# Fiber - Optic Communication Systems and Techniques

**Prof. Pradeep Kumar K**
**Department of Electrical Engineering**
**Indian Institute of Technology, Kanpur**

## Lecture - 13
## Concept of waveguide Modes

Hello and welcome to NPTEL MOOC on Fibre Optic Communication Systems and Techniques course. In the previous modules we discussed slab waveguides and, then we talked many times about something called as a mode. We talked about TE modes of different orders, we did not derive anything for those modes, or we did not derive any equation for TM modes, but we did talk about TM modes as well.

So, we use this word mode repeatedly and I promised you that the actual concept of a mode, I will discuss it later and in this class or in this module I would like to discuss what exactly do I mean by mode ok. So, this is something that must probably the familiar to you, if in case you have studied electromagnetics and, have dealt with waveguides it is in that context that one normally talks about and mode, but I will briefly recapitulate that more theory for you.

So, that we can understand what exactly are we talking about when we specify something as a mode and, then we talk about fibre modes in the later models ok. So, let me introduce the concept of a mode by looking at physical situation that is something very familiar to you ok, at least from the earlier study we should probably have seen this one many times, and even in are earlier module we have actually dealt with this case right.

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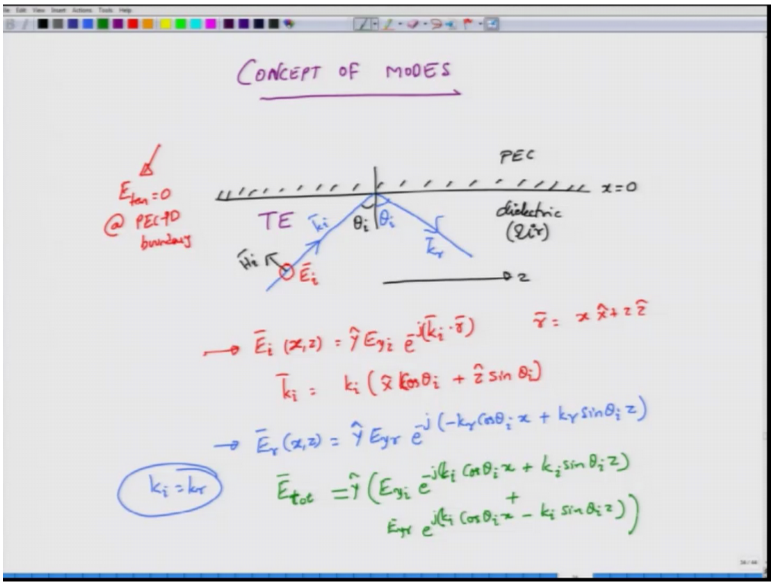


And that happens to be that offer, simple metal a perfectly electric conductor for which metal is a very nice approximation, but just for mathematically being consistent I am going to consider these to be a perfect electric conductor right. Now, you know that some boundary conditions that we discussed in one of the earlier modules, for this perfect electric conductor and I am considering dielectric medium.

So, for example, this could be simple thing like air ok, and we assume that this is perfect lost less dielectric, we know that the tangential electric field component must be 0 at the PEC and dielectric perfect dielectric boundary correct. So, whatever the waves that may have been incident on this particular boundary, it is always necessary that the tangential component must go to 0. Of course, if I assume that the PEC is slightly thick and, you know then there is essentially no wave outside the PEC as well. So, essentially all of the electromagnetic wave that is incident on to this boundary would be reflected back and you would not really find any field on the other side of the PEC boundary right.

So, that is something that we take it based on our experience of whatever that we have studied earlier, but the crucial condition that we are looking for is that the tangential component must be equal to 0. And because, I have not specify in the nature of electromagnetic wave, it does not mean that I am actually excluding light from this discussion, it is perfectly valid for us to include light as the wave ok. Because, light is also electromagnetic wave and then incident light onto this perfect dielectric and PEC boundary ok.

But normally metals are very very lossy at optical frequencies and therefore, there use as waveguide is not so much. Therefore, we do not really look at light all though technically it can be used to describe this mode; I mean describe this physical situation, or can be used in this physical situation ok. So, anyway that was a small digression. So, let us come back to electromagnetic wave perhaps at frequencies which are more suitable for, sustaining these waveguide modes you know by metals, which happens to be a few gigahertz to 10's of gigahertz ok, or maybe 100's of gigahertz anyway. So, I have this perfect electric boundary, now let me put down some coordinate system to work with. So, let me say that this is x equals to 0 boundary.

And of course, the parallel direction to the boundary that I am considering will be the z axis ok. So, my goal would be to kind of understand the wave propagation along the z axis. But, I am going to see what exactly is going to happen, when I send an electromagnetic wave at an angle ok, which is say some angle theta i theta i being the incident angle as measured with respect to the normal, to the perfect electric conductor and the perfect dielectric boundary which is located at x equal to 0 ok.

Now, when I send light in this way you will immediately ask, this is not like this is actually electromagnetic wave at a lower frequency ok. So, microwave field for example. So, you will immediately ask whether I am considering a transverse electric polarized wave, or transverse magnetic polarized wave. To keep the maths simple ok, I will assume that this is the transverse electric wave, which means then this would be the incident magnetic field, which anyway I am not going to look at it for now, but this could be the electric field incident ok, which could be polarized along the y axis ok.

So, electric field will be along the y axis so, you can write down this incident electric field as we have written many many times earlier, as a function of both x and z, which has an amplitude of say E y i where E y i is the amplitude of the electric field, which can be considered to be a constant and of course, this wave is polarized along the y axis. So, this is the transverse electric polarization that I am actually considering so; the incident wave is actually transverse electric polarized wave. So, anyway so I know that this is essentially a plane wave that have been considering.

So, I will have the exponential of the face factor of the form k i dot r ok, where k i will be the incident wave vector which itself has components along x and z, clearly this is given by k i which you the magnitude of the incident wave vector and the x component is given by x hat cos theta i plus and the z component is given by z hat sin theta i and I know that r position vector in this plane that, I am considering is x x hat plus z z hat. So, if I can go back and substitute into this expression, I would have figure out everything that is needed to describe this physical scenario, where the incident electric field is transverse electric polarized ok.

Now, what happens at this boundary clearly this boundary at this boundary the electromagnetic field would necessarily have a reflected field correct. So, I will have a reflected field which from Snell's law we know that makes a same angle theta i. So, wave theta r is essentially equal to theta i so, what changes here also we know it is the reflected vector k r which is in a direction are is different. So, it will still have a positive z component, but it will have a negative x component because it is moving away from the interface.

So, the reflected field if I were to write down which would again be function of x and z will still be polarized along Y direction I am assuming that the incident and reflected waves are both transverse electric polarized ok. This is slightly common since we do not expect the polarization to change, once the wave actually is incident on the PEC and get is reflected from that one ok, mathematical of course, you can show that the reflected wave will also have the same polarization as the incident wave.

Anyway because, the metal does not lead to anything to that; so, you have e r as a function of x and z which will have some amplitude let us say E y r and, then I will write down the full face of expression out here instead of writing separately k r dot r, I will write down the expression because we have seen this many times. So, this would be minus k r cos theta i x plus k r sin theta i z of course, being the two mediums being the same for the incident and reflected wave, I am just going to write this as k i equals k r indicating that the magnitude of the reflected wave vector is the same as the magnitude of the incident wave vector.

So, I have now are reflected electric field I have a transmitted, or other incident electric field, clearly there is no transmitted electric field, because this is the perfect electric conductors so, there are no fields on the other side ok. So, far it seems that what we have done is very very similar to a I mean to a slab waveguide, in the slab waveguide analysis also we started out with a single boundary right.

And then we said that you know I am going to send in light at an angle, which at that time would have to be greater than the critical angle and, then the light would be reflected of only when there is a critical angle the light would be completely reflected off, there will be only a vanish send fields on the other side. But, the reflected field would then begin to propagate in the other direction the polarizations could essentially remain the same the only difference is that, when it bounces of a dielectric medium it acquires a phase shift, which depends on whatever the polarization of the incident light is.

So, if it was a transverse electric polarization it would acquire phi TE as the phase shift upon reflection, or other 2 phi TE upon as phase shift upon reflection. And if it was the transverse magnetic wave, it would have acquired 2 phi TEM, but in the case of a perfect electric conductor I mean wave impinging on a perfect electric conductor the phase shift regardless of whether you are looking at TE or TM will be equal to 180 degrees ok.

So, for that reason I could have equally worked with transverse magnetic polarized waves here, but that transverse magnetic polarization would also right, you know requirement to decompose the electric field into two component. So, therefore, I have decided to avoid let us write mathematical you know extra hard work to simplify the work, I have chosen the transverse electric, but the ideas that I am talking about will apply equally to transverse magnetic fields as well ok.

So, anyway so I had this incident electric field, then there was a reflected electric field the magnetic field would also have similar changes, but at this point I do not really need to know the magnetic field ok. So, the concept of a mode for the TE case can be explained only with the incident and reflected electric fields, clearly there is no transmitted electric field into the perfect electric conductors so, all of the electromagnetic wave as actually reflected back. So, what is the total electric field in the first medium, in the first medium the total electric field which I will denote it as say E without capital T ok, or maybe if we want to specify more and not confused ourselves with a transmitted field, I will write this as E with a subscript of t o t ok.

So, this total electric field is given by so, this total electric field is given by the sum of incident and reflected fields luckily for us both are polarize is in the same direction and, we will now right this as E y i e power minus j k i cos theta i x plus k i sin theta i z plus E y r. So, I am going to write a y hat outside here to indicate that this is actually the y directed field. So, have E y r e power j k i cos theta i x minus k i sin theta i z ok. So, please note the signs, that I have written this is very crucial.

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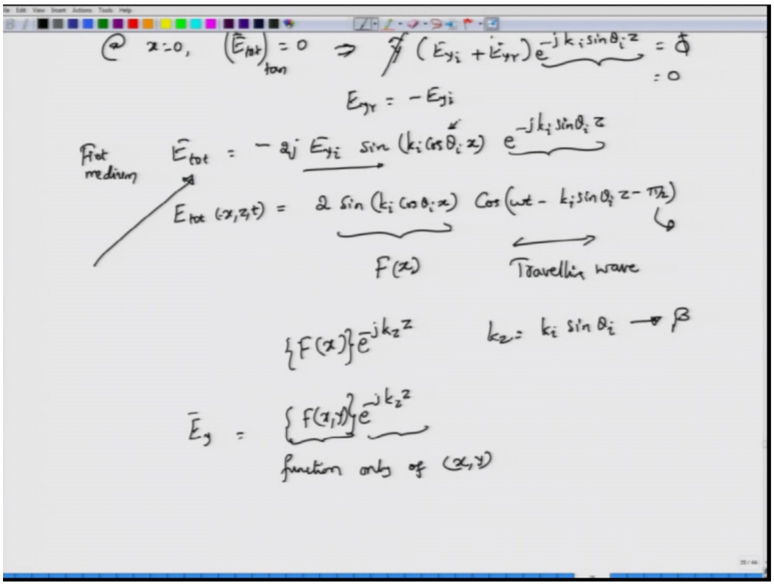

And now what is the condition the condition that we were looking for is that at x equal to 0, the total electric field must be equal to 0, actually the tangential component of the total electric field, must be equal to 0, but luckily for us the total electric field is already tangential, because it is in the y direction. Therefore, we simply set this equation to 0 so, what I have this will be 0 at x equal to 0 so, you can see the expression over here, when I put x equal to 0, in this one this entire term will become 0 ok. So, I do not need to consider that one and, what I have on the total the electric field is that are the condition is that I have Y hat E y i e to the power minus j, or other since this e power minus j k i sin theta I will be a constant.

So, I will have E y i plus E y r e to the power minus j k i sin theta i z, this will be equal to 0. So, this will actually be equal to 0 vector, but if you drop the vector thing this will actually be equal to 0 right. Clearly this exponential term is not going to 0, because this expression has to be valid for whatever value of z that I take.

So, there might be some values of z for which this are exponential might be equal to 0, but if you move away from that z, then this exponential will not be equal to 0. So, the only conclusion that you can draw about this expression is that the reflected field amplitude must be equal to the incident field amplitude.

So, once I know that the reflected field amplitude is equal to the incident field amplitude. Now, I can simplify the total electric field which I am going to do now here. So, the total electric field that I have will be in the y directions. So, I am not going to indicate the direction of this one simply write this as E total this is in the first medium ok. So, this is important this is in the first medium that I am considering.

So, this will be equal to you can show that this would be minus 2 j E y i sin of k i cos theta i x ok, times e power minus j k i sin theta i z this is the phasor form of the electric field, if you were to write down the electric field as a function of z, I am in the coordinates as well as time, what you get is 2 sin k i cos theta i x there is no change with this one, but when you multiply this one by e power j omega t and then take the real part of it, what you get here is cos omega t minus k i sin theta i z minus pi by 2.

So, forget about that initial phase pi by 2, if you forget about that one that is not really important, what you have is a travelling wave. So, this is actually travelling wave, but its amplitude also depends on x So, there is some sort of a x dependence function and, then there is a propagation term in the phasor form this would be even better. So, in the phasor form this would be F of x e to the power minus j k z times z, where k z is my shorthand notation for writing k i sin theta i sometimes I will also write this as beta.

So, I go back between k z and beta depending on the context, or depending on the simplified simplification that I would like to right ok. And this F of x will be functions only of x coordinate value right, in general of course, it will be a function of both x and y coordinates. And, then there will be along the z coordinate a factor which could account for exponential phase that is this is actually the travelling wave and, whatever that you have is a function only of the transverse coordinates and which transverse coordinates I am a talking about it is a transverse coordinates to the direction of propagation right.

So, I can write down my electric field in general ok, whenever I am considering this type of problems ok, I should be able to I mean write down this in the form of product of two functions, one will be the function only of the transverse coordinates x and y and the other one will correspond to the travelling wave part, anyway we will now discuss some very very interesting features of this expression. So, please keep this expression in mind.

Now, let us look at this when I said interesting features I mean only with I mean with respect to x mainly. So, let us look at this expression clearly, when I set x equal to 0 in this expression E total or electric field in the first medium will be equal to 0, right regardless of the value of z at x equal to 0 this electric field will be equal to 0 and it will be equal to 0, simply because k i cos theta i times x will be 0 so, this entire thing is 0. So, sin of 0 is of course, 0 is there any other value of x for which the sin function becomes 0 of course, yes when the argument of sin function becomes phi, then that sin of phi will again be equal to 0 and the first time.

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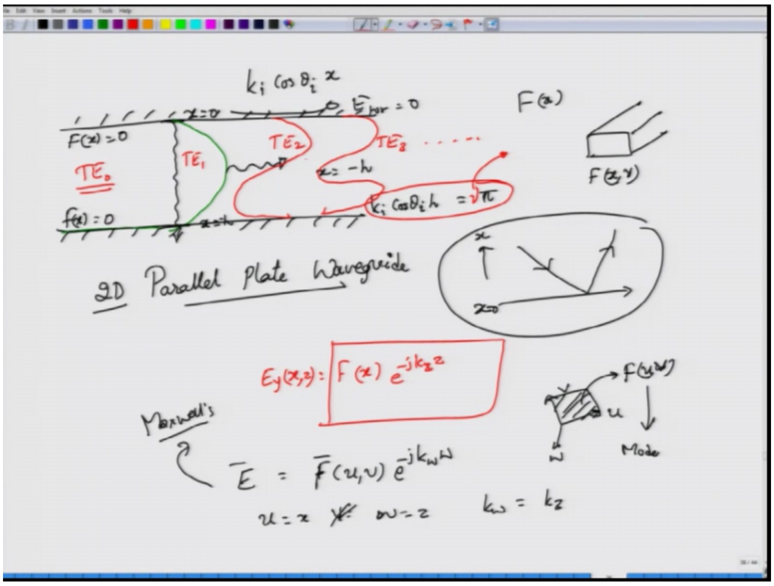


And this pulse broadening eventually that it actually happens right so, you have k i have fixed k i of course, because I have fix the medium, I have fixed theta i, because I have sent in a wave at a certain angle theta i and as a vary x by varying x means I am actually going down in this direction.

So, this is my x equal to 0 boundary at which E total was of actually equal to 0, but as I go along x at a certain value x equal to minus h such that k i cos theta i h will be equal to some pi ok. The first time it happens it would be pi at that point because, this the electric field F of x function is actually sin of this argument here, and the argument going through pi means that that function F of x at x equal to so, at x equal to h such that this condition is actually satisfied will be equal to 0.

Right so, at x equal to h this would be h equal to h the function again, or the sin function goes to 0 and the electric field actually goes to 0 ok. You can show that this continuous again and at x equal to 2 h, or rather x equal to minus h and minus 2 h the reason I have put in a minus sign is because you know the fields are propagating in the negative x direction. So, I have except changing the sign of x it does not really matter.

So, you could equally have considered a wave no initially in the in this direction and, then the wave would have bounced stops. So, that could also have been done. So, this would have been x equal to 0 and this would have been the positive x direction. So, you could have done the same analysis with this coordinate system, but because I chose the other coordinate system the values of x turn out to be minus h minus 2 h and so, on at these places the electric field actually goes through 0 ok.

So, for example, this might be one such way in which the field could be distributed that is electric field E y is distributed in this particular manner, which in the maxima at the centre and going through 0 at x equal to minus h ok, if I restrict myself only to that of course, I could also have the you know the field extending beyond this value of course, it does extend beyond this value. And go through 0 at x equal to minus 2 h and so on right.

So, at every multiple of h this goes to 0, this is one possible solution right which satisfies the boundary condition that the field go through 0 at all these multiples. The other possible solution is that I actually have double maxima here and, then I have this type of a scenario. So, in terms of frequency this is at a double frequency corresponding to this one, but this is also an equally valid solution right of course, I do not have to restrict myself only to 2, I can have 3 maxima and so, on and so, forth all these different field configurations, where we have written this F of x and remember F of x is actually the way in which the electric field component, depends on x right.

So, I have fixed z once I have fixed z the dependencies essentially on x in the form of F of x ok. So, this is e power minus j k z z and this F of x can have multiple solutions, and all these solutions are equally valid because in all those cases at x equal to minus h this would go to 0, at x equal to minus 2 h go to 0 and so, on and it so forth.

Now, let me do one thing I already know that I have a metal surface over here right. So, this is a perfect electric conductor which in fact, allowed all this field quantity is to actually start appearing right. So, this is how the field looked ok. So, the field is actually 0 at x equal to 0 and because of the perfect electric conductor. Now, what happens if I were to place another perfect electric conductor here. So, if I put another perfect electric conductor these fields will not be sustained ok.

So, clearly these fields are gone because of the perfect electric conductor and what I have actually achieved, is a very interesting thing have achieved in terms of x write, a function of x for a given constant value of z, but this pattern of half sinusoidal wave would actually move along the z axis. So, for example, if I got another z axis this would actually moved right of this would be again in the same way and this would keep on happening.

So, what you actually have in this kind of a scenario, where I have to perfect electric conductors ok, with this kind of a boundary that I have. So, the pattern whatever that I have between the two perfect electric conducting walls is that this pattern which is the function of x direction, which is this or no vertical direction that I am showing, on this pattern actually kind of moves along the z axis and it is not just one particular pattern.

So, it is not only one half sinusoidal cycle, it could be one complete sinusoidal cycle, it could be three sinusoidal half sinusoidal cycles four half sinusoidal cycles so on and so, forth. And all these patterns are equally valid solutions for this problem that we have considered ok. So, all that we did was to send in electromagnetic wave at an angle theta i and, what we manage to obtain is a functional dependence on x, which is a standing wave and this standing wave will go to 0 at many places, but if I know what to place a perfect electric conductor at one of the places where the electric field is going to 0.

Then what I have done is to essentially bound by electromagnetic wave, between the two walls, or bind the electromagnetic wave between the two walls. And this particular pattern which is the function of x, would move along the direction z indicating travelling wave along the z. So, all that is two perfect electric conductors did, was to establish the condition in such a way that they can guide the electromagnetic waves along the z axis.

So, this is a very very important thing and you must have been familiar with this analysis, this is called as parallel plate waveguide ok. Sometimes also called as parallel plane waveguide and, these different patterns which are functions only of x in this particular case so, this is a two dimensional parallel plate waveguide, you must also have studied a three dimensional, you know rectangular waveguide in which case the function instead of being just a function of x, it could be a function of x and y and these fields in general.

So, I am writing only for the electric field, but magnetic would also have the same kind of dependence, these fields in general are functions of transverse coordinates ok. So, maybe I can write down in a general u v w coordinate system. So, these are functions of u and v whereas, along the w direction there will be travelling wave. So, I have considered a general u v and a w direction so, this u and v or the plane that is here this function F of u comma v will actually be a vector function correct.

So, this would be a vector function and this vector function is called as the mode ok, this vector function has to satisfy boundary condition. So, F of x equal to 0 here F of x equal to 0 at these two planes is actually the boundary condition. So, together with the boundary condition this entire electric field is a solution of Maxwell's equation ok. So, any solution of Maxwell's equation together with the boundary conditions, and the function which depends only on the transverse coordinates while being guided along the other coordinate which is perpendicular to these transverse coordinates is called a mode ok.

So, mode in is nothing, but pattern of electric field and magnetic fields which satisfy boundary conditions and of course, they are the solutions of wave equation and, such then the solutions are such that the overall electric field and magnetic field pattern can be written in the product form of a function that is dependent only on the transverse coordinates and, this transverse coordinate function is propagating along the other coordinate axis ok. So, in the case of a parallel plate waveguide u was x v was really not required for us, because this was a one dimensional or a two dimensional parallel plate waveguide and then w was equal to z.

So, k w which is the propagation constant is actually equal to k z right and these patterns which I do as I told you, I can draw one pattern that way, I can also draw another pattern right. So, this pattern would also be another mode and, then I cannot draw three patterns these are all different modes of the waveguide ok. It turns out that the first waveguide mode is TE 1, the next waveguide mode is TE 2, then you have waveguide mode TE 3 and so, on ok.

So, TE 0 mode does not really exist and, this mode values that I am writing are simply the condition instead of k i cos theta i h being equal to pi, if this is equal to sum nu pi where nu is an integer. So, nu equal to 1 nu equal to 2 nu equal to 3 all these correspond to these different modes, one final point here before we complete this discussion.

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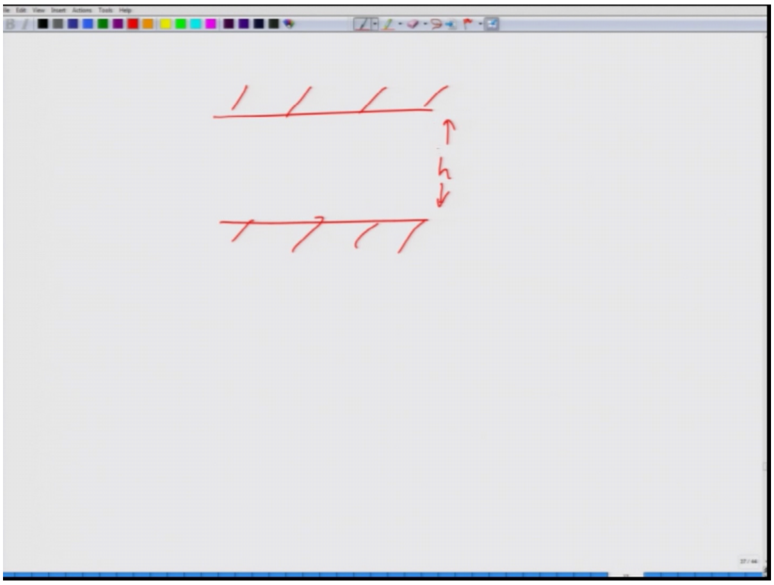


So, far what I did was to consider this parallel plate waveguide and, determine h according to the equation that I wrote this is some sort of transverse resonance condition if you would remember it. So, according to this transverse resonance condition what if I fix theta i, then h get is fixed ok, but it is possible for us to start off with a value of h here and, then explore what possible theta i values are required to satisfy this equation. So, if I were to fix h right, then not all theta i values are allowed only certain theta i values are allowed which satisfy that resonance condition k i cos theta i h must be equal to some integer multiple of pi ok.

And this comes because I have fixed h, then its shows that not all values of theta are possible only those that are possible are given by the transverse resonance condition, and corresponding to different theta I which are allowed solutions, you have a mode. So, corresponding E theta i you have a corresponding F of x function and these are the correspondence between the angle of incidence and the modes.

So, what we have discussed is the concept of a mode, which is the pattern of electric and magnetic fields ok. In the next module we are going to look at, how we can systematically analyze this waveguide structures, we are going to look at a systematic procedure to start from Maxwell's equation and end up having these model solutions is obtained.

Thank you very much.