Fiber-Optic Communication Systems and Techniques
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Lecture – 10
Transverse resonance condition for slab waveguides

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques course. In this module, we continue the discussion of slab waveguide and if you remember at the end of the previous module we wrote a condition called as transverse resonance .condition.

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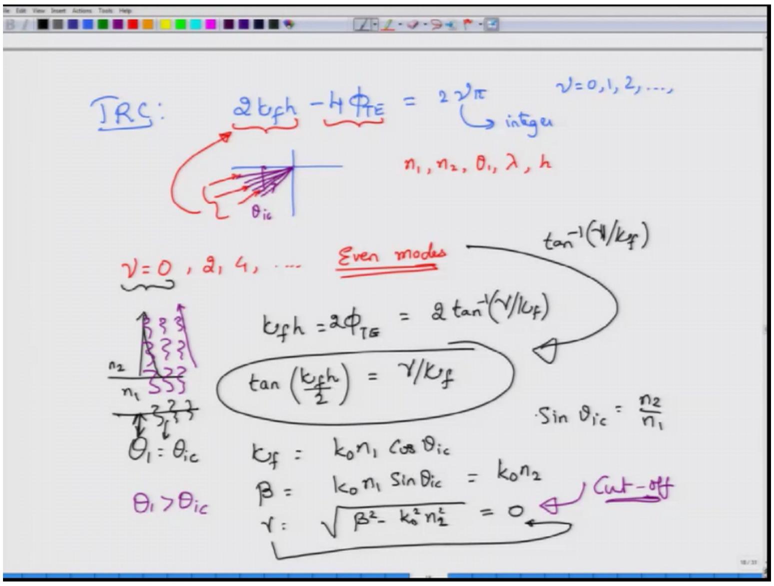

Where this transverse resonance condition is given by 2 kappa f times h minus 4 phi TE equals 2 times nu pi or other nu times pi where nu is an integer.

So, the value of nu is usually taken to be positive integers. So, it is nu equals 0, 1, 2 and so on. Each value of nu satisfies a condition and then gives the different value of incidence angle theta 1. So, it will not be a continuous value of theta 1 from say being right at the critical angle to all the way up to pi by 2 that we have.

So, not all these angles right launched at these angles will be guided by the waveguide. Although each of them will experience total internal reflection, but because they do not satisfy the transverse resonance condition many of those angles will actually experience what is called as destructive interference and eventually die out ok. Only those specific values of theta you know maybe this angle, this angle, this angle which satisfies the transverse resonance condition will survive. And this transverse resonance condition which we have written here relates all these parameter.

So, it relates n 1 which is the refractive index of the film n 2 the refractive index surrounding the film. The angle of incidence theta 1; the wavelength of light lambda and h because this h will be present is part of this one right the TRC. The sign of this 2 kappa f h is opposite to 4 phi TE and this point will be made clearer when we discuss the electromagnetic wave theory of slab waveguides or at least indicate the electromagnetic wave theory of the slab wave guide.

So, for now just remember that the phase shift that will be experienced along the x direction, which is captured by the first term will be opposite to the phase shift that is experienced by the reflected light or reflection phase upon reflection. But these 2 together should be equal to some integer multiple of 2 pi ok.

Now let us write down this equation slightly in different form; assuming condition that nu is equal to 0 ok. Or you could assume the condition that nu is equal to 0 2 4 and so on which we call as the even modes of the solution ok. It is even because the value of nu which is an integer is taken to be an even number ok. So, it is 0, 2, 4 and these can be easily simplified for the fundamental case of nu equal to 0.

So, when nu is equal to 0 the right hand side of TRC will be 0 and on the left hand side what I have is kappa f h is equal to 2 times phi TE, but I also know what is phi TE which is ratio of gamma to kappa f correct. So, this we have seen in the previous module.

So, this would be equal to 2 times tan gamma by kappa f this is not tan, but this is tan inverse. So, this is 2 times tan inverse of gamma by kappa f; so, put the tan on the other side after dividing both sides by 2. So, the condition that you would have is that tan kappa f h by 2 must be equal to gamma divided by kappa f ok.

So, this is the condition for the simplified condition for not only nu equal to 0 although we derived this one for the case of nu equal to 0, it is true for all even modes ok. So, for all even modes the resonance transverse resonance condition can be simplified by writing tan kappa f h by 2 equals gamma by kappa f.

Now let us consider what happens when theta 1 is equal to critical angle theta ic what happens to the value kappa f? Kappa f will be k 0; n 1, remember kappa f was actually k 0 n 1 cos theta ic right it was cos theta 1, but theta 1 is now equal to theta ic. And what about beta? Beta f equal to k 0 n 1; sin theta ic.

But I know that under the total internal reflection; the critical angle is actually given by n 2 by n 1 right, when n 1 is of course, greater than n 2. So, plugging in the value of n 1 sin theta c from this equation; you will see that beta is actually become k 0 n 2.

And gamma which is given by square root of beta square minus k 0 square n 2 square ok; it is n k 0 beta square minus k 0 square n 2 square actually becomes equal to 0. And gamma was the rate at which this evanescent mode was actually decaying in the medium which is outside the film right.

So, this was the direction perpendicular to the film and it is in that direction; the evanescent wave is actually decaying with an amplitude decay constant of gamma. So, the wave outside this was e power minus gamma x and it now sees that with gamma equal to 0 right at the critical angle when gamma is equal to 0; the evanescent wave does not really decay along the direction perpendicular to the film. But rather remains constant and of course, there is a propagating wave on the interface.

So, the there is a propagating wave along the interface here, which would not decay at; all it would remain constant ok. Only when we increase the angle theta 1 to a larger angle corresponding; I mean if I increase the angle beyond theta ic, then the propagation constant gamma will be non zero and it would then start to decay out the evanescent wave actually starts to decay.

So, this is barely at the position of what is called as a cut off. So, cut off conditions are always characterized by having this gamma, which is the decay constant outside the film. Of course, it is not only on the upper interface; a same thing happens in the lower interface as well right.

So, there is a lower interface or other there is a lower interface wave which is propagating which of course, would decay in the direction downward and perpendicular to the film right. So, whatever condition that we are talking about the upper interface is same condition that will also hold for the lower interface because we are considering a symmetric slab waveguide. So, cut off condition is when gamma actually becomes equal to 0 ok.

So, gamma is 0, kappa f is k 0 n 1 cos theta ic; you can either you know convert this sin theta ic into or rather extract what is cos theta ic from this equation or you can go to the more defining equation or the relationship between kappa f and beta.

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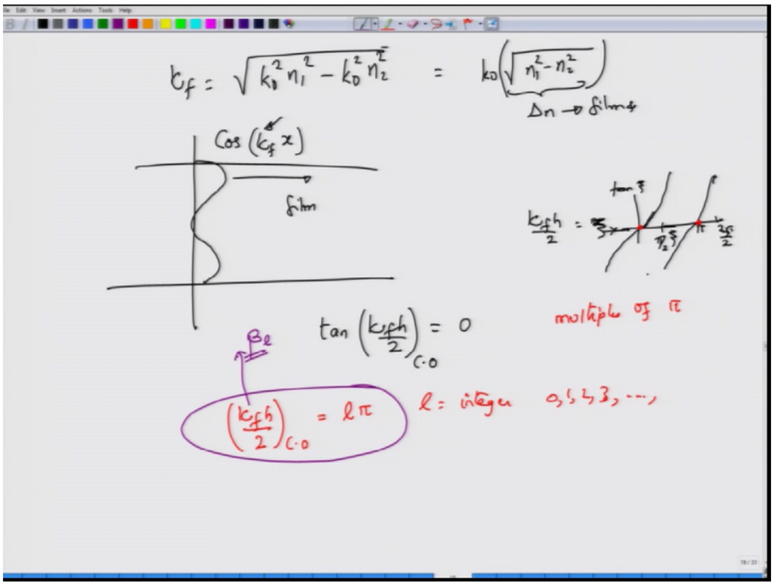

This is given by square root of k 0 square n 1 square minus beta square, beta at cutoff will be k 0 square n 2 square right.

And kappa f will therefore, be given by k 0 times square root of n 1 square minus n 2 square ok. This n 1 square minus n 2 square under root is sometimes denoted by delta n which indicates the refractive index difference between film and you know outside of the film.

So, this refractive index difference is directly proportional to or at least will be the square root of this refractive index difference or if I define the entire quantity of as a measure of refractive index difference will be directly proportional to kappa f ok.

And what is kappa f? Or why is kappa f important? Because kappa f corresponds to the way in which the waves are actually standing inside the film right. So, this is the film region and this is the pattern that would you would see transfers to the direction of propagation and it is this pattern which would actually propagate along the z direction.

So, that is why kappa f is actually important. So, on unreal value of kappa f means that this pattern would also be real and it would be this pattern which would propagate further. So, anyway; so, this is what happens at the interface and what we have just shown is that the condition for the cutoff for any particular mode would be that gamma must be equal to 0 ok.

Now if I substitute the cut off condition into the transverse resonance condition for even modes, which I have written out here what you would find is a very interesting expression. So, what I have is tan kappa f h by 2 will be equal to 0 ok.

I will write c o to denote that this is a cut off that we are considering. So, if you think of kappa f h by 2 as some variable x or maybe some other variables zeta and as zeta keeps on increasing from tan right. So, at I mean if you were to plot tan of seta with respect to zeta; this is how you would find.

So, at pi by 2 this would go off to infinity at 0 it would be 0 and then in the negative side at minus pi by 2 this would be going away to minus infinity. Again it would stats from infinity at this point right and then move through 0 at pi and then go off to infinity again at 2 pi something like that or 3 pi by 2 not 2 pi 3 pi by 2.

So, this is what we get when you plot tan zeta versus zeta and what this equation is asking you to do is that find out those values of zeta where this condition can be satisfied. So, if you were to go to that condition that condition is satisfied here, the condition is satisfied here and the condition is satisfied at other places.

So, it is in fact, satisfied at all odd multiples or maybe not odd at even multiples as well; so, at multiples of pi. So, this is this are the conditions for that kappa f h by 2; I mean tan of kappa f h by 2 being equal to 0 because it goes through 0 at all these multiples of pi.

So, remember tan is actually sin by cos; so wherever sin goes to 0 and cos is non 0 that is where tan actually goes to 0. Now, is this any helpful to us? So, let us look at this, so I have kappa f h by 2 undercut off being equal to some integer l times pi ok, where l is an integer and this integer is 0, 1, 2, 3 and so on and different modes that are possible with this one is also written out here ok. So, I am now looking at this condition and all I have found is that if I start giving different values of l; then I will actually obtain different values of kappa f. But getting kappa f is not really my goal, but from kappa f I need to extract the value of beta ok.

Because beta corresponding to that particular l will be the propagation constant, which is actually the important parameter for us;. Because that will tell us how the wave is actually propagating along the film. So, this cut off condition that we have written can be used to first find out what is kappa f; from kappa f you can find out what is the value of beta l ok; because they are actually related by related with respect to each other. So, this is the even condition that we have written and for all these values where this equation is satisfied you obtain a set of modes ok; so, where this is a integer.

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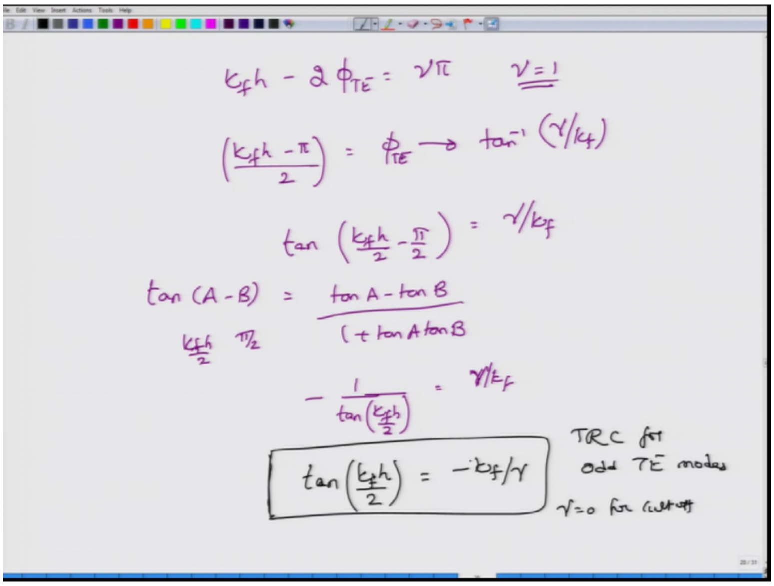

Now let us get back to the transverse resonance condition which we wrote earlier which is kappa f h; I believe the condition that we wrote here was 2 kappa f h minus 4 phi TE is equal to 2, nu times pi. So, let us get back to this condition and then write this way phi TE is equal to nu h after dividing a sorry nu pi.

So, this is nu pi of and now we consider the case where nu is odd. So, for example, if I consider the simplest case of nu equal to 1; then what will happen to this expression well I will have kappa f h minus pi; the entire thing divided by 2 is equal to phi TE, but I know that phi TE is tan inverse of gamma by kappa f. So, putting all this condition together I get tan of kappa f h by 2 minus pi by 2 is equal to gamma by kappa f.

Now you can use an equation which says tan of A minus B. So, simplify this expression and when you know if you do not remember that this is the expression tan of A minus B is equal to tan A minus tan B divided by one plus tan A times tan B with A equals kappa f h by 2 and B equals pi by 2.

What you get here would be minus 1 by tan kappa f h by 2 which would be equal to gamma by kappa f. So, this is gamma and you know interchanging all this you will actually end up with another equation; this time valid for odd modes of this waveguide that we have considered, which is tan kappa f h by 2 equals minus kappa f by gamma.

So, this is a condition that is valid for odd modes; odd TE modes because we have been considering only TE waves. For TM waves you can derive similar expressions, but they will be slightly different. So, there will be some n 1 square n 2 square also multiplying this tan inverse thing ok. So, that I will leave it as an exercise for you, but look at this expression again right now the cut off condition remains the same.

So, when I am considering the odd TE modes the cut off condition again means that gamma has to be equal to 0 for cut off. But the moment I said gamma equal to 0 while kappa f being a finite quantity, the right hand side of this fellow becomes equal to infinity right. And when will kappa tan of x go to infinity? When x goes to nor multiple of pi by 2.

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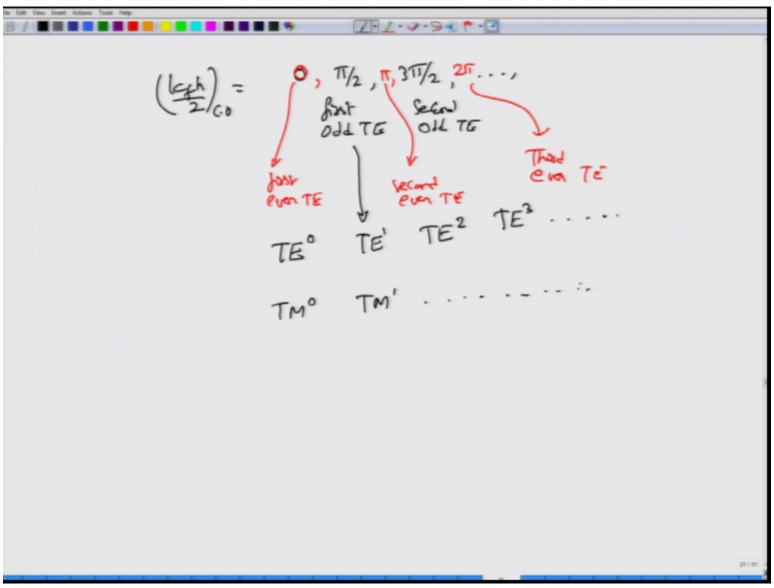

So, the first time it happens is at pi by 2 then there will be another thing that happens at 3 pi by 2 and so on. So, these are the conditions; so, this will be the first odd TE mode, this will be the second odd TE mode. Somewhere here you had 0 pi; so let me indicate that one with different color. So, this is 0 pi and then you had; so after 3 by 2 you will have 2 pi and so on.

So, this would be the first even TE mode, this would be the second even TE mode and this would be the third even TE mode and so on ok. So, these are the even modes for which kappa f h must equals when kappa f h by 2 under the cutoff equals 0; then you have the first TE mode even TE mode represent that one as TE 0 mode.

Then you have the first odd TE mode which you will write it as TE 1, then you have a second TE even TE mode which is TE 2 and so on ok.

Similarly, you will also have TM 0 TM 1 and so on based again on the values of kappa f h by 2.

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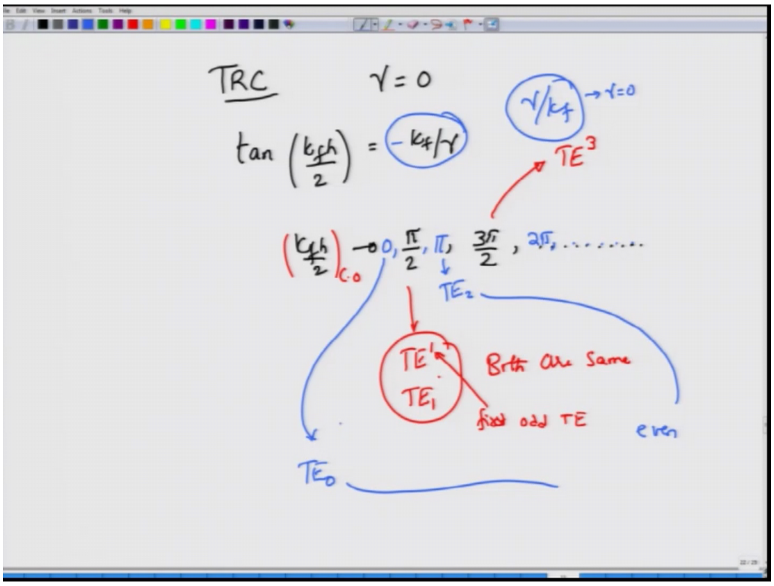

So, if you look at the transverse resonance condition that we derived for the odd mode and for the even mode and for the odd mode it is a reasonably simple way to derive that one which we will do it later. So for the transverse resonance condition if you look at that and then demand that for cut off; since gamma must be equal to 0 for cut off what are the different values of kappa f and hence what is the different values of beta that you would find?

You will be solving the equation remember the transverse resonance condition if you go back and then look at it would look something like this; this is for the even TE modes that we are considered. A similar equation of course, we will have to be written for the transverse magnetic modes as well or the TM modes as well. And what you see here is that the left hand side is a periodic function and since gamma is equal to 0 in this particular right hand side 1 by 0 will be quite a large quantity; in fact, it would be infinity.

And it does so, when this kappa f h by 2 goes through different values right. So, when kappa f h by 2 goes to pi by 2 then you will find out that this is the first solution that is available for us. So, sorry this is a transverse resonance condition for the odd modes that we have written. So, for the even modes of course, is the other way round.

And what you see is that as it goes from say pi by 2 then you have a 3 pi by 2 and so on there an infinite number of such solutions. The first solution that you will get will be the one that we will call as TE 1 solution; sometimes I will use a superscript sometimes, I will use subscript both essentially mean the same. So, do not worry about whether I have used a subscript or a superscript.

In this module both are essentially same mode; they are representing the same mode. And what you find is that this is the first odd mode odd because the number here is 1. So, this is your first odd TE mode ok, the next odd TE mode also will be obtained when you when your argument kappa f h at the cut off condition will increase beyond pi by 2 and actually go to 3 pi by 2.

So, what you get here will be the next odd mode which of course, will be TE 3. Now you might also have guessed that for the even mode condition instead of them being at pi by 2 3 pi by 2 when this left hand side goes through 0 right and for that one for the TE for the even modes you will have something like gamma by kappa f right. So, for the odd modes I think there is a minus signs.

So, do not worry about that one, but because at gamma equal to 0 the right hand side any goes off to infinity, but for the even modes the right hand side goes to 0 because gamma will be equal to 0 causes the right hand side to go to 0. And once that goes to 0 then you have different or in fact, you have an infinite number of solutions.

The first solution occurring at 0 which we will allow to be a valid solution, then you have solution at pi, then you have a solution at say after 3 pi by 2 you will have a solution at 2 pi and so on and so forth right. So the solution corresponding to 0 will be written as TE 0 mode, the solution corresponding to pi will be written as TE 2 mode.

And these are the even modes even because the numbers 0 and 2 and 4 and so on will correspond to even numbers. What is even more interesting is that once you solve this transverse resonance condition and then find out an appropriate value of kappa f and plot on the x axis a value of beta of course, can be obtained from kappa f.

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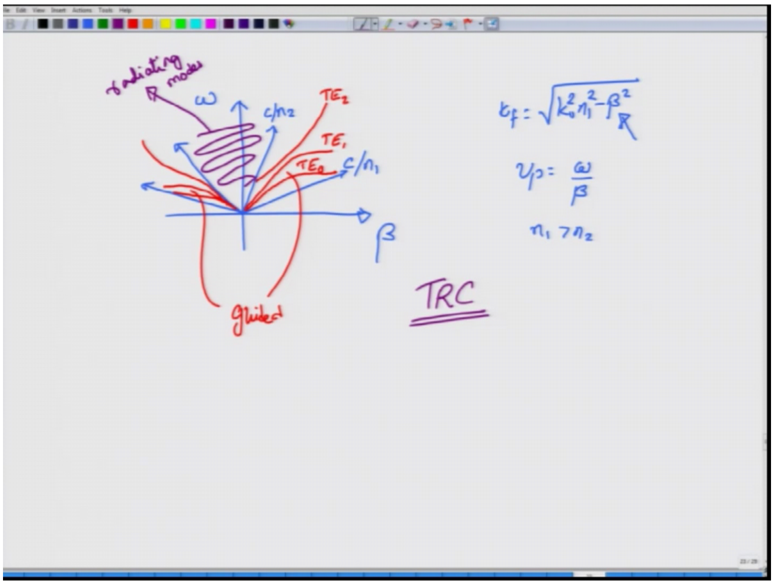

Remember that kappa f is given by k 0 square n 1 square minus beta square. So, once you know what is kappa f, then you can find out what is beta. And then once you solve the transverse resonance condition and then do so and plot for different frequencies what are the different allowed values of beta.

You will see that this omega beta diagram which is called as the dispersion diagram will be bounded by 2 lines. One line will have a slope of say c by n 2 and the other one will have a slope of c by n 1. So, what is the slope of c by n 1 and c by n 2? Remember that the phase velocity is given by the ratio of omega by beta and for the modes which are propagating in the slab and if the material were to be only slab; then the phase velocity would be c by n 1.

For the material if it was to be only cladding the slope would be c by n 2 and of course, n 1 is greater than n 2 there for the slope of c by n 1 is smaller compared to the slope of c by n 2. And then once you start; so, this is of course, is the material were to be made out of anisotropic climbing I mean just a homogeneous cladding or core separately, but what you have in slab waveguide which means that the value of beta will not be equal to c by n 1 or be equal to c by n 2, but it will depend on the value of kappa f via this particular equation.

And as you start looking at the first mode it will start at 0, but then eventually goes off to c by n 1. So, as omega increases the phase velocity of this fundamental mode starts to approach c by n 1. Then you will also see that the next mod will also do something like that, the other mode will do a different kind of a curve. So, you can find out how these modes actually you know a vary with respect to omega as you change the value of beta or alternatively as you are vary beta with respect to omega.

So, what you observe is that all this red lines are between the 2 blue lines which kind of gives you the bounding. Again if you were to consider backward propagating waves and then look at the negative values of beta; you will find very similar lines for the fundamental the next order mode and then the other higher order modes. If you have modes which are guided then guiding can only happen between these regions, this is called as the guided region.

And the region where the modes do not take up any value of omega or any value of the combination of omega and beta in the region in between; outside this cone like structure is where the modes are actually radiating. So, these are called as radiating modes; so, this is the region where we have guided modes, this is the region where you have radiating modes. And all this information can be obtained by solving the corresponding transverse resonance condition. Of course, the exact way of solving this transverse resonance condition is something that you will be seeing in the later modules.

Thank you very much.