Fiber – Optic Communication Systems and Techniques
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Lecture - 34
Basics of lasers-III (Population inversion and rate equation for lasers)

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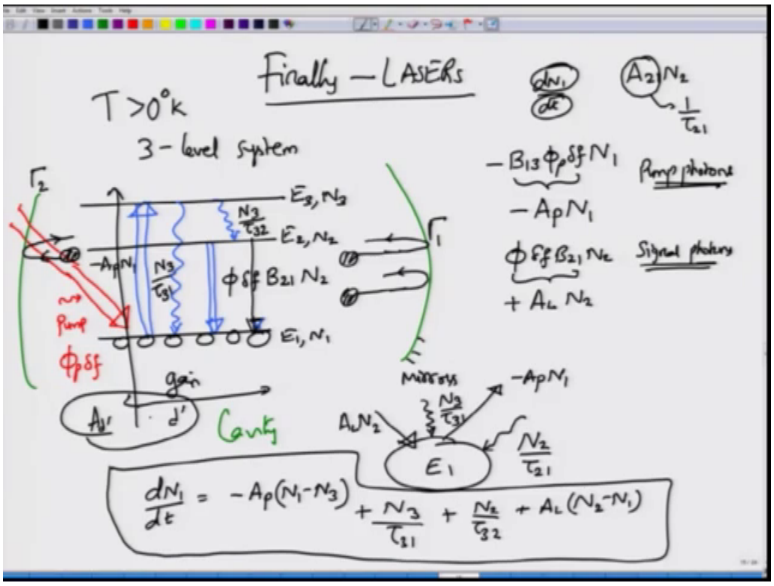

Welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module, finally we are going to apply the ideas that we talked about in the last two modules to understand lasers or to finally discuss what makes a laser. Well, we have all the ingredients here; we have a 3-level system. So, I am not being very specific about what kind of system this is ok, because these ideas apply to any system that can exhibit these 3 levels. And these levels are with the energy E 1, E 2, E 3.

This is the material or the gain medium, which is contained in the cavity. So, these green curves are actually for the mirrors, and they serve to kind of give you the feedback, so that is the photons that are emitted will be made to go back into the gain medium again and again and again, so that you can have this huge buildup of stimulated emission. And of course, you cannot just keep making them go back and back without actually supplying an external pump, and that is what you are actually going to do. So, we are going to supply an external pump, which is shown in this red line.

And what does this pump do, this pump allows the atoms in the energy level E 1 to get excited to energy level E 3. So, please note, this is very important. You are not sending in the pump of an energy such that it is actually going to get to E 2; you are sending the energy, such that you are giving it more energy, so that these atoms in the ground level do not go to E 2, but directly jump up to the level E 3.

And the interaction that governs which goes from energy level E 1 to E 3 is something that we already know it is given by minus B 1 2 phi P delta f N 1. And instead of writing this as phi 1 P B 1 or sorry this is B 1 3, because this is the level you are going from 1 to 3. So, instead of writing this as B 1 3 delta f phi P, I am going to introduce a simplified notation called as A P. Please do not confuse A with the earlier Einstein coefficient A, this is a different coefficient, this is the absorption coefficient or this is actually what we call as the transition probability.

And this probability why it is so or the transition cross section or the absorption cross section, why this is called as a cross section, why this is the probability is all something that we do not want to worry about, you just take this A P as a shorthand notation for B 1 3 phi p delta f. And the absorption process is governed by minus A P times N 1, which of course will deplete the population at the energy level E 1, while increasing the population at the energy level E 3 ok.

Of course, once these electrons go to the energy level E 3, they have two possibilities. Some of them spontaneously come down from E 3 to E 1, we will call that as N 3 divided by tau 3 1. Now, what is this tau 3 1, well we know that spontaneous emission is you know governed by the coefficient A 2 1 times N 2 for a simple 2-level system, where A 2 1 was called as the Einstein coefficient.

Instead of writing this as A 2 1, we simply write this as 1 by tau 2 1, it makes sense, because N 2 by tau 2 1 will tell you the rate at which N 2 is decaying because of the spontaneous emission. Numerator is the density population density and denominator is the time, which is what the left hand side was actually right. So, it was change in N 1 2 or the rate at which N 1 was changing with respect to times. The numerator was density; denominator was time.

So, all the spontaneous emission terms are actually written as 1 by tau with the subscript telling you which energy level to which energy level we are considering. Therefore, this particular interaction this spontaneous interactions from E 3 to E 1 can be written as N 3 divided by tau 3 1 ok. Similarly, and this is much more probable, because atoms go or like to go from E 3 to E 2, and the reason why I have put a wavy line here in these two, because these reactions are non-radiative that means, they do not actually give out any photons, but they serve only to deplete the population ok. This reaction on the other hand will give out spontaneous emission, so I will have to change this one. So, instead of a wavy line, I have to make this a straight line ok.

So, now, this interaction from E 3 to E 2 is a non-radiative one, so that would be N 3 divided by tau 3 2. So, notice that we are writing this as tau 3 2, because you are going from energy level E 3 to energy level E 2. From E 2 to E 1, we have two more interactions, one is that E 2 atoms in E 2 drop down to energy level E 1, and that one will be phi P delta f B 2 1 times N 2 ok. So, I have written this as phi P delta f B 2 1 as the phi P delta f B 2 1 times N 2 as the stimulated emission case. And just like previous this one, this is not phi P, because this is not the pump photon density, this is just phi delta f B 2 1 N 2, and phi delta f B 2 1 or B 1 2, because B 2 1 is exactly equal to B 1 2 can be written as A L ok. So, what you have is A L times N 2, and this is a plus as far as the energy level E 1 is concerned right.

So, for the energy level E 1, it loses population because of the absorption, which is given by minus A P N 1, it gains population because of stimulated emission, which is written as A L N 2. And then you will also have spontaneous emission from N 2, which would be N 2 divided by tau 2 1. And then you will have spontaneous emission down from E 3, which would be N 3 divided by tau 3 1 ok. So, this is how the total population density is actually changing.

So, if you write down the equation as to how energy or population level in energy level E 1 changes, you can write this as d N 1 by dt is actually equal to the pump absorption, so that would be minus A P N 1 minus N 3. I will tell you why I am writing this as N 1 minus N 3. So, this is the absorption pump absorption plus spontaneous emission down from energy level E 3 to 1 plus you have N 2 by tau 3 2 which is spontaneous emission plus you have stimulated emission. So, emission is represented by L, which is the lasing action and this is N 2 minus N 1.

The reason why I have written as N 1 minus N 3 right, is simply because since this laser is operated at a temperature greater than 0 degree Kelvin right, there will be a thermal energy. And because of thermal energy, the initial populations at N 3 and N 2 are non-zero ok. And you are not interested in the actual energies or actual populations, but you are interested in the population difference that comes about.

So, what is the contribution of the absorption, how does this go away, because these photons can be absorbed and some photons can be absorbed reabsorbed to make them go from E 3 to E 1. The actual change will be from N 1 instead of N 1, it will be N 1 minus N 3 times A P; A P stands for the pump absorption ok. And it is in the normal case, N 3 will be much smaller than N 1 ok. And of course, this is what equilibrium actually tells you that N 3 is much smaller than N 1. Therefore, the effective change at the level N 1 that is happening is given by minus A P N 1 minus N 3 ok.

However, for the spontaneous emission, spontaneous emission does not depend on N 1 when you look at spontaneous emission from E 3 to E 1, so therefore this is just N 3 by tau 3 1. Similarly, N 2 by tau 3 2, but for the lasing or the stimulated absorption or stimulated emission, what you have is the effective population difference N 2 minus N 1 times whatever the pump or the emission cross section is what would count. So, the equation that we have written you know in this box equation will tell you all the different terms that will serve to effectively change the population level at energy level E 1 ok.

So, d N 1 by d t consists of these four terms, where A L is defined as phi delta f B 2 1, and then A P is defined as B 1 3 phi P delta f, of course B 1 3 is the same as B 3 1, and B 2 1 is the same as B 1 2. And there is an important thing here that I want to mention. Phi P stands for the pump photon density ok. So, this is the external energy coming in because of the pumping process ok. So, this is pump photon energy density or the pump energy density to be this one.

Whereas, the photon density phi without the subscript is actually the signal photons right, because these photons are the result of stimulated emission between the levels E 2 to E 1. So, there is no P here, because P would refer to interaction between E 1 and E 3 ok. So, there is a difference between phi P delta f and phi delta f. So, please keep that difference in mind.

Of course, what will happen these photons, which are now stimulated emission effectively would start to travel, they would travel, they would hit the mirror. Assume that the mirrors have reflection coefficient of say gamma 1 on one side, and gamma 2 on the other side, and after they have traveled. Once they have hit the reflection, they these photons would travel back towards the gain medium, where they will further cause stimulated emission to occur.

And these stimulated emission photons or stimulated emission photons would again travel towards the second mirror undergo a reflection say at gamma 2 and then come back. So, inside the cavity, they are circulating back and forth back and forth, thereby increasing the total population density of the signals ok. But, because you want to take this thing out also right, you do not want all the radiation or the all the power to be there inside the cavity, you want some of that power to actually come out of the cavity.

What you do is you take gamma 1 to be around say 99 percent ok, so that 1 percent of the radiation is actually tapped out, and you take gamma 2 to be 1, which means there is 100 percent reflection on one facet, and there is about 99 percent reflection on the other facet. There is nothing sacred about 99 percent you can take whatever the percentage. Of course, what you have to ensure is that if you start taking all of the power out, then there would not be any power inside to sustain the stimulated emission. So, you have to just simply avoid that condition ok.

So, typical numbers of 1 percent, 10 percent or 30 you know 35 percent out and 50 percent out are quite common. There is of course, a certain relationship, which tells you that you can extract certain maximum power at an optimum power coupling ratio ok. So, you can maximize that power coming out of the laser by carefully choosing that gamma 1 ok.

Higher values of or higher tapping out or the higher coupled numbers like 50 percent or 90 percent simply does not mean that, you will be able to always get more power. Because, remember any power that you take out of the cavity, means there is that much amount of power less in the cavity to sustain this stimulated emission ok. So, you do not want to get too greedy, get some power out by 1 to 10 percent and then leave the remaining, so that it can sustain this stimulated emission anyway.

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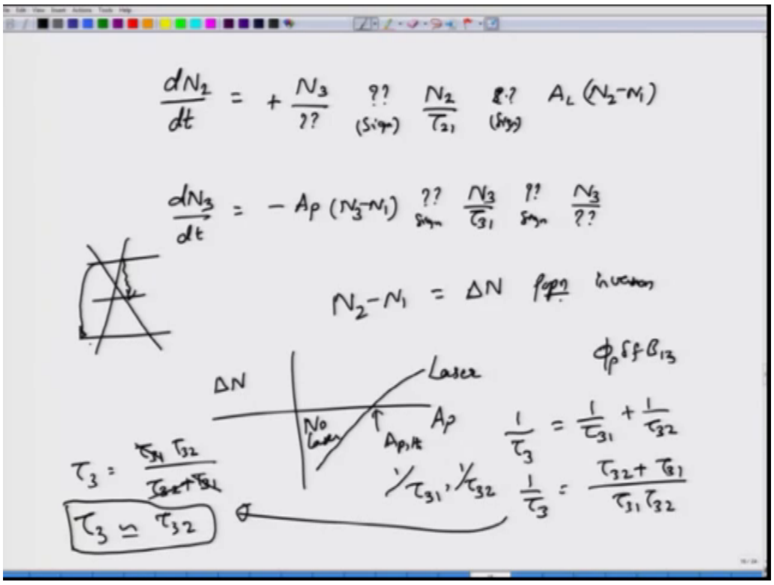

Any way so, I have written the equation for N 1 and I would leave as an exercise for you to write down the rate equation for population level N 2 and N 1, I will give you hints ok, so that you can write this. And then I will you know the sign here please you have to write down the sign here. And this is N 2 by tau 2 1 and again, the sign here you have to write down whether it is plus or minus. And then I will say this is A L N 2 minus N 1 ok.

And then finally, you have d N 3 by d t. And we will again write down this one, so minus A P N 3 minus N 1. I will leave the sign here for you to pick. This would be N 3 divided by tau 3 1. And I will again leave the sign for you to write down. And I will say N 3 divided by what ok. So, please fill up these equations, they will allow you to understand how the population levels are changing in energy levels E 1, E 2, and E 3.

And once you have written down these equations, the next step would be to try and find out what would be the minimum pump power required, so that you can actually have population inversion. So, the goal of this exercise was to ensure that you are pumping in certain amount of photons, such that the population level at N 2 will increase compared to N 1 and N 3. So, when you have strong pump or sufficiently strong pump just after the critical pumping, then N 2 will be greater than N 1. And the difference N 2 minus N 1, we will denote it as delta N and call this as the population inversion ok.

So, what you actually expect is that this delta N will be you know negative, when the pumping is not very strong. And as the pump increases at a certain critical or the threshold pump power, delta N just becomes equal to 0, which means N 2 will be equal to N 1. And once you know pumping increases beyond this threshold value, remember A P is proportional to phi P delta f B 1 2 or rather B 1 3 or B 3 1 whatever that is right.

So, it is phi P delta f, it is the pump photons that we are talking about. So, once the pump photons sufficiently exceed then exceed the threshold value, then delta N becomes positive, and you will actually have laser actions here. Unfortunately, there is no laser action, when A P is less than the threshold value.

Finally, in all those equations that you have written, you will find out that at population level 3, there are two different spontaneous emission factors that are present, so one is level 3 to level 1, so that is governed by 1 by tau 3 1, and then you have 1 by tau 3 2 ok. So, you can define the effective lifetime of a photon that is the characteristic time scale over which an atom at energy level E 3 would decay down to energy level E 2, or would decay down to energy level E 1 as 1 by tau 3 equals tau 3 1 by tau 3 2. This is like your parallel resistances being added. And this is given by tau 3 1 tau 3 2 divided by tau 3 2 plus tau 3 1, this is 1 by tau 3, or the effective lifetime tau 3 is given by tau 3 1 tau 3 2 divided by tau 3 2 plus tau 3 1.

Now, if you look at a typical laser, what you want is most of the atoms, which are at the ground level E 1 to end up in ground level E 2 ok, but because of a absorption E 1 atoms in E 1 go to E 3, now you have two options right. So, E 3 to E 1 can drop down, and that characteristic time scale is about tau 3 1, and the characteristic time scale from 3 to 2 is tau 3 2.

If you ensure that this tau 3 2 is very small that is why we had put them closer together indicating that. These two actually are quite close in the sense that the lifetime from 3 to 2 transition is much much faster, or in the sense that it is much smaller, the it would not spend much time to or it would not take much time for the atoms in a energy level E 3 to drop drown to E 2 ok, but it will take a long time for them to drop down from t 3 to or E 3 to E 1 ok.

So, if you say if you take this condition, then what you see is that tau 3 2 is very small compared to tau 3 1 ok, so because that is very small. So, tau 3 1 is much larger than tau 3 2, the effective lifetime tau 3 can be approximated as tau 3 2 itself, because tau 3 1 is larger in the denominator compared to tau 3 2. So, this will go away, and then tau 3 1 cancels out, and then you get tau 3 effectively as tau 3 2. And this is desirable this is in fact, the critical condition that is necessary.

If you have a matter in which this transition time is much faster compared to this transition time, which is much slower, then you would not be able to sustain population inversion. So, this is bad for our laser application ok, so that is one condition that we wanted to talk about. Of course, you could generalize these results to 4 level system I am not going to do that one.

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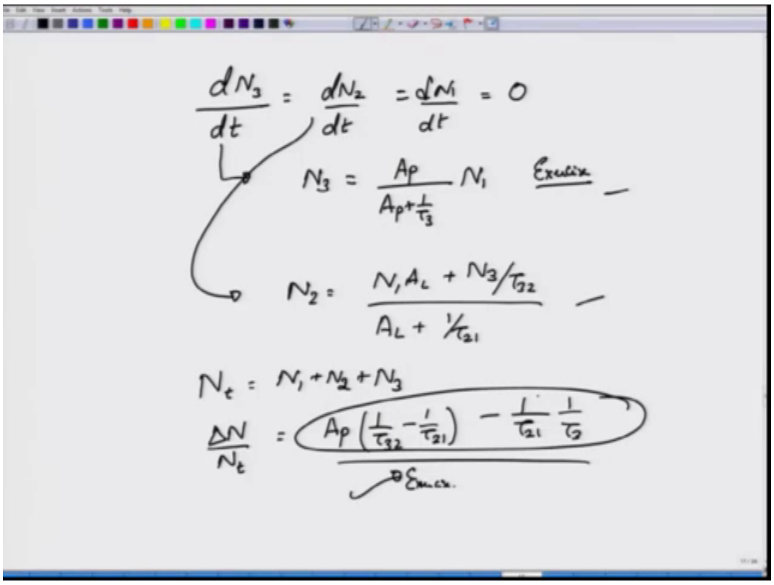

But, what I am going to do is that I want to consider what happens in the you know a steady state condition in which all of these populations d N 3 by d t, or d N 2 by d t, or d N 1 by d t all these terms are actually equal to 0. And when you set the first one to 0, what you will find is that you can write down N 3 in terms of N 1, so you will have A P divided by A P plus 1 by tau 3 times N 1 ok. So, this is a simple exercise again, you can show this, you just have to set d N 3 by d t equal to 0. And once you have done that, when you can get a relationship between N 3 and N 1.

Now, typically what happens is that once the pumping is critical ok, N 3 will be much smaller than N 1 of course, N 3 was always smaller, but you would have expected that, because there is absorption that N 3 can become higher than N 1. But, in the steady state condition, that would not be so, because most of them would be absorbed atoms from E 1 to E 3 would have dropped down to E 2 because of the spontaneous decay from 3 to 2.

So, effectively in the equilibrium condition, the population level N 3 is quite small compared to population level N 1, but N 1 itself should be smaller than N 2, so that delta N, which is the population inversion will be larger. And you can actually show by setting d N 2 by d t equal to 0 that N 2 is given by N 1 A L plus N 3 divided by tau 3 2 divided by A L plus 1 by tau 2 1.

If there are no additional losses, then total population density will be the sum of the population levels at 1, 2, and 3 levels ok. And this should not change, this should remain constant. And when you look at delta N and normalize it with respect to t, you will see that by using this equation, this equation, and the other equation for N t, you can see that this will be given by A P times 1 by tau 3 2 minus 1 by tau 2 1 minus 1 by tau 2 1 1 by tau 3 divided by I will leave this as an exercise to find out what is the denominator. Of course, we do not really need to know the denominator as importantly.

But, this equation delta N normalized with respect to N t is very critical for us. Why, because it allows us to look at the relationship between tau 3 2 and tau 2 1 ok, and it will also allow us to find out what is the critical pump pumping rate or pumping required, so that this population level delta N, or the population inversion delta N will be greater than 0, so that is the sufficient or that is a necessary condition is that A P should be greater than a threshold quantity. And what is the threshold quantity. Well for delta N to be positive, the numerator has to be greater than 0.

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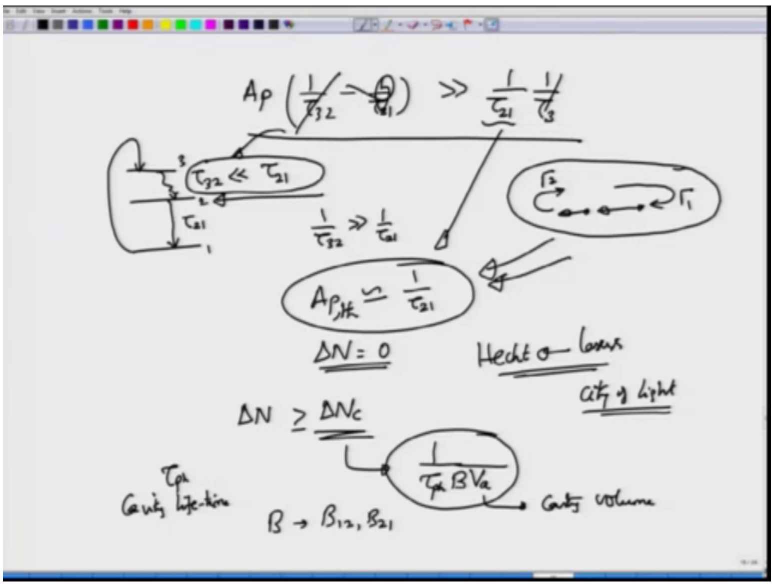

So, once you set the numerator greater than 0, you will see that A P 1 by tau 3 2 minus 1 by tau 2 1 must be far greater than 1 by tau 2 1 divided by tau 3, you can write down these equations. And what you would actually find out is that from energy level 3 to energy level 2 tau 3 2 is much much smaller than the lifetime from 2 to 1 ok. So, you have this tau 2 1 which is much longer, so which means that most of the atoms which go from 1 to 3. And from 3 to 2 would remain in this energy level E 2 ok, so that the population inversion of course, can take place.

If they do not remain and then just keep falling down to N 1, then there is no population inversion, no laser that can be sustained right. So, clearly tau 3 2 must be much much smaller compared to tau 2 1 ok, so because tau 3 2 is much smaller compared to tau 2 1. 1 by tau 3 2 will be much larger than 1 by tau 2 1, so that you have A P being much larger than 1 by tau 2 1, because 1 by tau 3 2 will actually sorry this is 1 by tau 3, but, anyway this can be approximated and into tau 3 2, and then can be cancelled out.

So, what you actually see is that the condition on the pumping grid is that it has to be much larger than 1 by tau 2 1. So, this is an important condition. And if you have not clear about one or two points about this one, please go back and solve this equation for yourself you will. And impose a condition that tau 3 2 is much smaller than tau 2 1, and you will actually be able to get this expression ok. Sorry this is not greater than greater than, this is actually approximately right.

So, A P threshold we will call this is 1 by tau 2 1, because tau 3 2 is much smaller than tau 2 1. And in this equation 1 by tau 3 2 being lesser than 1 by tau 2 1, means A P divided by tau 3 2 is much larger than 1 by tau 2 1 divided by 1 by tau 3. And you can cancel off this guy with this one, this anyway goes to 0, and A P will be approximately equal. The critical value will be equal to 1 by tau 2 1, but you of course need to pump beyond this particular thing.

So, what you want is population inversion to be sustained, which means that population level N 2. Most of the atoms, which are exerted from E 1 to E 3 and then drop down to E 2 must spend most of their time in the energy level E 2, they should not drop down quickly, they should spend their time here. And your pumping you know rate must be sufficient, such that this condition is satisfied. And the critical value for that should happen is approximately given by 1 by tau 2 1.

So, now you know that if you take a helium-neon laser, you take a indiag laser, you take an erbium doped fiber laser, you can then go and find out what is the lifetime of E 2 to E 1, the two energy levels that are involved ground and the next higher level ok. And then make a guess as to what would be the pump rate required for that you know for the laser to be sustained ok. So, this is very interesting calculation.

And the history of this calculation is also very interesting I would not be able to go to that history. But if you are interested, you can look at a book called Lasers by Hecht ok, he discusses this theory. This is entirely, I mean he discuss this ideas or the history of how these ideas of laser came about in a very very interesting manner, so you can take a look at that one. Sorry this is there is two books; one is Lasers, and the other one is City of Light. So, you can refer to either of the two books for a fascinating history of these ideas.

Of course, what we have is the threshold value. At the threshold value of course, delta N will be equal to 0 ok, which means you have just gotten about the value of delta N. But, it turns out that the value of delta N just being equal to 0 is not really the minimum required value. What you want is delta N to be greater than certain critical value, and we would not derive what is that critical values here if that derivation is little too long. But you need to see that just because you make delta N equal to 0, does not really give you a laser right away, you have to make this one to be greater than or equal to a certain critical value ok. I will give you the expression for this critical value that is given by 1 by tau p h times B V a.

If you are wondering what these terms are B is just a term, which we have written for B 1 2 or equivalently for B 2 1, V a is the cavity volume ok. So, you have this cavity, which has a certain area. So, this is the cavity right. So, this cavity has a certain cross sectional area, and it has a certain depth d here or d prime here, and this cross section is say some A d prime. So, this product of A d prime to d crime is what we have called us the cavity volume. And B of course, is this one and p h tau p h is the lifetime, which is approximately equal to tau 2 1 in our case ok.

So, this is the cap, this is also sometimes called as the actually this is not equal to tau 2 1, this is slightly different, we will simply call this as cavity lifetime ok. This cavity lifetime has to do with how much you know it has to do with the reflection coefficient in the 1st mirror, the passage, or the gain here, the attenuation in the medium, which is not gain, and then there is a reflection back on to the 2nd mirror gamma 2 right.

So, cavity lifetime is essentially the effective lifetime of a photon, which is stimulated emitted ok, but it would kind of statistically remain in that cavity before escaping out from the 1st mirror ok. So, mirror with the reflection coefficient that is less than 100 percent. So, it is not exactly tau 2 1, I am sorry tau 2 1 is the lifetime of the photon or the characteristic time of the photon, where it will drop down from energy level E 2 to the ground level E 1. And this critical value which is also equal to the steady state value surprisingly is something that is necessary, so that lasing cap happen ok.

So, we have discussed these 3 level lasers, there are a couple of additional things that we could discuss, but unfortunately I do not have time in this course to discuss those things. What I would now like to do is to look at basics of semiconductors in the next module, so that we can apply these ideas that we have talked about population inversion, rate equation you know, and how exactly or what exactly will be absorbed, and is that absorption process same as what we have been talking about. There you will see that in semiconductors, the concepts of electrons and holes become very important, and they have these different characteristics that go with them. And how they would affect lazing is what we are going to see in the next module.

But, for this module, if whatever the exercises that I have given you, please solve them, they are simple, they have to just you know rearranging the terms as an equation. And solving them, will allow you to understand these basic principles of the lasing that would happen, and that would be applicable for any laser that you can consider with some modifications of course, which are specific to the lasers themselves. So,

Thank you very much.