Fiber - Optic Communication Systems and Techniques

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Lecture - 20
Properties of modes of step-index optical Fiber (continued)

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module, we will complete the discussion of step index Fiber modes of a step index optical Fiber. We stopped at the right moment in the previous module, where we had written down a matrix which connects the unknown coefficient A B C and D and the coefficient matrix that multiplies this unknown vector A B C D was actually the result of applying boundary conditions on the 4 horizontal components or the longitudinal components or the tangential components and this is the equation that you had obtained.

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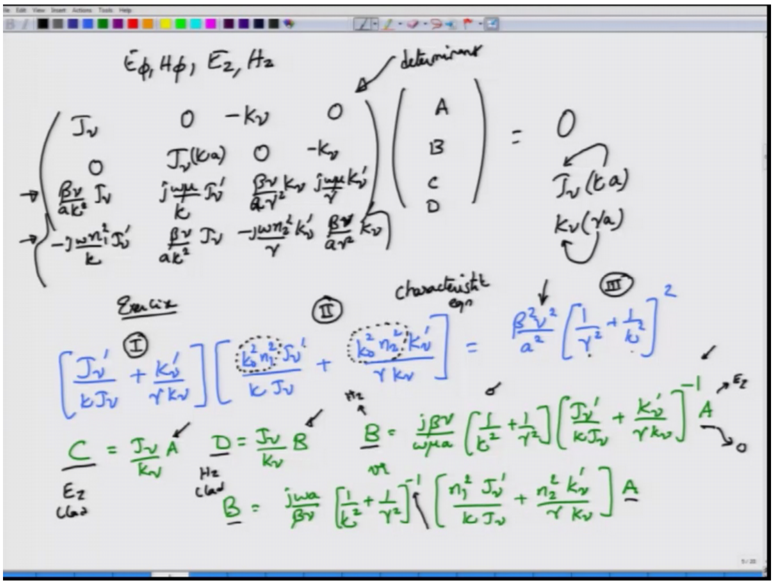

The right hand side of course is 0 and if you look at the left hand side please also remember that whenever I have mentioned only j nu, I actually mean j nu as j nu kappa A similarly k nu will be k nu with the function of gamma A.

So, kappa and gamma you have to be careful that kappa goes with j and gamma goes with k. That is just to simplify the notation and once you simplified the notation this is the big matrix that you are going to get and for this matrix or for this entire system of equations to actually have nontrivial solutions, it is necessary that the determinant of this matrix must be equal to 0 and when you write down the condition of determinant of this matrix to be equal to 0.

You will end up with an equation which is shown in this blue line you can derive this one ok, this is the small exercise for you to calculate the determinant and then show that this is the equation that you are going to get. It is a complicated expression or it is a complex looking expression no doubt about it.

But if you look carefully you will notice some kind of a structure, for example, the term that I have here in the first bracket which I will call as term 1. I will put a small circle around it just to show that this is not I this is term 1 and the 1 that I have written here is term 2 ok. So, I have term one and term 2 here and if you want I can write this one by gamma square plus one by kappa square as term 3.

So, I have this equations and what you as see if from this one now is that, if you look at term 1 you have the derivative of the Bessel function of order nu. So therefore you have j nu prime divided by kappa j nu and similarly you have the derivative in the I mean in the cladding case which is kappa nu prime divided by gamma kappa nu and the term 2 that you have is essentially the same as term 1 accept that j nu prime is getting multiplied by k 0 square n one square, where as kappa nu prime term is getting multiplied by k 0 square n 2 square.

So, expect for this multiplication these 2 terms are essentially kind of similar to each other, on the right hand side of course what you have to essentially noticed is that there is a nu in the denominator. In fact, there is a nu square in the denominator and beta is also present. Of course, what is the equation used for this the characteristic equation. What is the characteristic equation it is the equation whose solution is supposed to give you beta.

But this is a very complex equation the solutions are also not so simple, even with good numerical methods solving this complicate expression. Especially for different orders and other things and get the essential numerical accuracy is kind of difficult not impossible, but it is difficult and this was very difficult in the 1960. Late 1960 when this type of equations for first derived rather they were derived in the 60 early 60, but they are real use was in the late 60 at that point the computing power was not so high. So, solving this equations was not so easy at that time, even now solving these equations with the resulting numerical accuracies is kind of difficult.

But anyway this is the equation that you need to solve if you want to find out the value of beta and this is again much more than a transcendental equation because, beta is not just explicitly sitting here, but beta is also sitting in gamma square it is also sitting in kappa square and furthermore this kappa and gamma themselves or the arguments of the Bessel function, so it is really complicated is what I want to emphasize.

There are couple of additional relationships that you need to know, so you can see that C and D which are the amplitudes of the electric field in the cladding and D which is the amplitude of the magnetic field in the cladding are not really independent, but they can be related to the amplitudes in the core itself.

So, A and B are the core electric and magnetic field amplitudes and in terms of those we can write down the amplitudes of the electric and magnetic fields in the cladding. But there is a relationship between B and A which comes from third line or the fourth row of this matrix.

So, you can either look at the third row which will give you some term times A this term times B some term times C some term times D, the sum of all that will be equal to 0. But then you know that C and D themselves can be written in terms of A and B and therefore you can actually obtain one set of equation which relates B to A. Please remember B is the amplitude of the magnetic field Hz A is the amplitude of Ez ok.

Similarly, by following the fourth row of this matrix, it is possible for you to obtain another equation or another relationship between B and A. So, in one equation you have the term 3 appearing is proportional to 3 but term 1 is actually appearing in the denominator, because there is a minus 1 I mean this term is actually raise to minus 1. In the second relationship you have this term 3 appearing in the denominator, again because look at the sin here it is minus 1 and then you have a term which is the term 2 appearing in for it is in appearing in the numerator term.

Now, let us look at some special cases, the general case of nu we will come to it but let us look at a very special case of nu equal to 0. From our previous experience we would hope or we would think that nu equal to 0 would correspond to certain modes which would essentially be fundamental.

That is these are the modes that will first start to propagate, but as we will see very shortly that this is not true nu equal to 0 does not give you the fundamental mode. What kind of modes to you get when nu equal to 0? Where we go back to the characteristic equation and then we look at what happens when nu equal to 0, when nu is 0 the entire right hand side disappears. Since the entire right side has disappeared because I am assuming that this term 3 is not going to 0, then I have either 2 choices the first term going to 0 or the second term going to 0.

If I assume that the first term that is j nu prime by kappa j nu plus k nu prime by gamma k nu is actually equal to 0, then if I impose this condition back here in the relationship between B and A, I see that if this term is 0 in the relationship that I am showing here ok. So, this is the term that is 0 and this term unfortunately occurs in the denominator right and because this is occurring in the denominator and nu equal to 0 is also occurring in the denominator. These 2 are essentially going to be something like 0 by 0 form ok, but if in addition A happens to be equal to 0 then B will be finite for this 1.

So, what you will actually have is A equal to 0 from this relationship and if term 2 goes to 0 then because nu appears in the denominator in the second relationship and this term 2 is appearing the numerator in this case it must be the value of B which must go to 0.

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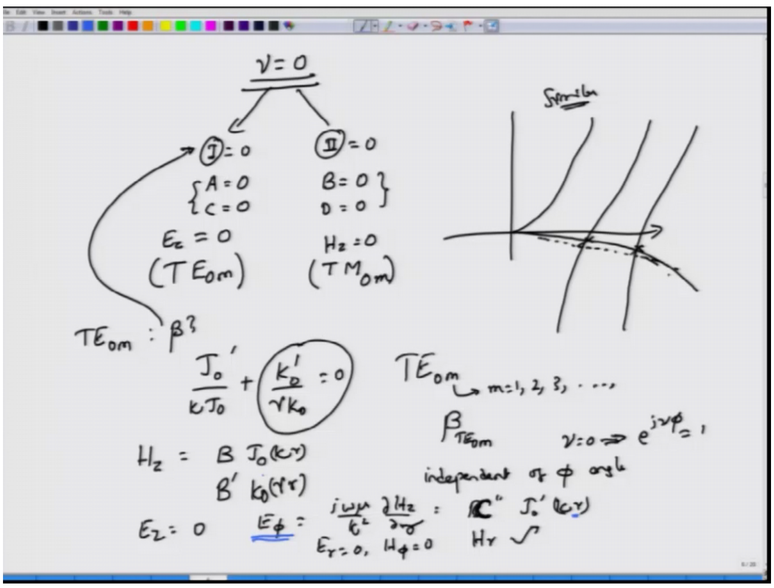

So, you can look at this particular thing and then when nu equal to 0 you have as I told you one term that is term 1 can be 0 or term 2 can be 0. If term one is 0 then it also necessarily means that A equal to 0, but equal to 0 also means C equal to 0. But we remember that A and C are the amplitudes of Ez, so what we have seen is that when nu equal to 0 Ez will be completely 0.

If you assume first term to be 0 Ez will be 0 both in core and cladding, in fact Ez will be entirely equal to 0. Since Ez equal to 0 such modes are called as TE modes and the order of this TE modes always has to be 0, I mean that is the first order the order will come out ok.

So, you have TE 0 and m I will shortly tell you what is m ok, so these modes are essentially called as TE 0 m modes. Similarly when term 2 equal to 0 this leads to the condition that B equals 0 which also means D is equal to 0, which further implies Hz equal to 0 and what you get are the transverse magnetic order 0 small m modes.

So, what you get are the TE and TM modes remember during the start of this discussion we had said that there is no TE and TM mode in general for a step index Fiber and it is true this is only for the special case where nu equal to 0, that you can divide the total solutions in terms of transverse electric and transverse magnetic. It would have been nice if this were to be the fundamental modes, but turns out the these are not the fundamental modes.

So, what are the field components when you actually have TE modes or rather how do I go about finding the propagation constant for TE 0 m modes. So, how do I find the propagation constant, well the propagation constant comes from looking at I equal to 0 that is the first term equal to 0 as the Eigen value expression or the characteristic expression. So, you have j 0 prime divided by kappa j 0 plus k 0 prime, so this is k 0 prime divided by gamma k 0 ok. So, this will be equal to 0 the solutions actually can be mapped in terms of this type of an equation, where you will see that j 0 prime by j 0 can actually exhibit oscillatory behaviour. In fact, it does exhibit an oscillatory behaviour that goes something very similar to the tangential 1 ok.

Similar, but not the same as the tangential equation the reason is j 0 prime goes to 0 at some points j 0 goes to 0 some point, so it is actually switching between plus infinity and minus infinity ok. So, it does look very similar to the tan function, whereas k 0 prime by gamma k 0 k 0 prime is the derivative of an exponential which would look like an exponential, especially because is k 0 has to be applied away from the core right.

So, the 0 points are all excluded from the k equations right. So, near 0 the k 0 solutions are not going to be looked at because, they are not part of the solutions anyway because this k solutions are to be in the cladding itself. So, in that cladding k 0 prime and k 0 both look like a exponential functions and therefore the ratios could only either decay or increasing. In fact, that is precisely what happens it would go something like this wherever the intersect, you will actually have a TE mode.

So, for the same TE first order 0 you will have multiple modes m starting with m equals 1 2 3 and so on and correspondingly you will have beta for TE 0 m mode. So, what are the field components for TE mode well for the transverse electric mode it is Hz which must of course, be some constant B which you do not know.

But anyway so the field fundamental Hz field should be j 0 kappa r and is there any dependence on phi well there is no dependence on phi that is a beauty of having nu equal to 0, because nu equal to 0 implies e to the power j nu phi is equal to 1 with nu equal to 1 and therefore these fields are independent of phi angle ok.

Independent of phi means that there azimuthally symmetric. So you go around the Fiber in a circular manner at a constant radius A you will see that the field essentially looks like same, is the magnitude of the field does not really change as you circle around the optical Fiber.

So, you have Hz which is some j 0 kappa r with respect to z anyway you know this is in the core region and in the cladding region you will have some constant D which of course, can be rewritten in terms of constant B. But anyway I will put all of them into A constant B prime and then I have k 0 gamma r ok.

So, this is how the Hz field would look like, Ez of course will be 0 you can show that e phi will be equal to which in this particular case will be j omega mu by kappa square del Hz by del r, this fellow will be equal to some constant. So, maybe that constant could be B or C double prime something like that, but this is j 0 prime of kappa r.

This is the core region and the cladding region they will be k 0 prime and then show that Er will be 0 similarly h phi will be 0. Because, r and phi are anyway related and Hr will also be like j 0 prime and it is this Hr and e phi. So, phi cross r which would essentially go in the z direction positive or negative z depends on the amplitude actually it would be positive z only.

So, this e phi cross Hr are will be the power that is or power density which would be pointing in the z direction. So, in the nu equal to 0 case ok, we have seen that the solutions can be divided into TE and TM and in TE you can have solutions which are in TE modes we can have multiple solutions starting with say m equal to 1 m equal to 2 and so on. They all represent the different intersection points of the characteristic equation or the solutions of characteristic equation. So, you have multiple values of beta for the same nu equal to 0 value and each of those propagation constant would be further included as the allowed mode of the waveguide. The similar ideas also are applicable for transverse magnetic mode or they exist for transverse magnetic mode their the Eigen value equation will be different because, the condition is that term 2 must be equal to 0.

So, that n 1 square and n 2 square which go and multiply the individual terms of term 2 will actually push the TE modes to be slightly away from the cut off for the TE modes. So, the TE modes would actually be very similar, but they would actually cut off at a slightly higher values of this one. So, they would actually slightly cut off at a higher values compared to TE modes ok, that is because of the n 1 and n 2 difference. If you were to sketch this TE mode field at least the electric fields if you were to sketch.

You would find that the field is directed along phi means it is circling right and at r equal to 0, it would be like j 0 prime and at r as it increases it would be and different things. So, you will actually see that the h field lines would actually be like j 0 yeah the e field lines would actually be directed in this manner.

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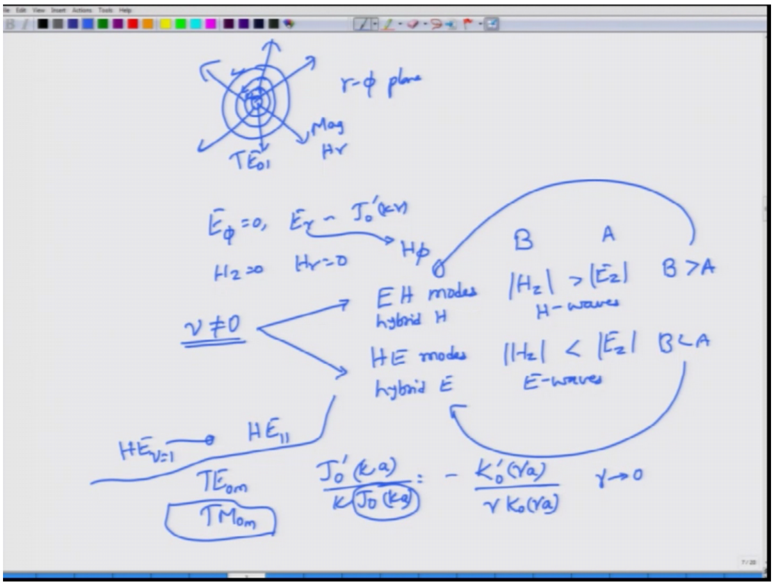

So, these are the e field lines for TE 0 1 mode and these are the magnetic field lines that you are going to get, because magnetic field lines will be directed in z as well as r direction. So, you have Hr which goes as j 0 prime kappa r and this is how the fields lines would actually look for magnetic field in the direction Hr, I am plotting this one in the r phi plane so electric fields will be circling in.

For the transverse magnetic fields it is e phi which is equal to 0, Er which would be something like j 0 prime kappa r of course and then you have Hz equal to 0 and you have Hr equal to 0. But because Er is present there will be h phi, so the field lines actually just change. So, what you would have called the field line for e in the TE case becomes the field line for the magnetic field in the TM case.

So, these are the solutions which are very special solutions for the case of transverse electric and transverse magnetic modes for the case where nu equal to 0. But in general when nu is not equal to 0 you will have other solutions and the solutions again can be classified into 2 groups, one is called as EH modes and the other is called as HE modes, HE stands for hybrid electric modes and EH stands for hybrid H modes.

Meaning in EH mode the magnitude of Hz will be greater than the magnitude of Ez, where as in the hybrid electric modes the magnitude of Hz will be greater less than the magnitude of Ez fields and since we know that Hz will have an amplitude B in the core Ez has an amplitude of A. The relationships are essentially telling you that if the amplitude B is greater than A, then you get what I called as EH modes and when B is less than A you get what are called as hybrid electric modes. Sometimes these are called as e waves and the corresponding hybrid H modes are called as H waves.

It turns out that the fundamental mode is actually 1, when you set nu equal to one. So, in that case you will have the first solution whose cut off happens to be right at 0 ok. We will talk about the cut off conditions you know in the next class, but if you are not really happy about the cut off condition you already know that.

So, you have j 0 prime kappa a divided by kappa j 0 kappa a being equal to this for the TE 0 m or TM 0 m modes. For TE 0 m modes the equation will be slightly different, but you might want to take this TM 0 m as an exercise. So, for TE 0 m modes this must be equal to k 0 prime gamma A divided by gamma k 0 gamma A and we know that this TM TE 0 m mode will experience a cut off when gamma goes to 0 right.

So, when gamma is 0 you can see that this fellow here will actually be. So, this gamma going to 0 means that this term here will be infinity on the right hand side and on the left hand side what if you want to match the left and the right hand side the only way to do that 1 would beta actually set this j 0 kappa a equal to 0.

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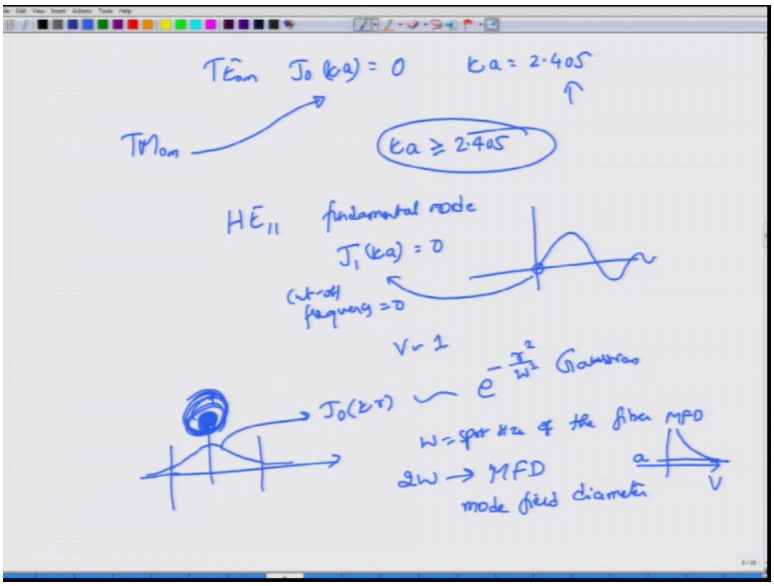

So, for the TE 0 m mode the cut off frequency is given by j 0 kappa a equal to 0 and the first such solution happens when kappa a equals 2.405. This is the magic 2.405 value that you keep hearing in optical Fiber, there is a general tendency to think that if v number is less than 2.405 the Fiber is single mode which is true. But where is that 2.405 coming from it is coming from this cut off condition. Similarly for TM modes the you can show that this is the same equation that is valid for the cut off as where for the cut off of TM 0 m modes as well.

So, the first mode won't appear until kappa a until is exceeds or just equals 2.405. It however turns out with a little bit of a lengthy explanation that HE 11 mode which is called as the fundamental mode in step index Fiber.

So, in general the optical Fiber, so this is the fundamental mode simply because the cut off condition turns out to be j 1 of kappa a equal to 0. Now j 1 is the Bessel function of the first kind and first order which actually starts at 0. Of course, something like this so because it is starting at 0 and this is completely allowed you will actually have cut off frequency which is 0.

So, cut off frequency of HE 11 mode is actually equal to 0 right; however, until v numbers becomes appreciably equal to 1 the corresponding propagation constant will not be that high.

So, even though technically you could take DC signals and then just directly launch into an optical Fiber by some method. They would not propagate to a large distance because their propagation constant will be very small. How does this HE 11 mode look like well the HE 11 mode actually looks very interesting at the centre it is quite intense, but then the intensity kind of drops off as you go away from the centre.

So, if were to plot the 2 D plot for this 1, so at the centre it would be max and then it would go away like this on both sides of course right this is a core and when you hit the cladding they would actually continue and drop out or evanescently decay. So, inside this would actually go as j 0 of kappa r and that is precisely what was important about the HE 11 mode, but it is intensity is actually maximum at the centre. You can actually express this j 0 kappa r instead of a Bessel function you can do a approximation in terms of Gaussian function with w called as the spot size of the Fiber ok.

So, the field can be written instead of j 0 kappa r you can express them in terms of e power minus r square by w square which is a Gaussian function. This is done purely to simplify calculations or to just get some number out of this one. Working with Bessel functions is slightly difficult, so if you approximate with the knife Gaussian then you can say w correspondence to spot size 2 w, then corresponds to the twice spot size and field would decay out by about 4 spot sizes and this spot size is important.

Because, if the if one Fiber has an extremely large spot size and you then go with a Fiber whose spot size is very small when you try to couple them right. The light is actually spread out much widely in this 1 because there is a larger spot size, where as the light collecting capacity will be smaller for this Fiber because, it has a smaller spot size and there will be a spot size mismatch. Twice the value of spot size is what we call as mode field diameter and when you match 2 Fiber this mode field diameter must be match as accurately as possible otherwise you will actually have lot of problem.

Interestingly this mode field diameter depends on the v number of the particular mode that is propagating, at least for the fundamental mode the v number has to be very close to 2 in order to have well confined field. If you look at how mode field diameter itself changes with respect to v for the fundamental mode, will actually see that initially the confinement is very small and then the mode field diameter approaches the diameter of the Fiber ok.

For larger values of v that is closed to 2.2 2.3, it would approach the value of A but always being slightly larger than 2 A, so the mode field diameter is you know indicative of the amount of confinement of the field inside the core of the optical Fiber and this mode field diameter actually is very large meaning very less confinement when v is very close to A 0.

So, this is another reason why you cannot just take a direct TC signal and then launch it into an optical Fiber. So, let me summarise we started off with the step index profile, we followed the systematic procedure to analyse the modes and derive the expression of characteristic equation to obtain the values of beta. Although that was a complex expression complicated expression, never the less for the special case of nu equal to 0 we found that you can separate the modes into transverse electric and transverse magnetic.

This transverse electric and transverse magnetic mostly look like circles intersected by radially outgoing lines depending on whether your TE or TM modes, circles will be either e phi or h phi and in general for nu greater than this one or nu greater than or nu arbitrary. The modes cannot be separated into TE and t m, but the modes become what you called as hybrid modes and depending on whether Hz is larger than Ez or Hz is smaller than Ez you have hybrid H modes and hybrid electric modes.

The cut off condition for TE and TM modes are the same, they have to be j 0 kappa a equal to 0 is the characteristic equation the solutions of those corresponds to different modes. Where as HE 11 cuts of at 0 and it has therefore the ability to be guided technically guided at frequency which are much less than the ones that are required for TE and TM and therefore HE 11 is the fundamental mode or the dominant mode of the Fiber. In the next module we will look at what is called as linearly polarized modes, which are the modes that are actually widely studied in literature because, of the practical you know help that the fibers actually give you.

We have assumed n 1 to be greater than n 2, but since the difference between n one and n 2 is so small you can assume n one to be approximately equal to n 2 and when you do that you will get the linearly polarized modes, which we are going to study in the next module.

Thank you very much.