Fiber - Optic Communication Systems and Techniques

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Lecture – 17
Further discussion on slab waveguides

Hello and welcome to NPTEL MOOC on Fibre Optic Communication Systems and Techniques. In the previous module, we discussed a systematic procedure to derived properties of modes of a slab waveguide, we first applied that systematic procedure for a metallic waveguide and since metallic waveguides are not used at optical frequencies, for the reasons that we mentioned, we then discussed slab waveguides and then we showed that the expectable modes of a slab waveguide can be partitioned into transverse electric and transverse magnetic mode.

Of course, in a real world scenario, when you launch on mode all these modes can excite, can be excited depending on how you launch a particular mode, some of the mode launching procedures for fibre optic, optical fibres were going to consider later on. We will not consider such a mode launching; you know discussion for the slab wave anyway. We saw that when we solved the equations right. So, we obtained a certain equation, which related the transverse wave number kappa f and the decay constant outside the slab waveguide gamma into a transcendental equation right.

So, this relationship as we told, you was what is called as the eigenvalue equation, because hidden in this kappa f and gamma is the propagation constant beta, because kappa f and beta are related to each other. Gamma and beta themselves are related to each other by solving this characteristic equation, we can obtain what is the corresponding propagation constant of that particular mode, which is propagating in a given waveguide.

The systematic procedure also had this advantage, that in addition to just knowing the value of beta, which anyway you could have found from the transverse resonance condition, you now, also have the capacity to actually look at how the modes themselves look like or essentially, what is the functional dependence of the modes of the different components electric field and magnetic field, corresponding to whatever components that go for the transverse electric and transverse magnetic modes right.

We did all these analysis only for transverse electric modes for transverse magnetic modes, the steps are very similar, except that for the transverse magnetic mode you start with the z component of the electric field and then derive the other components in terms of that z component, solve the wave equation for e z, obtain a characteristic equation, find the propagation constant and from the relationship of e z with other. Other components you will then be able to calculate the dependence of the modes on the coordinate x ok.

What I want to do now is to kind of reconsider that characteristic equation tell you a couple of properties of that one and then, show you couple of methods to solve that transcendental equation, one method would be what is called as the numerical method. Numerical method is fastest, gives you accurate answers, much more than what you would get, but for many problems or at least, until you know, the advent of computers, people used what is called as the graphical approach?

For most cases, graphical approach does yield very close answers to numerical and it is far for simpler to use the graphical approach and of course, it also provides you in addition to just the number. It also provides you the intuition as to which mode starts where and which mode ends, or what is the, dependence of a given mode with respect to the frequency or wavelength, which is very important when your analyzing the pulse propagation.

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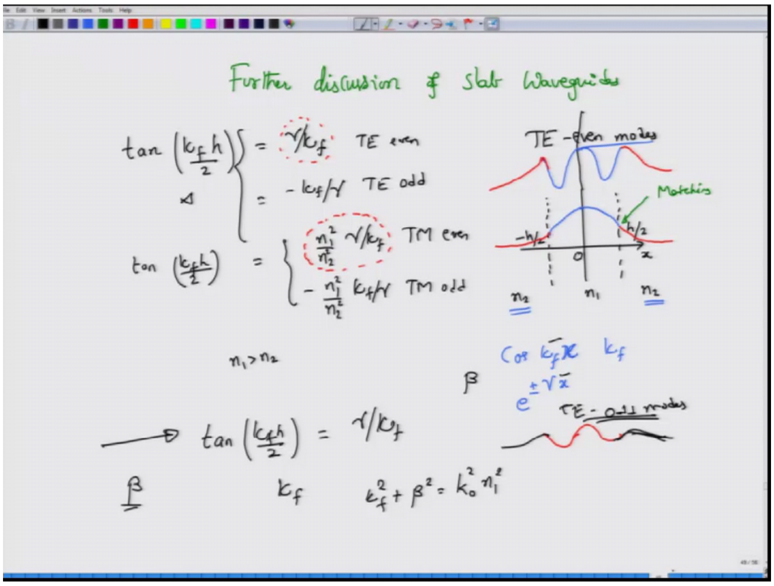

Through this slab waveguides we start from something that we already know, what I am going to do now is just to recapitulate those equations, which you already have seen earlier so, for that transverse electric even modes right. So, this is for the transverse electric, even modes, when we say even modes what we actually mean by that? Is that, if you were to look at the x axis, you know the slab waveguide of course, is propagate. The modes are propagating around the z axis, but is bounded by two planes right ok.

So, inside this plane you have n 1, outside this plane you have n 2 and this plane will be 1 plane kept at minus h by 2.The other plane is kept at plus h by 2 as we have seen. So, you can see that, I have kind of only being considered, the considering the x component in this one, because along the z component, you know that whatever the basic mode, it would simply propagate along the z axis.

So, when we say even modes, what we mean is that from the centre of the axis which of course, will be from the centre of the axis, which will be at x equal to 0, if you were to plot the modes you would have modes in the form of cos kappa f h and we have seen that this kappa f has to satisfy a certain relationship in order to. So, we need to solve this equation to find out kappa f, but once you find kappa f, then it is possible for you to sketch this modes right.

This is not kappa f h, this is kappa f x inside the film and outside the film or that is outside the slab, you have decaying modes right. So, you have either e power plus gamma x, which would of course, decay outside or in the cover region, because x is positive in that region. You will have e power minus gamma x for that one and you have e power plus gamma x for x less than minus h by 2, which essentially is the substrate and please remember all are analysis has been done for what is called as symmetric waveguide.

Symmetric waveguide means n 2 on both sides is the same ok. We will mention couple of points about asymmetric modes, later on. But for the symmetric modes that we have considered, we have done a fairly systematic analysis to obtain the equations, the characteristic equation. So, as I was saying what is an even mode for us? An even mode is one which has cosine dependence, because as the mode changes you know it would be cos kappa f x.

If we did not have this planes at plus h by 2 and minus h by 2, then this mode would actually have gone and become a cosine like this correct, but unfortunately that is how, that is not possible, because you have this planes, which will stop your cosine function. Just at this boundary right, at this point, it would have stopped.

Now, we have seen that, when we apply the boundary conditions what essentially we are doing is to match the fields, which would be decaying outside the film and at the point where at x equal to h by 2, where the film meets the cover or at minus h by 2, where the film meets the substrate, this decaying, exponentially decaying solutions, must smoothly merge with the solution in the film right.

So, you can see that there is a single maxima here and this essentially is what is called as the fundamental mode of the, fundamental TE mode of the symmetric slab waveguide. So, all that we did the complicated math that went into the systematic procedure was to essentially get this point matched.

So, this was the matching condition that we had to impose by all that mathematics, but what usually was happening was, you had a field in the film and then that would smoothly change to a decaying field in the cover or in the substrate higher order, TE modes are equally easy to sketch or probably difficult to sketch depending on how good you are with your cosine functions.

So, let me try and you know a sketch that one. So, this was half cycle. So, the other solution that might be possible is this one right. So, in this case I have, one half cycle, another half cycle. So, actually I have and almost complete cycle here, but then the field must decay outside. So, it is not really going over nicely. So, please excuse that one, but the field outside would still be decaying of course, the decay rate depends on what is the value of gamma.

So, in this way you can actually build up higher order modes. So, this could be called as TE 0 mode, this could be called as TE 2 mode, you know in that way you can actually keep building up your modes right. So, this is something that we have seen, in the previous class, but the physical picture, you might not have seen. What would be call as the odd modes, the odd modes would essentially be like a sin kappa f x right. So, they would actually go like this.

So, you have the odd, this one and then outside it would decay in the substrate as well as in the cover region. So, this would be the TE odd modes. I am not now, nice in sketching all these things, but it is essentially cosine and sin functions outside, it is the exponentially decaying functions. So, if you can, you know use MATLAB or any other, software that you are comfortable with, you can actually keep sketching all this, modes.

Except of course, that you need to know what is the value of gamma and you need to know, what is the value of kappa f of course, this can be found by finding beta right, but beta is actually hidden in this equation, what was the equation let us complete this tan kappa f h by 2 was given by gamma divided by kappa f, this is for the TE even modes and for the odd modes.

The left hand side will say would essentially remain the same, but the right hand side becomes minus kappa f by gamma for the TE odd modes ok. In both cases the solutions are not simple, because you have a transcendental equation, for the TM modes the left hand side essentially is the same. So, you have tan kappa f h by 2 being equal to n 1 square by n 2 square gamma by kappa f and this would be for the TM even modes and a minus n 1 square by 2 square kappa f by gamma for the TM odd modes.

Now, you can actually see that the right hand side is essentially same as the case for the TE 1, but these values are slightly larger corresponding to the TE case, because n 1 is usually larger than n 2.Of course, you need that, scenario, because you want waves to be guided in the film region. So, for n 1 being greater than n 2, n 1 square by n 2 square will be greater than 1and therefore, this term, that you have here right. So, n 1 square by n 2 square gamma times kappa f is slightly larger than gamma by kappa f of the TE modes.

So, now I have not derived the TM even and TM odd mode characteristic equation I will leave that for your consideration and now, what I want to do is to considered one of this equations, because the procedures for the other equations would essentially be the same. So, I want to consider one of these equations which I will consider for the transverse electric, even mode and I want to solve this equation ok.

So, how do I solve this equation, my goal to solve this equation would be to find out beta, but I am not going to do this in a straight forward manner that is I am not going to find out the value of beta, but I will first find out the value of kappa f, which satisfy these two equation. I express everything in terms of kappa f and then I will be able to compute beta, because the relationship between beta and kappa is, kappa f square plus beta square equals k 0 square n 1 square.

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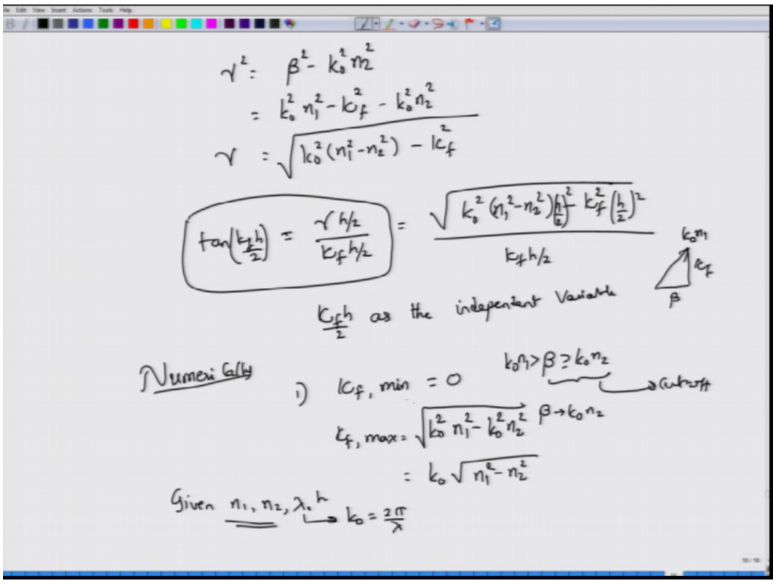

So, now, my job of course, would be to express gamma in terms of kappa, that is rather very easily done, because gamma square at least is given by beta square minus k 0 square n 2 square of course, beta square is nothing, but k 0 square n 1 square minus kappa f square kappa f will be treated as the variable, in this equations.

So, minus k 0 square n 2 square as it is. So, you can group the terms and then essentially obtained gamma in terms of kappa f. So, now, you can substitute for gamma in this expression ok, in this expression is substitute for gamma kappa f anyway is kappa f on the left hand side, you have kappa f h by 2 just to consider the solutions, you might want to scale up the right hand side as well by multiplying, by h by 2 on both sides.

So, on the right hand side you have, gamma kappa f h by 2 divided by kappa f h by 2.This is just to you know kind of replace all of those variables by kappa f h by 2 ok, that is all I am trying to do here, you can even do that by writing. This h by 2 into square root operation for gamma, because gamma is as we have seen from the above equation will be k 0 square n 1 square minus n 2 square minus kappa f square.

So, when you take h by 2 inside the variable out there or inside the square root, this becomes h by 2 square, this will be h by 2 square right and then denominator is essentially kappa f h by 2. So, what you have is a solution in which I can treat kappa f h by 2 as the independent variable, because all the equation out there, I mean the equation that we have written essentially depends on kappa f h by 2 ok.

So, what is the method of solution well, you can do a numerical solution. So, how do I numerically solve this equation, first I will figure out what is the minimum and maximum value of kappa f, what would be the minimum value of kappa f well the minimum value of kappa f will be when you remember, this is a triangle right. So, there is a beta here, sorry that is the transverse thing.

So, this is kappa f, this is beta and together is the incident wave vector or the wave vector inside the film, which is kappa 0 times n 1. So, you can actually, have the length of, kappa f becomes 0, at which point; k 0 n 1 will be exactly equal to the propagation constant beta right. So, if you start shrinking this height right then k 0 n 1 actually falls on to beta1, the projection of k 0 n 1 will then be exactly equal to beta, when kappa f, which is the height of this triangle will be equal to 0.

So, that is the minimum value that one can actually have. So, the minimum value of kappa f is actually equal to 0 right. What about the maximum value of kappa f, well we know that beta has to be there within say k 0 n 2, write a minute has to be greater than or at most equal to k 0 n 2, when this happens then you are essentially into cut off of that particular mode.

Therefore, the minimum value of beta will be k 0 n 2 at which point kappa f max will actually be equal to k 0 square n 1 square minus k 0 square n 2 square under root, you can rewrite this one by taking k 0 out and then you have square root of n 1 square minus n 2 square. So, we have determined what is the minimum value of kappa f; we have determined what is the maximum value of kappa f and within this region or within this range is what we are actually looking to plot this equation.

So, or numerically, solve this equation right. So, all that you have to do is you find out given n 1 n 2 lambda. You can calculate from lambda k 0 value, because k 0 is basically, to 2 pi by lambda and lambda will be usually specified in the free space region. So, given these parameters n 1 n 2 lambda, I think these are the parameters that you need oh, you also need the height of the waveguide.

Let us say that is h, then what you have to do is to first determine the minimum and maximum values of kappa f ok.

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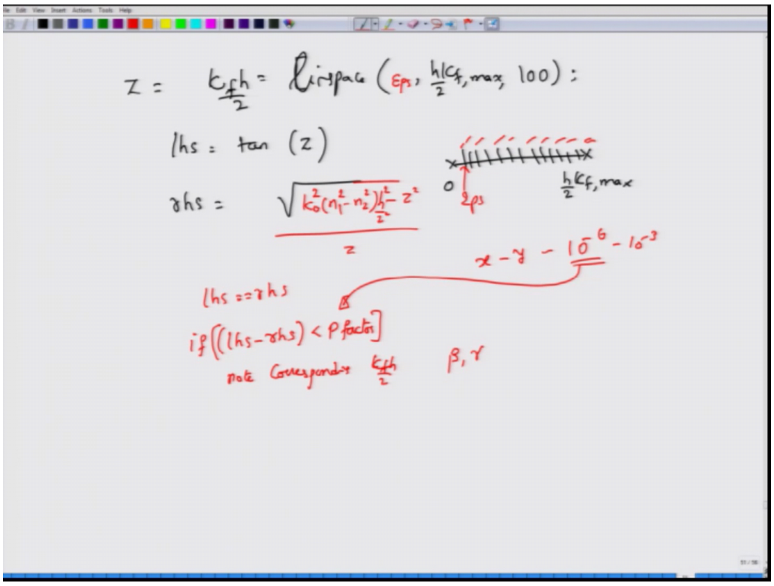

Once you have determined that you write a code right. So, let us say you have kappa f being assign you know some sort of an array, which you know you take some kind of, linear spaced array from 0 to kappa f max. I am using 0, because that is the minimum value and you create some 100 points or maybe 1000 points, depending on what points you want to create. So, all that this has done is that, it has taken this range from 0 to kappa f max right.

This is kappa f max, what you want to do is to actually multiply this 1 by h by 2, because that is your independent variable. So, you have this range of 02 h by 2 kappa f max and in this range you basically, create some 100 points ok. So, this can be done numerically and then numerically, obtain L H S, which is basically tan of kappa f h by 2.

So, you can of course, give a certain variables say z equals kappa f h by 2, in that case you are simply calling this, routine in whatever the language that you have as tan z. Similarly, what would be the right hand side? The right hand side is essentially, what we have seen square root of. So, this is the right hand side here, right and kappa f h by 2 is what we have called as z therefore, denominator is z here, numerator is z by 2 whole square.

Now, if you directly put z equal to 0 numerically, you will be having a problem therefore, you do not normally start your vector at 0, but you actually start your vector at a small value. Let us call that small value as epsilon ok. So, epsilon, epsilon may be at this particular point it could be just a non zero value, it, it will not be very large value, because you will then be moving away from the minimum value, but it will not be also so small that you will end up with numerical errors.

So, you choose your value depending on your, you actually look at what you are getting from your MATLAB code or whatever the code that your using to find out the appropriate value of the small non zero value ok. Epsilon s right and once you have found that one you can, you know write down all whatever that you had. For example, I mean you write down the right hand side thing. So, k 0 square you had, I think and n square minus n 2 square minus z square then there was h by 2 square divided by z.

So, you find out and z is the array that contains this value. So, these are the different values of kappa f h by 2, which is your independent variable. So, you create the left hand side array, the right hand side array and then what you can do is to find the points, where L H S is equal to R H S. Now, equals is a very interesting condition therefore, what you normally do is, you subtract L H S from R H S and then say if L H S and R H S right, there are difference between these two values is less than some predefined factor.

So, I will call this as p factor, if p factor then note corresponding kappa f h by 2 that would be the value of kappa f h by 2 from which you can then calculate beta, you can calculate gamma. What is a p factor? Let us say you are looking at, in numerical solutions you are looking at the difference between two vectors x minus y and you want the error to be in the order of say something like, one or 10 to the power minus 6.

So, this is of course, just my condition, this p factor could be any value that is dependent on the application that you have, but in general looking at the range of kappa f and error difference of about 10 to the power minus 6 or 10 to the power minus 3 whatever, that range would be sufficient right. So, once you have figured out that p factor then you can stop the program and note down the corresponding value of kappa h by 2 and that could give you beta versus gamma.

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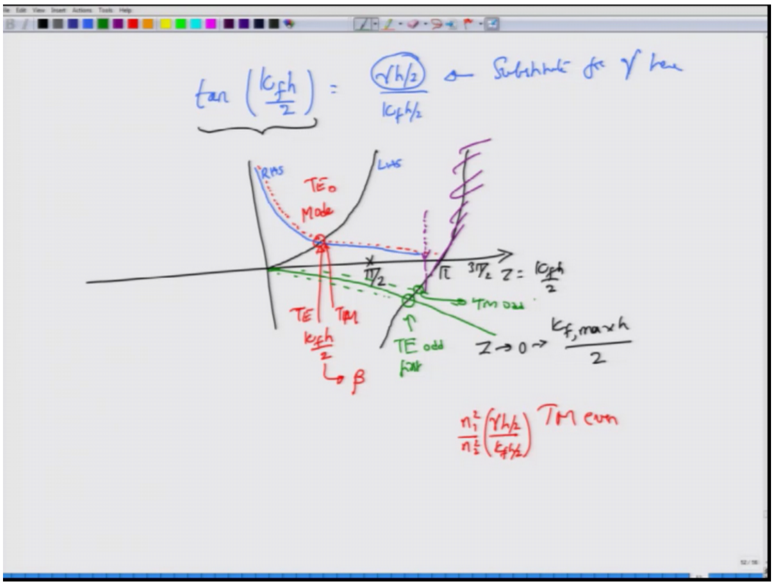

So, this is the numerical way of solving for the characteristic equation that will give you the number without any doubt. It will give you the number, but it does not give you the, you know what is that, the physical intuition has to what is happening right, to get the physical intuition. We go back to the equation. Here again, I am going to write down everything in terms of kappa f, but please remember that you know what is gamma times h by 2. So, it is all lengthy expressions.

So, I do not want to write that lengthy expression, but anyway I have written that one, that is why I written down this as gamma h by 2. So, substitute for gamma here, this is gamma. So, you can substitute it here, I mean get that lengthy expression that you have.

Now, the graphical approach is actually very interesting, what it does is that you plot the values of you know the left hand side and the right hand side. So, I define z right as kappa f h by 2. I have defined this one right and then I am now, going to plot this tan kappa f h by 2. So, how is the function, be a function of z like this where z is kappa f h by 2 well at 0 at z equal to 0 tan function will be 0, but then it increases steadily and of course, goes off to infinity at pi by 2.

So, when kappa f h by 2 equals or nears pi by 2, the corresponding value of tan function has actually gone up 2 infinity. It kind of asymptotically goes of to infinity then of course, as you increase this value of kappa f h by 2 further, it is start at pi by 2 and then goes through 0 at pi and then goes again back to infinity at 3 pi by 2, this is pi. So, this is pi and then you can keep moving. So, how far do I need to move this one well, I know what is kappa f max correct, I know this kappa f max beforehand.

So, all have to do is to take the argument of z from 0 to kappa f max times h by 2right. So, let us say kappa f max h by 2 happens to be at this point ok. I am just basically, taking some random numbers, but in the exercises, you will actually be getting the correct value. So, you can go back and put the exact value of kappa f max h by 2 here and then complete this problem or complete the understanding of this one.

So, let us say this kappa f max h by 2 happen to be here ok. So, all this region beyond this is really not important for us. So, you can scratch this out or we can remove that one. Now, let us look at the right hand side, what is the right hand side look like, when kappa f h by 2 or z is 0. Right hand side actually shoots up, because numerator will be finite, but the denominator will be 0. So, the right hand side shoots up. So, as you move from kappa f h by 2, you will actually see that it starts from infinity and then starts to go to 0 like this.

Of course, the rate at which this falls depends on what values of n 1 n 2 and lambda that you taken, but no matter what it is, it starts here at near 0 and then as it move goes down to 0 that is clear that the right hand side and left hand side have to cross at a particular point and this crossing point is the solution the corresponding value of kappa f h by 2 from which you can easily find out, what is the propagation constant beta. So, this is what we would call as the fundamental mode and this mode happens to be the TE 0 mode.

What would be the situations for the TM even modes ?Remember, for the TM even modes are right hand side is basically, n 1 square by n 2 square and then the corresponding gamma h by 2 by kappa f h by 2 is essentially, the same thing right. So, all you have to do is to now, take the right hand side and then draw another curve, which would be slightly away from this one. So, it would actually, you know depending on how far n 1 and n 2 are, but it would actually be slightly away like this and it will also go down to 0.

It starts at infinity and goes down to 0 and you can see that the mode is just around the TE mode. So, this would be the TM mode and this would be the TE mode ok. These are the even modes right. How about the odd modes? Well, for the odd modes right, you will have to start with minus kappa f h by 2 by gamma h by 2 correct. So, because there is a minus to the odd modes, actually occur in the fourth quadrant and, because the numerator is kappa f h by 2.

They would actually go something like this right. So, it will not touch the first branch here do not worry about the points being touched here, that is not of consequence. So, the first intersection happens in the region when kappa f h by 2 is greater than pi by 2. So, this would be the first TE odd mode ok.

Similarly, for the TM even mode, you will have to start somewhere here and probably meet or maybe you have to start somewhere here and then probably meet here. So, this correspond to the TM odd mode ok. Again, please note how closely this TE even and TM even are situated depends on the index values of n 1 and n 2.

In fiber optics n 1 or in optical fiber n 1 is very close to n 2.therefore, there ratio is almost one, which means TE and TM modes are kind of degenerate that is they have the same propagation constant where has in an integrated waveguide of which slab waveguide is an example, n 1 and n 2, contrast is usually larger. So, these two modes are not the same, but they are slightly different. So, they actually are to different modes in usually, in the case of for integrated optical circuit.

So, we have seen how to go about solving this characteristic equation for, you know in the graphical method or in the numerical method, there is one additional method, which is again graphical method, which, in which we instead of considering, kappa f and the corresponding values of beta.

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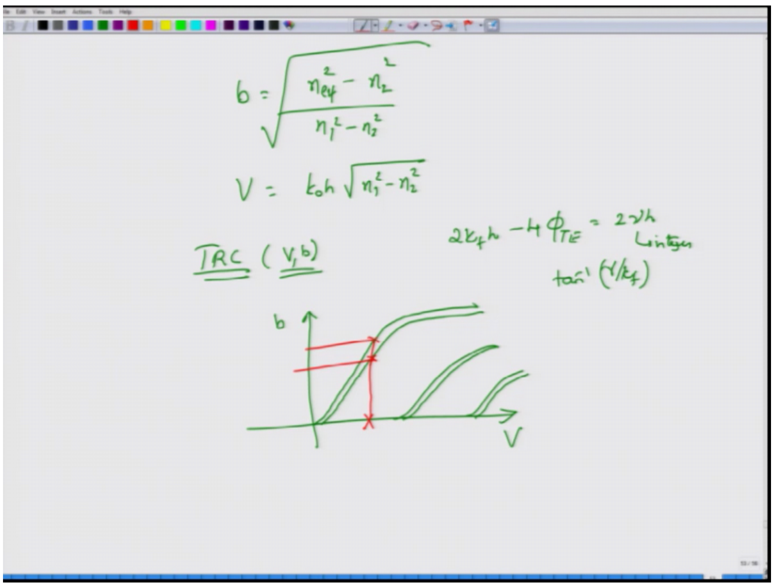

What to do is, we normalize these values by defining certain values called as b, which is essentially defined as n effective square minus n 2 square divided by n 1 square minus n 2 square and then we define v as k 0 h square root of n 1 square minus n 2 square, sorry, b is basically square root of this 1.

I forgot the square root and then from the transverse resonance condition, you express a transverse resonance condition in terms of v and b now. The transverse resonance condition, if you remember was 2 kappa f h minus 4 phi TE for the TE modes being equal to 2nu h, where nu was the integer. As we have seen earlier right and phi TE was essentially, tan inverse of gamma by kappa f correct.

So, you can actually express with the help of this v and b, you can transform the TRC equation, which would have dependent on gamma and kappa f into an equation, which is dependent only on v and b and the universal graph that you can actually plot means that you do not need to worry about which waveguide that you are actually calculating.

You can calculate this normalized way and then apply it for your waveguide, you can actually scale this value. So, this b versus v curve, which we will see for fiber optical, fibers as well would look something like this. So, you would have, TE modes and then you have the next group of modes like this and then the next group of modes.

So, given the parameters you can actually first find out, what is the corresponding v number for your waveguide and from there, find out what are the different propagation constant for different modes. We will discuss this when we talk about optical fibers in more detail.

Thank you very much.