I p h bar is actually the spectral density energy spectral density or the power spectral density of the short noise which we will write it as S with a subscript of s h and if you plot this S as a subscript, I mean S s h as a function of the frequency f, then you will see that it will be completely flat and therefore this is called as white noise, but your bandwidth will be only of certain range right. So, you are interested only in the range of bandwidth of plus B e to minus b e and over this range what you get will be the total noise power, ok.

So, the spectral density will have to be written as q I p h so that when you take the two sided power total power it will turn out to be two times q i p h bar times B e which is exactly the expression that we have derived from a heuristic understanding. So, please note that this is not completely rigorous derivation, there are rigorously derivations available we do not want to go into that.

What if you have low pass filter, but that is not ideal. So, for example, one of these low pass filters let us say has a nice roll off in this manner. So, in that case what would be the current then you have to go back to this expression and then relate this value of T to the value of this low pass filter characteristic. Alternatively, you can simply take the non ideal low pass filter characteristic and then multiply and integrate over the bandwidth so as to obtain the total short noise power. So, we assumed the short noise to have this spectral density which is perfectly fine.

Now, instead of integrating over this bandwidth which is rectangular bandwidth, you now have to integrate it over the actual non ideal low pass filter and when you integrate that one, you will get the total short noise that is contained in this bandwidth ok. So, that is important and there is another peculiarity that you must have noticed at this point.

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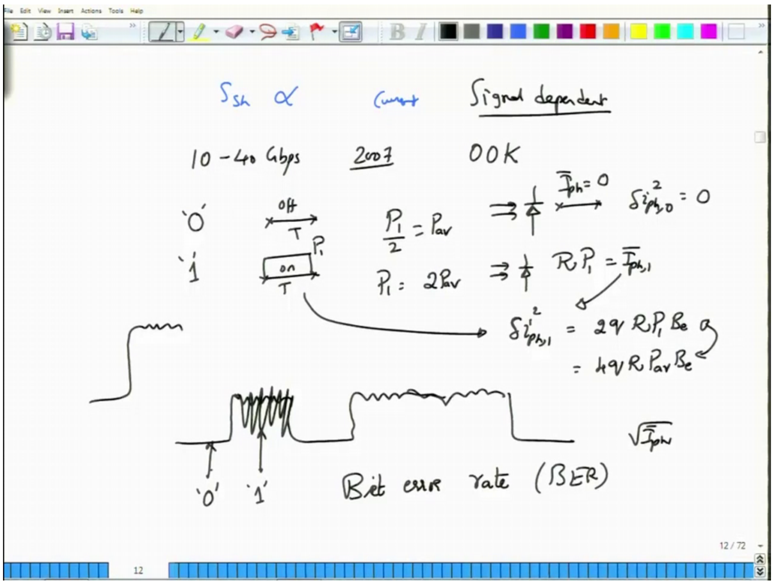

Short noise spectral density is dependent on signal power signal being the current here and therefore, it depends on the signal or the current itself which is very interesting because there is a certain communication system that has been widely used in optical fiber communications for a bit rate of up to 10 to 40 Gbps ok. This now is kind of out of fashion this went out of fashion in about 2007 and that is called as on off keying. We will of course, see more about this one off keying later on, but for our purposes what does on off keying mean is that if you want to send in a symbol 0 on an optical fiber using light means, you send a pulse of duration T seconds, but do not send any photon in this. So, which means that you simply turn the laser off during this time and when you want to send in a pulse of 1, you send in over the same time duration some amount of optical power. So, we will call this optical power as P 1.

So, on an average of course, you are sending P 1 by 2 which is my average power. So, in terms of the average power I can write down P 1 as 2 times P a v. So, what you see is that this is one that is laser is on to transmit a symbol 1 and the laser is off to transmit a symbol 0. When you send a 0 and put a photo detector of course, to detect and obtain the electrical pulse over that duration, you would not see any photo current because assuming all the other noise sources of the photo detector and the system are 0, then I p h will actually be equal to 0. On an average, you do not get any photon I mean you do not get any current because there has been no photons send when symbol 0 was transmitted.

However, the same photo detector we will generate a current of R P 1 when you send in a pulse of power P 1 over a certain duration T. So, this current will now be the current. So, I will call this as I p h 1 comma 1 bar just to denote that this is the average photo current and this is the current that would actually go into the photo current in term I mean that would actually be generated in the photodiode and because there is this current, there will be short noise which we will write it as delta I p h 1 s h square ok. So, this is the shortness or maybe we will remove this s h because we understand that this is because of the photo current. So, we can write this as delta I p h comma 1 this comma one indicates that this is the noise that is generated when you transmit bit one and of course, this has a variance of 2 q r P 1 over the bandwidth B e in terms of average power, you can write this as 4 q r P average times B e, but expressions are essentially the same because average power is P 1 by 2.

So, this is very interesting. Delta I p h 0 that is short noise will be 0 when you transmit 0 symbol and it will be non 0 when you transmit 1. So, this is actually signal dependent noise. So, this is very important. This is called as signal dependent noise and what is the implication of the signal dependent noise if you transmit 0 and 1, the 1 will have lot of noise; 0 will have no noise. 1 will have noise, 1 is noisy compared to 0. So, this is how the bits would actually be received and if this noise exceeds a lot then there might be some situation where when you sample the output these sampled outputs will be so close to each other that you would not be able to tell whether this came from a symbol 0 or it came from a symbol 1 and therefore, you commit errors called as bit errors and the rate at which you commit this error is called as bit error rate, B E R that characterizes this on off keying system.

The only way to actually counter this on off keying or bit error rate is to actually increase the power so that while noise increases, noise increases only as square root of I p h 1 average right the R M S value will increase only as square root of I p h 1 whereas, the power will increase as I p h square. So, let me just do that.

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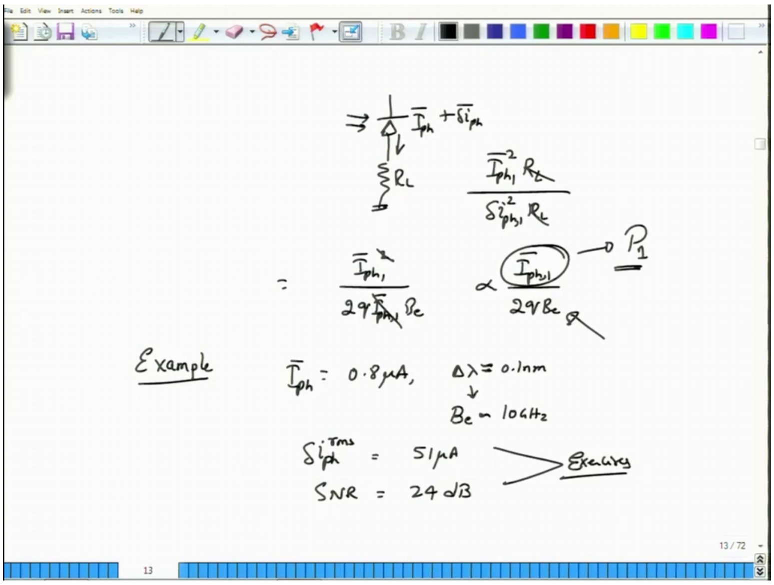

I have a photodiode I have a load resistor R L. So, if there is an average current I p h because of the excitation of the photodiode or by incident light onto the photodiode, it generated a current of I p h and this current passing through a load R L would generate a power average power of I p h 1 bar square R L. Now, this is simple I square R kind of an equation whereas, the noise power that is generated. So, your actual signal will be delta I p h as well. So, the noise that is generated will be delta I p h 1 square R L ok. So, I p h 1 bar square is anyway the same thing. So, this is bar square R L divided by of course, R L cancels out. So, we do not have to carry this R L around. So, R L cancels out, but what is the variance of the short noise the variance of the short noise is 2 q I p h 1 on an average times B e.

So, you see that the SNR is actually proportional to the current that you are generating in the photodiode and of course, this current is proportional to the optical power that is launched when you send a bit 1. So, the only way to increase the signal to noise ratio is to actually raise the input signal power. So, that the current raises and therefore the short noise although raises only by square root and therefore, you are saved by this mechanism.

So, to counter or to reduce the be here you can increase this one. I will give you an example a numerical example I will leave the calculations to you. Suppose, the average photo current turns out to be some 0.8 micro ampere and a bandwidth delta lambda is about 0.1 nanometer ok. You can calculate and then show that the bandwidth is approximately 10 gigahertz. This is not 0.1 about slightly less than that, but anyway. So, you this is approximately 0.1 nanometer, this is approximately 10 gigahertz. If you assume that this is the electrical bandwidth B e, calculate what would be the R M S value of the short noise and this R M S value will be approximately 51 micro ampere and also calculate the signal to noise ratio which will turn out to be 24 Db. I will leave these two as very simple exercises to be performed. The equations are there in front of you. This is an expression for the signal to noise ratio. You calculate what is the current that you have transmitted or given the value of the current, you can plug in to those values. You know the value of q, you know the value of B e and then, you can find out that this signal to noise ratio is about 24 Db.

This is not the only noise source that your that is that you will be seeing in a photo detector circuit.

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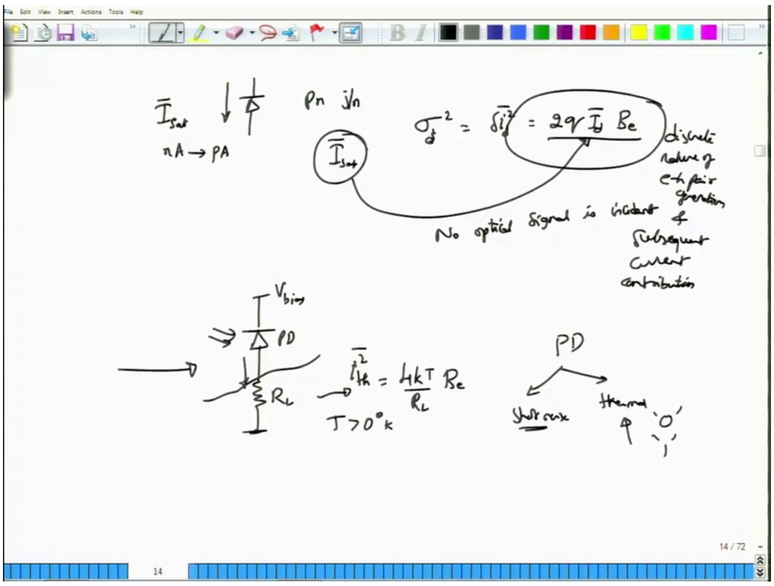

In fact, the photo detector construction itself if you remember is actually a p n junction right and then there is a p n junction when you reverse bias it, there is a reverse saturation current and this reverse saturation current is not out very small usually in nano ampere, but very good photo detectors actually pull this noise down to all the way down to pico ampere and they do this by actually reducing the surface because it can be shown that this reverse saturation current is proportional to the surface area. So, if you reduce the shrink that, then you can bring down the currents by order of magnitudes. So, from nano ampere to pico amperes, you can bring this down, but this is also a noise or rather this is also a current right and because of this current, there will be on an average current there will be a noise that is generated which we will call this as sigma d square which is the variance of this one or if you are not comfortable with that I can write delta I d square on an average.

And I will give you the value of this one as 2 q I d bar times B e because the terminology in photo detector is slightly different. We do not call it as reverse saturation current, we call it normally as the dark current with the assumption that no optical signal has been incident, no optical signal is incident on the photo detector and assuming of course, the photo detector is now operated in the reverse bias which is any other requirement. You will see that the noise that is generated because of the dark current or the saturation current is given by this expression.

It may seem very strange that dark current actually has the same expression, but the source of this one is not the light source that is the noise here is not because of the lights or because there is light source is not present in this case. In fact, this is because of the discrete nature of the electron hole pair generation and subsequent current contribution. What we mean to say is that, when the photons are by themselves coming in like the form of bullet us you shoot bullet sometimes, you do not shoot bullet us sometimes, you shoot bullet us one after the other continuously or maybe sometimes three bullet us come out together. So, it is kind of a short discrete nature.

A same situation occurs in a p n junction as well in the p n junction, the electron hole pairs that are generated they are also created discretely and when they move, it is like you know shooting a gun into an empty can. So, there is a short noise because of the source there is short noise because of the detector process itself usually when light is turned on this detector short noise can be neglected, but when light is turned off, the detector short noise cannot be neglected and that is what is the source of this expression to q i d bar B e which is exactly the same expression as the input.

So in fact, in any short noise you know situation when there is any discreteness situation and you are doing the counting application you would essentially encounter short noise kind of expressions in all these scenarios ok. So, that is what the source short noises and this is the detector short noise and these two are all also present together. Of course, when there is actual current because of the light then that would be added to the dark current, but this value is, so, small that you normally neglected.

Then there is another source of noise and that source of noise is the photo sorry the load register that we actually keep because the load resistor registers themselves or noisy this is the bias voltage, this is the photo detector, light is shining here, current is coming through. Even if you assume that all these currents are ideal there is no short, noise nothing because the resistor is made out of an actual material which is kept at a temperature T greater than 0 degree Kelvin that is thermal energy and this thermal energy contributes to the current noise of 4 K t by R L that is the into times B e of course, where B e is the bandwidth of the electrical filter and this is the thermal noise generated by the photo detector given by and this is the current thermal noise that is generated by the photo detector.

So, this is this is also something that would be added. Now, usually photo detectors are operated in two limits; one is the thermal limit and the other is the short noise limit. Of course, most photo diodes today are operated in the short noise limit because there you get excellent sensitivities and other things the problem in thermal noise is that this thermal noise is independent of the bit transmitted. So, it will be the same for bit 0 as well as same for bit 1. Of course, this is not the photo receiver at all because I mean photo detector receiver that is normally used because this has lot of problems.

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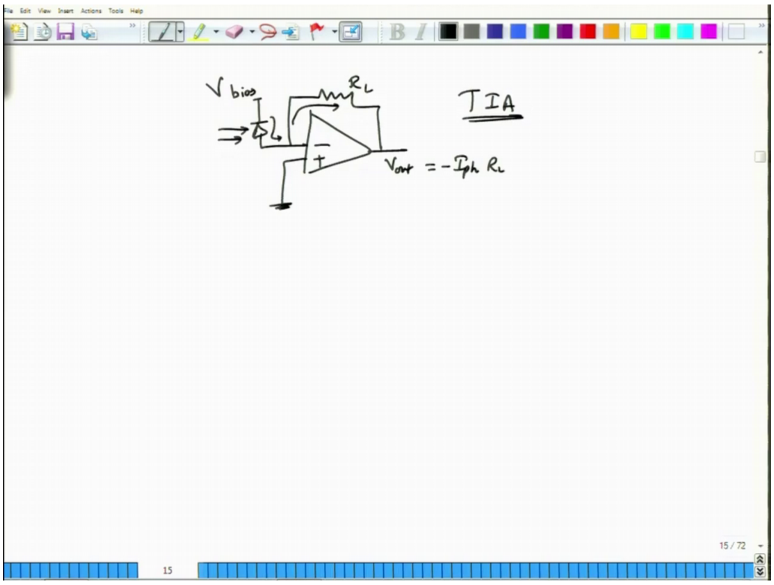

So, the actual circuit which is quite popular is what is called as the trans impedance circuit in a trans impedance circuit, you take the photo detector connect directly onto the non inverting terminal bias the photo detector in the reverse manner so that when light is incident there will be photo current generated.

If you assume that the op amp is ideal, then this current instead of going through the op amp will actually get routed and pass through the load r here and then of course, whatever the output voltage that you would see would of course, be given by I p h R L itself. So, this is V out is equal to minus I p h R L; minus sign is simply because you are operating a operational amplifier in it with by connecting the input to the non inverting terminal. So, this circuit is called as transimpedance amplifier and the actual noise analysis of the transimpedance amplifier is quite complicated, but this is a topic that is thoroughly covered in many analog circuits courses and I would actually request you to find a course. NPTEL is also running on analog circuit's course. So, you can actually look at the noise analysis. So, here noise is because of the short noises and the thermal noise and the amplifier itself is going to generate noise because it is actually made out of transistors and other devices.

So, this completes our noise analysis in photo detectors. The implication of noise and there is also some interplay of this noise with the erbium doped fiber amplifiers that we will consider in the next or next come upcoming modules. The implication of this on to optical communication systems is what will occupy our interest after a few modules where after we talk about the optical components.

Thank you very much.