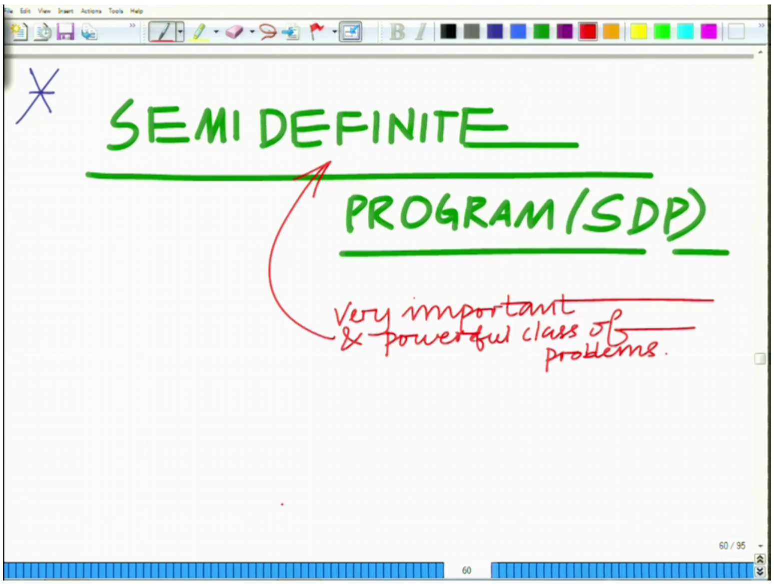
Applied Optimization for Wireless, Machine Learning, Big Data
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 75
Semi Definite Program (SDP) and its application: MIMO symbol vector decoding

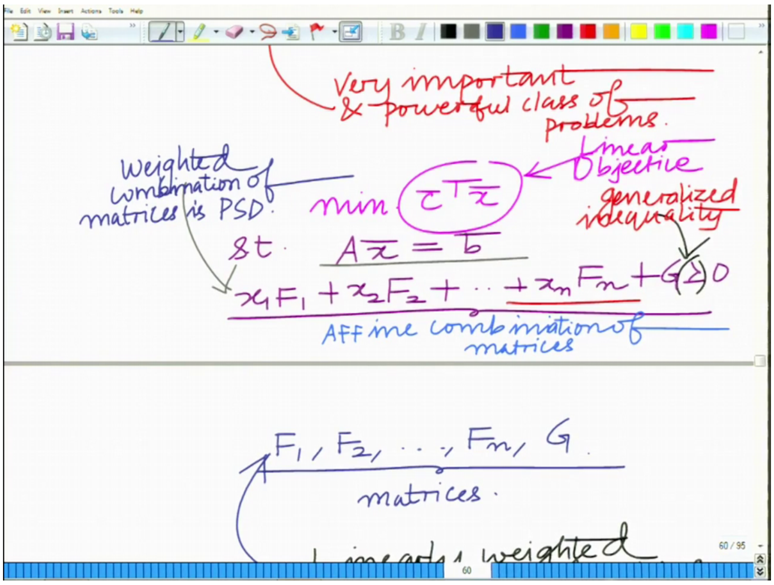
Hello. Welcome to another module in this massive open online course. So, we are looking at convex optimization, various problems and their applications. Let us start looking in this module at different class of problems and which is very interesting that is the same time very powerful and also understand the various applications. This is known as Semi Definite Programming.

(Refer Slide Time: 00:43)



So, it is a slightly more advanced and as I said a very useful class of course. So, this is known as Semi - Definite Program, simply termed as an SDP for Semi - Definite Program and as I said this is a very important and powerful, very interesting as well as and very important and powerful class of problems. And what is the semi definite program?

(Refer Slide Time: 01:51)

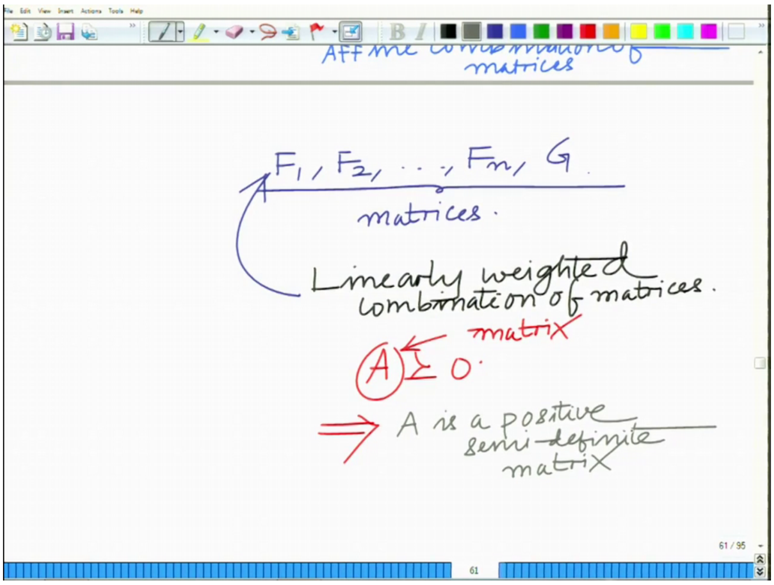


Put simply a semi definite program is the following, where you are minimizing a seemingly simple objective that is objective was still a linear objective.

So, you have a linear objective. However, the constraint is something interesting. The constraints are well, first let me write the equality constraint that is A x bar equal to your again this is something that is very similar. But the inequality constraint is something very interesting, you have x 1 F 1 plus x 2 F 2 that is a linear combination of a weighted combination of matrices x n or rather an affine combination of it x n F n plus G.

These are matrices this is less than or equal to 0 or you can also say this is greater than or equal to 0. So, you have x 1 F 1 plus x 2 F 2 x n F n and so on. So, what is this? This is a weighted combination ok; linear on affine combination that is you are combining.

(Refer Slide Time: 03:25)



These matrices F 1, F 2, F n comma G; so these are matrices ok,these are matrices and what you are doing is you are performing a weighted combination, a linearly weighted combination of these matrices ok. You are performing a linearly weighted linearly weighted, linearly weighted combination of these you are performing a linearly weighted combination of these matrices and at the same time ok.

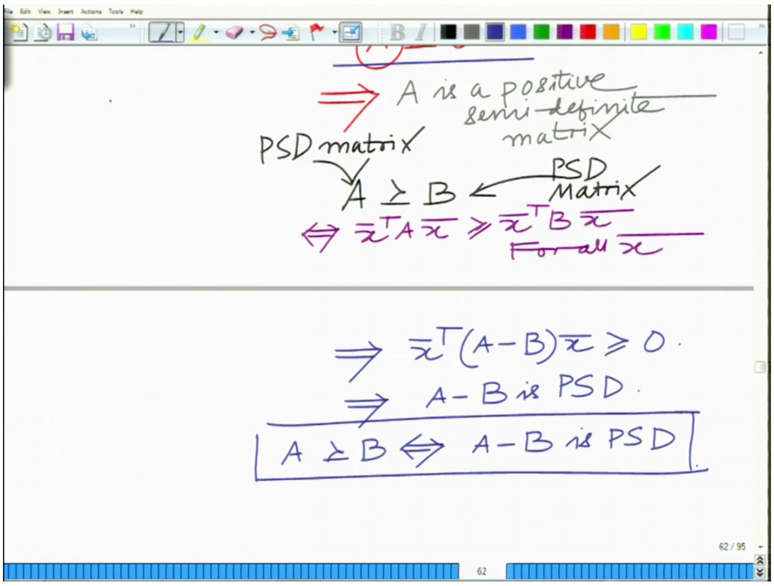
So, you are performing linearly weighted combination of this basis, and also if you look at this inequality here, this is a generalized inequality. You can see what this means now? This is very interesting because what we have on the left is a matrix ok. We are saying this matrix has to be greater than equal to 0 which implies that this matrix has to be positive semi definite.

So, this inequality for a matrix where A is a matrix, this implies that A has to be a positive semi definite matrix which in itself is an interesting, because now we are defining an ordering or an inequality on the set of matrices which is rather unusual because typically you cannot compare 2 matrices.

So, this inequality this generalized inequality basically specifies that the matrix A has to be positive semi definite. So, this optimization problem the first constraint is an equality constraint. The second inequality constraint is a generalized inequality.

What it says is this, weighted combination of matrices; this says that this weighted combination of matrices has to be positive; this weighted combination of matrices has to be positive semi definite ok. So, this has to be positive semi definite, alright.

(Refer Slide Time: 06:50)

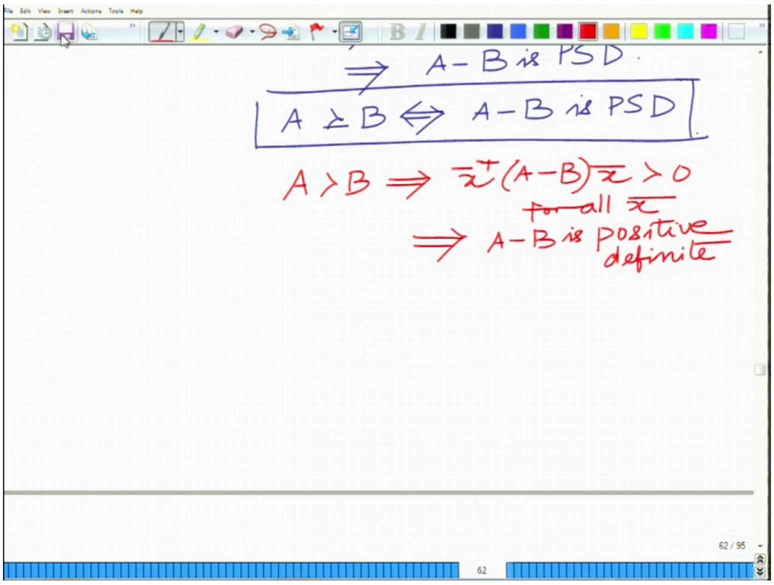


And in fact, this inequality itself this generalized inequality of the set of positive semi definite matrices is interesting. Let me just briefly describe this. We say 2 positive semi definite matrices that is A is greater than equal to B that is where A is a positive semi definite matrix, this is a PSD matrix and this is also a PSD matrix. We say a is greater than equal to B if and only if x bar transpose A x bar is greater than or equal to x bar transpose B x bar for all x bar.

So, if any vector x bar, x bar transpose ax bar remember for positive semi definite matrix x bar transpose A x bar has to be always greater than or equal to 0. So, if x bar transpose x bar is always greater than equal to x bar transpose B x bar. Then, we say that the matrix A is greater than equal to matrix B and note that this also implies that x bar transpose A minus B x bar is greater than equal to 0.

So, this implies that A minus B is positive semi definite. So, A greater than or equal to B implies that A minus B. In fact, you can put this as implied as in is implied by A minus B is positive semi definite.

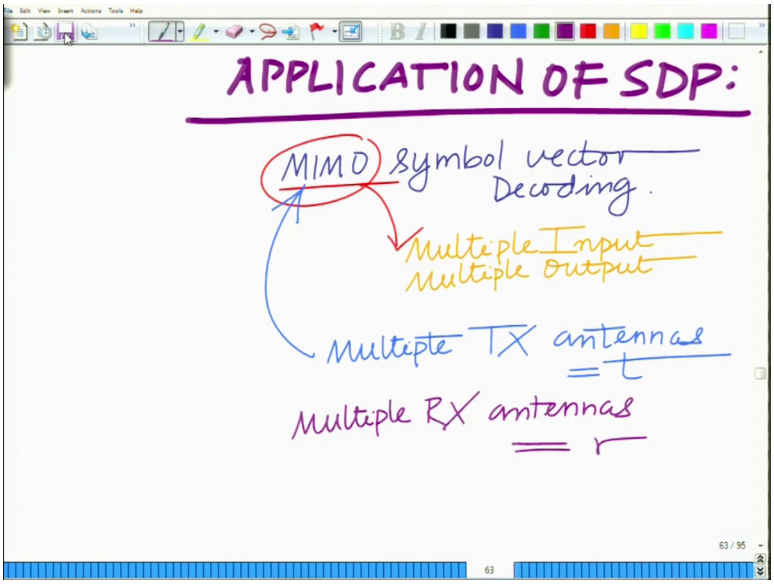
(Refer Slide Time: 08:45)



This is another way of defining the same thing ok. So, A minus B is positive semi definite. Now, similarly for positive if you have a strict inequality A greater than B, naturally this implies x bar transpose A minus B x bar is greater than 0 for all x bar this implies A minus B is positive is positive definite with this implies A minus B is positive definite.

So, my A is strictly greater than B, if A minus B is positive; definite A is greater than equal to B, if A minus B is positive semi definite. This is the notion of this generalized inequality on the set of positive semi definite matrices ok. Now, let us look at an interesting application.

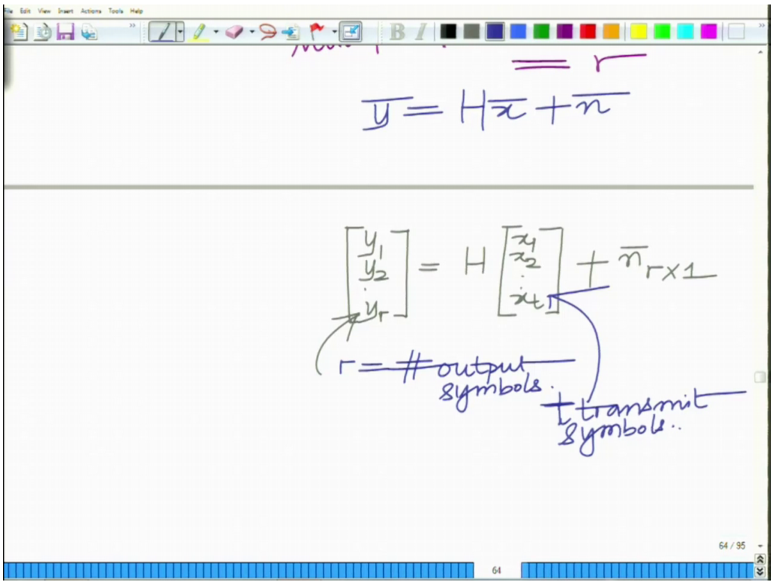
(Refer Slide Time: 09:43)



For this semi definite programming, this has as I said its very interesting its powerful it has several applications. So, let us look at an application of SDP and the application is as follows. Again let us go to our MIMO wireless system. So, let us consider a MIMO system and we want to perform MIMO, we want to perform MIMO symbol decoding as we know as you well know by this point of time that MIMO stands for Multiple Input is something that you are all very familiar with.

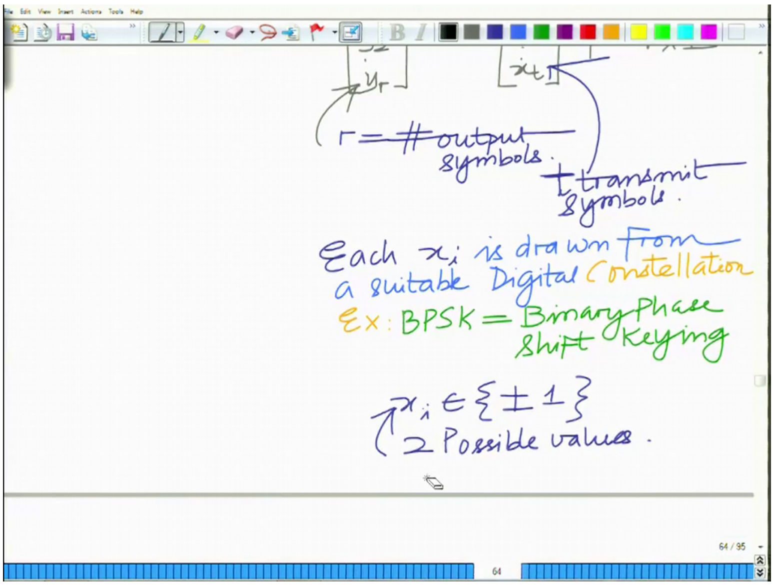
Multiple Output communication system which is very popular in the context of wireless to achieve high data rates ok. And what do you mean by Multiple Inputs? Multiple Inputs means multiple transmit antennas. Multiple Output means the multiple receive antennas. So, in this MIMO system, you have multiple TX antennas and you have also multiple RX equals r.

(Refer Slide Time: 11:46)



And I can represent this MIMO system model as y bar equals H x bar plus n bar, where you have this vector of received symbols y 1, y 2, y r equals H times x 1, x 2, x t. These are t transmitted symbols plus n bar which is an r cross 1 vector ok. Now remember you have r output symbols on the r antennas and you have t transmit symbols on the t transmitted antenna.

(Refer Slide Time: 12:49)

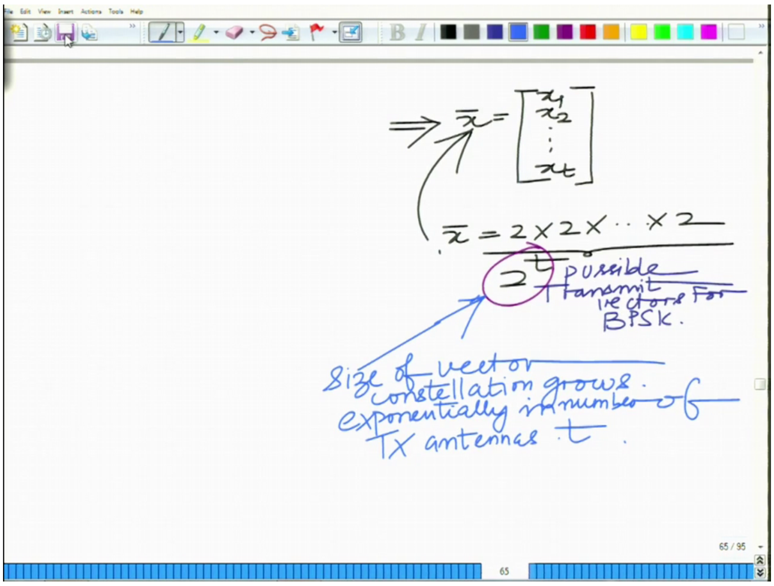


Now, the point is each of these symbols right, if you look at each x i in the digital communication system, each symbol x i has to be drawn from a constellation that is it cannot take any possible value. For instance: if it is BPSK it has to be plus or minus 1 if its QPSK it is plus or minus 1 plus or minus j that is you can have only 4 possible symbols 1 plus j, 1 minus j, minus 1 minus j, minus 1 plus j.

So, each x i is you have to draw it is drawn from a suitable digital constellation for a digital wireless system. Example, BPSK: this is basically your binary phase shift keying which implies there are 2 phases as you are seeing this is a binary phase shift keying, this implies each x i belongs to plus or minus 1 ok. So, which means each x i can be plus or minus 1.

Now that means, each x i has 2 possible values ok. So, each x i can be plus or minus 1. Now, that means, each x i has 2 possible values ok. So, each x i can be plus or minus so on that means, each xi has 2 possible values implies if you look at the symbol vector x bar which has x 1, x 2, x 2 up to x t.

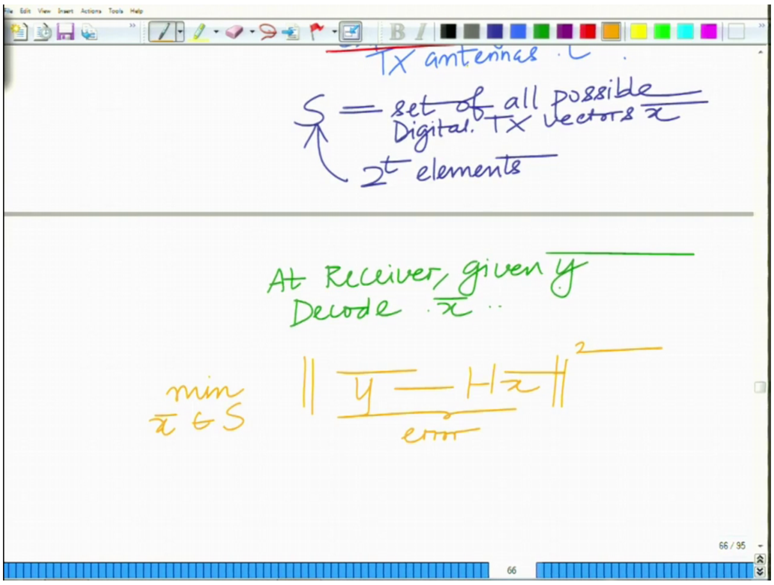
(Refer Slide Time: 14:43)



So, if you look at x bar which has x 1, x 2, x t each has 2 possible values implies x bar 2 times 2 times so on up to 2 which is 2 to the possible t transmit vectors for BPSK. So, basically now you observe something interesting, the size of this set; the set of the vector constellation like corresponding to each digital constellation right of x i which is for which for instance belongs to BPSK.

You have a corresponding vector constellation way to which these vectors x bar belong and that is of the size 2 raise to the power of t which is growing exponentially in the number of transmit antennas t that is the problem. So, this is size of the vector constellation in the number of transmit antennas t.

(Refer Slide Time: 16:42)



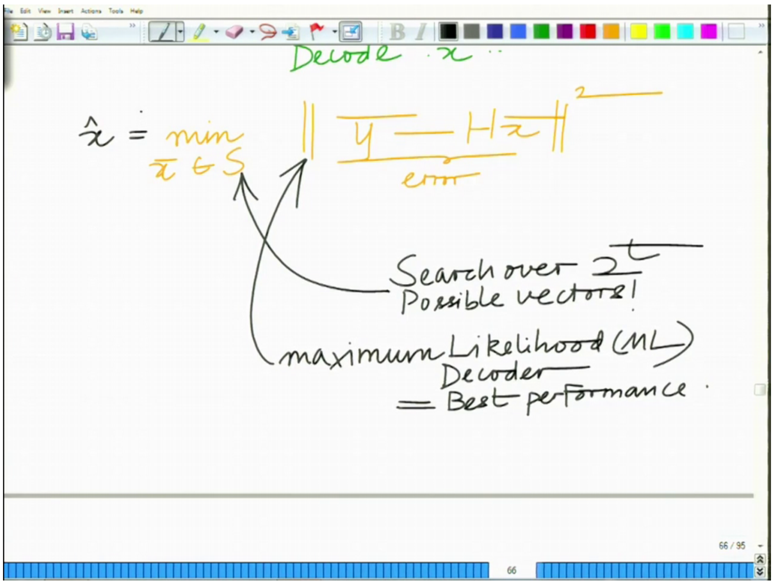
And therefore, what is the problem? So, this is growing exponentially ok. Now what is the problem? So, where is the problem? The problem is ok. So, now, let us say we have the set S. S equal to set of all possible transmit vectors or possible digital transmit vectors; set of all possible digital transmit vectors x bar and now this sector this set has 2 to the power of t elements and therefore, now what is the problem? The problem is at the receiver once you receive y correct, we have to find the x bar that has been transmitted, the vector x bar that has been transmitted like ok.

At receiver given y bar, we have to find x bar that is our estimate or estimate is not a good one. Let us say we have to decode x bar, find x bar or let us say one has to decode. So therefore, now the typical decoder that you use or the best possible decoder is what is known as the ML decoder that is you look at this error y minus H x bar.

This is the error, you look at and the norm square of the error and you minimize the norm square of the error, you find the vector x bar which minimizes the norm square of the error that is known as Argument ok. Or you find the vector x bar which minimizes the norm square of the error; but the problem is now this x bar must belong to this one of this possible 2 raise to the power of t vector. That is this x bar must belong to this set S or which denotes a transmit vector constellation.

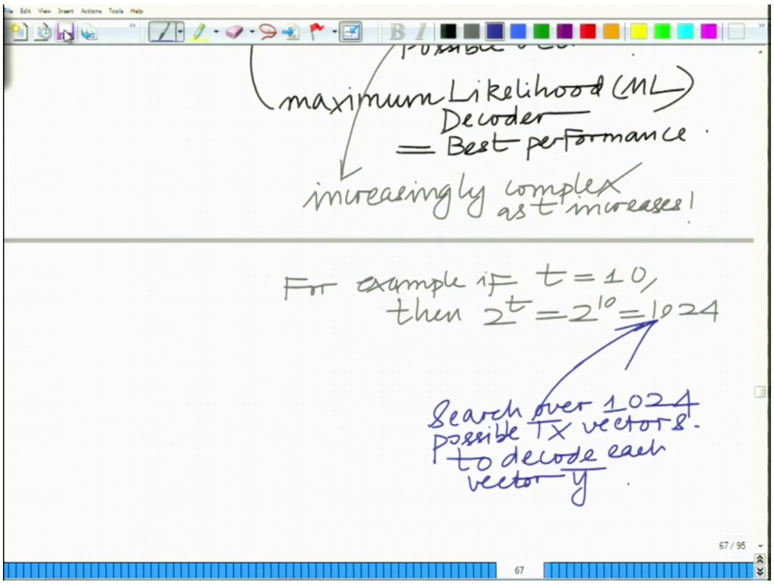
So, you have to minimize this with x bar belonging to S and now them can see the problem.

(Refer Slide Time: 19:10)



The problem is you have to search over all t possible vectors, search over 2 to the power of t possible vectors that is a problem ok to find x hat which basically minimizes this. Now this decoder this has a name, this is known as the maximum likelihood decoder which is the best performance ok.

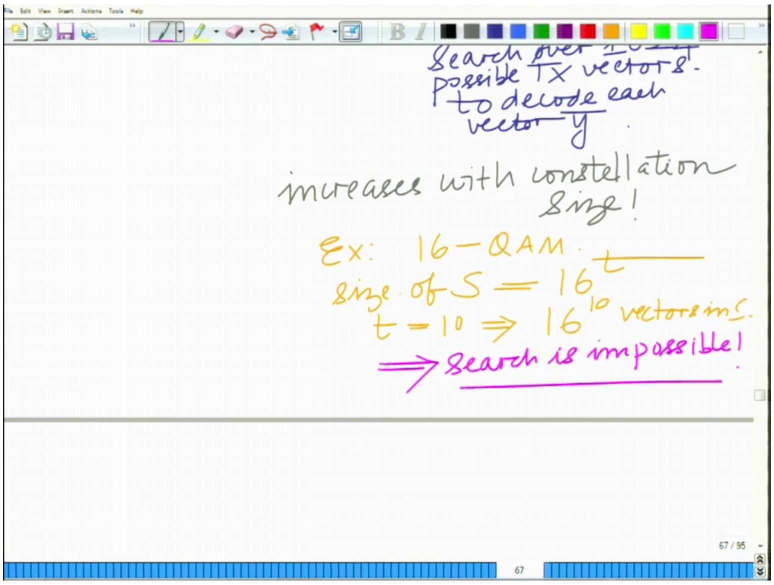
(Refer Slide Time: 20:07)



Now, you want to apply the maximum likelihood decoder, but you have to perform search over 2 raise to the power of t vectors which is increasingly complex as t increases ok. For example, if t equal to 10, then 2 raise to the power of t equals to raise to the power of 2 equals 1024. So, in fact that you have to search for over a set of 1024 possible transmitted symbol vectors for each received vector right.

So, you have to search over was able TX vectors to decode each vector y to decode each vector and further this observe that this increases with the constellation. For instance: if you have 16 QAM, then the number of vectors becomes 16 to the power of 16 raise to the power of 10.

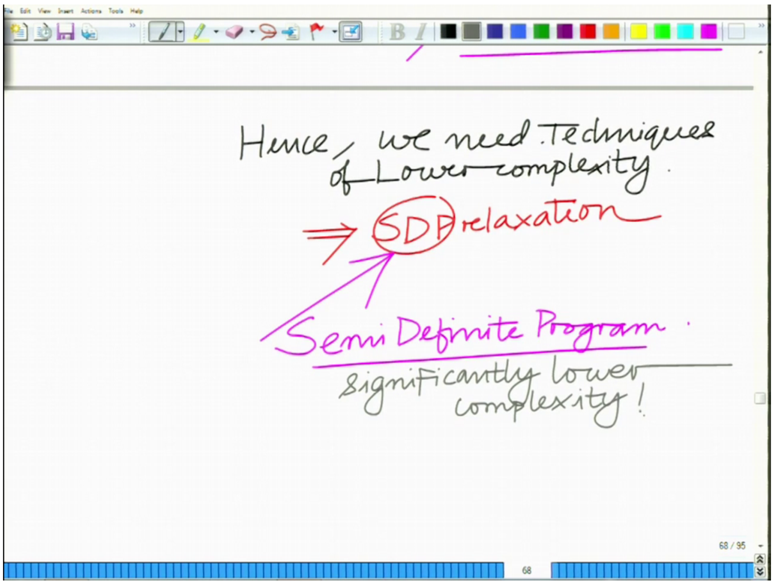
(Refer Slide Time: 21:44)



For example ad this increases with the number of vectors example this increases with the size of A constellation size example for 16 QAM and this becomes size of S that is a transmit vector constellation equals sixteen power t.

Now t equal to 10 implies you have 16 to the power of 10 vectors in H over which you have to search and this is impossible. Implies the search is impossible or next to impossible. Implies the search is impossible ok. And therefore, we have to come up with low complexity. Therefore, what we have to do? We have to come up with; hence, we need to come up with low complexity techniques.

(Refer Slide Time: 22:58)

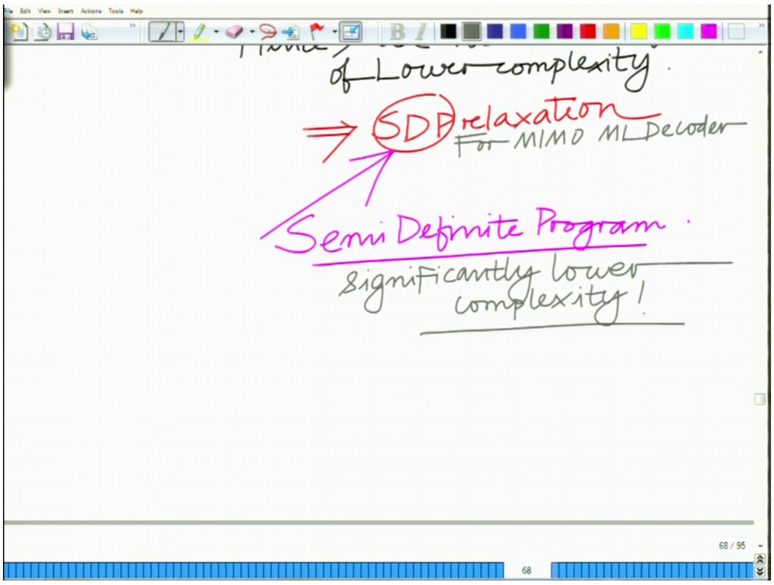


Hence, we need and one such technique is basically what is termed as SDP relax. We have seen what is SDP already. SDP, Semi - Definite Program. So, one such technique is SDP relaxation, we relax it relax this ML decoder problem as a Semi - Definite Program ok. So, we can perform we can formulate this problem as a semi definite which we are going to see subsequently shortly.

We are going to formulate this MIMO ML decoder as a semi Semi - Definite Program which has a significantly lower complexity and that is the important thing this is a. So, this is the SDP relaxation for ML decoding or for I would say rather MIMO ML; this is for the MIMO ML decoder which has a significantly lower complexity and this is a very interesting.

So therefore, this is you very useful for practical implementation.

(Refer Slide Time: 24:22)



So, this is significantly lower complexity in comparison to the one the full ML decoder. The optimal ML decoder which is exponential complexity which is virtually impossible for a large number of transmit antennas and large constellation sizes. So, we perform an SDP formulate this is an SDP semi definite program which can be solved very efficiently using modern convex solvers. And therefore, the resulting ML decoder or what we can say that it is an approximate ML decoder has a significantly lower complexity.

So, we will stop here. And this SDP relaxation and how is SDP exactly used for ML decoding what is a procedure for that we will look at it in the subsequent module.

Thank you very much.