

**Fiber – Optic Communication Systems and Techniques**  
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**Lecture – 37**  
**Optical properties of semiconductors-I**  
**(Direct bandgap and indirect bandgap, Density of states)**

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques course. In this module we will look at semiconductors and we look at the Optical Properties of Semiconductor. We have been seeing semiconductor basics in the last module, we have seen that you have intrinsic semiconductor, you have extrinsic semiconductors, extrinsic semiconductors are those which are doped by additional atoms. These doping can result in the semiconductor being turned into n type semiconductor where, the majority carriers will be electrons and the p type semiconductor where, the majority carriers will be the holes and then you can also form pn junctions.

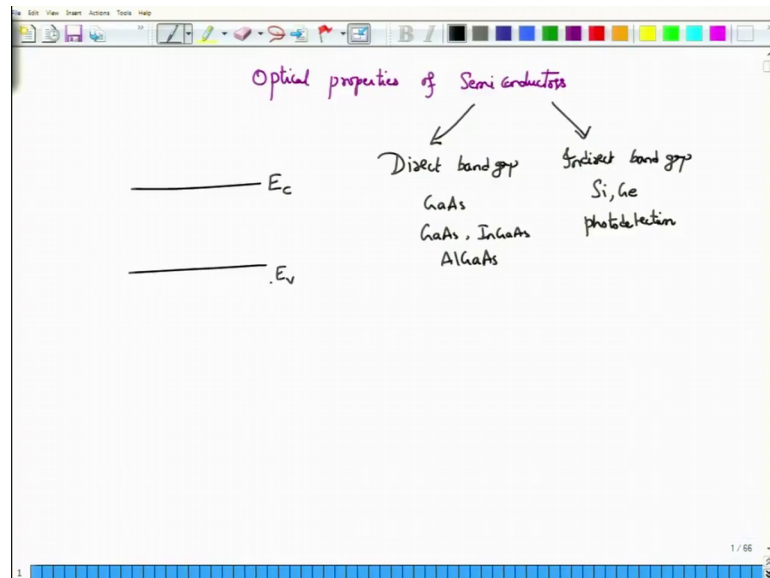
But before we look at junctions and the how to make a laser, we need to understand the processes that are involved in typical lasers, how they are affected or how they are actually implemented or how they are related to semiconductors. Please recall from our laser basics that we need, 3 elements to make laser one is pumping, which moves atoms or molecules in the ground state or the energy state even to a higher energy state into creating what is called as a population inversion.

So, pumping is the first element of a laser then, to achieve high gain it is necessary to incorporate optical feedback in the form of mirrors or cavity mirrors and it requires a separate cavity design as such, but that is a second essential element of a laser. And the third is the active medium itself, the active medium is the one which possesses the property of being able to create a population inversion and when there is stimulated emission that happens, then this active material can sustain the stimulated emission and therefore, form what is called as a laser.

Now, pretty much the three elements are that are necessary for a laser can be found in semiconductors as well. Here of course, there are some differences in the actual generic laser versus a semiconductor laser, which gives semiconductor lasers their own

advantage of being compact and high efficiency and reasonable gain in the semiconductors. So, let us tackle the first problem of optical absorption and optical emission. Of course, we will come to these topics in detail slightly later, but we will begin with understanding when optical absorption is possible and when it is not possible. So, to start with let us first identify 2 types of semiconductors.

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Now, I am talking about semiconductor as the active medium and population inversion is actually created slightly differently from the laser basic. We will not cover in this course how to make the feedback although, I will touch upon it in very short in the next few modules as to how to actually make the feedback in a semiconductor laser to improve its gain and its wavelength or frequency selectivity. And the pumping in this case does not rely on external radiation, there if it is not optically pumped laser, but it is electrically pumped laser. So, what we mean by this is that, all that is sufficient to you know create pumping in semiconductor laser diodes is to actually have a current and forward bias the junctions that are involved ok.

So, from this you know big picture it is very clear that, junctions are going to play an important role and we will look at what happens when you bring two different materials together and how they can be used for lasing. But before that, let us first look at what types of semiconductors are actually suitable for I know making themselves as a laser material. In that case, we distinguish 2 types of semiconductor, the first one is called as

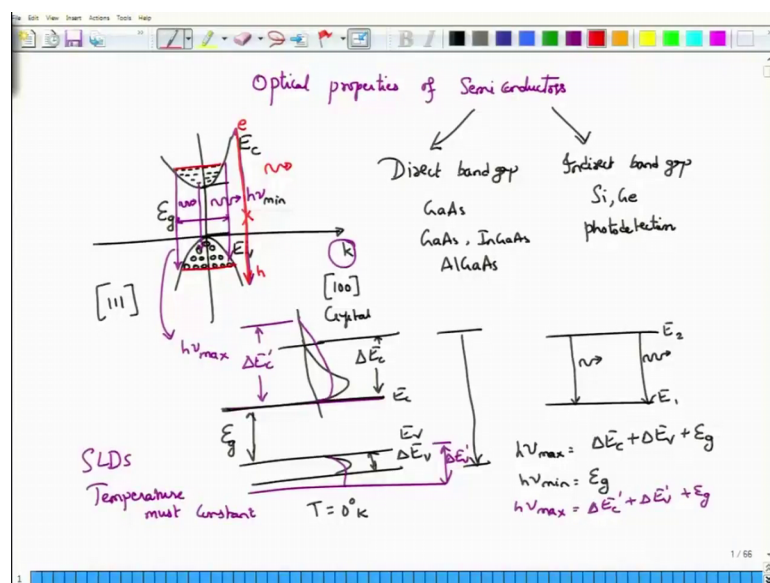
the direct band gap semiconductor and the other one is called as an indirect band gap semiconductor.

The popular semiconductor that we know of namely silicon and germanium actually fall in this band and therefore, they are actually good for photo detection process, unfortunately they are not good for lasers for the reason I will just explain to you now. Whereas, direct band gap semiconductors of which the prominent examples are the gallium arsenide and the lattice matched gallium arsenide, indium gallium arsenide or the indium phosphate gallium arsenide or aluminum gallium arsenide or sometimes called as AlGaAs.

So, these are the different material semiconductor materials of course, under certain conditions they do can become indirect band gap materials but in most cases they are going to be direct band gap material. So, what we mean by direct band gap materials? Now, recall that semiconductors are distinguished by having a valence band and having a conduction band I am not displaying the vacuum level in this diagram, so please take care of that one.

But it turns out that, instead of looking at the valence band and the energy band in this way, what really determines the lasing efficiencies are the two conditions; one is the energy condition and the other is the conservation of momentum condition, we may have looked at in the previous module, so we will again just very briefly look at it.

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So, you have the conduction band not actually flat as we have shown, but that flatness is actually along the length of the crystal, but what we are going to plot in this one is the horizontal axis, which corresponds to the momentum vector  $k$  along specific direction. So, for example, for the gallium arsenide this might be the direction crystal axis as we would call them. So, this is the crystal direction and along this crystal direction as the momentum keeps going, this is how the energy versus  $k$  diagram would look like. So, for a direct band gap material this is how the energy band gap would look like roughly ok.

This is there are various intricacies involved in this one, we will not going to all the details there and the idea is that once this type of a material is there then, if you forward bias a junction, it will result in having lot of electrons occupying the conduction band and there will be lot of holes which we will denote by having lot of 0's here. So, what you see is that, all the holes collect more or less at the top of the valence band and all the electrons collect at the kind of a well that is formed in the conduction band. And what happens is that, it is now possible for an electron to actually transition or recombine from the where conduction band to the valence band, in that process it can easily emit light ok, off the frequency  $h\nu$  which is determined by the energy level transformation.

Now, it is to be slightly so, you have to look at this one in a slightly different manner compared to the typical laser basic in the modules that we talked about, how the transition happens. There it did not matter, whether we had a transition from this location in the crystal to this location in the crystal or not the crystal location in the material because, both processes would essentially result in identical photons being emitted ok. So, in this process which there would be call it as a energy transition from the energy level  $E_2$  to  $E_1$ , which creates a photon is slightly different inside rather quite different in the semiconductor case because, now the frequency of emission is dependent on the energy difference of the transition right.

And it is what you can actually see is that, this energy transition will be having the minimum energy gap which of course, corresponds to the energy gap of the semiconductor itself and this energy gap, so if photon emit at the bottom of the conduction band to the just at the top of the valence band, then their energy will be minimum energy. However, electrons are also collected far away from the band and there are holes which are also available. Therefore, transition can happen at the band edge in this way, provided we satisfy a certain conditions which I am going to talk about it now.

But you do have transitions happening at the band edge maybe beyond that, it will not be possible because, the conservation of momentum will not be achieved, but let us say over this range if conservation is happening then, the band edge transition will result in a photon ok. This will also result in a photon, but that would result in a photon with the different frequency which we will call as  $\nu_{\max}$ .

So, what you see is that, compared to a typical atom lasers or some other material lasers, the lasing frequency and the spread around that lasing frequency is actually quite wide for a typical semiconductor material. That is not just the only thing that you need to take away, there is another thing that you need to remember, what happens as you increase the temperature slightly from the operating temperature of the semiconductor. Now to be specific, let us start with temperature  $t$  equal to 0 degrees right.

So, when temperature is equal to 0 degrees then, you know that the conduction band edge would go something like this right and the maximum spread is the energy spread of these electrons measured from the band edge. Of course, there is a similar measurement on the or similar spread of the holes in the valence region as well. So, you have  $\Delta E_c$  as well, of course, the region in between is the energy gap ok. And the maximum transition that you can now have, will be the one that would go at the edge, top edge of the conduction band where there is no spread because of the temperature. So, this is at  $T$  equal to 0 degree Kelvin. So, this is the transition that can happen.

And therefore,  $h\nu_{\max}$  will be the energy difference from the top. So, this is  $E_c$ , this is  $E_v$ , the total energy that change has occurred will be  $\Delta E_c$  plus  $\Delta E_v$  plus  $\Delta E_c$  plus  $\Delta E_v$  plus the energy gap  $E_g$  ok. Of course, the minimum  $h\nu$  that can occur will be between this 2 band edges which is basically, the energy gap itself ok. But the problem here is that or the picking a property of the semiconductor is that, this spread actually increases as you go to higher temperature.

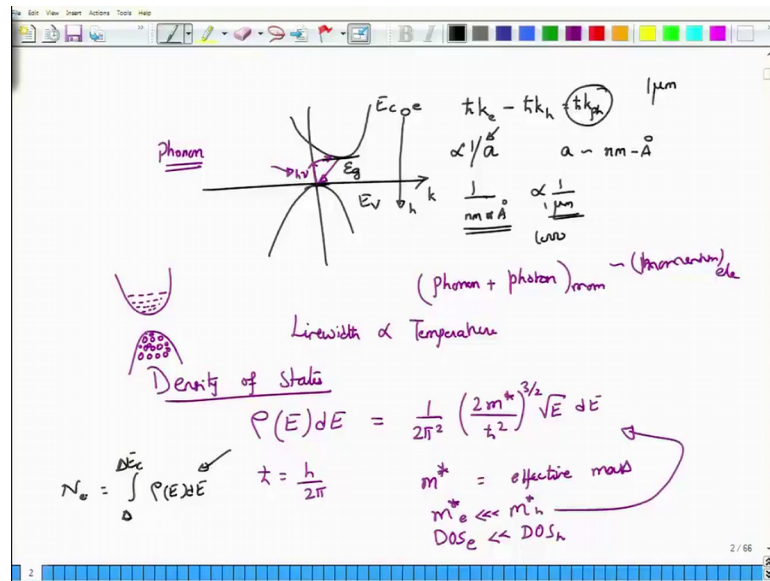
So, as you go to higher temperature both the values of  $\Delta E_v$  as well as the value of  $\Delta E_c$  will increase. So, they will be called let us say  $\Delta E_c'$  and  $\Delta E_v'$ . Now the frequency spread will go from  $E_g$ , which is the energy gap to the  $\nu$  values  $\Delta E_c' + \Delta E_v' + E_g$  assuming that, the energy gap itself is not affected by the temperature change by that much. Why should it increase? Of course, we know that, this is all controlled by the temperature and the Fermi

Dirac statistics that is obeyed by the semiconductor. And because, with increasing temperature there is a lot of kinetic or thermal energy available, so the electrons can spread out and the holes can also spread below the valence band thereby increasing the spread  $\Delta E_c$  and  $\Delta E_v$  and thereby increasing the line width of the laser now this is very crucial ok.

So, in most semiconductor laser diodes, temperature must be maintained very accurately or with as much accuracy as possible, temperature must be maintained to a given value. Any increase in the temperature causes the line width to spread and increase line width is absolutely bad for communication purposes ok. So, this is essentially the idea. Of course, in all these cases you might ask, why did not we have a transition from say at the extreme bandaged say at this point to this point. So, why did we not have this transition let me draw it in red color so that, we know that this is not allowed here, the reason is as you move away you know along the horizontal axis, the momentum actually starts to change ok.

For a direct band gap material, this is perfectly fine. The reason why we do not have this extreme edge is because, the electrons have not filled up ok. But because the transition is still happening with you know vertical transition is still happening with an electron which is sitting here to a hole which is sitting here, the  $k$  vector for these 2 are essentially equal to each other ok. The difference in the  $k$  vector will be in the form of the  $k$  vector of the electron that is emitted, but because we have not filled up to this level, the spread of the line width is given only by the level at which up to this is spread ok. But this is not the same case in an indirect band gap material ok.

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In an indirect band gap material what you find is you have again the  $k$  vector and then you have a  $k$  vector against which we are actually looking at the energy level ok. So, this is how the conduction energy band would look like and this is how the valence band would look like ok.

One of the conditions that the absorption must satisfy is of course, that energies have to match. So, here now the transition energy will be governed by this energy gap, if at all there will be a transition, but most importantly there is a momentum condition as well. So, if the electron must fall and recombine with the hole then, the electron momentum which is given by  $\hbar k_e$ , the difference between these 2 must of course, be equal to the momentum that the photon has taken place.

So, it would be  $\hbar k_{\text{photon}}$ , so  $h$  stands for hole,  $e$  stands for electron. Now in a typical system the momentum right is governed by you know or is proportional to inversely proportional to the lattice width. And, this lattice width will be in the form of a few nanometer to angstrom, which means that the corresponding electron and the hole of course, the hole momentum here is almost 0, but then the electron momentum that you are seeing here right because of this particular point will be very very large, it will be one by angstrom.

Whereas, photon is quite small because, this lasing is happening at around say 1 micrometer, so the momentum will be proportional to 1 by 1 micrometer whereas, the

electron will of or the electron momentum will be proportional to  $1/\lambda$  by nanometer or angstrom right. And this factor is about 1000 times at least larger than the photon momentum therefore, in a semiconductor material especially, in the indirect band gap material, transitions are usually not allowed because of the large mismatch in the photon momentum as well as the electron and the hole momentum or at least the electron momentum which is very crucial in this process. That gives how can we still have transition, when it is possible to have a transition, but that transition is not favored because of this problem with the momentum mismatch, but that transition is possible; provided you supply certain energy first let us look at absorption.

So, I supply a certain energy ok, let us say I bring the energy level up to this much and then I need to have what is called as a phonon ok, so to make a transition onto the horizontal side. So, you can see that there is one vertical transition line and then to make the further transition happening on to the conduction band, you need to have a phonon ok. Not a photon, but a phonon, which is the material quantized particle, do not worry about what that is, it is just the material that has to assist the photon thing right.

So, suppose I have somehow certain supplied an energy and this energy is sufficient for me to lift up to this vertical distance but this corresponding horizontal distance must come from a phonon ok. And in therefore, you need to have phonon plus photon and as you know photon momentum is quite small, so this total momentum will be then equal to momentum of the momentum of the electron ok. So, this is how by having an external or by having the material itself assist in the transition then, is possible to absorb energies and launch electron into the conduction band of course, the same process would actually follow in the recombination process as well.

So, you need to have a phonon given out while, you want to obtain a photon that comes out. And most of the cases in this indirect band gap materials this transition is not preferred simply because, the transition requires a photon assistor. And therefore, this a second order transition whereas, in the direct band gap it is the direct first order transition and if you go to quantum mechanics it will tell you that second order event for second order transition events are much much more rarer compared to the first order effects ok. To overcome that lower probability of the transition you need to actually start with a heavy and large material.



So, it may be possible it may be possible in near future or may be short future that silicon and germanium may actually be used to create laser diodes, but the efficiency of those lasers is still not going to be very compatible with the current efficiencies of the laser diode which are made out of direct band gap materials ok. So, this is some absorption characteristic and what is important about this one is that, the line width is essentially proportional to the temperature because, increasing temperature creates or increases the spread  $\Delta E_c$  and  $\Delta E_v$  which increases the range over which emission can happen.

Now, let us get back to an interesting and important parameter called as the density of states right. We have looked at density of states earlier, but this time we will be looking at it much more carefully because, this is very important ok. So, what is this density of states, what we mean by density of states, is an estimate of how many electrons are there in the conduction band and how many holes are there in the valence band. Not actually the estimate by itself, but what we actually estimate is the number of allowed states in the conduction band and the number of allowed states for the holes in the valence band; not that it will always be occupied, but at a given temperature at a given this one what is the number of allowed state for the density of energy states that can be in principle occupied by the electrons and in principle occupied by the holes in the valence band ok, so that is the density of states.

Now, without actually giving you the or derivation of this one, it is sufficient for us to note that the density of states per you know, the number of states per volume per volume because, I multiplied this one by energy is actually given by  $\frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2}\right)^{3/2} \sqrt{E} dE$ , please note the star  $\hbar$  square,  $\hbar$  is the Planck's constant. So,  $\hbar$  is Planck's constant  $h$  divided by  $2\pi$  to the power  $3/2$  square root of  $E dE$ .

Now, this is important equation ok. Of course, this equation may be no not I mean, you might ask where is the semiconductor in this one; the semiconductor part of the part of the input into this equation comes because of this  $m^*$ . This  $m^*$  is called as effective mass of the carrier ok. If it is an electron then, we would write this as  $m^*_E$  and we would write this as  $m^*_H$  for the holes and it turns out that, the electron effective mass is far less compared to the hole effective mass ok. The effective mass means that, if you take an electron outside of a material and it is a free electron then it has a certain mass which you know is given by roughly  $9.1 \times 10^{-31}$  kilograms right.

So, that is the mass of an electron when it is free and it is you know outside of any material you know influences.

But when the electron goes into the material or when you are looking at the electron mass in the material, it so happens that if, you are trying to measure the mass of this electron because, there are multiple electrons in front and in back and so on, they would essentially create a screening effect ok. And this screening effect should be accounted in your calculations and that will that is accounted in the calculations by looking at the effective mass and this effect mass will be much smaller than the actual mass of the electron or the hole, but in this density of states it is not the actual mass that matters, but it is the effective mass that matters.

And the effective mass of a semiconductor in a in a semiconductor is such that, the effective mass of the electron is much smaller than the effective mass of the holes. Now, what is the implication of this result on to the density of states, what it means is that the density of states for the electrons will be much much less than the density of states for the hole ok. Why, because the density of states is directly proportional to how much effective mass that is sitting in here ok. Now, let us do a calculation, which will be interesting and it will tell you how the magnitude of the density of states for electrons and holes will differ.

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The image shows a handwritten derivation on a whiteboard. On the left, there is a diagram of energy bands. The conduction band is at energy  $E_c$  and the valence band is at energy  $E_v$ . The bandgap is  $\Delta E_c + E_c$ . The effective mass of an electron is  $m_e^*$  and the effective mass of a hole is  $m_h^*$ . The thermal length is  $\tau_{ph}$ .

The electron density  $N_e$  is calculated as:

$$N_e = \int_0^{\Delta E_c} \frac{1}{2\pi} \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} \sqrt{E} dE$$

$$= \frac{1}{3\pi^2} \left( \frac{2m_e^* \Delta E_c}{\hbar^2} \right)^{3/2}$$

Given  $\Delta E_c = 0.1 \text{ eV}$  and  $m_e^* = m_e$ , the electron density is:

$$N_e \sim 2.5 \times 10^{24} \text{ electrons/cm}^3 \quad \tau_{ph} \sim 1 \text{ ns}$$

The valence band width is  $\Delta E_v \sim 0.012 \text{ eV}$ . The hole density  $N_h$  is:

$$N_h, N_e \sim \frac{E_v}{E_v - \Delta E_v}$$

The drift current density  $J_{drift}$  is:

$$J_{drift} = \frac{N_e}{\tau_{ph}} = 2.5 \times 10^{24} \times 1 \times 10^9 = 2.5 \times 10^{33} \text{ electrons/cm}^2/\text{sec}$$

Suppose I have some conduction band and I want to fill a certain energy band  $\Delta E_c$ , so  $\Delta E_c$  plus  $E_c$  is the next I mean range. So, I want to fill this entire range of  $\Delta E_c$  above the conduction band with electrons ok. So, I need to know, how many electrons are necessary. So, let me put a small  $e$  to that one to indicate that, I am going to fill this region up from  $E_c$  to  $E_c$  plus  $\Delta E_c$  by the electron. So, what is the number of electrons that I require? It turns out that you need to go back to this density equation and then integrate this density equation over the energy level difference.

So, if you integrate this one over 0 to  $\Delta E_c$  because, that is the range of the energy that is where going to fill this one and then you put row of  $E dE$  this will be equal to the density of electrons that are available or the number of carriers that are available. So, this would be integral that you would carry out and this is a fairly simple integral. So, I will leave that calculation to you. So, you have  $\hbar^2$  to the power 2 square root of  $E dE$ .

So, integrate this one, this is all constant, I will give you the values later on, but integrating square root of  $E dE$  will give you  $E$  to the power  $3/2$  divided by something. So, effectively what is get, here will be given by  $1/3 \pi^2 m_e \Delta E_c$  right, that is the range over which we are filling up the carriers to the power  $3/2$  ok. And given that  $\Delta E_c$  is about 0.1 e electron volt, which is not a large number, but it is reasonably a small number and the value of  $m_e$  which I forgot to write down in this one so, but I will give you the value of  $m_e$  later on for you ok.

So, when you put these values into this expression and calculate what would be the value of  $N_e$  turns out that,  $N_e$  is roughly  $2.5 \times 10^{24}$  electrons per centimeter cube. So, the number of number density or the number of electrons that are necessary to fill up an energy level between  $E_c$  to  $E_c$  plus  $\Delta E_c$  will be in this range  $2.5 \times 10^{24}$  electrons per centimeter cube.

Now, if you start with the same number, that is you assume that the holes also are equal to the number of electrons then to what distance  $\Delta E_v$ , so this is  $E_v$  and this is say  $E_v$  minus  $\Delta E_v$ , so over what distance do these fill up the holes. Please remember that, the density of states for the holes is much larger compared to the density of state for the electrons. Therefore, if I am giving a certain energy level then I get a certain energy

density for the electrons, but if I give you that the electron has to be density has to be same for electrons and the holes then because, a density is larger the spread around  $E$  will be smaller ok, so that is the basic idea.

So, because density of states for the electrons is smaller, they will fill up or they will require a larger spread to fill the same amount of charges whereas, the same amount of charges or same amount of holes can will fill only a smaller fraction of  $\Delta E$  v ok. So, you can actually instead of carrying out this entire equation or entire this one, you can relate  $N_h$  and  $N_e$  to  $m_h$  and  $m_e$  or stars of this one. And then show that,  $\Delta E$  v will be roughly 0.012 electron volt. So, for a given same number of carriers in both cases, the number of carriers are the same, but in one case the energy spread is about 0.1 electron volt and in other case it is about 2 orders of magnitude smaller compared to the holes.

And the reason that electrons have a smaller mass or not the reason, the implication that electrons have a smaller effective mass also means that, when you forward bias a junction electrons will more quickly because of their higher mobility lower masses into the hole region. And therefore, it is always or it is in most cases desirable to pump electrons from the n layer onto the p side layer. So, you normally dope your semiconductor laser diodes with larger values of n doping compared to the values of p doping ok.

So, what we see is that, for the same charges the or the same concentration  $\Delta E$  v is smaller compared to  $\Delta E$  c of the larger this one ok. Now we need to look at how much current is required to maintain this amount of carrier ok. Before we do that we need to know little bit about the lifetime of the carrier. So, what we mean by lifetime is, what is the time characteristic time over which electrons here can combine with holes and form the or emit a photon ok. This is of course, applicable in the direct band gap cases as we have emphasized repeatedly. This lifetime is let us say,  $T_{ph}$  and the current required to maintain let us say this amount of carriers will be about  $N_e$  divided by  $\tau_{ph}$ . And this is the current density, so this is the current required and you will see that, if  $\tau_{ph}$  is about 1 nanosecond ok, which is very typical for a semiconductor material then, the required current density will be quite high.

So, you have some  $2.5 \times 10^{24}$  into  $1 \times 10^9$ . So, because, you know  $1 \text{ nanosecond}^{-1}$  will become  $1 \times 10^9$ , so this is about  $2.5$

into  $10$  to the power  $24$  plus  $9$ , there is about  $33$  electrons per centimeter cube per second. That is a huge amount of current density. So, all we said about semiconductors is that they do have reasonable the small amount of current requirement, but if you see that this calculation they seem to have a large electron density or current density required. So, where is the catch here is what we are going to see in the next module.

Thank you very much.