Fiber - Optic Communication Systems and Techniques
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Lecture – 26
Pre-chirped pulses and Inter- and Intra-modal dispersion in optical fibers

Hello and welcome to, NPTEL MOOC on Fiber-Optic Communication Systems and Techniques course. In the previous module we were discussing dispersion and in this module we will continue to discuss dispersion ok. And we first consider some topic that we were just about to finish in the previous module, that of chirping introduced by the dispersion in the fiber as the pulse propagates through the optical fiber ok.

And we have said that when the pulse is actually initially unchirped, that is there is no phase variation across the pulse that phase variation is actually induced chirped ok. Because of dispersion and this can be overcome by using what is called as Pre-Chirped pulses.

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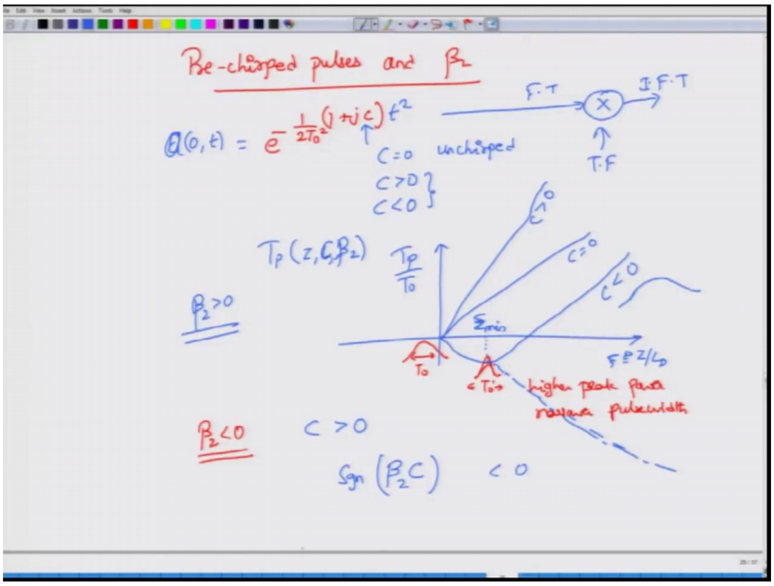

The general expression for the electric field of a pre-chirped pulse or at least the pulse envelope will be apart from some constant which have denote taken to be equal to 1, is that you do not just have e power minus 1 by 2 t square by 2 t 0 squared. But you actually have this parameter c which is called as the chirp parameter. Of course, when you put c equal to 0, you obtain no chirp that is when you actually call the pulses to be un chirped.

And when c is greater than 0 or c is less than 0 you get 2 different types of chirping. I am not going to carry out the mathematical details. The mathematical details are essentially the same you have to take the Fourier transform of this one and then multiply the Fourier transform with the transfer function of the pulse. And then you have to take the inverse Fourier transform this is tedious, but I am not going to do that one for you.

But the idea of or at the basic result of all this propagation equation solving or how doing all these operations, is that the pulse width T p continues to depend. Now in addition to z it will also depend on the chirp parameter c, and beta 2. Of course, the dependence on z and beta 2 we already knew. So, the longer the propagation the pulse would normally have become flattened and flattened or broader and broader. But now there is something very interesting that goes on. First let us consider the case where beta 2 is greater than 0. I am going to assume that we are working with normal dispersion regime ok.

And then plot this T p with respect to or T p by T 0 and let us say which is kind of normalizing the pulse width with respect to the original pulse width as a function of z z or z by L D. So, many many dispersion lengths have to be allowed or the pulse has to propagate so that these effects can become more pronounced. In the case when c is equal to 0 the pulse width increases linearly. So, this is the case when and initially un chirped pulse is launched then its pulse width T p continues to increase linearly with respect to the propagation distance ok. If c is positive then you will see that the slope actually increases much more rapidly so, this is the case when c is greater than 0 ok.

And the pulse width continues to increase of course, I have it is not really nice straight line, but please excuse that this is actually straight line. So, these are straight lines and how to get these straight lines is something that you are going to do it in the assignment or in the exercise. Now let us makes c negative so we have exhausted c equal to 0 and c greater than 0 cases so, now we make c negative.

What could you expect; it turns out that the pulse width actually decreases below that of the initial pulse width t 0 for a certain distance. This distance we call this as z min ok. So, of course, normalized means so you can write this as zeta mean where zeta is by definition z by L D. So, at a certain distance of the propagation initial distance of the propagation the pulse width actually reduces. And then increases this is a very interesting phenomena ok.

What it means is that if you actually had a pulse which had a certain width here of some you know proportion to T 0, the pulse width here actually is smaller. So, this T 0 prime which I have written is actually smaller and the pulse amplitude also would have increased. Because in the lossless system the pulse width and the pulse height both have to be the product of these two have to be constant so that the total energy is always conserved.

So, as the pulse broadens the amplitude drops, but as the pulse compresses the amplitude actually increases. So, what you are actually getting are higher peak power pulses. So, the power the pulse this one is higher in peak power. Plus it has a narrower width or narrower pulse width compared to the initial pulse width.

So, what we have actually accomplished is not so much as the pulse expansion, but so much as the pulse compression. And pulse compression is a very very important topic in almost all ultra fast optics applications where the goal is to go from say nano second pulses to femto second to auto second pulse width. So, to obtain smaller and smaller pulse width it is necessary to keep compressing the pulses.

Of course, this is one of the techniques, but this is a very very important technique. This is the technique that takes you from nanosecond to femtosecond. So, all you have to do is to either preacher up the pulse and then pass it through a fiber or any other material whose beta 2 is positive. Alternatively the same conditions will hold when beta 2 is less than 0, but this time you change the sign of c. So, earlier you had c less than 0 correct.

So, earlier you had beta 2 to be positive and you started seeing this pulse compression when c was less than 0. But now when beta 2 is negative that is we are working with a material or you are working with a fiber whose dispersion regime is anomalous dispersion regime then you just make c to be positive. So, essentially what you are looking for is the product of beta 2 and c. That is sign of the product of beta 2 and c and if the sign turns out to be less than 0. There is both have opposite signs then you will actually see pulse compression.

Of course, you do not see pulse compression throughout I mean you do not see a curve which would go like this right. It would continue to compress out what happens is that after a certain compression this one then the pulse width actually starts to increase again. So, eventually you would have pulses that are broader at the output, but at least up to a distance of some zeta min the pulse actually compresses.

And the zeta min can be controlled by controlling the dispersion length L D, via that parameter beta 2. So, you can actually engineer the value of beta 2 that you want so that you can actually accomplish this pulse compression. And this pulse compression is a very important topic as I told you in; pulse shaping at ultra fast optics. This pulse compression can also be put to some use for dispersion compensation. How, all I have to do is or not all I have to, of course, I have to do lot of design ideas.

But if I have a fiber whose beta 2 is positive then a pulse that is launched into this fiber would actually have expanded the pulse width right or expanded the pulse. So, you would have had some chirping, plus the pulse width would have expanded when the pulse has propagated through the normal dispersion fiber. So, this is what would have happened when you have taken the information and then transmitted say from Delhi to Chennai ok.

So, you have covered such a long distance. Now, at Chennai you do not want these pulses to be very broad I mean they are of no use when the pulses have become so broad because they are going to talk to the other pulses. So, you want to actually pull the pulses back into their original time slot. And you can do so by actually putting in a fiber whose beta 2 is opposite to that of the beta 2 of the transmission fiber.

So, you have one leg where beta 2 is positive, you have another leg where beta 2 is negative. All that you have to ensure is that the total distance beta 2 times L 2 will be equal to beta 1 times L 1, where beta 1 is the you know rather beta 2 L 1 must be equal to beta 2 L 2; where L 1 and L 2 are the lengths of the first leg of transmission and the second leg of transmission.

In fact, this is what is done in legacy systems in many systems where at the end of every span you actually put in a fiber whose beta 2 is actually negative sign as that of the forward or the span fiber. So, the span cyber usually is beta 2 with a negative value then you put this beta 2 with a positive value. Or if beta 2 is positive here you put a negative value effectively the pulse which has started off with some t 0 expanded out.

And because it has now seen and another you know an opposite sign of beta 2 it could actually start to compress. Of course you have to adjust all the parameters correctly so, that the compression exactly cancels out the broadening induced by the forward fiber. And this technique is actually called as Dispersion Compensation in Optical Fiber Communication Systems ok; so, very important technique something that we are going to see later in our course.

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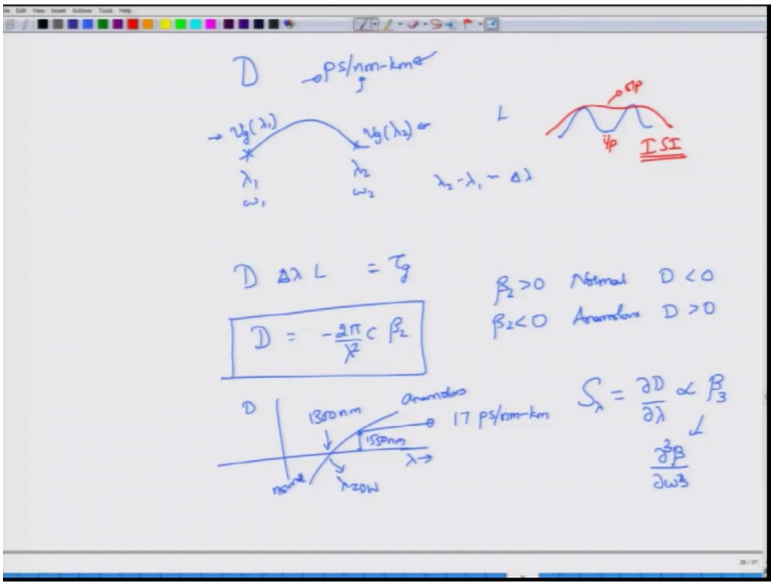

One final topic of dispersion before we actually look at the causes of dispersion; this term instead of using this group velocity dispersion and then using beta 2 to explain what is happening ok. Manufacturers of optical fibers use another term called as Dispersion Parameter; which is measured in picoseconds per nanometer kilometer ok.

What is the strange unit of picoseconds per nanometer kilometer, what this unit simply tells you is that; if your pulse actually has two extreme frequencies or extreme wavelengths. That is the Fourier transform of the pulse I am talking about what is the group velocity of this one: what is the group velocity of this other end of the spectrum right. The difference between these two over a length L or over a unit length L would actually correspond to the dispersion parameter.

It would actually tell you: what is the delay between the two edges of the pulse spectrum ok. In arriving at a common point or arriving at the output of the fiber. So, how much delay is measured in pico second and what is the distance over which they have propagated is measured in kilometers. And what is the spectral width that is lambda 2 minus lambda 1 which I will denote it as delta lambda is what is measured in nanometers.

So, in fiber optic communication systems and in most cases where you are dealing with light, it is quite common to use wavelength rather than frequency to describe the width ok. And this delta lambda is called as the spectral width this spectral width of course, is because of various reasons you have your initial laser itself having this delta lambda or you have your, you know you are going to modulate the pulse.

And therefore, you kind of effectively expand or broaden the pulse width that will have a certain delta lambda. So, all that delta lambda what is the delay in arrival time between the two edges of the pulse band width of the pulse spectrum is measured by this (Refer Time: 12:13) parameter D ok. And this dispersion parameter D is related because see D delta lambda L will tell you the total delay correct. So, you have picosecond per nanometer kilometer multiplied by nanometer, multiplied by kilometer.

So, this will be the total delay and inside this delay is basically equal to tau g ok. And this tau g of course, we also have seen from our earlier this one that it can be related to beta 2 omega square z right or beta 2 omega square. So, we have seen this one earlier as well, but without going into the details at this point we want to give you too many mathematical details.

This D can be rewritten in terms of or this can be related to beta 2 by this expression. This involves converting delta omega 2 delta lambda something that you do not need to worry about it at this point. And the main reason that I am introduced this dispersion coefficient or dispersion parameter D is because I want to show that D and beta 2 are related with an opposite sign.

So, when beta 2 is greater than 0 you have what is called as normal dispersion parameter, when the normal dispersion so, in that case D will be less than 0. But when beta 2 is less than 0 you have what is called as a normalize dispersion for which D is greater than 0.

And if you look at this D you will see that the overall D actually goes something like this; it will go through 0 in the fiber at around 1,300 nanometer and at 1,550 nanometer ok. At 1,550 nanometer the value of this dispersion D is roughly 17 picosecond per nanometer kilometer ok. So, what I have plotted is lambda on the x axis and D on the y axis. Of course, because after this 1,300 nanometer which is called as lambda ZDW. Where ZDW is called as a 0 dispersion wavelength the region here is actually anomalous ok, whereas; the region here is normal.

And of course, this D with respect to lambda is not a straight line, D actually with respect to lambda is kind of a curve. And you sometimes introduce what is called as slope of the dispersion parameter which I denote this as S lambda given by del D by del lambda. And in fact, this can be related to beta 3, beta 3 is basically the third order dispersion which is derivative of beta with respect to omega the third derivative with respect to omega.

So, these are some of the important parameters regarding dispersion when you look at the data sheet of an optical fiber you will see all these terms being used. But now the big question is what is causing this dispersion. I mean we have seen how to model dispersion, we have seen the effect of dispersion. So, if you actually send in 2 pulses then both pulses essentially broaden out and then kind of overlap.

So, this is what you would get at the output while this is what you have sent at the input. So, you cannot read distinguish one pulse with another pulse. And if you actually send it through an eye diagram, you will see that the eye diagram would be completely closed. So, you do not really see the system out there because of what is called as Inter Symbol Interference.

So, this interfering of one pulse with respect other pulse because of the broadening is called as inter symbol interference. And that is the major effect of dispersion in communication systems. Of course, dispersion has other effects such as; chirping, or pulse compression, pulse expansion, there are other uses of dispersion as well. especially when you combine it with non-linearity those are all some different applications that we do not want to go. But we want to understand where this dispersion is coming from ok. To do so I will do a little bit of a hand waving ok.

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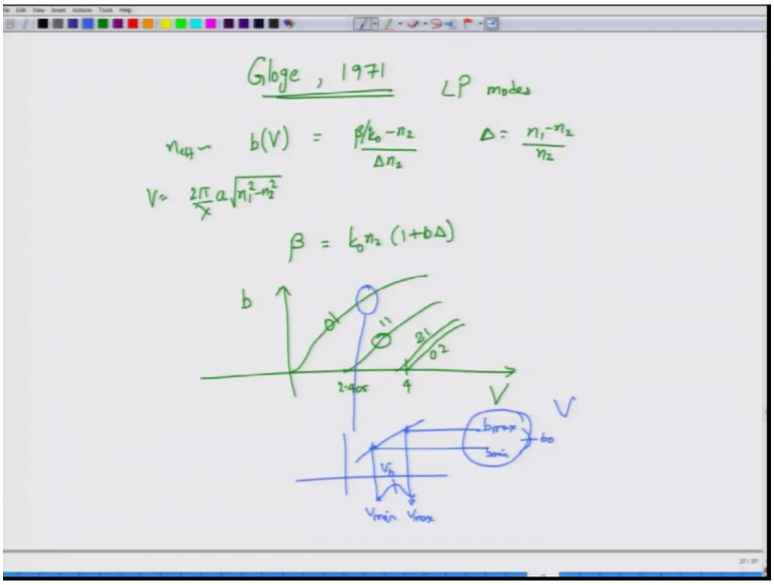

These results are actually taken from a paper which I mentioned to you earlier ok, by Gloge I think it was 1971 and do not really remember I will give you the details later on. But what he showed in this paper is that for the linearly polarized modes that we are talking about; you can define a propagation constant a normalized propagation constant. As beta over k 0 minus n 2 divided by delta times into ok where, delta is being redefined by this paper as n 1 minus n 2 by n 2 ok.

Where n 1 is the core refractive index maximum core refractive index, n 2 is the cladding refractive index. And the difference delta is n 1 minus n 2 normalized with respect to n 2 and this b is the effective index related parameter. And V of course, you already know is the normalized parameter. So, normalized frequency parameter or normalized frequency 2 pi by lambda a times n 1 square minus n 2 square.

What he did was to actually rewrite this beta which is the propagation constant in terms of the refractive indexes of the fiber it's self. So, we are now looking at an expression where beta can be written as k 0 n 2 times 1 plus b times delta, where b is the propagation constant of the mode that is propagating. And of course, you know from our earlier discussions of L p modes that there are many many modes. And therefore, you have to be careful as to which mode is propagating.

So, of course, if you look at that famous diagram that we have actually plotted for b versus V. You will see that this is for the case of 0 1 mode and then you have a group of modes or which are essentially 1 1. And then you have kind of 2 1 0 2 modes which are very close to each other. Of course, this cutoff is occurring at 2.405 and this cutoff is occurring just about 4 so do not worry about the actual numbers.

But if you look at these different modes, that we have right and if you were to zoom out in any one of those mode; so, let us say I zoom in on this one what I actually see is this curve ok. And I would actually have sent in a pulse whose spectrum would be this way right. it has some v min it has some v max and because of this V min and max there is a difference. So, you will now have b max and b min and you see that this is not a linear relationship.

What you actually see the relationship here which say b 0 with this as v 0 as the center is not an exact replica of this one it's not a linear relationship. And because of this being non-linear in relation what happens is that the propagation constants actually depend on what lambda that you are transmitting or on what no the pulse width of that you are transmitting. And most importantly these parameters are now further functions of V and the derivatives of these are what is going to give you the dispersion in the fiber ok.

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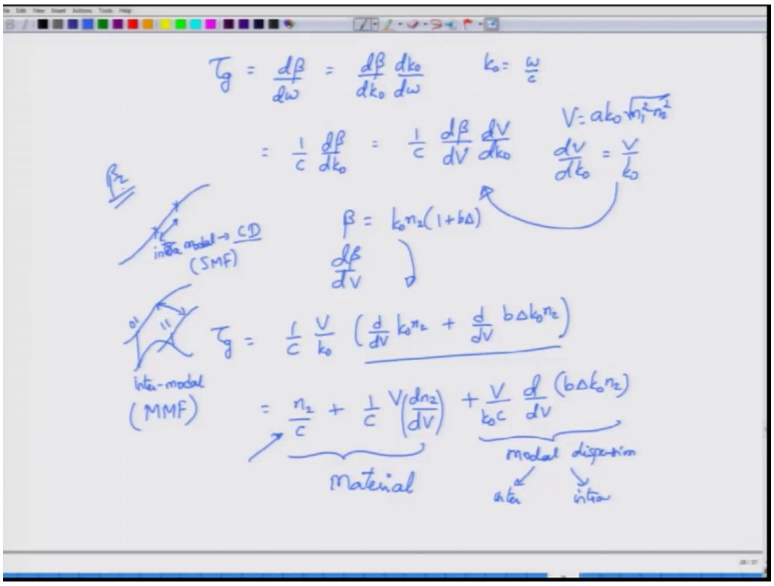

But we do not normally do it in that way we actually go back to the delay tau g that is that we express. And this tau g we write it in terms of you know the simple D beta by D omega that we already know which is of course, beta 1 parameter. You can adjust this expression d beta by d omega by differentiating beta with respect to k naught and then differentiating k naught with respect to omega. Where k naught of course, is given by omega by c, where c is the phase velocity. And you are simply writing this dk 0 dk 0 on both sides for the day entire expression is still the same. And, but of course, because k 0 is in this space or dk 0 by d omega will be 1 by c. So, this is actually equal to 1 by c d beta by dk 0 ok.

And now what I am going to do is to readjust this equation because I want to bring in the normalized parameter or the normalized frequency parameter. So, I am going to consider this as d beta by d v time's dv by d k naught. And I know that v is given by k 0 square root of n 1 square minus n 2 square ak0 square root of n 1 square minus n 2 square. And I can rewrite that one so, dv by dk naught will actually be equal to v by k naught ok, I can substitute that into this expression. And since I also have written beta as k 0 into 1 plus b delta I can find out what is D beta by dv. And therefore, write tau g as please verify this expression v by k naught d by dv k 0 n 2 plus d by dv b delta k 0 n 2.

I have just expanded this beta into this one or you know expanded that out and then differentiated with respect to v. And dv by dk naught is basically v by k naught so I have just considered that 1 by c comes in with 1 by c. Now when I actually carry out these derivatives what I get is that I get three terms ok. I will tell you the importance of these three terms very shortly so, what you actually see here is this one right.

So, I know what is m 2 by c that is simply the phase delay kind of a thing right, but then I also have d n 2 by d v what does that mean; v is the frequency and with respect to n 2 when you are looking at the derivative of n 2 with respect to frequency. This would mean that the material property that you are considering off the fiber the cladding material into itself is dependent on the frequency. So, n 2 is not the same for all frequencies, but n 2 changes of course, it changes for frequency means it is also different for different modes.

So, this change of n 2 with respect to frequency is not really in my control because this is the parameter property itself. So, these 2 parameters the first and the second term are actually what is called as material dispersion. Because they relate how beta or the refractive index n 2 is changing with respect to the frequency as the consequence of the material property.

So, if you want to change this d n 2 by dv or you want to shape that 1 you have to actually experiment with different materials. Because each material has its own dispersion which is you know its own material dispersion. And therefore, this term is something that comes because of the material itself and that there is normally not a lot of control once the material has been selected. But the term that is there on this right hand side is what is called as the modal dispersion.

Modal dispersion in turn can be broken up into inter and intra; which means that if I am going to consider only a single mode fiber the propagation constant b will be different and it will be non-linearly related. But it will still be with the same single mode right so this is called as intramodal dispersion. But if I have launched my optical pulse such that the spectrum goes over like this.

Therefore, it is propagating in some mode as 0 1 some portion is also propagating or the next mode is also excited; then because the propagation constants are different for the 0 1 mode and 1 1 mode. The part of the light that is excited or exciting into the 1 1 mode will travel with a different velocity as the part of the light that is excited as 0 1 mode. And this difference is called as intermodal dispersion.

Intermodal dispersion is quite common and in fact, is one of the major problems in the multimode fibers. Whereas, intra modal dispersion sometimes is called as simply chromatic dispersion is an important parameter in SMF ok. But our formalism would simply want you to know what is beta 2. And therefore, does not really depend on whether you have applied it for a single mode fiber or you have applied it for a multimode fiber. But it is interesting to see that this modal dispersion comes because of two effects as I have told you. So, is there anything that we can say more about the modal dispersion.

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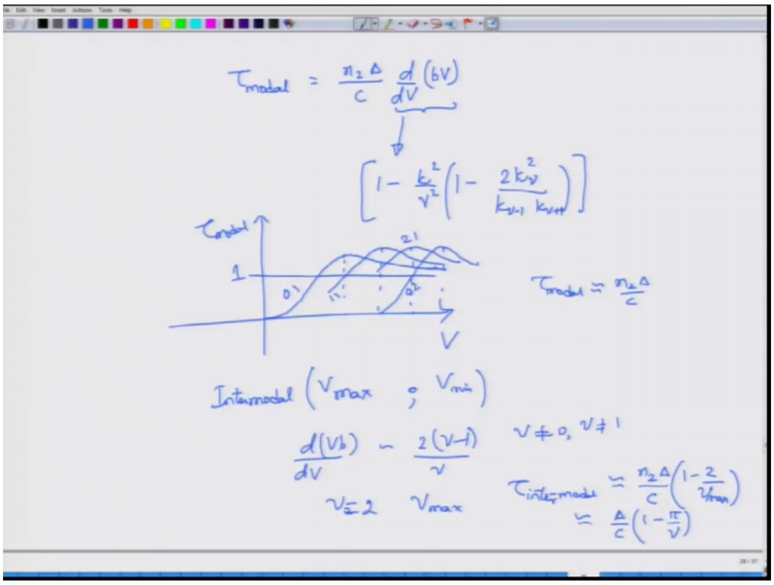

Yes, you can show and I will give you the notes for this one in the class by actually carrying out the derivative. That you can write this as n 2 delta by c and then d by dv times bv ok. Where b is the normalized propagation constant as you already know and further expression for d by dv times bv is something that is not you know easily obtainable.

But there have been approximations that people have used for example, one approximation is 1 minus kappa square by gamma square. I do hope you remember what kappa and gamma are times 1 minus 2 kappa nu square divided by kappa nu minus 1 kappa nu plus 1. Please note that nu minus 1 and nu plus 1 are the orders of the modes that we are talking about.

So, this derivative itself can be approximated this way ok. And if you actually plot this tau model as Gloge has plotted in his paper and you look at that when with respect to V. You will see that for 0 1 mode you will actually see something like this, the normalized model will be 1 most of the modes actually you know asymptotically reach this 1.

But if you go to different modes for example, 1 1 mode would look something like this, and then you have a 2 1 mode which goes something like this then you have, a 0 2 mode which goes something like this. So, this is for 0 1 this is for 1 1 this is for 2 1 this is for 0 2 and so on. So, these modes actually have further away of course, from the cut off the model dispersion or the model delay can be approximated as n 2 delta by c.

And they actually have a max model delay at different values of V number. So, far away from the cutoff you actually have this approximation for all of them ok. And this intermodal delay that I talked about essentially asks you to find out what is the maximum excited mode. So, for which you have a max value of v and then you have a min value of V.

So, what is the delay in arrival between the max and min values. And you can actually use another equation for that which will give you approximately this condition. Of course, this condition is not for nu equal to 0 and nu equal to 1, but for other modes this is reasonably ok. And when you look at this expression and calculate what would be the difference between say nu 2, that is the second nu equal to 2 and say some nu max ok.

You will see that this inter modal delay ok; this is intermodal delay can be approximately written as n 2 delta by c 1 minus 2 nu max. So, when you start exciting larger and larger number of modes, then you will also see that intramodal delay depends on this one ok. For and this further can be approximated as delta by c 1 minus pi by V, V being the parameter that you already know.

So, this is an expression for intermodal dispersion, but for a single mode operation there is no intermodal delay. Because there is only a single mode, but in that case you are dealing with intra modal dispersion either intra modal dispersion also can be obtained with some approximations which; we are not going to discuss now.

So, the net effect is that in dispersion in optical fibers comes because of materials and because of the waveguide itself. All these terms that intermodal interim modal depending on v depending on b actually came from the wave guide properties. Therefore, the total dispersion is the sum of material and wave guide dispersion. We will have to say little bit more on dispersion in the next module and then consider what is called as polarization mode dispersion.

Thank you, very much.