Fiber - Optic Communication Systems and Techniques

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Lecture – 27
Beam propagation method

Hello, and welcome to NPTEL MOOC on Fiber-Optic Communication Systems and Techniques. In this module, we will study pulse propagation through optical fibers. Now, you might ask we have been studying pulse propagation through optical fibers in the last two three modules, what new thing are we going to discuss in this module. It turns out that pulse propagation through optical fibers is such an important topic for it has many, many applications which maybe some of them may be good, some of them may be bad ok.

But, it is very important to understand, how the pulse is affected as it propagates through the fiber. For example, in communication systems that is when we use optical fibers to communicate information in the form of pulses right, so each pulse may be representing an information in a certain manner. For example, the presence of an optical pulse of a certain duration t 0, may represent bit 1 being transmitted in a digital optical communication system; and an absence of a pulse may represent a bit 0 being transmitted ok.

And if the fiber were to be an ideal channel, it would only it would not actually change the amplitude as well as it would not distort the pulse, it would not broaden the pulse or it would not compress the pulse. But, unfortunately as we have seen an optical fiber is not an ideal channel, although it is much better channel compared to other types of channels that are available say copper cable, satellite, earth communication and so on and so forth.

But, nevertheless there are certain things that the optical fiber does do the pulse that is propagating which causes errors, when you start detecting them at the receiver. So, leading to some errors which is quantified usually in the form of a bit error rate. So, you want to keep the system BR bit error rate to be small, then it is necessary to understand, how these pulses actually propagate through the fiber.

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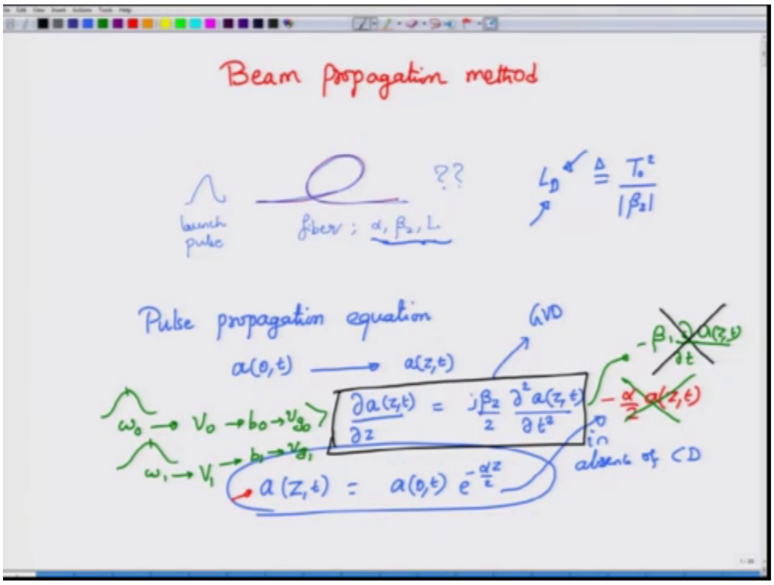

We have already seen how pulses propagate through the fiber in some sense, because we derived what is called as a pulse propagation equation correct. So, this pulse propagation equation, we derived it which governs how in the typical propagation problems, how a pulse whose amplitude let us say or envelope, let us say is given by a 0, t to remind you again 0 is for z equal to 0 that is at the input of the fiber.

How this would change to a z, t right, so in fact this change is governed by a certain equation, which is the pulse propagation equation, which says del a of z, t by del z that is the rate at which this a is changing with respect to z is directly proportional to the dispersion parameter beta 2, which is which we called in the last module as group velocity dispersion correct, it is technically group delay dispersion. But, these two terms are sometimes used interchangeably, because delay and velocity are inversely related with respect to each other. And what was the other term here, it was del square a z, t by del t square.

So, you might say that well we already have a pulse propagation equation. So, it would actually make our life very easy to understand pulse propagating through the fiber. Unfortunately, this equation can be solved only for very special cases; that is it can have solutions closed form solutions, as we would call them only for very special cases. And one such special cases that of the Gaussian shaped pulse, which was propagating through the fiber.

And we saw in the last module that when you take an unchirped Gaussian pulse propagated through the fiber. Then at the output of the fiber, the pulse will become chirped that is its phase will change across the pulse. So, different parts of the pulses are experiencing different phases. And this phase change is dependent on the position of the pulse that you are actually looking at. And therefore, this gives you an unwanted frequency modulation, because change in phase or rate of change of phase will directly impact the instantaneous frequency of the pulse.

So, the spectrum you know or the pulse will actually undergo chirping as we would call and depending on the sin of beta 2, it would either be linear up chirp or linear down chirp. And this linear chirp is simply, because we have assumed the Gaussian pulse to propagate in the fiber ok. However, that so that that special case of Gaussian pulse is not really true for all you know for all practical applications, I mean yes. Most of the pulses that are you know generated by lasers, both from semiconductor lasers and you know fiber lasers, can be approximated to have pulse shape, which is Gaussian or even if it is not Gaussian, it is close relative that of the secant hyperbolic function, which also has certain analytical solutions, closed form solutions.

So, you can use these equations, solve the equation to know precisely, what happens as the Gaussian pulse or the hyperbolic secant pulse passes through these fibers. Secant hyperbolic pulses and Gaussian pulses themselves are very good approximation to pulse shapes that are actually produced. But, in many situations in communication systems in order to combat what is called as inter symbol interference you want to shape the pulses that are propagating in the fiber ok.

So, you will actually shape the pulses using certain filters. And this shaped pulses will not be so easy to solve analytically. To understand, what is happening to those pulses as they propagate. For example, the simple thing that if I were to take an arbitrary launch, I mean pulse and launch it into the fiber, and ask how does the pulse width change for this pulse. I would not be able to find out by solving this equation in the analytical sense right. So it is not like ok given this pulse, use this equation, solve write some mathematical equations, and then finally obtain the expression for the output pulse. It just does not happen for arbitrarily shaped pulses. And arbitrarily shape pulses are very important in all pulse processing applications including communications and certain other applications, which are related closely to non-linearity in the fiber something that we have not really discussed yet.

So, if we cannot really use this pulse propagation equation, then what good is this pulse propagation equation. It turns out that the equation is more or less fine ok, except that we cannot solve it analytically for all arbitrary cases. Therefore, we will solve it numerically, because I can use a computer. And then, I can change this equation or solve this equation on a computer. And then study different or arbitrary pulse shapes that are propagating in the fiber. And what is the effect of dispersion on the fiber ok, so that is what we are going to do.

And one such technique in order to do that that is numerically solve these equations is what is called as beam propagation method ok. It is actually a subset of another method called a split step Fourier method, which is used to solve even more generalized pulse propagation equation that generalize pulse propagation equation is called as non-linear Schrodinger equation. And that will account for the non-linearity in the optical fiber that non-linearity in the optical fiber is not of immediate concern to us, it will be discussed in some later modules ok.

So, but the basic idea of beam propagation method is what we are going to discuss, in this module. And it is such a nice and simple method that you can write about 20 or 30 lines of MATLAB or Scilab code to actually implement this beam propagation method. And study how different pulse shapes actually propagate through the fiber, given the parameters of the fiber right.

So, what parameters of the fiber do you have, you have attenuation, you have beta 2, which of course in the second order dispersion or the group velocity dispersion. And L being the length of the fiber, you need to know this. Sometimes in the problem or sometimes in practice you do not really know what is the length of the fiber or you are not really interested in the actual length of the fiber, but you are interested in the dispersion length of the fiber.

And if you recall what dispersion length of the fiber was, it was actually defined as the ratio of the square of the pulse width t 0 or rather t 0 square and the magnitude of the dispersion coefficient beta 2 or rather magnitude of the dispersion parameter GVD parameter beta 2 ok. Because, you know that significant changes in the pulse shape happens only for fiber lengths, which approach or exceed this dispersion length ok. So, this is what you have, of course you might observe that in these equations that we have written in the pulse propagation equation that we have written. We have not really included attenuation alpha.

Now, it is a simple phenomenological way of including alpha that is possible. For example, suppose the fiber has no dispersion, which means that beta 2 is equal to 0. Then what do you expect the pulse to actually look like, when it propagates through the fiber. The pulse envelope would simply be whatever the pulse that you have at the input, except it is now attenuated by a factor of say e power minus alpha z where alpha is the attenuation parameter or the attenuation of the fiber that we have taken.

Of course, in many cases alpha represents the power attenuation that is attenuation measured with respect to launch power, and the power that is available at the fiber. Therefore, to take into account that we are dealing with fields and not really with powers, you can just divide this alpha by 2. So, you actually have field attenuation of alpha by 2, whereas power attenuation of alpha. And this is what you are going to obtain, in the absence of dispersion right.

So in the absence of dispersion which we will call as say C D just as a general place holding convention so, CD is not really the only dispersion that we are considering, it could in fact be material dispersion, it could be wave guide dispersion, in that it could be inter model, intra model all those dispersions can be are all different, but we will put all of them together in the heading of CD ok.

So, in the absence of CD, this is what we expect that they will simply be a amplitude change. And this amplitude change can be incorporated by modifying our pulse propagation equation by simply writing this as minus alpha by 2 a z, t right. I hope you see that how simple it is to write this one, because you can differentiate the left hand side with respect to z, and the right hand side with respect to z. And then, you will see that this minus alpha by 2 term will drop out. And then, it will be proportional to on the right hand side will be proportional to minus alpha by 2 times a z, t.

So, addition of this minus alpha by two a z, t will help you to take into account the attenuation, but because attenuation is usually not so frequency dependent for the region of operation that we are considering. We do not really worry about attenuation ok, because whatever that we have at the output of the pulse. We will finally, multiply that 1 by e power minus alpha by 2 times z, so because you can account for attenuation in a very simple manner.

Usually, this term is not included in solving the pulse propagation equation. Of course, the equation that we have written is very nice ok. It helps you to understand how the pulse propagating through the fiber, but it does not really tell you. What happens when you have two different pulses, one pulse having a frequency or centered at frequency omega 0, and the other pulse centered at frequency omega 1.

We know that when you have two pulses at two different frequencies, then these two would correspond to two different values of V 0 and V 1. Assuming, we are looking at single mode propagation. This would correspond to two different V parameters V 0, V 1, which in turn would correspond to two different normalized parameters V 0 and V 1. Of course, these are only at the center any other frequency components will have to be looked at whether deviation about omega 0 and about omega 1. And the fact that b 0 and b 1 are different leads us to know that the group velocity will also be different. So, group velocity for the pulse which is centered at omega 0 will be v g 0, whereas the group velocity for pulse which is centered at omega 1 will be v g 1.

Therefore, you should technically to account for the fact that you can have different group velocities, add a term that would also include this group velocity term. And since, we know that group velocities are inverse group velocity is related to beta 1. The term that you want to add actually will be first order with respect to del t, and it would be del a z of t by del t ok.

So, this is the term that you need to add, if you are going to consider group velocity as a separate term, in that case the time that you are representing t here will not be the delay time or the retardation time, but it will actually be the or it will be the actual time of the pulse that you are considering ok. However, again unless you are dealing with multiple pulses, which are all centered at different frequencies. You do not usually consider solving this pulse propagation equation by including the term beta 1.

However, when you have multiple pulses, you have to write an equation, which will include the individual group velocities. So, you will have beta 1, 0 beta 1, 1 and so on for different frequency components. And those also should be reflected in the pulse propagation equation ok. The point of for the last 5 10 minutes of what I am saying is that we actually have this equation. And we are going to solve this equation using some numerical methods ok, because other terms although are important are not necessarily important.

At this stage in since it is our first introduction of a numerical method for solving pulse propagation. The other terms become important, when you allow for pulses to interact with each other. And this interaction is actually brought out by the non-linearity in the fiber, so because we are going to postpone non-linearity for quite some time. We are not going to deal with pulse to pulse interaction.

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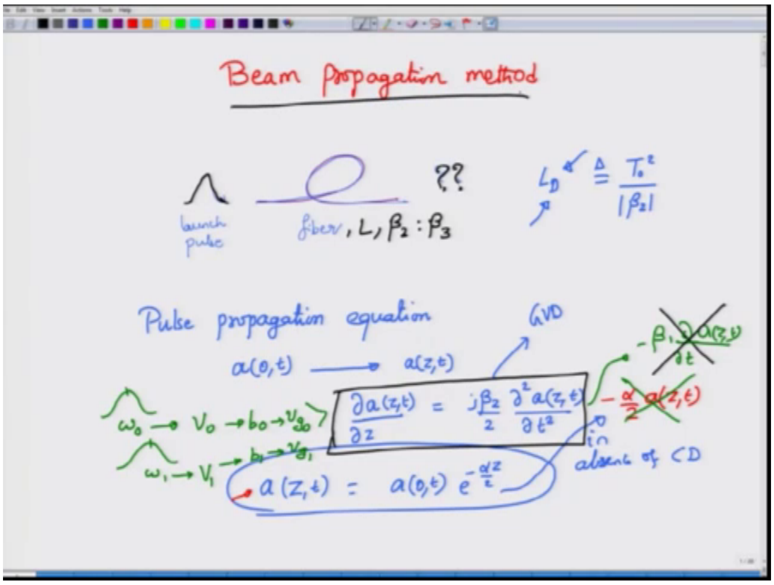

We simply assume that you have a pulse or a sequence of pulse, which is propagating through the fiber. And that can be studied by solving this pulse propagation equation. So, the idea is that we have the fiber of certain length L has a parameter beta 2. Of course, beta 2 can be positive or negative and moreover beta 2 itself can vary over the frequencies. So, in that case you will have to also deal with beta 3 ok, which is much more complicated. And it will modify the pulse propagation equation in a slightly different manner, which we will see in the end of this module ok.

And the goal here is that you have a certain launch power ok. So, you have a certain launch power here, and you want to know what happens to this launch pulse not launch power, it is launch pulse. You want to know, what happens to this pulse which is launched into the fiber, how does it show up on the output. And you can you are going to do that one or you are going to know that one by using what is called a beam propagation method.

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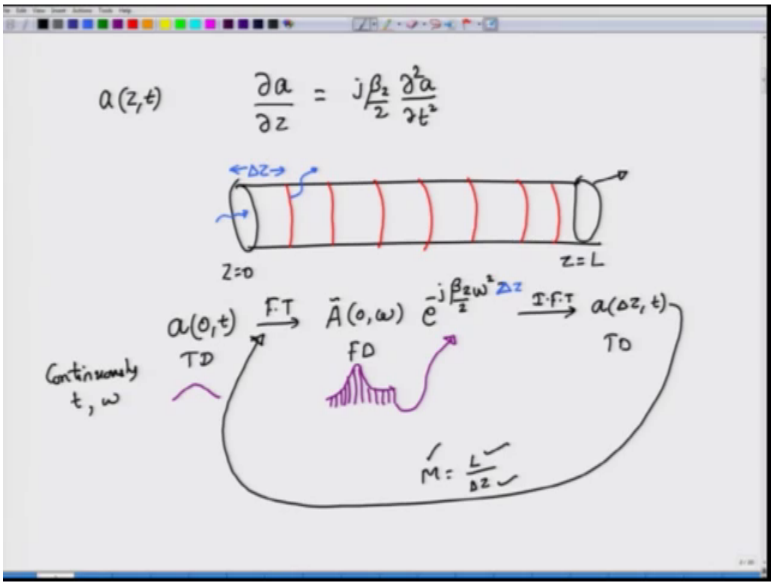

Now, what is beam propagation method, let us start with the equation that we are considering, I am going to suppress the dependence on z and time of the pulse a here. So, actually a means a of z, t, I do not want to write z and t every time ok. So, this is the simple equation that you have please note that this pulse envelope a is a function of both z as well as t. z of course being the distance along the fiber, which I am going to consider it like this. Let us say this is the fiber, this is z equal to 0, this is z equal to L ok.

You can of course normally certain parameters, and obtain what is called as a normalized pulse propagation equation. We are not going to do that one here ok. How do we solve this equation well; how did we actually derive this equation. Well we kind of derived this equation by a three step procedure. We actually said that you start off with the pulse envelope right at z equal to 0, you have some a of 0, t. You then Fourier transform it, because you want to study how the propagation of individual frequency components is being you know affected by the fiber. So, you Fourier transform it to obtain a of 0 omega.

And I hope that you remember that this omega that I have written is actually the frequency deviation or in other sense that I am actually looking at the baseband evolution ok. But, when you actually go and look at this beta 2 term you need to keep in mind that this beta 2 is actually parameter that is measured around the center of the carrier frequency or rather at the central frequency or the carrier frequency ok.

Anyway, so you start with the pulse in the time domain. So, I will call this as TD just to indicate that this is in the time domain. This is in the frequency domain, and then what we do we multiply individual frequency components by this term right. So, e to the power minus j beta 2 by 2 omega square is known, but what else am I going to measure. Of course, I should write z or you know z equal to L, which will cover the propagation from input to the output.

But, it turns out that numerically when you are solving these equations, it is nice. Because, otherwise numerically it will the numerical solutions will become unstable to avoid that numerical unstability or instability what you do is you section up this fiber into small sections of some delta z long ok. It is not required that you section them up into equal parts, but it is usually easier to code that way, when you do it with equal parts. And what you are actually looking at is how the pulse in the frequency domain, this starts out at say z equal to 0. We look at z equal to delta z, and to do that one instead of multiplying it by z, you multiply this one by delta z ok.

So, here you go you started off with pulse at z equal to 0 then propagated over a distance delta z propagation in this case simply means that you are multiplying the phase factor here. And every frequency component omega that you have in the frequency domain of or the spectrum of the pulse will be multiplied by delta z. So, pictorially if I want to depict this one, this was some launch pulse let us say. After going to the frequency domain, let us say it this was the spectrum. And these are all the different frequency components that I have right. And each frequency component will be multiplied by this phase factor e to the power minus j beta 2 by 2 omega square times delta z.

Now, what is the next step? After doing this, I need to take inverse Fourier transform. When I do that in the time domain, I obtain the pulse, which has now propagated a distance delta z away from z equal to 0. And I know how it looks in time. And how do I obtain what would be the output at z equal to L, I simply repeat this process of taking the Fourier transform propagating through a distance delta z, and then taking the inverse Fourier transform, again I do it, again I do it. So, I keep doing this one until I reach certain steps. So, I reach say M step, where M is about L divided by delta z or maybe L by delta z minus 1 does not really matter.

So, you need to repeat this step until you finish propagating through all the small delta z sections such that once you have propagated through M such delta z sections, you would have reached the final length L ok. So, maybe in that case M is actually equal to L by delta z, it does not really matter ok. So, you can of course, you know given the values of L. And you decide the value of delta z you will know what is the number of sections, and then how many times you have to repeatedly follow this procedure.

Now, at this point, all these procedures that we have written time domain, frequency domain, and then back to time domain. They all have a problem in the sense that they all depend continuously on t and omega right, but that is not how a digital computer would work. A computer or a laptop or whatever that you have a computing device would not recognize variables, which are continuous right. So, they will only recognize variables, which are discrete correct. To deal with this problem, what we need to do is to replace these continuous operations by discrete operations. What do I mean by that?

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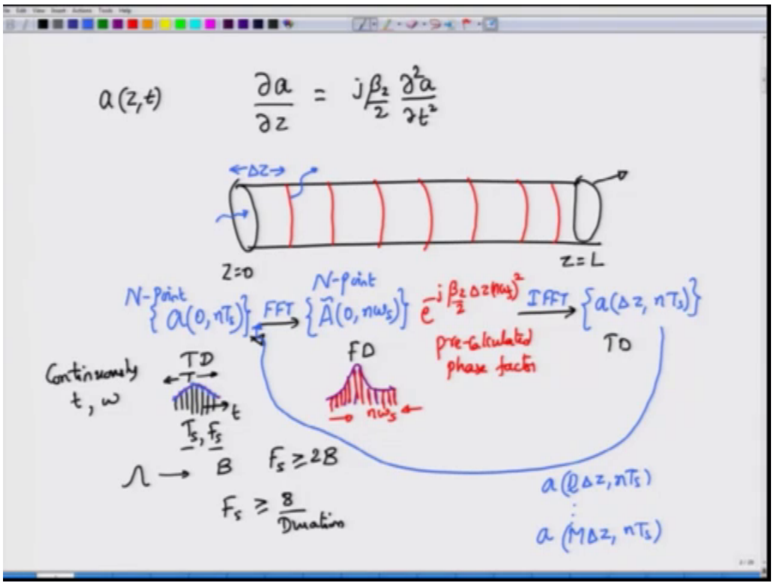

Well, I know a of 0, t has a certain expression. And you know, when you plot it is how does it look, you would have plotted it with a continuous time t here. And this value of t over this range would actually be infinity, because you can know you have t is actually continuous right. But, on a computer you can not represent that entire pulse in that manner. So, what you do is you actually sample the pulse ok you sample these pulses at a sampling period of T s.

Alternatively, sampling rate of F s, how should I choose this sampling rate F s, well you know that if I have a pulse right, then this pulse has a certain bandwidth, let us say B. Then your sampling rate F s should be at least greater than 2 times B right. Of course, in a general pulse shape, you would not know what is the actual spectral width or the bandwidth of the pulse. So, in that case you kind of do a hit and trial or a trial and error. You assume a certain bandwidth, which is reasonable. So, if you have been given a pulse of certain duration, you assume that the spectrum to the first order has a width of 1 by duration.

And then, you take the sampling rate to be at least twice of that. In practice, you will have to over sample this one by a factor of 3 or 4, which means that in practice F s will be at least about say 8 or maybe even more than that 10 times the duration or 10 8 times the inverse of pulse duration ok. So, if the entire pulse is given which is say t second pulse, then 8 by t is what we would actually take the sampling rate of this particular pulse.

And of course, F s is equal to 1 by T s right. So, sampling interval and sampling rate are inversely related to each other. These are very conservative numbers as I said ok. If you have some idea of what is the bandwidth, then you have to take the bandwidth into account and sample the pulse sufficiently ok. Because, if you under sample, then you will get into little bit of a trouble later on ok. So, it is little bit of a trial and error at this point. But, once you have solved it with one or two values of F s, you will know what value of F s can be considered to be optimum ok, so that is the first step.

So, you have to convert this a of 0, t, which is a continuous pulse into a discrete set of numbers, which are actually obtained by sampling this one at intervals of T s. So, you have actually create an array of numbers, which would be the sample. So, if these are the samples, which are all taken at integer multiples of T s, and you would have created that particular array.

Now, this array is stored in a computer. And ready for Fourier transform, but on a computer I cannot take a Fourier transform, which will result in a continuous frequency. I have to replace this continuous Fourier transform by a discrete Fourier transform. And I can do the discrete Fourier transform numerically, efficiently by taking what is called as FFT. FFT is Fast Fourier Transform that will allow you to go from discrete domain of the pulse to the discrete frequency domain.

So, what we actually get after this FFT is not really this you know 0 comma omega. What you will actually get is n omega s ok, where omega s is the sampling frequency. So, you will actually get discrete array here ok, which would be the Fourier transform of the input ok. If you have taken this original input to be of N-point array, then the FFT will also be of the same N-point thing.

And then next step would be to multiply this one right, so you have to multiply this phase factor. And multiplication by a phase factor is very simple. All you have to do is you write down this e power minus j beta 2 by 2 ok, and delta z of course you know. But omega is not really the actual omega, but this is n omega s; where n goes from say 0 to n minus 1 or from minus n by 2 to plus n by 2 minus 1. So, whatever that is that is actually that n omega s is what you are looking for; and it is not really omega s, but it is omega square right, so that would be n omega s square.

In other words, you have the frequency array here ok. And once you have the frequency array or the frequency resolution, then you find out those particular frequencies. And then put that frequency here, and that is where the change here is right. So, if you initially take the carrier e power j omega 0, t, then the actual frequency that you are looking at will be the deviation frequency ok, but as long as you are working in the base band do not worry about it.

You simply take the Fourier transform. And from the Fourier transform you multiply that one with the phase factor. And this phase factor can be calculated beforehand, so that is why, this is sometimes called as pre-calculated phase factor ok. And this pre- calculated phase factor does not really change every time you run this algorithm. Because, once you fix delta z, the frequency array is fixed. And all these things are also fixed.

And final step would be to replace the continuous inverse Fourier transform by discrete Fourier transform inverse Fourier transform, which is implemented by what is called as Inverse Fast Fourier Transform or IFFT. And when you do that, what you get is the discreet array right, which would have now propagated over a length delta z. And this is the array that you are going to get of course.

We are now going to go back, and then start the process again. So, FFT, so that you next time, when you run it, the pulse would have propagated 2 delta z and then so on so forth up to you have propagated M delta z, where M is the number of times or the number of sections, you actually have considered propagation of the fiber. So this method where in we start with the input in the discrete form, because that is what the computer will recognize and then take the fast Fourier transform, which is a way of discrete Fourier transform, and then multiplied by the phase factor. Then take the inverse Fourier transform to obtain the pulse in that time domain back. And then continuously repeat this process, until you go to the output is called as beam propagation method ok.

This is applicable for equations that we have discussed the pulse propagation equation. And when you have to consider beta 3, then you know the situation will be slightly different. And we will discuss that effect of beta 3 sometime in the later modules ok.

Thank you very much.