Fiber - Optic Communication Systems and Techniques
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 14
Systematic procedure to obtain modes of waveguide

Hello and welcome to NPTEL MOOC on Fibre Optic Communication Systems and Techniques. We were discussing in the previous module and the mode, the concept of a mode and then, indicated the end of the module that we are going to look at a systematic procedure to understand these waveguide modes.

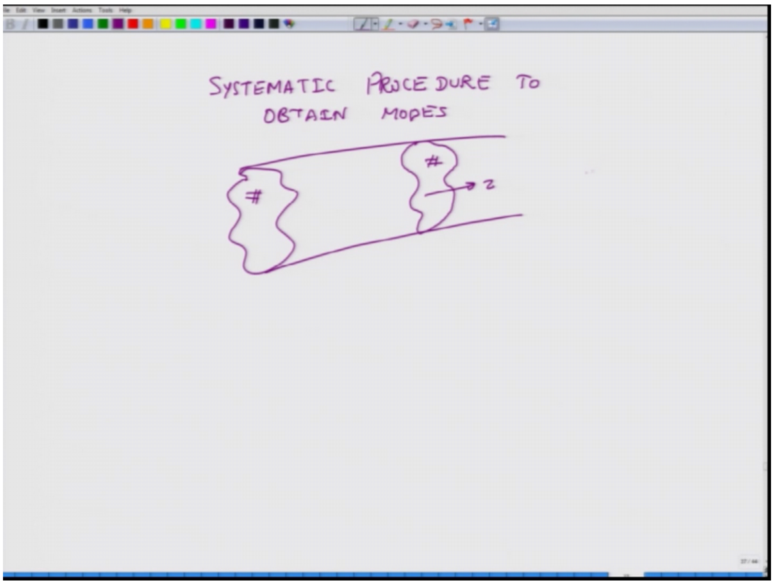
There are a couple of assumption that one talks about or one makes when we start understanding the modes of the waveguide. The waveguide mode or the sorry, the waveguide structure is assumed to be uniform along z axis. And, the cross-section could be, any irregular cross-section, but the cross-section would remain the same along z axis. That is what we mean by when we say that the waveguide is uniform along the cross-section.

For simpler treatment, we will also neglect any material losses. So, all of the materials that are waveguides are made out of, are considered to be looseness. These two assumptions are of course very unrealistic because you have in many situations are requirement that your waveguide needed to be bent around, ok. For example, in integrated optics you may want to connect your input to multiple outputin which case it is necessary that you use couplers and those couplers have to be specially separated by bending appropriated, at appropriate angles bending of waveguide appropriate angles.

So, clearly this assumption is unrealistic, but it is sufficient for us to actually I mean that assumption what we use in understanding the modes of a waveguide is because, you know its mathematical simplicity. So, we do not want to complicate our analysis by allowing the waveguide to be non-uniform. Moreover, we do not want to allow the cross-sectional variations that naturally occur when you manufacturer, you know manufacturer of fibre or fabricate a waveguide to actually cloud. Cloud are understanding of the fundamental transmission problems.

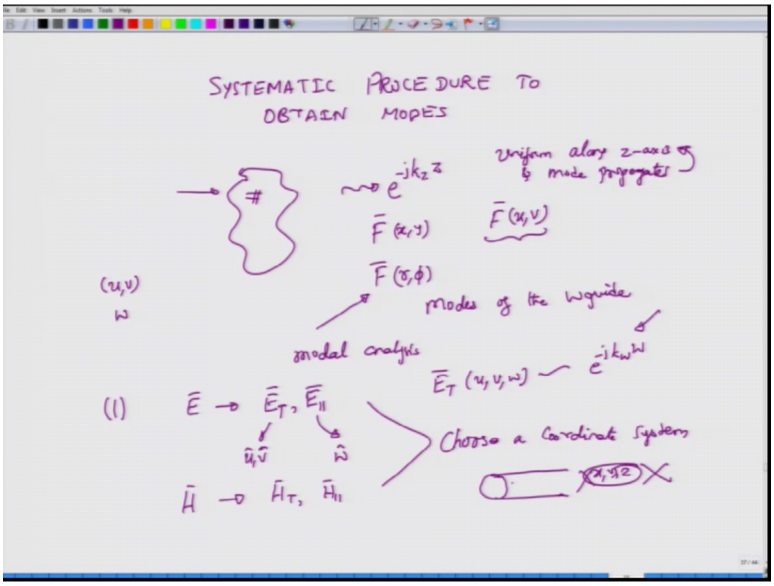
So, purely for mathematical simplicity we are going to neglect any of the non-uniform that might appear and for the same reason, we also go to neglect the looses. If you take advanced courses in waveguide, you would know how to incorporate the non-uniform waveguide structure as well as looses and many other things that you may not really like to consider in the practical waveguide.

(Refer Slide Time: 02:43)



Anyway, so the idea here is that I have waveguide of some arbitrary cross-section, and this cross-section remains the same as I go along z axis which is the direction in which the waveguide modes are assumed to be propagating, ok. This cross-section although I have shown it to be very irregular remains consistent at all points. So, I mean hopefully this cross-section is the same as this cross-section, although the figure may not really show it, but the cross-section remaining uniform is a key which allows us to forget about z part because I can then express all of z variations simply by writing this as e per jk z z, k z z as the propagating fact along z.

(Refer Slide Time: 03:28)



Then, focus my attention to obtain F of x, y for the triangular waveguides or F of r, phi for a circular waveguide or in general F of u, v for a general cross-section or general coordinates u, v and w. So, these F of u, v or F of x, y appropriately F of r phi r called as the modes of the waveguide and this is the mode that we are trying to find out, ok. So, modes of the waveguide this is called as Modal Analysis. So, the procedure that we are going to follow to obtain this F of u, v or F of r, phi or F of x, y is called as Modal Analysis of the Waveguide.

So, did you understand why we removed any z axis from our discussion or other we assume z axis discussion to be in the form of e for minus jk z z because we have assumed the waveform; I mean the waveguide to be uniform along z axis and more propagating along z axis. So, more propagates along z axis. So, what is the systematic procedure? Well, the procedure seems to be quiet straight or you know is actually straight, not really complicated, but of course the actual calculations may become tedious and complicated. Because, if you allow for very general cross-sections to consider, the idea here is that I know that electric field actually has two components, right.

So, I have what is called as a transverse component and then, I have what is called as a parallel component. The parallel component is the component that would be you know along z axis for the case that we are considering or in general, it would be the component that would be along the direction in which the waveguide itself is extended, and this E T corresponds to the transverse components that is components which are in the plane that is transverse to the direction of the waveguide.

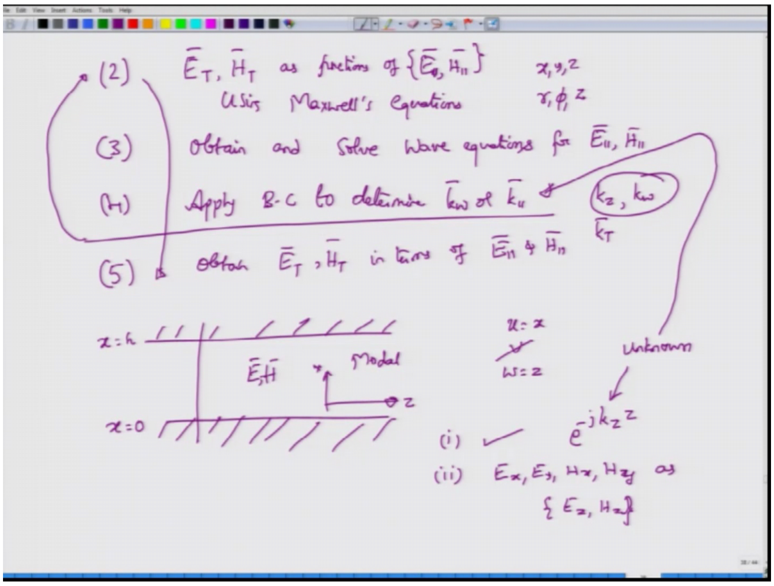
So, in the general case u v are the transverse coordinates, w is the parallel or the longitudinal coordinates. So, E T will be a vector in the u and v plane whereas, I mean having components u and v and E parallel will be a component that would be along w direction. Of course, it is important to realise that the transverse component could be function of all 3 coordinates, right. So, it is not necessary that the transverse coordinate be a function only of u and v. Of course, this coordinate dependence is on w axis is what we have captured by writing as E power minus jk w times w. So, we do allow for dependence on w, but we imposes specific dependence on w in the form of travelling way.

So, all the components that are there as the electric field and the magnetic fields can have transverse components or can be separated to transverse components and the longitudinal component and each of the transverse components and the longitudinal component by themselves can be a function of all 3 coordinates u, v and w, but along w which is the direction of the waveguide, we impose the very specific functional form which is E power minus jk w times w. If you are confused by u v w, you can replace u v w by x y z and work with planar cross-sections to simplify your you know understanding, but it is not really necessary to think always of the Cartesian coordinate system, right. The electric field we have decomposed into transverse and parallel components. We will similarly decompose H field also as transverse and parallel components with the same remarks that go with the electric field.

So, the first step is to understand that you have E and H and then, choose a coordinates system in such a way that your life becomes simple. For example, if the waveguide happens to be an optical fibre which you know as such as a circular cross-section, then choosing a coordinate system x y z will be a very big problem for you because, analysis becomes very difficult as x and y components couple to give rise to r and phi components or rather the components E x and E y will be in a very complicated manner depend on x y itself if you were to analyse them in the circular cross section.

So, it is always better to choose the coordinate system in which the analysis actually becomes quite simple, ok. That itself is an art, but luckily we have had a literature of about 100 years to guide us. So, we do have standard coordinate systems and if your problem happens to be not following into any of the standard coordinate systems, then good luck to you. Well, there are ways to now get on that problem, but that is not our concern here.

(Refer Slide Time: 08:41)



So, the first step would be to choose a coordinate system which is simpler for you to work. In this course, we will either be looking at x y z coordinate system which is the Cartesian rectangular coordinate system or we will be looking at r phi and z which is the circular cylindrical coordinate system. The second step is that remember we have decomposed the electric field and magnetic fields into those E T and H T E parallel and H parallel components, correct.

So, now what we do is, we express those transverse field in terms of parallel component. It is in fact possible to do so by applying Maxwell's equations. So, express E T and H T as function of E parallel and H parallel, that is longitudinal components. The reason for this is again mathematical simplicity. There is no specific reason. As to pick only the horizontal components or the longitudinal components, you could have done the other way around as well, but it is a simplified I mean this analysis or this method simplifies the problem. So, that is the second step.

So, express E T and H T are functions of E parallel and H parallel using Maxwell's equations. So, whatever the coordinate system that I have chosen, in that coordinate system I have to express Maxwell's equations and then, write E T and H T as functions of both E parallel and H parallel. In general they will be function of both E parallel and H parallel.

The third step is an interesting step which essentially means, which essentially calls for you to obtain and solve a wave equation. Remember these are all waves, right except that the wave is not uniform. It is a non-uniform wave having certain x and y dependence or certain u and v dependence, but it is your responsibility to obtain and solve wave equation for E parallel and H parallel. Usually the functional form of the wave equation will be same for both longitudinal components.

Therefore, if you solve one, you can easily guess the solution of the other. There will be some constants of solution which we will have to fix by applying the boundary condition, ok. So, apply boundary condition to determine the propagation constant. Remember one of the goals of your analysis would be to actually determine the propagation constant k z or in general the propagation constant k w. Of course, we will also have the perpendicular or the transverse wave number k t, k w will be function of the transverse wave number k t and whatever the medium k vector that is there anyway, but you need to apply boundary condition to actually determine these quantities, the longitudinal propagation constant.

So, I will just write this is k w, and this corresponds to the propagation constant or I could have written it as k parallel. The fifth step is to obtain that transverse and the transverse electric and magnetic field components E T and H T in terms of E parallel and H parallel. We have already written E T and H T in terms of this, but once I know the solution for E parallel and H parallel by following these 4 step procedures, the fifth step procedure is to go back to this step and then, obtain the solutions, ok. These solutions of course have to satisfy boundary conditions as well and this will allow us to obtain that transverse mode form f f u and v. So, this 5 step procedure is what we are going to follow in the next few module to understand different waveguides, of course are understanding of waveguides is for slab waveguides which we have already seen, not really in this using the systematic procedure, but using a big picture of the ray approach.

We will see that this approach also gives the same results and of course, this approach give you much more information in the rare approach. You won't know what is the way in the fields are distributed, where as this one will allow you to know what the fields are distributed functionally. I mean ray theory does not tell you the evanescent fields exist, but this theory will easily give you the evanescent fields that exist and the functional form of that as well. So, there is lot of merit in following this procedure and for optical fiber analysis. The simple ray approach is not satisfactory. So, you need to understand the systematic procedure to derive those mode fields of the waveguide modes of an optical fiber.

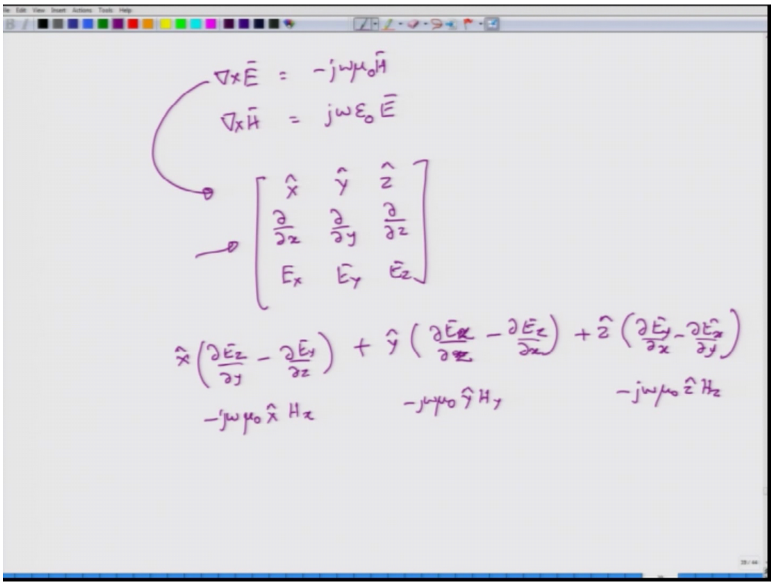
So, we are going to look at these systematic procedure and before I look at it for the slab waveguides and the optical fibers, in the next module what I am going to do is to reconsider the parallel plate waveguide, which consisted of two perfect electrics fields, right and then, see how far I can go with these steps to solve and obtain the corresponding electrics fields and the magnetic fields. The total electric and the magnetic fields are essentially do a modal analysis of this particular problem. So, here I can be little careful in choosing the coordinate system. I do not want to work with negative values of x or other x equal to minus h, I do not want to put in I will put x equal h and x equal to 0, ok. This of course will be the z axis. Luckily for me u is x v in do not need to be considered and w is z.

So, the first step of choosing the coordinate system I have already done it. This is z direction, this is x direction, ok. So, this is how x direction is and this is how z direction is. I have also you know assumed the uniform lossless property, so that the cross-section at any time is actually nice plane or a line here and this height would not change. So, this height of x equal to h, I mean in the height of h will not change as a move along the z direction.

So, the first step is done choosing the coordinate system, the second step of course when choosing a coordinate system have also assumed E power minus jk z times z dependence with k z being the unknown quantity that I need to determine. Remember that was the I know idea of step number 4. So, we have not reach step 4. I am not going to fully solve this case. I am going to leave most of it as an exercise to you. I will just indicate the procedure. I want you to use this procedure to solve for the case of a slab waveguide in the next module. These are basically indicative of what we are going to do, but the full use of the systematic procedure will not be for this one, but it could be for the slab waveguide.

So, first step is choose the coordinate system which we have done and along with that we have also put down the way in which these fields depend on z which is in the form of E power minus jk z times z. Now, the second step would be to express the transverse components which are E x E y H x and H y as functions of E z and H z. Remember these are the components that I have that the longitudinal components E z and H z.

(Refer Slide Time: 16:15)



To do that one, I need to apply Maxwell's equation. The equations that I am going to use are the Curl equations. These are the ones that will allow me to interconnect electric and magnetic field in the case of a free space medium that I am considering between the two parallel plate, between the parallel in a perfect electric conducting surfaces. This would be the curl expression and this would be the expression for H current axis in between. Therefore, this is simply j omega epsilon naught. I am also assuming that the medium in between the parallel plate waveguide is a free space medium. So, what I get is this expression, correct. This is in the frequency domain or the time harmonic expressions.

Now, what is curl of E? Curl of E is basically obtained by the determinant of this matrix which will be E x E y and E z, right and obtain a determinant of this one that would be x hat del E z by del y minus del E y by del z minus y hat del E z by del x. Since I am going to take the minus sign later on, I can actually rewrite this as del E x by del z minus del E z by del x. The z component of this field will be del E y by del x minus del E x by del y. This should be equal to minus j omega mu naught x hat H x minus j omega mu naught y hat H y and then, minus j omega mu naught z hat H z.

I do not really have time to carry on this analysis in this module. So, what I am going to do is to take this step in the next module. It is not the problem is not finish because we are just at the point of you know solving the step number 2, where we are trying to express the transverse field components, but these expression will also be useful for us to discuss the slab waveguide which we are going to do in the next module. So, we are going to use the systematic procedure to complete the problem for the parallel plate waveguide and then, use the same expression to see how we can obtain the module solutions for the slab waveguide as well.

Thank you very much.