Fiber - Optic Communication Systems and Techniques
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Lecture – 33
Basics of lasers-II (Einstein's theory of radiation)

Hello, and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module, we will further study the basics of lasers. And if you recall what we studied in the last module, we saw that when you considered matter ok, which typically is a laser gain medium or the active medium as we would call it, this matter has different energy levels, they are separated by a certain gap in the energy levels. The reason why you have gaps is something that we are not going to cover in this course, but I will give you a brief idea, when we talk about semiconductor basics in the upcoming module.

But for now just imagine that matter whatever the active medium that we are dealing with which of course is one of the first ingredients of what forms a complete laser. This particular active medium has multiple energy levels or actually what it has a energy bands ok. And we considered two such energy bands say E 1 and E 2 or since we are not going to over complicate ourselves. We will assume that instead of these bands being considered, we are considering the discrete energy levels.

So, in that particular case, we can consider two discrete energy levels separated by a certain energy gap or a band gap ok, or this case it could be energy gap. And in the equilibrium and especially when the temperature is equal to 0 degree Kelvin that is and when there are no thermal energy available to the electrons in the matter, what you would find is that almost all of these electrons are located in the energy level E 1, there is they occupy the energy states below that of the edge E 1 ok.

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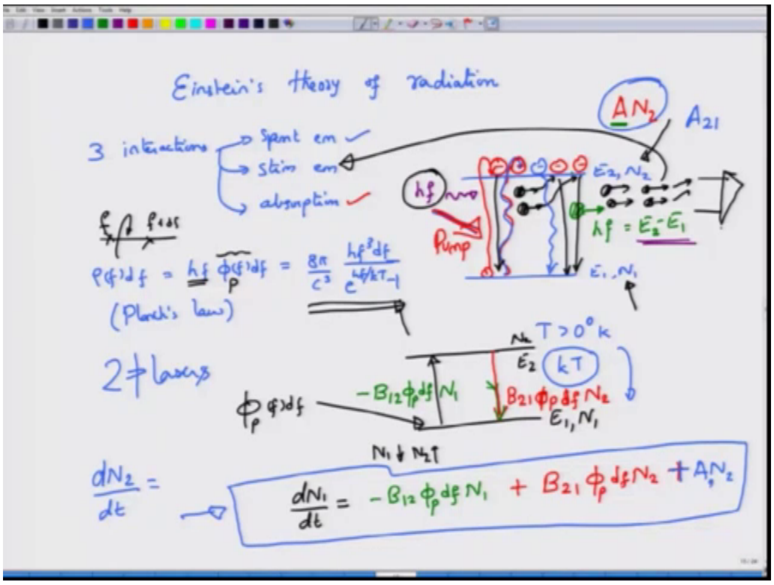

And if you now assume that the temperature is actually greater than 0 degree Kelvin, then each electron has on an average a thermal energy of k T, which causes a few of the electrons to just get excited and land in the second energy level. Of course, as we have already seen, this electrons which have landed or atoms which have landed in general, they do not stay there for quite long ok, and they actually spontaneously come down from the higher energy level E 2 to the lower energy level E 1 ok. And this interaction or if this reaction, where the excited atoms or the excited electrons fall down spontaneously on their own without any intervention or without any external energy that is being supplied is called as the spontaneous emission. And this is one of the interactions that we have already spoken about.

Of course, this excitation that has happened is also spontaneous in the sense that this excitation is not because of any external energy source, but the energy source coming simply, because the matter is now at a non-zero temperature. So, this thermal energy, which causes the electrons to just kind of go up or being excited into the next higher energy level also occur spontaneously. So, you could, in fact call this as kind of a spontaneous absorption, but this case spontaneous absorption of what, the spontaneous absorption of the thermal energy. But, you see that it is not very convenient way of talking about it. So, we do not normally attribute any specific name for this interaction of you know of electrons or atoms taking amount or absorbing a certain amount of kinetic thermal energy, and then jumping up to the next you know energy level.

But, it is possible to actually excite atoms in a much more controlled way by having external pump signal ok. So, having an external pump signal allows these atoms in the ground state to actually absorb the energy coming from external energy source, which we call as the pump, which is typically optical signal or photons or it could be simple current. So, both types are used in fiber optic communication systems to make different kinds of lasers, the one that is optically pumped is what is called as the erbium doped fiber, and you make a laser out of it, we call this as a erbium doped fiber laser. But, the semiconductor lasers do not really have an optical pumping, they have this electrical pumping.

So, the idea is that you create certain amount of energy or you actually supply certain amount of energy to the atoms, which are in the ground state E 1, so that when they absorb that energy, they can jump up to the higher energy state, or they can move to the next higher available energy state. And this process is called as absorption due to external pump being incident on the matter.

Finally, when you have created sufficient number of excited electrons, and all these electrons are essentially present in the form of excitation, very few of them start to drop on their own ok. And how much of excited atoms drop spontaneously is you know proportional to a certain constant A times whatever the density of population that exists in the energy level E 2.

So, we call the density of population, so as we have already seen the floor, ground floor, and the first floor analogy in the previous module, we can think of simply this as the population density the number of people so to speak, each carrying the same amount of energy, and located at the first floor. And they when they randomly drop down and combined with the vacant space below or they just go to the vacant space below that is called as a spontaneous emission ok. So, of course, when they do so, they would actually emit a photon of an energy that is the difference between the two energy levels. So, the photon energy is h f, where h is the Planck's constant, and this energy that is emitted will be equal to the energy difference between the two levels E 2 and E 1.

So, this spontaneous emission rate that is the rate at which these atoms in the excited state fall down on their own is kind of very small ok, which means that most atoms do not as soon as they land up on the higher energy level E 2, they would not automatically drop down. But, they do drop down after a certain characteristic life time, which is proportional to the constant A. Incidentally; this constant A is called as Einstein's A coefficient and it is really not a constant, it depends on several factors.

But, for our application or for our purposes, we will assume that this is to be a constant ok. And this is called as Einstein's A coefficient, because it is the coefficient A. And this idea or this theory of what is called a semi classical theory of radiation was first given by Einstein. And this is a very interesting and very simple ideas or simple way of tackling what happens with when the radiation interacts with matter ok, radiation by radiation I mean optical energy. So, or it could also be the energy that is released in the form of photons by the lasing action itself ok.

So, as we have seen very few of them drop down by spontaneous emission. But, when you now send in an external photon or it could be a photon that has been just generated with the noise, whose energy actually matches that of the energy difference E 2 minus E 1, then this photon will cause the electron here to drop down. So, this and once we drops down it of course, emits another photon of the same energy h f. And it turns out that this new photon that has been emitted will so, this photon which has been incident is not actually absorbed, it is just used to trigger the excited atoms, which are in the energies level E 2 to drop down to E 1, thereby releasing a photon.

Therefore, after the first interaction, what you will actually see is that you have two photons, which are identical in all aspects, and they have the same energy, they have the same polarization, and they have no they are in coherence with respect to each other. So, you will actually see two photons. Now, these two photons go and trigger additional electrons or additional excited atoms, which are present in the energy level E 2, which will then fall down to generate a total of about four photons right. So, you will now have instead of two photons, you now have four photons.

And as the external or as the noise induced initial photon kind of builds up, this process of emission continues to happen until all of the excited atoms in the energy level E 2 have completely dropped down to E 1, so as to give you a huge radiation. From using this big arrow to indicate that the radiated energy or the number of photons that you are getting at the output or the optical power or intensity is quite high. And it is being stimulated in the sense that it needs of course, some starting point of a photon, which could be generated because of the noise, or it could be just kind of seeded from external sources.

And once that has taken place, then the rest of the active material starts you know giving so many photons, because there is large amount of population at N 2 because of the pump absorption right. So, pump is incident, the atoms at the ground level E 1 absorb the pump photons, and then jump to the excited state, thereby creating a population inversion here that is the population N 2 will be larger than the population N 1. So, in equilibrium, it is the other way around right.

So, in equilibrium, when there is no interaction going on, most of the atoms are in the ground level E 1. But, when there is external attraction going on in this form of you know pumping and subsequent absorption of the pump photons or the pump energy, then the number of atoms in the energy level E 2 far exceed that of the atoms in N 1, which of course, is what we call as population inversion, and that is one of the key ingredient for a laser.

So, once population inversion is achieved, then any small trigger kind of a thing small in the sense that a very minuet amount of initial seed photons are necessary to trigger this action, which causes or which builds up through the laser medium. And this process of building up a coherent radiation by continuously stimulated emission is in fact called as stimulated emission that is this is called as stimulated, because it is being triggered by another photon let us say.

And how good are we going to absorb or in other words, if I am supplying pump a density of phi of f right or maybe because I am actually looking at the pump photons, I am going to write this as phi p. So, if I am supplying a pump with a photon density which of course, will be proportional to the intensity or the flux of the photon over the frequency interval f to f plus delta f, because these quantities are actually frequency dependent.

So, when I have this phi p of f d f times h f, then that will actually correspond to the total energy density over the frequency range that we have considered that is small f to f plus d f. In that range, whatever the energy density is actually proportional to the energy times the population, because each photon carries an energy of h f, when you have a number of photons.

So, this is the pump, specifically therefore I am writing this as phi p of f, and that in the equilibrium or for a blackbody radiation is given by this expression ok. In the sense that when there is an equilibrium achieved, then the radiation actually is quantized and the radiation quantized radiation is given by this particular expression. So, now as the pump photons are incident onto the material causing the absorption to take place, what would be the rate at which the population density in the two levels actually changes right?

So, remember in the 0 degree Kelvin case, when there are no external energies. All of the atoms are in the energy level E 1 itself, none of them are in the energy level N 2. But, we are not going to operate our laser material for active medium in the 0 degree Kelvin situation; we will be operating them at room temperature. And because of the room temperature, you know there is a certain non-zero thermal energy available to the atoms. And these thermal energies are actually taken or absorbed by the atoms in the ground level, then they get excited into the energy level E 2.

But, this process because of thermal energy being absorbed is not sustained, and the number of such excited atoms because of thermal absorption are actually quite small ok. So, in that room temperature also, most of the population is actually in N 1. But, once you start pumping in or send the external pump, then this pump will give you energy or pump gives the energy to the material, which then allows the atoms in the ground level E 1 to absorbed and move up to the next level E 2. And when they do so, they are going to deplete the population N 1 and increase the population N 2. So, as pump photons are absorbed, N 1 decreases, while N 2 increases ok.

So, if you know concentrate only on two levels E 1, E 2 with energy levels N 1 and whatever initial energy level N 2 that we have, and as the pumps are absorbed, the rate at which the population in the ground level is decreasing will be denoted by dN 1 by dt, correct. And this will be because of three processes, well first it will be decreasing, because you are sending in the pump and the absorption process is given by B 1 2 phi P d f ok. And I will put a minus sign here, the minus sign indicating that this is actually because of absorption the energy or the population density in energy level E 1 is decreasing. So, decreasing is represented by minus.

Of course, this is the photon density snd B 1 2 as you have guessed is actually called as the Einstein's B coefficient. Of course, this is only telling you how the population densities being or the pump photons are being absorbed, but how many actually leave the ground state N 1 or ground state E 1 is proportional to N 1 itself. So, the expression that I have written in green color here, which is minus B 1 2 phi P d f N 1 stands for absorption and contributes to decrease in the population level N 1 ok. So, you have minus B 1 2 phi P d f N 1 causing the absorption process.

But, once there you know electrons are at the top ok, and because we have considered only two energy levels here. The same photons which have you know cause initial absorption can also cause photons to actually come down that is they can cause the stimulated emission as well. But, we will not write down that as a stimulated emission, we will simply write it down as stimulated emission, but the idea of stimulated emission is that they will increase the energy level N 1 correct, so that is they will increase the population level of energy state E 1.

And therefore, that absorbed, so that stimulated process can be written as plus B 2 1, so this would be B 2 1 right. This is B 2 1, where 2 1 stands for interaction from 2 to 1 or emission from level 2 to level 1. So, this would be B 2 1, and of course it is proportional to phi P d f, and whatever the population level that exists at N 2. So, this is plus B 2 1 phi P d f N 2.

And of course, there is another term that we should not forget the other term is the spontaneous emission term right. Spontaneous emission term is as we have seen is simply proportional to A times N 2 or in fact, given by A N 2. And it is N 2, because it is spontaneously coming in from energy level E 2. So, these are the three interactions on the right hand side, which describe how the population density at E 1 is changing.

So, if you think of population density being decreased or increased as some sort of a current, then you can think of this equation that we have written as a Kirchhoff's current law, wherein the current here in the particular node will be given by outgoing current plus the incoming current ok. So, outgoing current is because of absorption; incoming current is because of the stimulated and spontaneous emission.

So, we have written this equation, and I will leave as an exercise for you to write down an equation for N 2 ok. Again with be careful, any decrease in the population will be represented by a minus sign, any increase in the population will be represented by a plus sign. And for now, consider only the two energy levels ok, I will tell you that only matters which have only two available energy levels will never allow laser to happen or whether they will never allow lasing to happen. So, two levels are not good for lasers. So, two levels are not going to give you any laser, so you need at least three levels, but most materials are four levels.

But, in some approximation, one can think of them as three level. So, In fact, the helium neon laser which operates at 632.8 nanometer or the Nd YAG laser which operates at 1064 nanometers, both these are four level lasers, even the semiconductor injection laser diode is also a four level laser. But, for some applications or for some with some simplifying assumptions, we can treat the four level systems as three level systems ok. So, three level gives you a nice, I mean it is a minimum required level for lasing, and also it is most applicable with some approximations of course, for the more practical four level systems. And we will of course, look at three level systems.

But before that, the reason why I am considering only two energy levels is very simple. We have seen how the population densities look like, when there is equilibrium or when there is no external energy being supplied in the form of a pump. In that case, we know that the population density N 2 will be much less than the population density N 1, and there is a relationship between these two, which we have seen in the previous module, so that relationship is an exponential relationship right.

Now, that we have some absorption and or we have some pump incident pump radiation being incident. And this we have allowed for stimulated, spontaneous and absorption three different kinds of interaction with the radiation. What do you expect the relationship between B 1 2 that is where the transition is from energy level E 1 to energy level E 2, and B 2 1 where the transition is from E 2 or energy level E 2 to energy level E 1 right. This is a stimulated emission, this is the stimulated or this is the absorption right. So, what do you think of the two you know relationships are they equal, are they different is B 1 2 smaller than B 2 1 in magnitude, or B 1 2 is greater than B 2 1 in magnitude, and what is the relation with respect to A.

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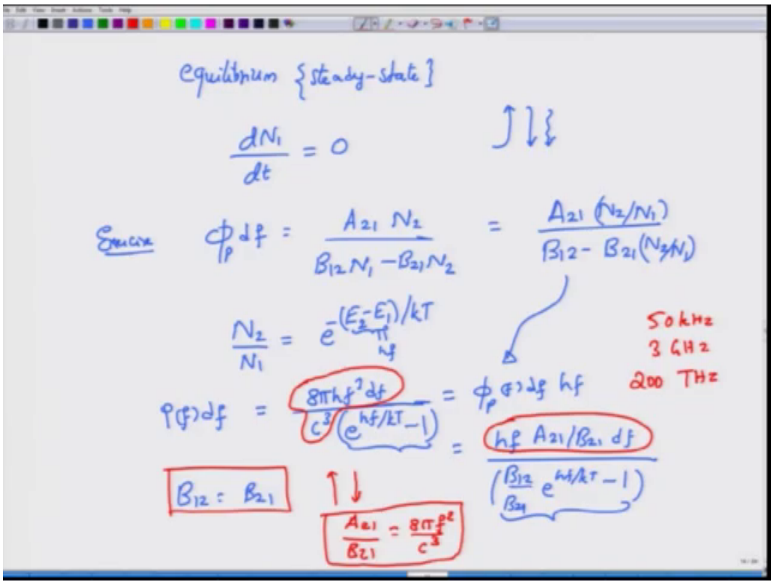

All these relations can be obtained by reconsidering a situation, where we are in the equilibrium, or I would actually call this as steady state ok. All that process of initially you know, pumping in an absorption and all the things have happened, and now we have reached a steady state or an equilibrium. In the steady state, what happens is that there is a balance between absorption and emission, so which means that the population density level dN 1 by dt does not you know change, and in fact that would be equal to 0 that is the rate will be equal to 0. So, what it means is that all absorptions are you know balanced by emissions, one will be stimulated, one will be spontaneous.

So, now if you said this dN 1 by dt equal to 0 in the previous equation that is in this equation, and then you know equate the terms, what you get is phi P d f being equal to A 2 1 N 2 divided by B 1 2 N 1 minus B 2 1 N 2. So, please take this as an exercise, you know in case you want to know how this equation is derived.

And this equation is interesting, because now what I am going to do is I am going to take this N 1 out, so that I can write this as A 2 1 N 2 by N 1, and then I have B 1 2 minus B 2 1 N 2 by N 1. But, I know the ratio of N 2 by N 1. What is the ratio of N 2 by N 1, N 2 by N 1 is E 2 the power minus E 2 minus E 1, which of course is the energy gap right, so divided by k T; k T of course, is the thermal energy.

So, now I also know what is the energy density right, so that energy density was rho f d f. And this was given by 8 pi h f cube d f divided by c cube e to the power h f by k T minus

1. Of course, E 2 minus E 1 is equal to h f, so I can actually you know go and write that one in that form right. And then equate this expression to the phi of x expression that we have written. So, this is of course, equal to phi P of f d f times h f. So, if you remove h f from both, then you will actually be able to write down the equation.

All alternatively substitute for phi P of f from the above equation, and then write down the equation here, and then you know equate the two, what you get is h f A 2 1, so sorry I change the notation here slightly. Instead of A, I have I have written it as A 2 1, just to indicate that this is interaction from second or energy level E 2 to E 1 ok. So, you have h f A 2 1 by B 2 1 d f divided by B 1 2 by B 2 1 e to the power h f by k T minus 1. So, you can you know see that these two are essentially same or equate the two terms out there.

And what you will actually see is that B 1 2 will be equal to B 2 1 ok, because you look at the denominator here, sorry this has to be in the bracket. So, you look at that equation here, so you have e to the power h f by k T minus 1 multiplied by B 1 2 by B 2 1. And you have e to the power h f by k T minus 1 that would be B 1 2 by B 2 1. Therefore, we are led to a very important conclusion that the coefficients Einstein coefficients of absorption is exactly equal to the Einstein's coefficients for stimulated emission.

And then you have all this term h f A 2 1 B 2 1 d f being equal to 8 pi h f cube d f by c cube, and then you can equate the two to obtain A 2 1 divided by B 2 1, which is the ratio here to be equal to 8 pi f square divided by c cube. So, I think there is a d f, no the d f anyway we will cancel. So, you have A 2 1 by B 2 1 being equal to 8 pi f square by c cube.

Now, we are at the end of this module. So, what I am going to do is that I will ask you to calculate this ratio A 2 1 by B 2 1 for two different frequencies. One the simple low frequency case, so let us say a 50 kilohertz case, and then you increase the frequency to RF ranges or microwave range, so let us say 3 gigahertz, and then you have you know and then you do the same thing for let us say about 200 terahertz ok. So, you are I will give you three frequencies, one is 50 kilo hertz, the other one is about say 3 gigahertz, so giga means 10 to the power 9, and then you do the same thing at 200 terahertz, tera being 10 to the power 12 ok.

So, we are increasing the order of magnitude in the frequencies here, and what you will actually observe is that this A 2 1 ok, because it is proportional to the frequency square increases as the frequency increases. And this is very important, because at low frequencies whenever you send in some light or shine light on to a matter right, you are still giving some amount of energy right. And then there is some you know movement from ground level to the other level, if there if the particular matter supports that one. But, the spontaneous emission is you know is so small in this case that you can completely ignore that. But, spontaneous emission cannot be ignored, when you start increasing to the frequency say that 200 terahertz. 200 terahertz is roughly equal to fifteen 50 nanometers, which is where you are operating in the fiber you know operating with the optical fibers.

So, to summarize what we have seen in this module, what started off was by looking at the two population levels N 1 and N 2 belonging to the energy states E 1 and E 2. We saw three interactions, spontaneous, stimulated and absorption. We wrote down the rate equation, this equation where you see how the population density is changing is called as the rate equation. I will leave the rate equation for N 2 to be written by you. And you can you know it will be essentially the same thing, but with minus signs replaced by plus signs.

And then finally, when we equated the two under the steady state condition, we found a very interesting and very important conclusion that the coefficient B 1 2, which takes atoms from energy level E 1 to E 2 is the same coefficient, which takes atoms from E 2 to E 1 ok. So, B 1 2 is equal to B 2 1. And finally, we also found out that spontaneous emission is proportional to frequency square, and therefore increases as the frequency increases. And at operate optical frequencies that is in a few terahertz scenario, this spontaneous emission will also become very important. What we will do in the next module is to apply these concepts and rate equations for a three level system, and then talk about how to actually make a laser.

Thank you very much.