Fiber – Optic Communication Systems and Techniques

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Lecture – 16
Systematic analysis of dielectric slab waveguides

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module we are going to apply the systematic procedure to study Slab Waveguides. So, this will be kind of a review of the procedure that we have already been seeing because, we applied it to parallel plate waveguide in the previous module. Now most of the techniques or most of the steps that we have done is already for the analysis of slab waveguide we have already accomplished that in the previous module itself ok.

So, we have chosen the coordinate system; which would be slightly revised for this problem to bring out couple of symmetric aspects of the problem. Otherwise we still going to we are still going to choose rectangular coordinate system, Cartesian rectangular coordinate system, then express the transverse electric and magnetic fields in terms of the longitudinal electric and magnetic fields, write and solve the wave equation for the longitudinal component, apply boundary condition to obtain the characteristic equation and other constants of the solution.

And finally, the step that you know you would be doing as an exercise would be to express all the other components or rather you know from the expressions of the other components or the transverse components you obtain the more functions of them ok.

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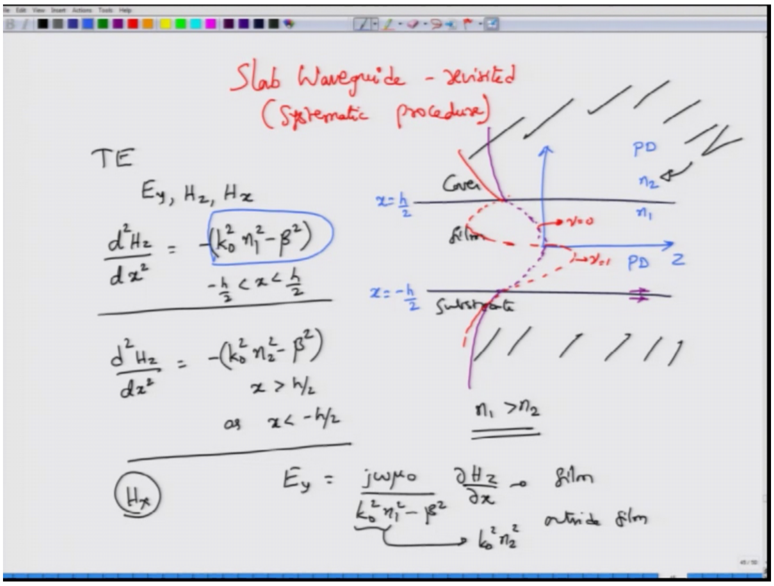

So, we have already seen the first step of choosing the coordinate system, but I am going to change the coordinate system slightly as I told you ok. We still have the interface or the waves propagating along the z axis, but now my x axis will start at the centre of the slab ok, of course, since this is slab waveguide there will be a refractive index n 1 here and a refractive index n 2 we are going to assume perfect dielectric in the second medium, as well as a perfect dielectric and the first medium. So, that we do not really need to consider the losses ok.

So, I have n 1 here and n 2 here and now this plate will be or this plane will be at x equals minus h by 2 and this would be x equals plus h by 2 ok, because see we have seen that the centre of symmetry would actually be at the centre as we know we saw an t n t m case. So, what we called as odd function or an even function did not really have the; same kind of a value of nu there ok. So, just to keep them redefine the symmetry properties. So, I am going to change the coordinate system by moving up this way.

Now, before we can even start the systematic procedure; I already know that if I am going to consider the transverse electric modes, than the only non 0 components. That I am going to have will be E y H z and H x of which H z will have the following wave equation which we have seen already d square H z by d x square equals minus k 0 square n 1 square minus beta square ok as long as your x is between; minus h by 2 to h by 2 ok, now this is the difference that you are going to get.

So, in the previous parallel plate waveguide we had only one equation which was valid inside the slab, but outside H z was assumed to be 0 far away from the interface, but now I cannot do that I have to assume d square H z by d x square will be equal to minus k 0 square n 2 square minus beta square, because outside medium is all a perfect dielectric of refractive index n 2.

So, this is the case when x is greater than h by 2 as well as n 1 x is less than minus h by 2 that is it would be the same case over here for the lower part of it of this is a so called substrate and this is a cover and in between the film as we have seen. So, in this film you have this equation with n 1 and in the substrate and the cover you have the equation here with n 2. And we of course, also know that n 1 is greater than n 2, you know this is from our simple ray picture, where we were looking for total internal reflection as the cause of the modes being propagated.

So, this is what the equations that I have of course, I also know how E y is related to H z ok. So, E y is given by j omega mu not divided by K 0 square n 1 square minus beta square del H z by del x as long as you are within the film. So, let be write down this one this is film, this is cover and this is the substrate. So, inside the film you have K 0 square n 1 square; outside of the film whether we are talking about substrate or the cover you have to replace K 0 n square by K 0 square n 2 square. So, this is our equation that we have of course, I am not written down an expression for H x, you can do that one as a part of the exercise of this problem ok.

So, what sort of solution should be considered, now we do know that if it was a perfect electric conducting walls then there would not be any field outside, whatever the continuity of the tangential magnetic field would be that would have given rise to the surface currents right. In this case there is no possibility of a surface current, but still it may be possible for us to have a solution in this form.

So, it is a sinusoidal solution, but the solution cannot go to 0 here at the boundary, why? Because, the continuity between two perfect dielectric is that the tangential components just be equal to each other. It does not say that the tangential component has to go to 0 here only that this to will be equal to 0 and we do know that there are evanescent fields outside the film, because of total internal reflection phenomena. So, these fields would actually decay in this exponential manner.

So, the solution would look more or less like a sinusoidal field inside, but outside it would be an exponential decaying fields these are the so called radiation modes or the decaying modes or the decaying fields actually, but inside it would look like a sinusoidal or cosinusoidal kind of a wave. And incidentally, this mode now will have what is called as TE 0 mode ok. So, it will we a solution even the mu is equal to 0. The field is not going to 0, when you substitute mu equal to 0 ok. The other higher order modes are equally possible so you might have the same kind of a decay constant or you might have a slightly different decay constant out there.

But, then the field itself could be something like this ok, and then the field would be decaying around in that manner or maybe decaying in this case. So, outside it would still be decaying, but inside it would be a sinusoidal kind of a field same thing as in the parallel plate waveguide this would correspond to mu equal to 0 this would correspond to mu equal to 1, but it is not so simple as I am writing it down ok. So, we are going to look at; why it is not so simple and the key to this kind of solutions lies in this expression ok.

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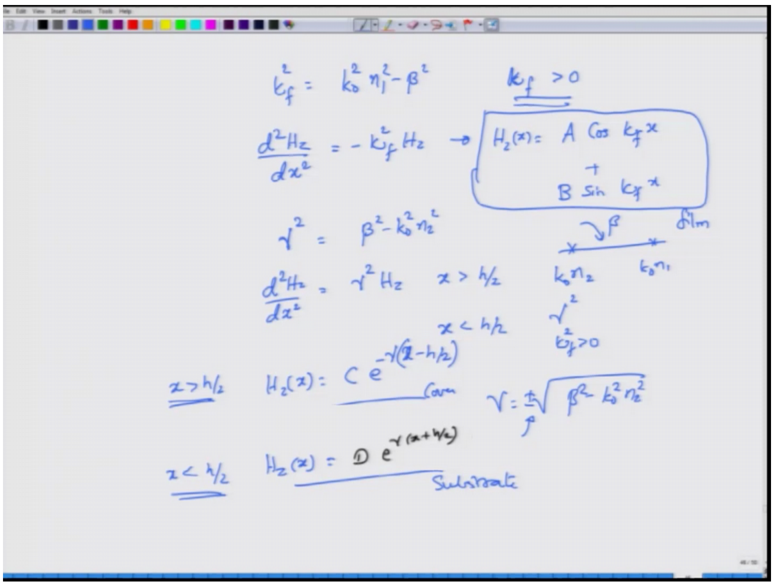

I am going to introduce couple of notations here; I will say kappa f square which is the transverse wave number to be defined as k 0 square n 1 square minus beta square and I clearly want kappa f to be a positive quantity ok.

So, only when kappa f is a positive quantity, I will have d square H z by d x square equals minus kappa f square times H z have a solution in the form of A cosine kappa f x plus B sin kappa f x. So, this should be the condition within the film. Now outside the film I am going to define a quantity gamma, now this should remind you of the decay constant; which is what I have is hoping that you would be reminded of and this gamma square will be defined as beta square minus k 0 square n 2 square right.

So, we have already seen earlier that the allowed values of the longitudinal component would lie between; k 0 and 2 to k 0 n 1. So, beta can lie somewhere over here we have seen this already the previous module. So, now, while once I have define this gamma ok, as beta square minus k 0 square n 2 square clearly gamma is positive kappa f is also positive.

But the solutions are going to be different ok, because, the solution here will be d square H z by d x square will be equal to so, this is in the outside the film that I am considering. So, outside the film this fellow will be positive now ok. And what you will have is gamma square the solution here was gamma square being positive in kappa square being positive.

So, this would be gamma square H z for x greater than h by 2 and for x less then h by 2. Now here is an interesting thing ok, the solution inside is fairly you know ok, but the solution for x greater than h by 2 must come by assuming gamma to be a positive quantity so, did you get this wrong here hold on a minute. So, I had minus k 0 square n 2 square minus beta square.

So, I defined gamma as so, gamma is beta square minus k 0 square n 2 square, the solution is in this form. So, when beta is greater than k 0 n 2 then I am going to get two solutions for this one right. So, gamma will actually be equal to square root of beta square minus k 0 square into square plus or minus correct. And I cannot choose the plus sign for the field outside or in the cover right.

So, when I am looking at x greater than h by 2 I cannot assume gamma square to be a positive quantity, in fact, gamma square is just defined in this way, but gamma can be plus or minus and I have to choose the minus quantity over there.

So, when I chose a minus quantity the solution for H z of x will be let say C e power minus gamma x minus h by 2. The reason why I have written x minus h by 2 is to simplify the boundary conditions. So, at x equal to h by 2, this exponential term will be equal to 1 and therefore, the value of H z as you come in from the cover will be equal to just C, but when x is less than h by 2, the field that I need to consider will have a positive gamma value, why? Because, there x will be negative ok.

So, I will have D e to the power plus gamma x plus h by 2. So, again x is negative, so please remember that; x is negative and x is greater than minus h by 2. So, because of that the term in this bracket in the exponential function will be negative and it would of course, be corresponding to a decaying field. So, unlike the previous case, where we had fields which were 0 completely outside the waveguide, now the fields are non 0 outside the waveguide.

So, we have three solutions now; one corresponding to the field in the film the solution corresponding to the field in the cover and the solution corresponding to the field in the substrate. As I told you I have assumed that this waveguide that I am considering the slab waveguide is symmetric waveguide. Therefore, cover and substrate both have the same refractive index.

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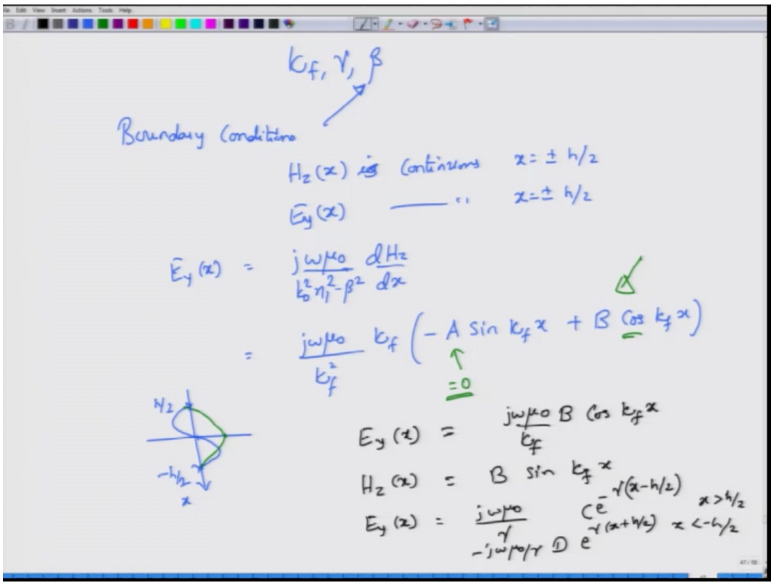

Now, our step would be to find out the relationship between kappa f, gamma, beta I mean are goal is to find out what is beta and I cannot get that one, unless I know how to obtain kappa f and gamma. So, somehow I have to get a relationship between these and that relationship will be obtained provided I know the characteristic equation for beta. And to do that one or into derived that one I need to know the I need to apply the boundary conditions ok.

So, I am going to apply boundary conditions and what are the boundary conditions? Well we had H z which is a tangential component. This should be continuous at the two interfaces; x equals plus or minus h by 2 and there is a second condition that I need to obtain. That would be in the form of E y right. So, I need to have E y of x also being continuous at x equals plus or minus h by 2.

So, what would be E y of x now? E y of x outside would be equation which would be a decaying and then within the film it would be sinusoidal. So, I know that E y is related to d H z by d x, that it is related to the derivative of that. So, if I write down E y of x that would be j omega mu not divided by k 0 square n 1 square minus beta square. When I am writing this equation; inside the film and then, you have d H z by d x ok. So, d H z by d x and K 0 square n 1 square minus beta square is nothing but, kappa square. And then d H by d x inside the film will pull kappa f out and then convert cosine to sin and sin to cosine just like the previous case.

So, I will have a kappa f outside and then I have minus A sin kappa f x plus B cosine kappa f x ok. I have to kinds of solution one is a sin solution one is a cosine solution. What I am going to do is that I know that this field that I am considering is actually even kind of a function right so, I have this as an x axis.

So, I have this as x equal to minus h by 2 and this is x equal to plus h by 2. If I were to assume this is solution of the sin form so, this would be the sinusoidal function like this; within the film. So, this would correspond to the odd modes, if my solutions are assumed to be a form of cosine then they would actually be something like this which would be the even modes ok. So, I am going to consider only even modes in this problem. So, I will retain B and I will set A equal to 0 by myself ok, I mean I can consider separately the solutions. So, I am going to first solve the problem with B and then leave the problem with A as an exercise for you.

So, I have kind of simplified my problem just to obtain an equation or just to understand the even terms. We will look at what happens for the odd terms also later on ok, until then you will be actually solving them as an exercise. So, E y of x is given by j omega mu not by kappa f, there is a constant B here then I have cosine kappa f times x.

This is for the field inside the film right. The corresponding H z of x will be of the form. So, let us look at H z of x. H z of x because, we have set A equal to 0 will not have this term. So, it will have only this B sin kappa f x terms so, this would be B sin kappa f x ok, this is in the film. What would happen in the cover region?

So, in the coverage E y of x will be equal to j omega mu not divided by K 0 square n 2 square minus beta square and then derivative of H z with respect to x. And in the cover we already know that H z goeses some C, but when you go to the field in the cover you will have and then you differentiate this one you are going to pull this minus gamma out correct.

So, I have going to pull this minus gamma out; which means that this would be minus gamma e to the power with the constant C minus gamma x minus h by 2 and K 0 square n 2 square minus beta square is basically minus gamma square so there is a minus and a minus sin on both sides.

So, I can remove this one and then instead of writing this k 0 square n 2 square I can just write this as gamma ok. So, this is when x is greater than h by 2 and similarly when x is less than h by 2, I know that you know the solution is in terms of D and when you differentiate this one with respect to x gamma will be pulled out. And I know that there is a minus somewhere out there so I will have a minus j omega mu not by gamma. And the constant will be D e to the power plus gamma x plus h by 2, when x is less than minus h by 2. So, for the fields in the substrate this is the equation ok.

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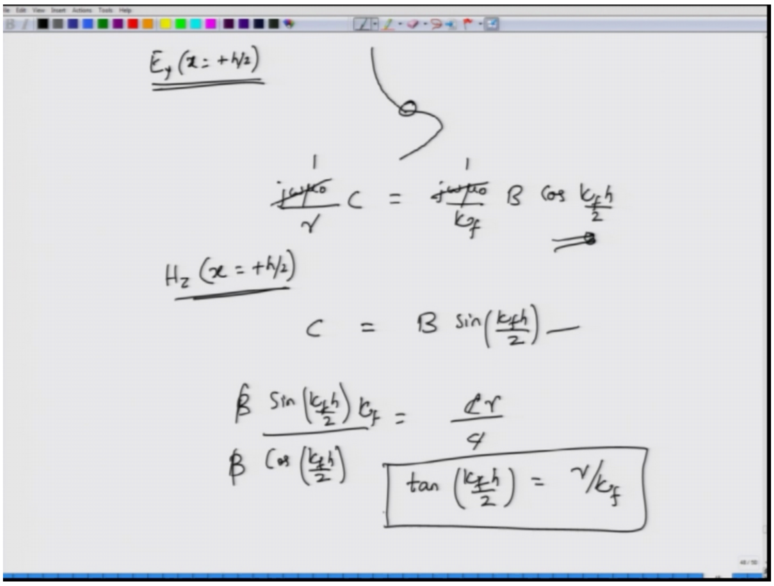

Now, apply the boundary condition one; that is H or rather lets write down the boundary condition first in terms of E y. So, I have E y at x equals to plus h by 2 being continuous, that is if I come from this exponential function. Then the solution here should naturally should be kind of continuing naturally into the film region; which means that the value of the field in the film must exactly match the value of the field outside at the boundary right.

So, you have two fields or two different solutions and these two solutions must match or match up at this particular boundary and that is what should happen, I mean that is, what is the boundary condition. So, you come from the top and you are within the film these two value should be equal to each other. Now you go back to the expressions that we have written and in the expression that we have written if you come from the field direction at x equal to h by 2.

So, you will have this cosine kappa f h by 2. This j omega mu not by kappa f will all be remaining in the same way and then you will have E y of x coming in from the whatever that evanescent side will be if C e power minus gamma at x equal to h by 2, this exponential function will be unity. So, the equations that I actually get from the evanescent side will be j omega mu naught let me go back and write down this one. So, j omega mu naught divided by gamma times C that must be equal to j omega mu naught by kappa f. So, I have j omega mu naught by kappa f; times B cosine kappa f h by 2 ok.

So, this is the equation that I am going to get from the of from applying the boundary condition at x equal to plus h by 2 on E y component of course, I do have a component H z which is tangential component for the magnetic field, because there are no surface current layers that we have assumed.

This component would also be continuous and coming in from the evanescent side or the cover side H z at x equal to h by 2 will have D sin kappa f this will be coming in from the film side but if you are coming in from the H z case, that would be C itself H z at x equal to h by 2, that would be C which will be equal to on the right hand side it would be equal to B sin kappa f h by 2.

So, I will have B sin kappa f h by 2. So, I have this two equations luckily, C is there in the left hand side, B is there on the right hand side, if I divide this equation by this 2nd equation out their what do I get so, I will get sin kappa f h by 2 divided by cosine kappa f h by 2 ok. That should be equal to so sin on there is also j omega.

So, j omega mu not is constant on both sides so I can actually remove them here itself so I will have 1 and 1 here. So, I will have B sin kappa f h by 2 divided by B Cos kappa f h by 2 divided by kappa f. So, kappa f goes on top this must be equal to C divided by C by gamma so, gamma goes on top B cancels out from this equations sin by cos is essentially tangential I mean tan functions so you have tan kappa f h by 2 being equal to gamma by kappa f.

Now, is not this the same equation or that that we obtained from transverse resonance condition, which would relate kappa f the transverse wave number to the decaying constant gamma. This is exactly the same equation and if you were to instead of assuming that electric field would be in the form of a cosine kappa f x.

If you had assumed it would be in the form of sin kappa f x, you would have obtained the equation that would have corresponding to the odd modes right, there you would have instead of tan you would have cot I mean instead of gamma by kappa f you would probably you would have kappa f by gamma with a minus sign right. So, something very similar to what you have seen from the ray picture you have the same equation.

Now, it is not only for the characteristic equation that we went through this step. It is because in addition to obtain the characteristic equation I also know how the field components E y h x and H z themselves vary. So, why is it important? Because if I were to change the constitutive and parameters of the slab waveguide for example, I make n 1 greater than n 2 already I know that, but if I were to increase the index contrast by a larger value, what would happen? If were to increase the contrast by larger value while keeping the same slab waveguide thickness, then the field gets more and more confined inside the inside the film itself ok.

So, you have the field otherwise so, what happens when n 1 and n 2 are not very different still maintaining n 1 greater than n 2. Then some of the field actually starts to decay outside or leak, outside the film. So, if you want a better confinement you want to make n 1 greater than n 2, while keeping the same thickness assuming right.

Of course, if you play around with thickness as well as with the index contrast, then you can you know essentially get whatever profiles you want, but the important point is that you would not have gotten this inside ok, unless you done known how the field components actually look like when their propagating inside the slab waveguides and so that is way of the systematic procedure is so useful ok.

Of course, as a mathematical exercise this is all fine, the physical intuition is still what we talked about in the ray theory approach, but mathematics or Maxwell's equations helps us to complete that incomplete physical intuitive picture ok. So, physical intuition picture is very good, it gives a basic characteristics, but it would not tell us, how the field qualities themselves will look like and that is perceive what this analysis has given you.

Now, although we have written down the equation, you know the characteristic equation tan kappa f h by 2 equal gamma by kappa f, please remember that solving these type of an equation is very difficult, in fact, it is not possible analytically, because kappa f contains beta; which is unknown gamma contains beta which is also unknown ok.

And then you have a tangential function of x being equal to some kind of a tangential function out there, which makes your life difficult, because this is called as a transcendental equation, and this equations are to be solved either by graphical methods or by numerical methods. We have considered only transverse electric modes, we have not considered transverse magnetic modes, but the procedure would be essentially same.

There you would in fact, start with I mean you would in fact, start with E z and then express H y and E x in terms of E z H z will be 0 in that particular mode for that for those type of polarised modes.

So, the procedure would essentially be the same and you would arrive at characteristic equation which again you have to solve either numerically or graphically ok. One final point before we close this discussion on slab waveguides of course, we will have one more module discussing the properties, but for today we wrote all those constant right a b c and d, but we never found out what those constants work.

In fact, to find those constants, you have to apply the boundary condition on the substrate and the film layer as well plus you have to impose the total power constraint ok. Which means that, if I am sending in certain amount of power into a particular mode, then that power will essentially determine one of those calls and so, it is possible for you to apply boundary conditions at the two layers eliminate two constants arbitrary constants from them and you will be left with only one arbitrary constant; which you will be fixing with the help of total power condition ok.

Since that is little complicated I mean the mathematical steps are tedious the procedure is not complicated. So, I have to decided to skip that step and instead give that as an exercise to you when we come to you know when we come to the assignment in this course. So, in the next module we will revisit these equations and then see how to solve them graphically and also study couple of properties of these modes and then we will be talking about single mode fiber until then.

Thank you very much