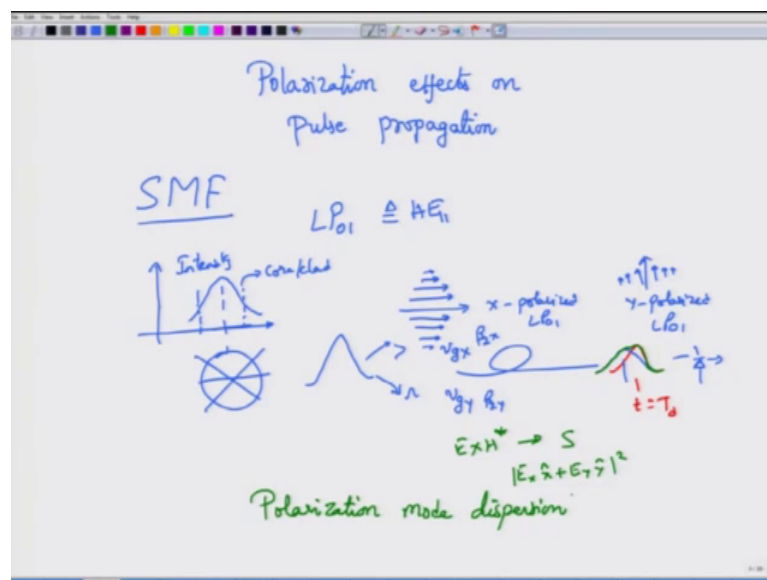


Fiber - Optic Communication Systems and Techniques
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 28
Polarization effects on pulse propagation

Hello, and welcome to NPTEL MOOC on Fiber-Optic Communication Systems and Techniques. In this module, we are going to study the effects of polarization dependence of pulse, which is propagating in the optical fiber.

(Refer Slide Time: 00:37)



What do we mean by polarization effect on pulse propagation? What we mean is well, let us focus our attention only on what is called as single mode fiber which means that we are dealing with only one mode at a given operating range.

And we know that in the weakly guided approximation the single mode fiber is you know supports the fundamental mode which is called as LP₀₁ mode which of course is the same as HE₁₁ mode right. But what you might not have realized is that if you look at the power, of course the power would show something like this I really do not show you, but you know maybe instead of trying this power in this 2D picture, I will show you the power in the 1D picture. So, if you take a cross section across the fiber, you will actually see that the most of the pulse power is concentrated at the center.

And it kind of Gaussian way falls off as you go away from the core; they are the center of the core. So, this is of course true when you take the cross section in any way, because this LP₀₁ mode is completely independent of the azimuthal angle. So, all different cross sections you know whether you cross section it this way or you take this way, this way would essentially exhibit the same type of intensity or power patterns.

So, if I were to plot this as intensity on the Y axis, and on the X axis let us say this is the center of the core, at the center of the core you have the maximum intensity, and as you go away from it you will have the intensity fall off. Somewhere here you would have the core to clad interface. So, there is some field in the cladding as well.

How good you have confined this pulse or how good you have confined this mode depends on various parameters of the numerical aperture, the value of delta and so on and so forth that is not our interest at this point. What is our interest is that although we have written the power profile, if you actually look for the electric field pattern, you will find 2 types of electric field pattern. You will see that the field lines could be you know organized something like this.

So, this is if this is this horizontally directed segments can be considered to be X polarized, then you have what is called as X polarized LP₀₁ mode. And then you can of course, have a polarization that would be vertical ok, and this would since this is X horizontal line, and this vertical line is the Y direction, so this would be called as Y polarized LP₀₁ mode ok.

So, even though you have a single mode fiber, and we say that it is actually single mode ok, because you have only 1 mode LP₀₁. In actuality what happens is that these 2 modes are I mean there are actually 2 modes at LP₀₁ ok. And these 2 modes have the same value of beta that is they have the same propagation constant.

And if the fiber were to be ideal in the sense that there are no distortions along the fiber, there is no other you know changes in the fiber along it. And the fiber is actually made out of nice isotropic material homogeneous isotropic linear material. Then if you launch an information or if you launch pulse, which is X polarized you know at the input of the fiber, then that will come out of the fiber with the same polarization.

And if you launch in you know pulse and which is initially Y polarized at the input of the fiber, then this Y polarized pulse would propagate and come out of the fiber at the same polarization. And if you were to launch input which has some portion in X polarization some portion in Y polarization, because these 2 polarizations are not talking to each other or interacting with each other. Then this amount of pulse which is in the X polarization would come out independently. And the Y polarization amount would also come out independently. And they both will arrive at the same point ok.

So, this is very important, because when you launch a pulse, and this pulse breaks up into X polarized pulse, and Y polarized pulse I am just showing this in a pictorial manner. Then when you have pass it through a fiber ok, and if this fiber somehow is not an ideal fiber, but it kind of distinguishes between X and Y polarization, it gives a certain group velocity $v_g X$ for the X polarized wave and gives your velocity $v_g Y$ for a Y polarized wave, what would happen at the output, well by now you already know. You have 2 pulses one of them propagating with a velocity $v_g X$, and the other pulse propagating with a velocity $v_g Y$.

And furthermore if you allow for some dependence with respect to frequency on $v_g X$ and $v_g Y$ itself, then you have $\beta_2 X$ and then you have $\beta_2 Y$. These 2 pulses can be considered as coming off from 2 different modes right. So, it is essentially like 2 different modes which are propagating with their own velocities. And therefore, they do not arrive at the same time.

So, you might let us say have a pulse, which arrives at some fixed time t equal to some T_d is my delay time that I am going to consider. So, let us say the X polarized pulse has come in, but because of the way that the group velocity of X and Y polarized modes differ in the fiber. The Y polarized mode might arrive slightly earlier itself or it may arrive later does not really matter,.

But, now when you put in through a detector you want to detect what would be the output, since the detector is only going to look at $E \times H$ right or $E \times H$ conjugate, which if you remember is actually the pointing vector correct, and since you are considering kind of scalar variables and transverse coordinates. What this actually means is that you are looking for $E_x^2 + E_y^2$ magnitude square. And this when you do this magnitude square, the detector, which is actually doing this right, the detector cannot

detect the fields it can only detect the power. What it actually detects is this combined pulse which would then be looking like this right.

So, you can clearly see that compared to the original pulse. The new pulse is now distorted ok. This is kind of 2 different modes interacting with each other, so you can think of this as intermodal dispersion. So, this kind of a dispersion, which distorts the pulse simply, because even though you considered single mode fiber, the fact is that single mode fiber, supports 2 fundamental modes. And if the fiber is made out of a material, if the fiber exhibits any property, which allows you to have different values of $v_g X$ and $v_g Y$ ok, and this phenomenon wherein the values of X and Y polarized modes actually travel with different velocities is called as birefringence ok.

So, and all optical fibers have some amount of birefringence. Some optical fibers have large amount of birefringence, because you want to create it that way, but in most standard single mode fibers the birefringence exists not because you want it, but because it is the material property. And you try to control it you try to make that birefringence small so that the two modes which are launched on the X and Y polarization or the pulse is launched on the X and Y polarization arrive at the same time.

But, when they do not arrive at the same time, because of this birefringence or the residual birefringence in the fiber. The overall pulse when it arrives, will be distorted will be more or less spread out, and in very severe cases they may interfere in such a way that they actually cancel each other. This phenomenon is called as polarization mode dispersion.

Polarization mode dispersion unfortunately is a very complicated phenomenon. So, it is that we really cannot do justice in 20 minute module, because the polarization mode dispersion in optical fibers in practical optical fibers that are laid out which has you know thousands and thousands of kilometers of these fibers that are laid out is not variously difficult to mathematically understand it.

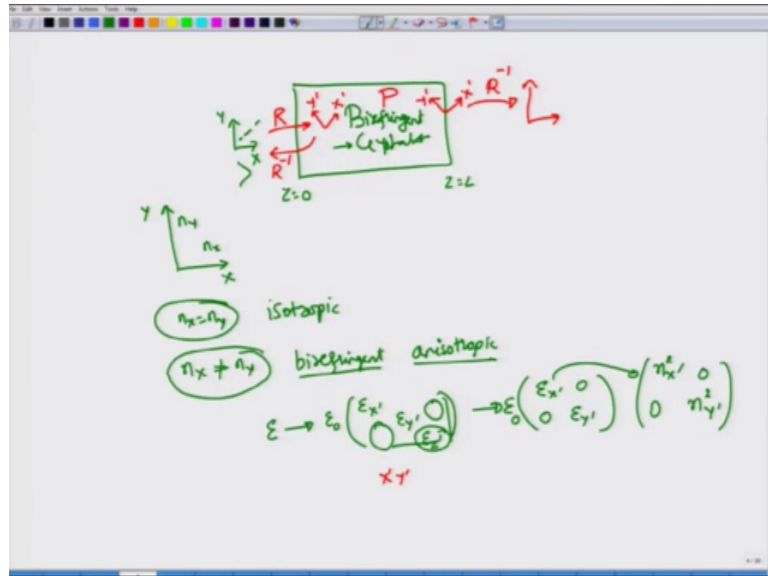
And moreover our understanding of polarization mode dispersion is also quite limited in the sense that there are at least 2 orders of effects of polarization mode dispersion there is a first order polarization mode dispersion which I described just now ok. So, you had these pulses at the same frequency, but launched onto the 2 different modes. And these different modes are misaligned in time, because of their group velocity difference.

And therefore, the overall pulse has, now just kind of expanded or distorted. If the first order p m d effect ok, and this first order p m d effect was discovered in the late 80s and early 90s and substantial amount of work was done to understand this p m d phenomena. And there have been some good mathematical models and numerical approaches to study this p m d which first order p m d which is which one of which is what we are going to discuss in this module, but it turned out that as the data rates were increased further. We actually had to deal with what is called a second order polarization mode dispersion.

In the second order polarization mode dispersion the birefringence axis which I am going to discuss shortly itself is not fixed or itself is dependent on frequency, which means that different frequency components are going to see different axis of birefringence, which means that the entire effect is kind of very complicated. And it is usually studied in the context of some statistical phenomenon ok.

So, we do not really have a good mathematical model for second order p m d. We mostly use statistics or Monte Carlo type of statistical analysis to deal with the effect of second order PMD on the pulse propagation, but nevertheless the idea is that pulse propagation does I mean the polarization mode dispersion whether it is first order or second order PMDs. There is also third order p m d which you might now guess that it is even more complicated than what we have allowed for. All these effects in result; in the fact, that the pulses are distorted and the pulse output polarization will not necessarily be the same as the input polarization.

(Refer Slide Time: 11:24)



So, how do we go about understanding this PMD, well to understand PMD we need to first understand how light propagates in the so called birefringent medium. Now, what is birefringent medium? Well, you know if you go to simple X and Y coordinate system, and you have a pulse which is propagating along the z axis ok..

The refractive index that the field sees along X polarization; let us call this as n x and along the Y polarization we call this as n y. If n x is equal to n y of the pulse which is propagating along z, but it has electric field which is polarized along x and it has electric field which is polarized along y.

So, if you have those 2 polarizations, and these two polarization see the refractive index n x and n y which are the same then we say that the medium is isotropic ok, which means that the refractive index that the polarized light sees does not really depend on the polarization whether the light is polarized along x direction or y direction or any other direction, the refractive index that the polarized light will see will be the same that is the isotropic material. But what if n x not equal to n y, do not ask me how it is possible it is a material property.

When some of those interesting things, that result with n x not equal to n y is this phenomenon. When you take a glass slide, which is actually made out of calcium, then you will actually see what is called as double refraction right. So, on roadsides you will see this simple artifacts being sold where it looks like there is a 3 D picture of you know

some Taj Mahal inside right. So, if you hold it up, you will see that there is a cut in the glass. So, those glasses actually are birefringent.

And the birefringence means that the light polarization determines what is the refractive index that the light freeze, so which means that they have different phase and group velocities depending on the polarization of the light. And that double refraction is quietly used to make those simple 3D looking glass pieces that you actually can buy along roadside anyway; that was just a digression.

What we really are looking at mathematically is that, if I have a pulse which is propagating, then this pulse would actually have n_x not equal to n_y ok. And such medium is called as birefringent medium or in general it is called as an isotropic medium. And the most common and isotropic medium that we are going to study is a birefringent medium, whose refractive index or equivalently its permittivity will no longer be a simple scalar permittivity, but it will actually be a matrix. And in a certain coordinate system which is called as the material coordinate system or the crystal coordinate system which let us denote it by a different coordinate axis.

So, you have a different birefringent or a crystal medium. And optical fiber does you know is actually birefringent to some extent. But we will continue to use the word crystal ok, because this is a common terminology in dealing with anisotropic materials. So, what we mean by crystal is that this is the lab coordinate X and Y wherein you are going to launch a light maybe light is propagating along Z . So this is Z equal to 0 this is Z equal to L . And light would be polarized along X direction let us say or Y direction.

So, this is how the world is outside this birefringence material or the crystal material. But inside crystal material there are only two axis on which the pulse actually are you know propagates without talking to each other. And this axis which we will call as say X' and Y' are not usually aligned to be the same axis of X and Y .

So, there is a certain offset angle between the two and you can actually relate this x y with 2 X' Y' via are what is called as a coordinate transformation or a matrix which transforms the X and Y axis on to X' and Y' . And of course you can go back from X' Y' back to X Y coordinate axis by multiplying the X' Y' vector by R inverse which is the inverse transformation coordinate transformation that you are going to do.

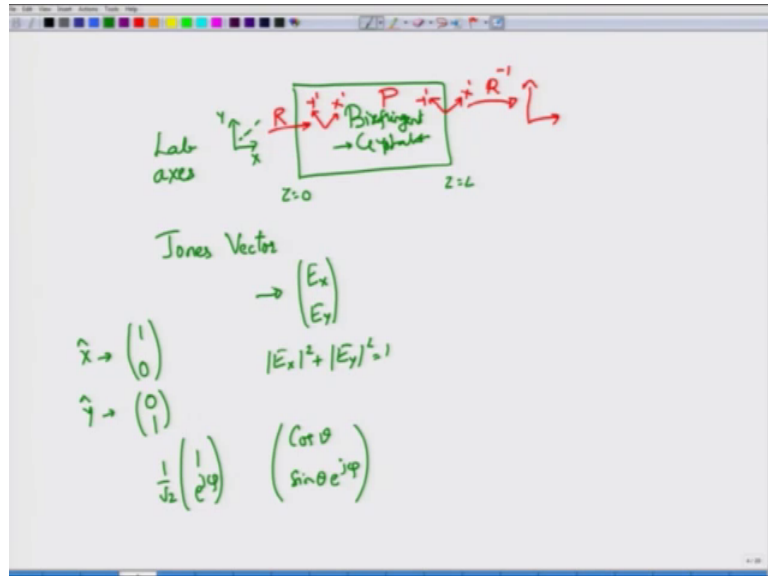
So, what you have to do is to first take the pulse, which is in the lab domain or in the lab frame of reference coordinates X and Y transform that into X prime, Y prime, which is the crystal axis or the crystal coordinate system. Propagate this one over whatever the length that you have which will be governed by a propagation matrix P I am going to show you how the propagation matrix looks like. So, just before the end of the crystal what you would have will be how the pulse has propagated in the crystal and is affected by the propagation matrix P , and then you go back from X prime, Y prime to your lab coordinate system by multiplying with the inverse propagation matrix.

And in this X prime Y prime coordinate system. So, in the x prime y prime coordinate system the epsilon matrix will actually be something like this ok. So, this is 0, this is 0 which means that this is a diagonal matrix with X prime and Y prime being different and Z prime really is not of our concern, so we do not really worry about this one, but the other two terms are actually 0. So, you are essentially looking at epsilon X prime 0 0 epsilon Y prime as a sub matrix, because the wave is propagating along Z . And therefore, I do not really need to know this row and that particular row.

So, what we actually think of epsilon as a simple material constant in our usual courses actually turns out to be not a constant, but is rather a more general quantity called as a tensor. And in this context for birefringent materials, this tensor can be replaced or represented in the matrix form by in this fashion ok. And of course, epsilon X prime is actually $n \times \text{square prime}$ ok, because the relative permittivity is actually proportional to or answer is given by the square of the refractive.

So, if you are getting little confused, do not worry what we are going to do now is to actually use or give you this procedure which will allow you to understand how light propagates through this birefringence material. More details on birefringence unfortunately I cannot provide you in this module. You will have to look at those details in some references which I will suggest ok.

(Refer Slide Time: 18:17)



The primary tool that we are going to use in understanding the propagation is what is called as Jones Vector. And I know that you know not I know I have discussed Jones Vector earlier in the course. What was Jones vector well the input Jones Vector was that you would represent the electric field components in terms of their X and Y coordinates or X and Y components by writing this as EX, EY right.

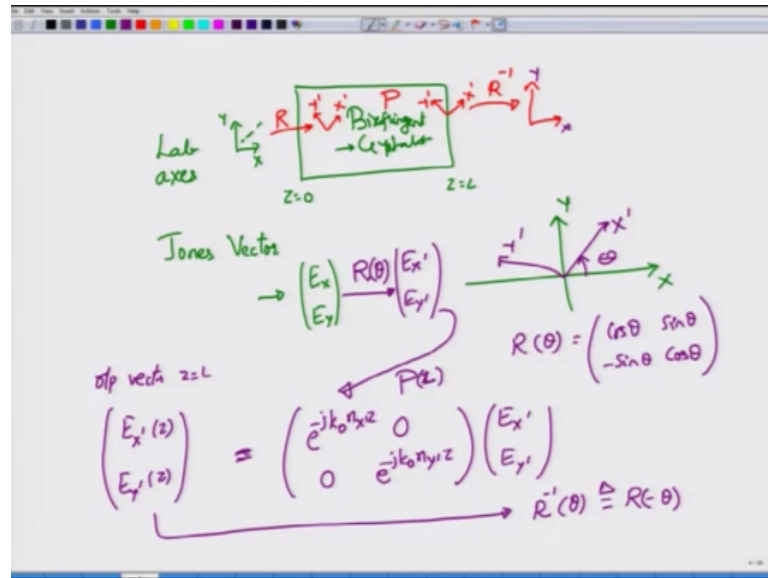
Of course, you would have normalized. This one in such a way that magnitude of EX square plus magnitude of EY square will be equal to 1. For simplicity an X polarized wave would be written as 1 0, so this is for the X polarized wave, Y polarized wave was written as 0 1.

And in general if you have a linear polarization you can write this as $\frac{1}{\sqrt{2}} e^{j\phi}$ but at any other angle you will have to write down the actual this one. So, sometimes this notation of $\cos \theta$ $\sin \theta e^{j\phi}$ is used with 2 parameters θ and ϕ ok. To account for not just linear polarization, but also for circular and elliptical polarizations.

So, we are going to use one of these forms. If your light is propagating in its in its polarized along X, then you can use the simple you know the actual Jones vector of 1 0. If it is a long Y, you can use 0 1 and so on, so whatever the input polarization is it is measured in the so called lab axis or the lab frame of reference and in that lab frame of reference this is the input Jones Vector which is normalized as I have told you ok.

The first step is that we recognize that the pulse that you have launch Jones Vector that you have described is only valid for the lab axis. The actual Jones vector that you need to write will have to be written for the crystal axis..

(Refer Slide Time: 20:15)



So, you have to go from X Y axis to X prime, Y prime axis. And these two axis are related like this. This is X and Y. And you have X prime and Y prime axis here. X prime and Y prime axis are at an angle of theta where theta is the rotation angle between X Y, and X prime Y prime is the offset angle between the lab and the crystal reference frame. And you can go you can transform any vector which is in the XY coordinate to the vector in the X prime Y prime coordinate by multiplying the vector by the coordinate transformation matrix R of theta which you can show is a 2 by 2 matrix given by this one ok.

So, when you multiply this E X, E y vector which is the Jones Vector in the lab axis with R of theta which is the vector in the birefringent crystal, then you multiply that one or you actually obtain this as E x prime, E y prime. Now, E x prime, E y prime have to propagate through the crystal which know has a certain length say L or maybe if you do not want it you can just write it as Z, and this propagation matrix will be e power minus j k 0 K 0 of course is the free space wave vector times n x Z 0 or n x prime Z e power minus j k 0 n Y prime Z. So, this is the propagation matrix.

So, we are going to multiply this propagation matrix to E_x prime E_y prime to obtain E_x prime at Z and E_y prime at Z that is right at the output or at any distance Z in the crystal axis. This is what you would have actually obtained right..

Now, what is the final step well you have to take this output E_x prime Z , E_y prime Z , because this is the output Jones vector. When Z is equal to L ok, and then multiply by the inverse matrix to go from x prime y prime axis back to X and Y axis. And this R inverse of θ turns out to be you can show this one as an exercise by you know can be obtained by the same R matrix by changing θ to $2\pi - \theta$ ok. When you do that one, the sine of these two elements will interchange the off diagonal elements whereas, the diagonal elements would essentially remain the same, because \cos minus θ is \cos θ and \sin minus θ is $-\sin$ θ .

(Refer Slide Time: 22:46)

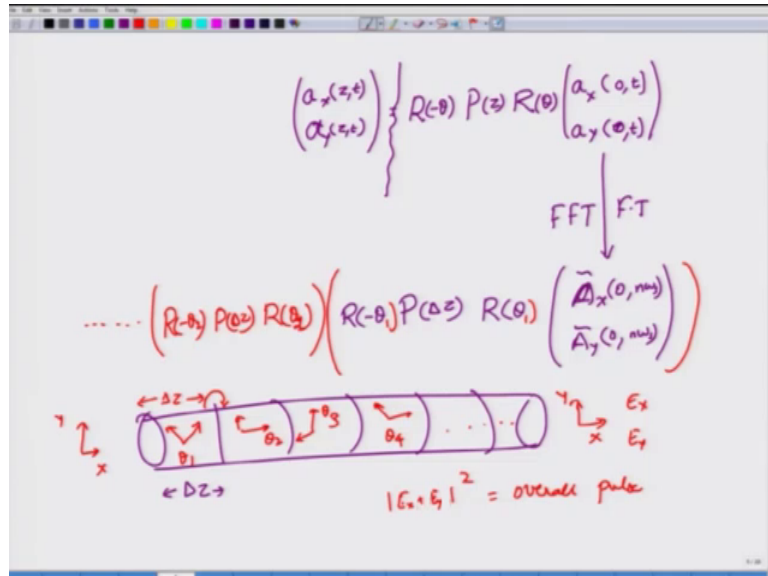
$$\begin{pmatrix} E_x(z) \\ E_y(z) \end{pmatrix} = R(-\theta) P(z) R(\theta) \begin{pmatrix} E_x(0) \\ E_y(0) \end{pmatrix}$$

So, this is what you have to do to go from the Jones vector which you of course, know at z equal to 0 which is at the launch point of the fiber or any birefringent material multiply it by the R matrix which of course with a certain value of θ then you propagate it with a distance of z and then you have R of minus θ to go from X prime Y prime back to X and Y .

So, once you follow this procedure what you get is a Jones vector at the output. So, instead of writing this at z equal to 0 , let me write it in this way, so E_x at 0 E_y at 0 . And

instead of dealing with electric field you may want to deal with the pulse on envelope that is another thing that you can do.

(Refer Slide Time: 23:34)



So, you can actually deal with a x of 0 a y of 0, a x of 0, and a y of z. Of course you do know that what you actually have is not a x of 0,, but rather a x of 0 and t correct what you had is a x of 0 t which is the input pulse launched which is x polarized and the pulse which is launched and why this is a y of 0 t. And of course, you know at the output also is that it is not really a x of z, but it is a x of z t, a x of z t and this is what you actually have. Of course, it is not really like in this manner.

So, this relation is not exactly correct. What you need to do is you have to take the Fourier transform. So, you have to first deal with A x of 0 omega, and then A y of 0 omega ok, this would be the frequency dependent Jones vector or Jones vector for each of the frequency components that you are looking at. And instead of taking ft there is Fourier transform in if you want to implement this numerically you are going to take first Fourier transform or DFT discrete Fourier transform.

And therefore, obtain this discrete array omega s or n omega s components right. And once you have this discrete Fourier transform you are going to multiply this one by R of theta and then you have p of delta z because you know you have a fiber of this section long then you are going to assume that this fiber can be sectioned into many sections of

delta z length. So, you multiply the propagation constant only over that delta z length and then you have R of minus theta.

In order to account for first order PMD, what we assume is that while the lab frame axis the lab axis x and y are fixed all that I know in the orientation the offset angle theta is different in different sections. So, let us say this is theta 1, and let us say this is theta 2 which is different. So, this is theta 2, and this is say theta 3 this is theta 4 and so on and so on up to the last section.

We assume that over the length delta z, the birefringence axis or equivalently the angle theta one is fixed, but once you go from one section to another section the angle theta actually changes. Therefore, if you propagate this over a length delta z, what you have to do is to propagate it with the appropriate angles as well. So, once you have done this one this is the output r delta z with a birefringence angle of theta 1. And then you further need to now start multiplying this by the same procedure.

So, you have to take this as R of theta 2 P delta z which of course, is kind of constant and then you have r of minus theta 2 and you again start multiplying and so on and so forth to obtain the Jones vector at the output. And once you obtain the Jones vector and output you can then take the magnitude square that is E x plus E y to obtain the overall pulse power.

So, this is what you actually have to do. And this method will account for as I have told you only the first order PMD. The specific distribution of theta one and theta two is subject to some statistical tests. And you will actually find models wherein you can you know find the you will find literature wherein you know what is a statistical distribution of theta 1, theta 2, theta 3, theta 4 and so on.

Ah In the next module we will see how to implement BPM, and what were the results of this BPM. And this method that we have just described is the for the polarization mode dispersion is exactly like the PMD meth or sorry is exactly like the BPM method except that this is now applied individually to the x and y polarizations right. And the coupling or the interaction between the x and y polarizations is happening because of the rotation matrix R ok.

If the rotation matrix were to be diagonal or identity matrix, then these two components would not actually talk to each other. But because this is not identity matrix, you have x polarization talking to y polarization, and messing up your entire pulse which is propagating. By the way these are fairly easy to implement in matlab, I will give you the matlab based code and which you can use to implement and study the pulse propagation both for BPM as well as for PMD.

Thank you very much.