Applied Optimization for Wireless, Machine Learning, Big Data
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Lecture – 55
Practical Application : Compressive Sensing

Hello. Welcome to another module in this massive open online course, in this module let us start looking at another new in fact revolutionary and path breaking development or technology and that is a Compressive Sensing alright; which is really revolutionized signal processing.

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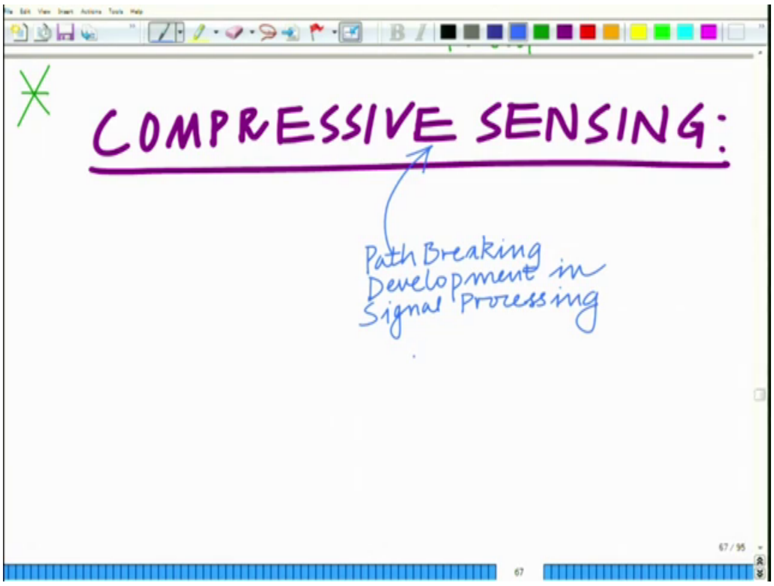

So what we want to look at is an overview of compressive sensing or also known as compressed sensing and it is relation to the optimization framework that we have looked at so far.

So this is, it is a latest and very significant path breaking development or path breaking technology in signal processing and it is applications are everywhere even in communication, biomedical signal processing so on in many domains. And to understand compressive sensing let us start by considering a signal x bar which is unknown.

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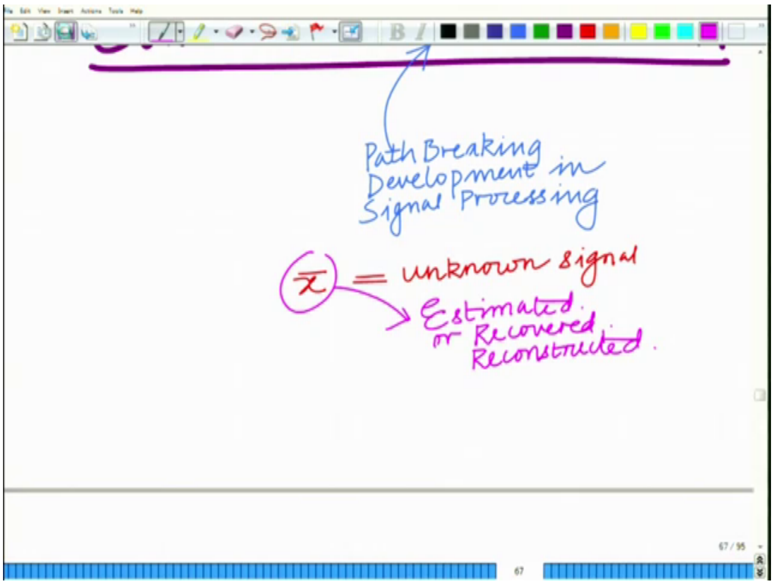

X bar is an unknown signal and this has to be therefore either estimated; since this has to be unknown either estimated or you can also say recovered or reconstructed there several words for the same. If it is an image there we say the image has to be the unknown, image has to be the original image has to be reconstructed ok.

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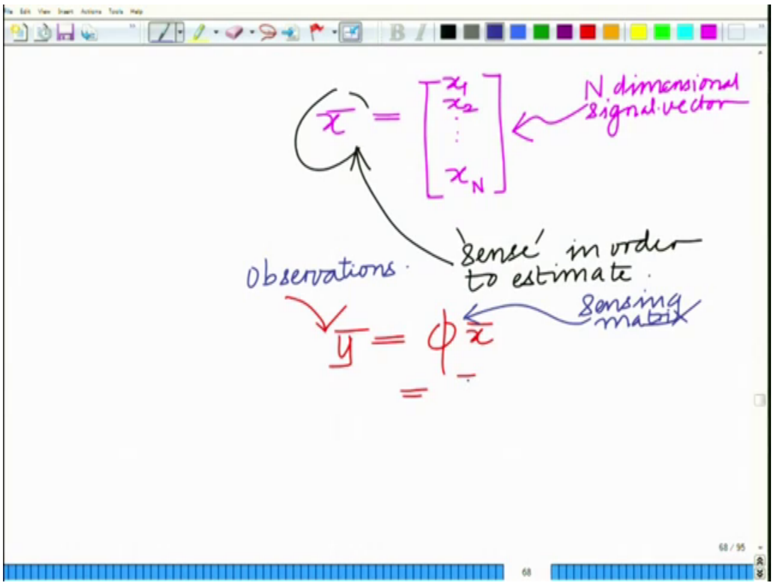

So, let us say this x bar is an n dimensional signal vector it is a signal vector which is x 1 x 2 up to x N this is an N dimensional signal vector and how do we estimate this N dimensional signal vector naturally we have to make some measurements. So, we make measurements for the signal vector this unknown signal vector x bar in order to estimate the signal vector x bar. So, that basically we are sensing the signal vector ok, so that is part of this compressive sensing.

So, we have to sense this in order to estimate. And therefore, what we have is we have this y bar equals phi times x bar. So, we are measuring or we are this is your observation vector or your sense. So, you are sensing this vector and these are your observations y bar is your and this becomes your sensing matrix.

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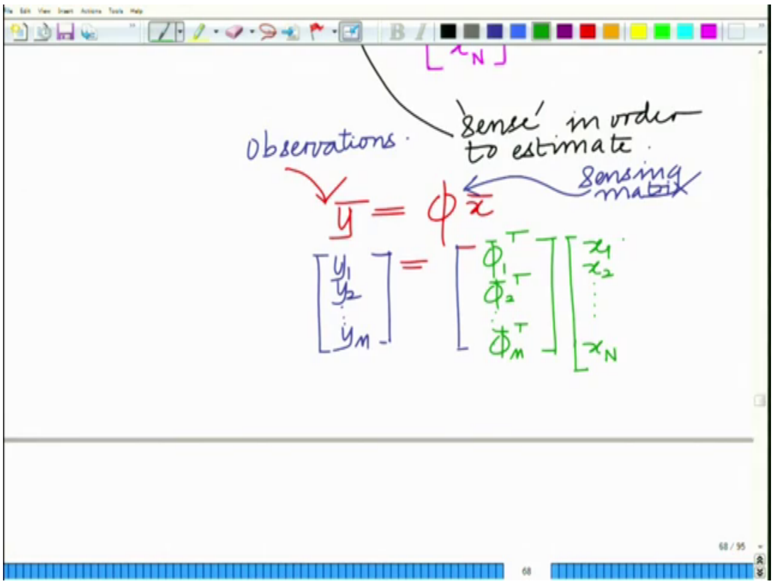

And this can be written as let us say we are making M observations so we have y 1 y 2 y M this is equal to phi 1 bar transpose phi 2 bar transpose so on phi M bar transpose times your let us write this dimensions suitably so that it is clear.

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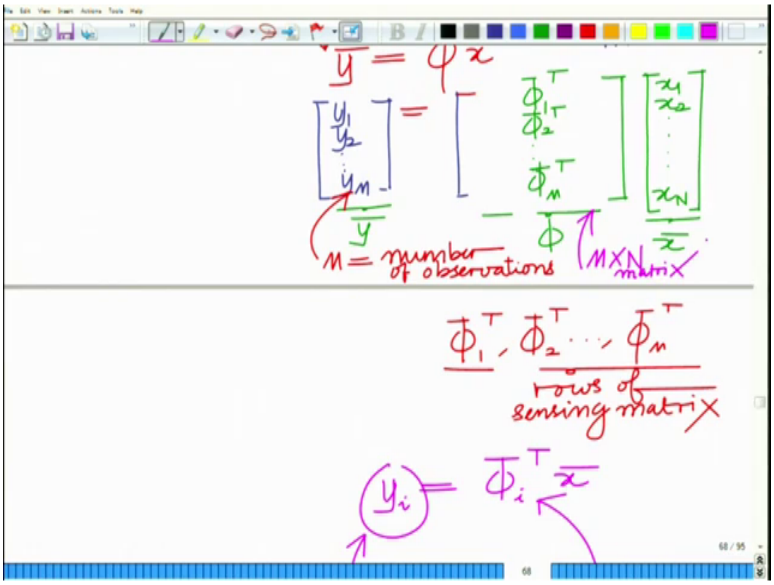

So, this is phi 1 bar transpose; the different rows are phi 1 bar transpose phi 2 bar transpose phi M bar transpose because this is M rows and this is the vector that is being sensed.

So, this is your matrix phi, this is x bar this is the observation vector y bar, so M is the number of observations; this quantity M equals number of observations. And x 1 x 2; and now these phi 1 bar phi 2 bar or if you call them phi 1 bar transpose phi 2 bar transpose phi M bar transpose these are the rows these are the rows of the sensing matrix.

And therefore now so what we order we doing we are making these M observations y 1 y 2 y M through this sensing matrix phi alright. So, each observation you can think of it as a projection of this vector x bar. So, what you are doing is your forming phi 1 bar transpose into x bar that is one observation phi 2 bar transpose x bar, so each of these observation is a projection of this vector x bar on a row of this sensing matrix phi.

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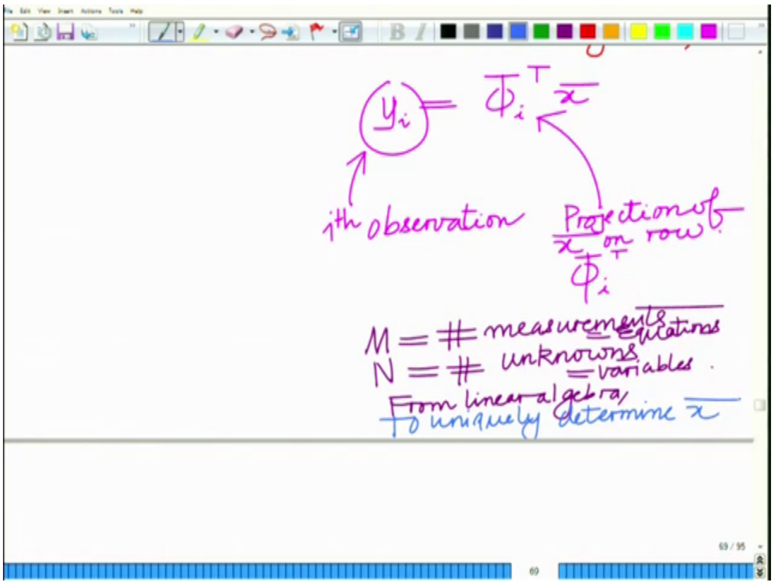

So, if you look at this you have and you can clearly see this you have y i that is i-th observation equals phi i bar transpose x bar. So, this is i-th observation; this is projection of x bar on row phi i bar or phi i bar transpose. So, each measurement is basically projection of the signal vector x bar on the made on a row of the matrix phi.

And now since there are M observations and N unknowns this matrix phi is an M cross N matrix this is an M cross N matrix and naturally if you look at this we are trying to estimate the signal vector x bar number of observations is y number of observations is M. So, you can say M is the number of observations or measurements, so M is the number of and N equals N equals the number of unknowns. And therefore, this is a system of equations with M observations or M equations M is the number of observations measurements or M is the number of equations and N is the number of unknowns.

And from linear algebra we know that in order to recover x which is vector of size N you need at least N equations to uniquely determine x. So, M is the number of measurements which is the same as the number of equations N is the number of unknowns or the variables from linear algebra; we know to uniquely determine x bar we need M greater than or equal to N.

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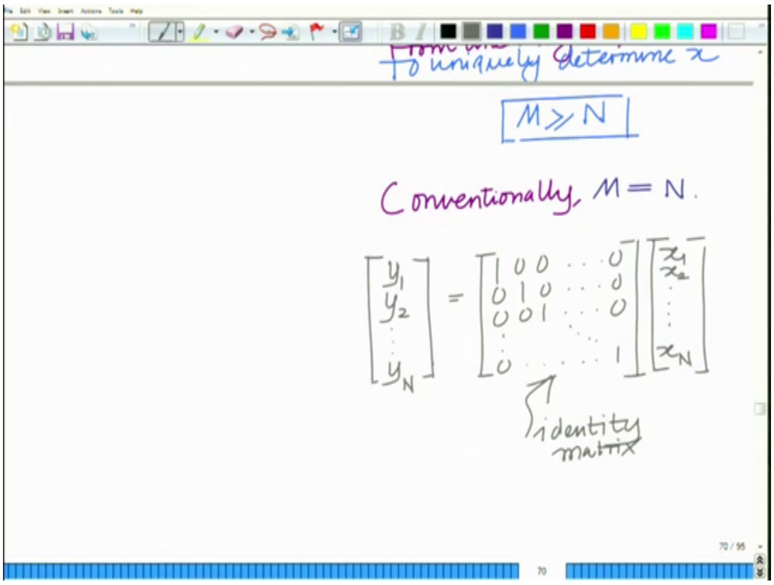

This is what we need and conventionally in conventional framework we have M equal to N ok. And how do we do that if you look at that that is we have these N observations y 1 y 2 up to y N which is simply the identity matrix. So, typically what you have is you are simply sampling the signal that is you are simply for instance it is your time domain signal you are simply sampling the signal. At each instant you are taking a sample or let us say your image you are sensing each pixel via a sensor.

So, for a signal with N samples alright you are making N measurements this is typically what we do we make we have a signal of length N we make N. So, for instance signals in time domain we make N different at N different instance we sample the signal at these N different instants N measurements and from those measurements we recover the signal.

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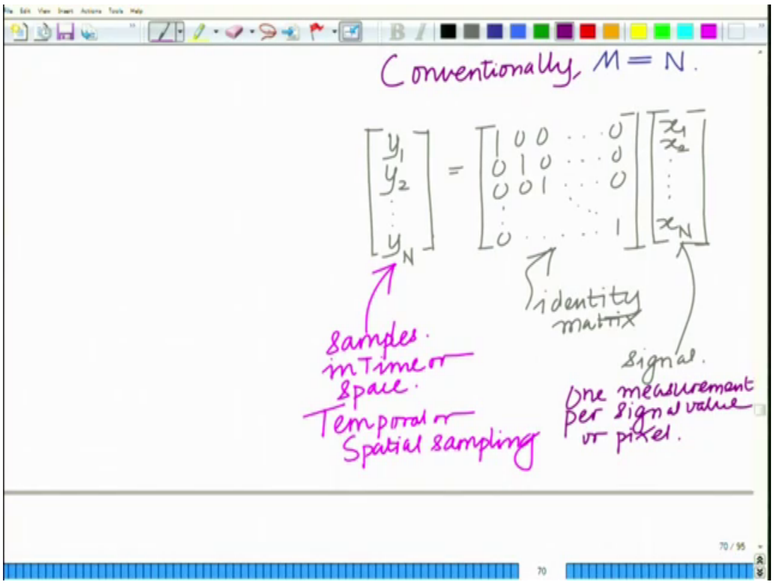

So, these are the samples, so you can say these are this is the signal and these are the samples in time or space for instance for an image these can be in space. So, these are sampling, so this is time or temporal or spatial sampling you can say you can say this is this is temporal or spatial sampling this is temporal or spatial sampling.

And we are making one sample per one measurement per sample or signals one measurement of per sample or signal value. And therefore, to uniquely determine is we said we uniquely determine the signal with N samples we need at least M greater than equal to N we can choose M equal to N. Now however, consider the following thing let us take a simple example consider a typical image for instance.

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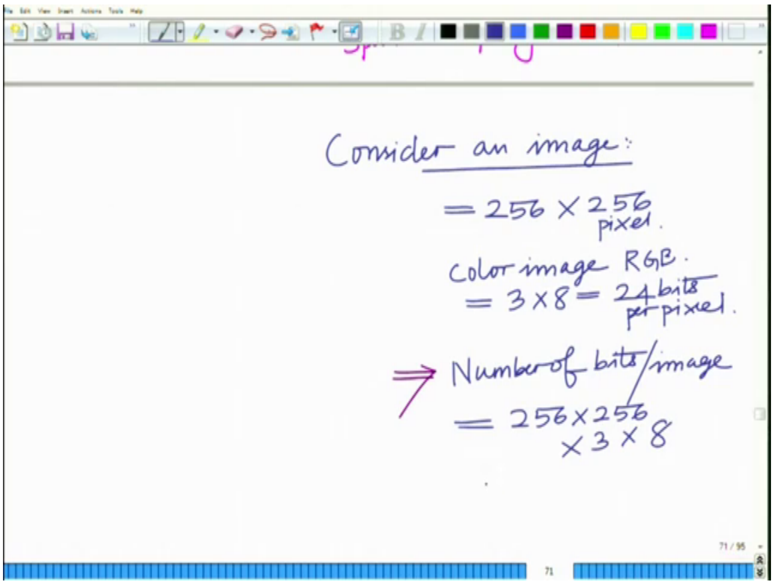

Now, the image equals let us say it is 256 it is a small image it is a 256 so 256 pixel image. And let us say it is a color image implies you have color image that is RGB implies for each of these RGB components you need 1 byte for each pixel which means 1 pixel 3 into 8, that is your RGB 3 into 8 equals 24 bits 24 bits per pixel. This implies the total number of bits per image the number of bits per image is 256 into 256 into 3 into 8 bits per pixel which is equal to if you look at this is up comes out to be roughly 1.58 Mb ok.

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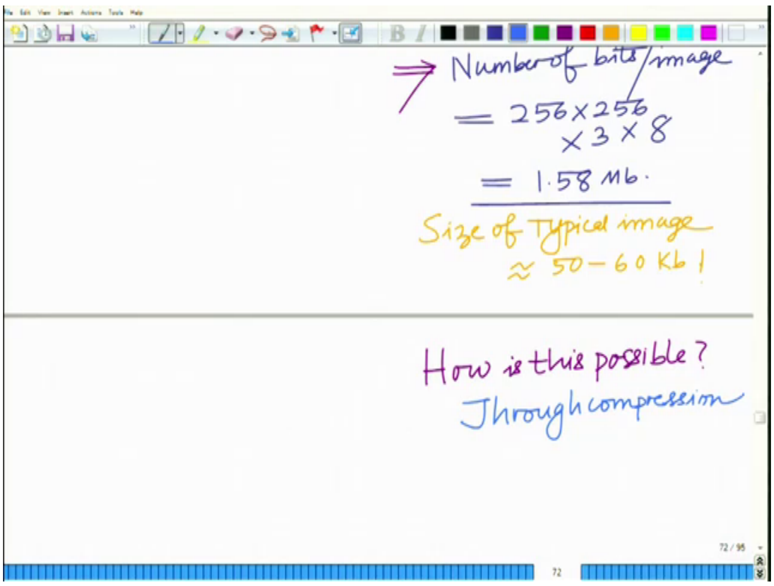

So, if you look at each image we just look at the raw image the number of bits required to store a raw image. Then you have about for a simple image that is 256 or 256 pixels of course, images nowadays have much higher resolution.

But let us consider a simple example a 256 cause 256 resolution which means you have and with color image that is you have 1 byte for each of the components RGB and so three components and 1 byte for it. So, 3 into 8 there is 24 bits per pixel into 256 into 256 that is a 1.58 Mb ok. So that is however if you look at size of a typical image on your phone or your computer the size of a typical image a typical image file would be around 30 to 40 Kb or at most 50 Kb. So, size of typical image this is let us say 50 to 60 Kb only.

Now, where is the difference coming that is where is the reference so, from a 1.5 Mb megabits you are coming to 50 to 60 Kb. So, what is happening to this huge size this image of huge size; how is this possible; how is it that you are able to get an image store an image at such a small size even though the raw image has so many bits. The obvious answer to this is that that image instead of storing a raw image this image is being significantly compressed in size in terms of the number of bits. So, typically for image so how is this possible this is possible through this is possible through compression.

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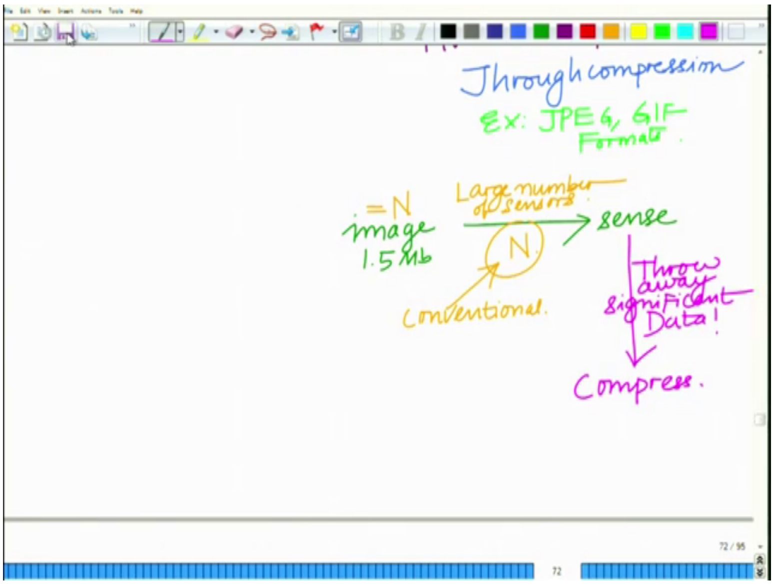

For example, you have J PEG G I F these are various formats for these are various formats for compression. So, what is this paradigm for signal processing or compression and this can be represented as follows let us consider a simple example. Let us say you have a image which is 1.5 megabits what we are doing is the following you are first sensing this image. So, this is a large image which implies you require a large number of sensors this is your number of observations also if you look at the earlier model this is basically N so basically this is conventional ok.

So, which means the number of sensors required is N so image is of size N you have a large number of sensors of size N this is your conventional sampling or sensing. That is you are taking one sample you require one sensor per sample. However, interestingly or rather strangely after sensing you are compressing that is you throw away throw away significant amount of data. So, what you are doing is strangely you are using a camera; for instance you have a camera alright in your phone or let us say separate standalone camera.

Then you are sensing it at this huge resolution alright. For instance you are sensing it in terms of the megapixels you have a camera which has resolution of several megapixels you are sensing it. And ultimately the image the raw image that you capture over the raw signal that you capture as a huge size we just captured using a large number of sensors alright.

But, after the capture process you are throwing away a significant amount of data to compress it which means basically you are using a large number of sensors that is your original device is very expensive because of the large number of sensors. But at the same time you are not able to leverage or you do not need this, because you are throwing away a large amount of data, because the compression is coming after the sensing process that is the important aspect. The compression here is coming well after the sensing process that is the important thing to realize here.

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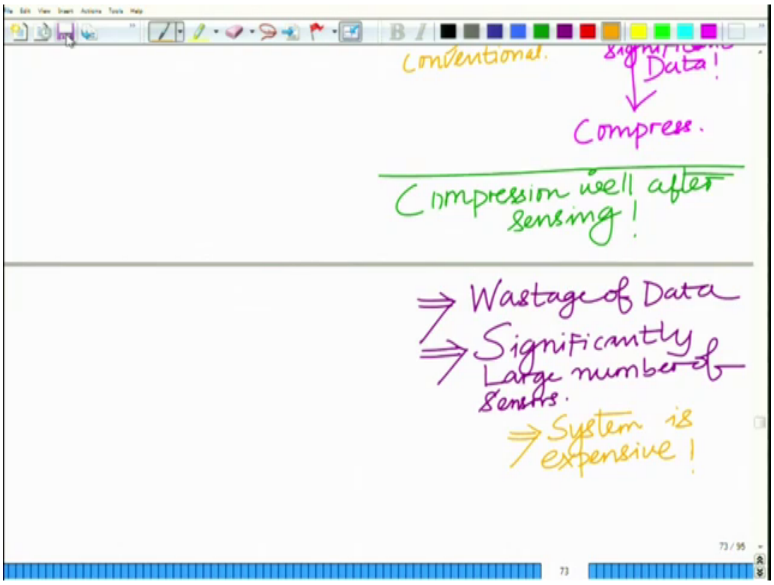

In this conventional paradigm you have compression well after the sensing compression is coming well after the sensing process. This leads to a wastage this leads to a large number of sensors wastage this leads to a large number of sensors significantly implies the resulting system is expensive implies the resulting system is extremely expensive.

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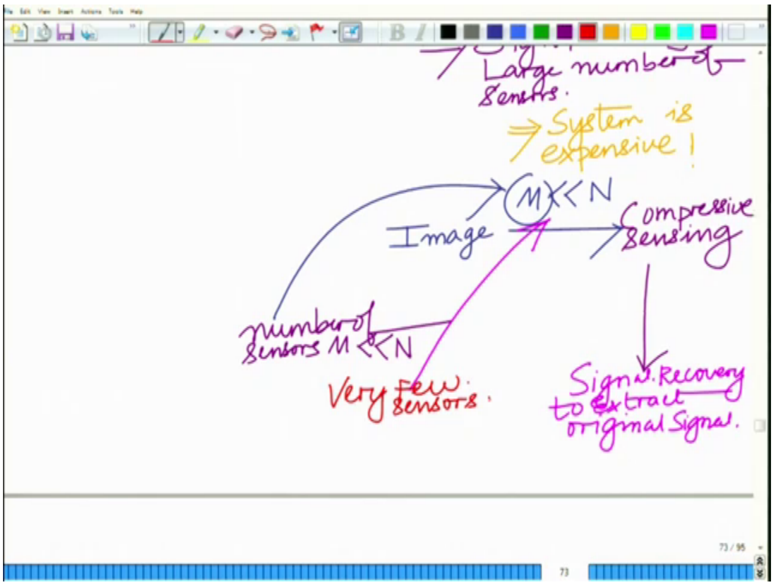

Now instead consider now this is instead consider this paradigm you have an image consider this paradigm you have an image you perform M measurements much less than N that is number of sensors M is much less than N this is term as compressive sensing.

So, you are not using number of sensors that is equal to the M is not equal to M; M is not greater than. In fact, the number of sensors M is much less than much lower than the number of samples or the size of the signal vector N.

So, basically while the sensing process itself you are compressing it is as if for a 1.5 megabit image you are making only 60 kilo bit of observations roughly alright. So you are not sensing and compressing, but compressing while sensing itself.

Now, therefore, now since you are compressing while sensing later you have to recover the signal or reconstruct the signal. So, first you are so instead of sensing and then compressing you are compressing while sensing, and then you can performs signal recovery signal recovery to extract the original signal. So, this is the idea so this is very few measurements followed by signal recovery very few requires very few senses.

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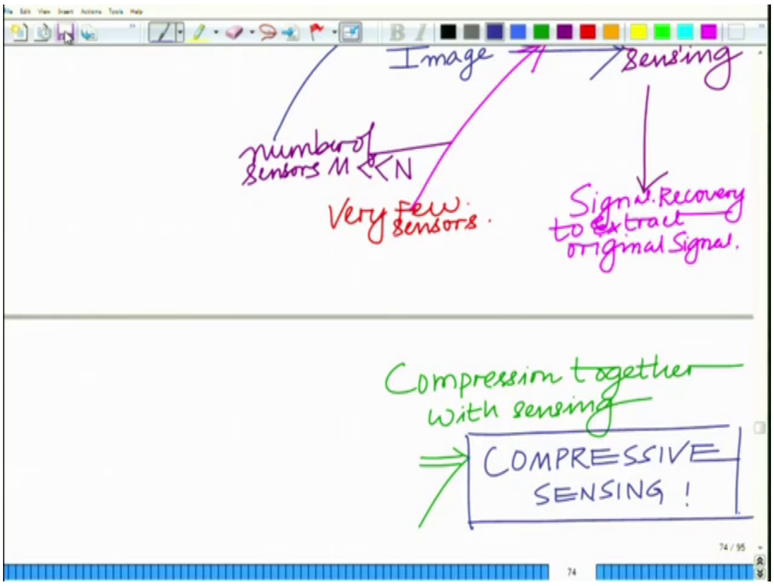

So, you are performing compression during sensing or compression together with sensing this is basically nothing, but the framework of or let me just write it in block letters. Emphasizes this is basically the framework of this is the framework of compressive sensing which results in a significant saving in terms of cost significant saving in terms of the number of sensors, because you are making very few measurements in comparison to the size of the signal.

Now naturally that implies also that since the number of observations is less than the number of signal samples. Remember, this is similar to I remember we said the sensing process similar to having solving a system of linear equations M is the number of observations N is the number of signal samples.

So, we need M greater than equal to N now naturally if M is less than N one cannot uniquely determine the signal vector x bar, because linear algebra results still remain the same. So, therefore, one has to come up with some engineers techniques or some new ideas to reconstruct the original signal x bar from this compressed or compressively sensed signal from this compressively sensed signal alright y bar.

Now, what are those techniques first how do you sense the signal in a compressive fashion and what are those engineers ideas that you use for reconstruction that forms the basis of comparison. In fact, these this is where the path breaking ideas come in that is how do you reconstruct this signal using a much fewer number of sensors or much fewer number of observations in comparison to the total signal size.

And it is not possible to do it using; of course, if you look at the results in conventional linear algebra it is not possible to do it therefore one has to come up with a new framework. And that is where the optimization aspect also comes in; which is what we are going to look at in the subsequent module.

Thank you very much.